
#### Abstract

Запропоновано метод формування унімодальної вирішальної функції при прив'язці кореляційно-екстремальних систем навігацї до поверхні візування з кількома яскравими об'єктами. Обрано моделі поточного та еталонного зображенъ, здійснено постановку і вирішено завдання виявлення та багатопорогової селекції об'єкта прив'язки на зображенні з високою об'єктової насиченістю. Одержано кількісні оцінки ймовірності локалізації об'єкта прив'язки на зображенні

Ключові слова: поточне зображення, виявлення та багатопорогова селекиія об'єкта прив'лзки, унімодальна вирішальна функиія


Предложен метод формирования унимодальной решающей функции при привязке корреляционно-экстремальных систем навигации к поверхности визирования с несколькими яркими оббектами. Выбраны модели текущего и эталонного изображений, осуществлена постановка и решены задачи обнаружения и многопороговой селекции объекта привязки на изображении с высокой оббектовой насыщенностью. Получены количественные оценки вероятности локализации обвекта привязки на изображении

Ключевые слова: текущее изображение, обнаружение и многопороговая селекиия оббекта привязки, унимодальная решающая функция

# A METHOD FOR LOCALIZING A REFERENCE OBJECT IN A CURRENT IMAGE WITH SEVERAL BRIGHT OBJECTS 

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## 1. Introduction

Correlation-extreme navigation systems (CENSs) of aircraft are auxiliary navigation systems. They are designed to correct (snap) the trajectory of an aircraft in preliminarily selected points. Any deviation from a given trajectory is determined by combining the image of the snap area or the image of the object itself on a certain background, formed by the CENS, with the reference image (RI). The image that is formed by the system during the flight of the aircraft is called the current image (CI). The errors found by comparing the CI and the RI to locate the sought-for object are used to correct the errors of the inertial navigation system that accumulate during the flight of the aircraft. However, the accuracy of determining the coordinates of the groundbased reference objects, which are the navigation reference points, does not always correspond to what is required. This happens due to the ambiguity in detecting and selecting the reference object (RO) under conditions of high object
density and, as a consequence, the inability to form a unimodal decision function (DF) of the aircraft control crew. Therefore, it becomes necessary to develop appropriate methods for ensuring the required accuracy in determining the coordinates of the RO. Moreover, the CI is formed under the influence of natural and deliberate interference as well as unfavorable weather conditions. The need to take into account the above factors further emphasizes the relevance of research to develop methods for the formation of a unimodal DF system.

## 2. Literature review and problem statement

A significant number of publications have been devoted to the formation of the CENS decision function. In [1], a method for the operational synthesis of reference images of the viewing surface (VS) is based on the formation of a field of fractal analysis. In [2], a method is suggested to base the
early formation of the RI on the formation of a correlation analysis field. The methods that are considered in [1, 2] help localize the reference objects in VS images with different object density and generate RIs, but there is no mention of the task of localizing objects in a current image where specially created false reference objects can be present.

In [3], a technology is proposed to create coatings of objects, which would result in changing the reflecting properties of the surfaces. This, in turn, leads to an ambiguity (multimodality) of the decision function formed by the CENS. The result of the DF formation is influenced by distortions caused by the environment of the electromagnetic waves band [4]; however, such distortions are mostly masking rather than leading to the appearance of false objects (FOs) in the VS.

In [5], approaches are considered to detect and localize the contours of objects in images by using the Hough transform, but the research is performed without taking into account the features and differences in the content of the images. In [6], there are findings of studies of the effect of scale distortions on the result of localizing objects, and in [7], there are findings given about the optimization of the procedure for detecting objects in images. At the same time, the proposed approaches have limitations as to localizing ROs in current VS images of high object density, since reference objects and false objects can have rather small differences in their informative parameters.

In [8], a method is suggested for detecting objects by optoelectronic systems. The peculiarity of the method is that the detection of an air object is performed in the absence of false objects, and a priori information about the back-ground-target situation is not taken into account.

One of the stages in selecting a given object in the image of a complex three-dimensional terrestrial scene obtained by the onboard sensor of an unmanned aerial vehicle is considered in [9]. However, the algorithm that implements a set of procedures for processing and analyzing images as well as deciding on the coordinates of the location of a given object in the CI is designed without taking into account the content of the image scene.

In [10], with the aim of forming the DF, it is proposed to use the minimax criterion for the similarity of two images. At the same time, there is no regard to the features of the back-ground-object composition of the viewing surface. In [1, 4, 11], it is suggested that objects in images of various types should be localized by using methods of the theory of fractal analysis. However, the use of these methods and the algorithms based on them require to estimate the object density of the CI and, if necessary, to eliminate the excess of object density.

Thus, despite the actively conducted studies to develop methods for the formation of a unimodal DF, it is not known how to solve the problem of localizing ROs in current VS images with high object density when several objects that are close in brightness and shape to the reference object are present in the image. Also, there are no known algorithms the application of which will ensure in such conditions the localization of the RO in the CI and the formation of the unimodal DF.

## 3. The purpose and objectives of the study

The purpose of the study is to estimate the probability of localizing the reference object in the current image with
high object density, using the detection algorithm, preliminary multi-threshold selection of the reference object, and specifying the maximum DF value.

To achieve this goal, it is necessary to solve the following research tasks:

- to choose the models of the current and reference images and to formulate the task of developing the method for localizing the RO of the CENS in an image with high object density;
- to solve the problem of detection and the preliminary multi-threshold selection of the RO in the current image of a viewing surface with several bright false objects;
- to solve the problem of clarifying the maximum DF value and determining the coordinates of the RO in the field of the CI matrix.


## 4. Selection of models of current and reference images. Statement of the task of developing a method for localizing the reference object in an image

The CI Model $\mathbf{S}_{\mathrm{CI}}$. To describe a VS, we will adopt a model in which the undistorted initial image $\mathrm{S}_{\mathrm{II}}$ is described by the brightness values of the corresponding objects and VS backgrounds in the resolution elements:

$$
\begin{equation*}
\mathbf{S}_{\mathrm{CI}}=\mathbf{S}_{\mathrm{II}}=\|\mathrm{S}(\mathrm{i}, \mathrm{j})\| \tag{1}
\end{equation*}
$$

where

$$
S(i, j)=\left\{\begin{array}{l}
S_{v}(i, j), \text { at } S(i, j) \in \mathbf{S}_{v}, \\
S_{w}(i, j), \text { at } S(i, j) \in \mathbf{S}_{w}
\end{array}\right.
$$

is the brightness of the image element of the v-th object of $\mathbf{S}_{\mathrm{v}}$; $S_{w}(i, j)$ is the brightness of the image element of the w-th background of $\mathbf{S}_{\mathrm{w}}$;

V and W are, respectively, the numbers of objects and backgrounds of different brightness and shape in the II.

In accordance with (1), let us make for the CI model of the viewing surface the following assumptions:

- the current and initial images have the same size of $\mathrm{N}_{1} \times \mathrm{N}_{2}$ pixels;
- the VS objects have significant brightness values relative to the background. The RO of the CENS has the greatest brightness;
- the RO and the background within the resolution element are uniform in brightness;
- each $\mathrm{i}, \mathrm{j}$-th element of the CI is a normally distributed value with the variance $\sigma_{\mathrm{ij}}^{2}$ and the average luminance value $S(i, j)$. In the absence of interference, $S(i, j)$ can take one of the two values: $\mathrm{S}_{\mathrm{v}}(\mathrm{i}, \mathrm{j})$ or $\mathrm{S}_{\mathrm{w}}(\mathrm{i}, \mathrm{j})$. The contrast of the RO relative to the ambient background is defined as $\Delta \mathrm{S}=\mathrm{S}_{\mathrm{v}}(\mathrm{i}, \mathrm{j})-\mathrm{S}_{\mathrm{w}}(\mathrm{i}, \mathrm{j}) ;$
- the dispersion of noise in the receiving channels of the CENS is the same, i.e. $\sigma_{\mathrm{ij}}^{2}=\sigma^{2}, i \in \overline{1, \mathrm{~N}_{1}}$, and $\mathrm{j} \in \overline{1, \mathrm{~N}_{2}}$;
- for the number of the background elements belonging to the set of $\mathrm{S}_{\mathrm{w}}$ and the objects belonging to the set of $\mathrm{S}_{\mathrm{v}}$, the valid relation is $\mathrm{V} \ll \mathrm{W}$.

Taking into account the assumptions made, the density distributions of the brightness $S$ of the background and object elements are determined by the expressions:

$$
\begin{equation*}
\mathrm{w}_{\mathrm{w}}(\mathrm{~S})=\frac{1}{\sqrt{2 \pi} \sigma} \exp \left[-\left(\mathrm{S}-\mathrm{S}_{\mathrm{w}}\right)^{2} / 2 \sigma^{2}\right] \tag{2}
\end{equation*}
$$

$$
\begin{equation*}
\mathrm{w}_{\mathrm{v}}(\mathrm{~S})=\frac{1}{\sqrt{2 \pi} \sigma} \exp \left[-\left(\mathrm{S}-\mathrm{S}_{\mathrm{v}}\right)^{2} / 2 \sigma^{2}\right] \tag{3}
\end{equation*}
$$

Concerning the signals of other objects of $\mathbf{S}_{\rho}$, close in brightness and commensurable with the reference object, hereinafter referred to as false objects (FOs), we make the following assumptions:

- the maximum size of $\mathbf{S}_{\rho}$ does not exceed the diameter $D_{e}$ of the resolution element on the terrain; otherwise, such an FO can be considered stable and as usable as a reference object;
- the equivalent diameters of $\mathbf{S}_{\rho}$ are distributed according to an exponential law.

The latter assumption makes it possible to take into account only one distribution parameter - the average diameter $D_{0}$ of $\mathbf{S}_{\rho}$ - in the formulation of the problem and also to assume that for a known mean value, the maximum entropy is revealed by the following exponential distribution:

$$
\mathrm{w}(\mathrm{D})= \begin{cases}\frac{1}{\mathrm{D}_{0}} \exp \left(-\mathrm{D} / \mathrm{D}_{0}\right), & \mathrm{D} \leq \mathrm{D}_{\mathrm{e}}  \tag{4}\\ 0, & \mathrm{D}>\mathrm{D}_{\mathrm{e}}\end{cases}
$$

The signal from $\mathbf{S}_{\rho}$ with the equivalent diameter $D_{e}$, taking into account the fill factor of the image resolution element, has a luminance value determined in accordance with the expression:

$$
\begin{aligned}
& S(i, j)=S_{\rho} \frac{D}{D_{e}}+S_{w}(i, j)\left(1-\frac{D}{D_{e}}\right)= \\
& =S_{w}(i, j)-\frac{D}{D_{e}}\left(S_{w}(i, j)-S_{\rho}\right)
\end{aligned}
$$

where $\mathbf{S}_{\rho}$ is the average brightness of an FO.
The density of the distribution of the probability of a signal from an FO with allowance for distribution (4) has the form:

$$
W_{p}(S)= \begin{cases}\lambda e^{\lambda\left(S_{w}-S_{p}\right)}, & S \leq S_{p}  \tag{5}\\ 0, & S>S_{p}\end{cases}
$$

where

$$
\lambda=\frac{\mathrm{D}_{\mathrm{e}}}{\mathrm{D}_{0}\left(\mathrm{~S}_{\mathrm{w}}-\mathrm{S}_{\mathrm{\rho}}\right)}
$$

Let us assume that the signals from an FO in the area of the CI frame are randomly distributed and represented by a Poisson flow, possessing the property of stationarity and ordinariness without any aftereffect.

Description of the RI. Because of the instability of both the absolute values of the brightness of the individual elements of the VS as well as the contrast of objects and the background, we will assume that the RI is given by the sign of the contrast and the geometric shape of the object. That is, we represent the RI as a binary image. The elements of the object have the value of 1 , and the background elements have the value of 0 .

Statement of the problem. To consider the CI model with several objects that are close in brightness and geometric form with the RO, it is necessary to solve the problem of localizing the reference object.

Let us denote by $\mathrm{F}_{\mathrm{p}}$ the number of cells in the frame with signals from the FO , so:

$$
\begin{equation*}
\mathrm{F}_{\mathrm{p}}+\mathrm{F}_{\mathrm{v}}+\mathrm{F}_{\mathrm{w}}=\mathrm{F}_{0} \tag{6}
\end{equation*}
$$

where $F_{0}$ is the total number of frame elements that hit the camera's field of view; $F_{v}$ and $F_{w}$ are the numbers of frame elements taken by the reference object and the background. Using the assumptions that $\sigma \ll \Delta \mathrm{S}$ and $\mathrm{F}_{\mathrm{v}} / \mathrm{F}_{0}>0.5$ makes it possible to split the solution of the problem of localizing the RO in the CI into several stages. The first stage is to detect the object, whereas the second consists in its preliminary selection against the background of the FO. The third stage consists in finding the maximum $D F$ value of $R_{j}$ from the aggregate

$$
\mathrm{R}_{\mathrm{j}}=\sum_{\varsigma=1}^{\mathrm{L}} \mathrm{R}_{\mathrm{i}}(\mathrm{i}, \mathrm{j})
$$

which is determined by a layer-by-layer analysis of the number of DF cross sections $\zeta$ and the search for its single value corresponding to the maximum.

During the detection phase, it is necessary to estimate the average background value without taking into account the spatial relationships between the image elements. Then, having established the quantization threshold with respect to the mean value of the background found in the previous step, it is required to convert the CI to a binary image. Further, using a priori information about the geometric characteristics of the object, it is necessary to solve the problem of selecting the RO in a binary image. It should be noted that some of the signals from bright FOs are perceived as the RO signals. Therefore, at the third stage, the unimodal DF is formed by searching for its largest value, which corresponds to the complete coincidence of the CI with the RI.

## 5. The solution of the detection problem and multithreshold selection of the RO in a current image with bright false objects

The current image in the line-by-line expansion in accordance with (6) represents a vector of the dimension $F_{0}$. As a result, we have a sample of the volume $\mathrm{F}_{0}$, which forms three disjoint classes of $\omega_{\mathrm{i}}$, corresponding to signals from the background $\omega_{\mathrm{w}}, \mathrm{MO} \omega_{\rho}$ and RO $\omega_{\mathrm{v}}$, and the density of the sample distribution is given by the expression:

$$
\begin{equation*}
\mathrm{w}(\mathrm{~S})=\sum_{\mathrm{i}=1}^{3} \mathrm{p}_{\mathrm{i}} \mathrm{w}_{\mathrm{i}}(\mathrm{~S}) \tag{7}
\end{equation*}
$$

where $p_{i}=\frac{F_{i}}{F_{0}}, \quad i=1,2,3$ are a priori probabilities of the classes; $\mathrm{w}_{\mathrm{i}}(\mathrm{S})=\mathrm{w}\left(\mathrm{S} \mid \omega_{\mathrm{i}}\right)$ means the conditional probability densities of a random variable $S$ under the condition that $S$ belongs to a class of $\omega_{\mathrm{I}}$; they are defined by expressions (2), (3), and (5).

In order to isolate the signals of the RO from the background signals, we split the sample consisting of the elements of the three classes into two classes with respect to the quality index, which is defined further.

Let us set quantization threshold 1 of the sample to two classes, according to which we assign the signals of the RO to one of them and the signals of the FO to the second class. In this case, the probabilities of errors of the first and second kinds are determined by the expressions:

$$
\begin{equation*}
\alpha=\int_{S_{w}-1}^{\infty} w_{v}(S) \mathrm{dS} \tag{8}
\end{equation*}
$$

$$
\begin{equation*}
\beta=\frac{1}{1+K} \int_{-\infty}^{S_{w}-1}\left[w_{\rho}(S)+\mathrm{Kw}_{w}(\mathrm{~S})\right] \mathrm{dS}, \tag{9}
\end{equation*}
$$

where

$$
\mathrm{K}=\frac{\mathrm{F}_{\mathrm{p}}}{\mathrm{~F}_{\mathrm{w}}} .
$$

By the probabilities $\alpha$ and $\beta$ in the second stage, we can determine the probability of the correct localization of the RO, which, in accordance with (8) and (9), can be considered a function of threshold 1 and can be maximized by choosing the corresponding threshold $\mathrm{l}=\mathrm{l}_{\mathrm{opt}}$. Since the distribution parameters of mixture (8) are unknown, the first step is to estimate the unknown parameters, which include $\Delta \mathrm{S}, \mathrm{S}_{\mathrm{w}}$, and $\mathrm{p}_{\mathrm{w}}$. The parameter $\lambda$ is uniquely determined by expression (7). The number of the RO elements $F_{v}$ is known; hence, the probability is known as:

$$
\mathrm{p}_{\mathrm{v}}=\frac{\mathrm{F}_{0}}{\mathrm{~F}_{\mathrm{v}}}, \mathrm{p}_{\mathrm{p}}=1-\mathrm{p}_{\mathrm{w}}-\mathrm{p}_{\mathrm{v}} .
$$

By applying a nonlinear transformation to the original sample, we can construct a histogram of the distribution of the transformed random variable. The nonlinear transformation best emphasizes the "center" of the background distribution $\mathrm{w}_{\mathrm{w}}(\mathrm{S})$. By comparing the central parts of the histogram and the theoretical probability density, we determine the mean value of the background $\mathrm{S}_{\mathrm{w}}$.

During the transformations, the dynamic range of $\Delta \mathrm{S}$ was divided into intervals of the $\sigma$ length, and the center of the interval with the largest number of sample values was taken as a rough estimate of the mean value of $\mathrm{S}_{\mathrm{w}}$.

By performing the appropriate transformations and substituting the parameter estimates in expressions (8) and (9), we obtain estimates of the error probabilities $\hat{\alpha}, \widehat{\beta}$ :

$$
\begin{align*}
& \hat{\alpha}=\int_{\widehat{S}_{w}-1}^{\infty} w_{v}\left(S \mid \widehat{S}_{w}\right) d S  \tag{10}\\
& \hat{\beta}=\frac{1}{1+\widehat{K}} \int_{-\infty}^{\hat{S}_{w-1}}\left(w_{w}\left(S \mid \widehat{S}_{w}\right)+\hat{K}_{w_{p}}\left(S \mid \widehat{S}_{w}\right)\right) d S, \tag{11}
\end{align*}
$$

where

$$
\widehat{\mathrm{K}}=\hat{\mathrm{p}}_{\mathrm{p}} / \hat{\mathrm{p}}_{\mathrm{w}}, \hat{\mathrm{p}}_{\mathrm{v}}=1-\hat{\mathrm{p}}_{\mathrm{w}}-\mathrm{p}_{\mathrm{\rho}} .
$$

For the given threshold 1, we transform the initial $\mathrm{S}_{\mathrm{CI}}$ of the CI into a binary image $\mathbf{H}$ according to the rule:

$$
\mathrm{H}_{\mathrm{i}}= \begin{cases}1, & \mathrm{~S}_{\mathrm{i}} \leq \mathrm{S}_{\mathrm{\rho}}-\mathrm{l} ; \quad \mathrm{S}_{\mathrm{i}}>\mathrm{S}_{\max } ; \\ 0, & \mathrm{~S}_{\max }>\mathrm{S}_{\mathrm{i}}>\mathrm{S}_{\mathrm{p}}-1 ;\end{cases}
$$

The quantization threshold determines the probability of occurrence of errors of the first $\alpha$ and the second $\beta$ kinds. In turn, the values of $\alpha$ and $\beta$ determine the minimum value of the signal-to-noise parameter $\mathrm{q}=\mathrm{q}_{\text {min }}$, at which the required probability of the correct localization of the RO is reached:

$$
\mathrm{q}_{\min }=\Phi^{-1}(1-\alpha)+\Phi^{-1}(1-\beta)
$$

where the probability integral is:

$$
\Phi(\mathrm{x})=\frac{1}{\sqrt{2 \pi}} \int_{0}^{\mathrm{x}} \mathrm{e}^{-\mathrm{t}^{2} / 2} \mathrm{dt} .
$$

Now it is necessary to solve the task of selecting an object in a binary CI against the background of the MO, using a priori information in the form of a binary RI.

The algorithm for processing the binary CI for the purpose of solving the problem of selecting the RO is as follows. For each fragment $\mathbf{H}^{i} \subset \mathbf{H}$ of the CI, having a certain configuration and size of the object, a comparison is made with the RI, which consists entirely of single units. The operation of comparing the binary images consists in adding, "according to module two," image elements and in forming the DF by using the formula:

$$
\begin{equation*}
\mathrm{R}_{\mathrm{i}}=\sum_{\mathrm{k}=1}^{\mathrm{F}_{\mathrm{p}}}\left(\mathrm{~S}_{\mathrm{RI}_{\mathrm{m}}} \oplus_{\mathrm{mod} 2}^{\oplus} \mathrm{H}_{\mathrm{m}}^{\mathrm{i}}\right) \tag{12}
\end{equation*}
$$

where $\mathrm{H}_{\mathrm{m}}^{\mathrm{i}}$ is the m-th element of the i -th fragment of the CI; $\mathrm{S}_{\mathrm{RI}_{\mathrm{m}}}$ is the m -th element of the reference image $\mathrm{S}_{\mathrm{RI}}$.
${ }^{m}$ The decision rule is that the fragment $\mathbf{H}^{\mathrm{j}} \subset \mathbf{H}$, for which

$$
\begin{equation*}
R_{j}=\inf _{i} R_{i} \tag{13}
\end{equation*}
$$

is declared to coincide with the RI. The index $i$ takes as many values of M as there are all possible fragments shifted with respect to each other with a given configuration in the frame $\mathbf{H}$. If the property of (13) is true for several fragments, then the decision to localize the RO is not accepted.

To compare $\mathbf{H}^{\mathrm{i}}$ with the RI consisting of units, it is more convenient to operate with numbers:

$$
\mathrm{z}_{\mathrm{i}}=\mathrm{F}_{\mathrm{p}}-\mathrm{s}_{\mathrm{i}}, \mathrm{i} \in \overline{0, \mathrm{M}},
$$

each of which represents the number of units in the $\mathbf{H}^{\mathrm{i}}$ fragment. Then the decision rule is that the fragment $\mathbf{H}^{\mathrm{j}} \subset \mathbf{H}$, for which

$$
\begin{equation*}
Z_{j}=\sup _{i \in, \bar{M}} z_{i} \tag{14}
\end{equation*}
$$

is declared to coincide with the RI.
To estimate the probability of the correct localization of an object, we proceed as follows. Let the size of the object be $T_{1} \times T_{2}$ cells. We divide the CI matrix into rectangular $\mathrm{T}_{1} \times \mathrm{T}_{2}$ submatrices. If necessary, we will slightly enlarge the frame size so that an integer number of submatrices fit into it, which we denote by $\mathrm{M}+1$. In this case, the estimates of the probability of the correct localization of the RO will be obtained as underestimated due to the increase in the frame. Suppose that the true position of the object falls into one of the submatrices; then we denote by $\mathbf{H}^{0} \subset \mathbf{H}$ the fragment of the CI corresponding to the RI and by $\mathbf{H}^{i}$, $\mathrm{i} \in \overline{1, \mathrm{M}}$ we shall denote the fragments placed in the remaining submatrices.

Let the probability of occurrence of 1 in $\mathbf{H}^{i}$ be equal to $r_{i}$. Then the numbers of $z_{i}$ are distributed according to the binomial law:

$$
\begin{equation*}
P\left(z_{i}\right)=C_{F_{v}}^{Z_{i}} \mathrm{r}_{\mathrm{i}}^{\mathrm{z}_{\mathrm{i}}}\left(1-\mathrm{r}_{\mathrm{i}}\right)^{\mathrm{F}_{\mathrm{v}}-\mathrm{z}_{\mathrm{i}}}, \quad z_{\mathrm{i}} \in \overline{0, \mathrm{~F}_{\mathrm{v}}} . \tag{15}
\end{equation*}
$$

The probability of the correct localization of the RO by using decision rule (14) is equal to the probability that the number of units in $\mathrm{z}_{0}$, corresponding to the coincidence of the RI and the object, will exceed all other numbers of $z_{i}$, $i \in \overline{1, \mathrm{M}}$.

We denote by $A_{0}^{j}$ an event consisting in the appearance of $z_{0}=j \in \overline{1, F_{v}}$ units in $\mathbf{H}^{0}$, and by $A_{i}^{j}, i \in \overline{1, M}$ the event
consisting in the fact that the number of units in $z_{i}$ does not exceed $\mathrm{j}-1$. The events $A_{\mathrm{i}}^{\mathrm{j}}, \mathrm{i} \in \overline{1, \mathrm{M}}$ are independent in aggregate because $\mathbf{H}^{\mathrm{i}} \cap \mathbf{H}^{\mathrm{m}}=\varnothing \forall \mathrm{i}, \mathrm{m} \in \overline{0, \mathrm{M}}$. In accordance with formula (15), the probabilities of the events $A_{i}^{j}$ are determined by the expressions:

$$
P\left(A_{i}^{j}\right)= \begin{cases}C_{F_{v_{0}}}^{j} r_{0}^{j}\left(1-r_{0}\right)^{F_{v}-j}, & i=0 ;  \tag{16}\\ \sum_{m=1}^{j-1} C_{F_{v}}^{m} r_{i}^{m}\left(1-r_{i}\right)^{F_{v}-m}, & i \in \overline{1, M}\end{cases}
$$

Then, by the probability multiplication theorem, the probability of the event

$$
L_{j}=\bigcap_{i=0}^{M} A_{i}^{j}
$$

is equal to

$$
P\left(L_{j}\right)=\prod_{i=0}^{M} P\left(A_{i}^{j}\right), j \in \overline{1, F_{v}} .
$$

Since the events in the aggregate are incompatible, the probability that the number of units in $\mathbf{H}^{0}$ exceeds the number of units in all other fragments of $\mathbf{H}^{\mathbf{i}}$ is determined by the expression for the probability of the correct localization:

$$
\begin{equation*}
P_{c l}=\sum_{j=1}^{F_{v}} P\left(L_{j}\right)=\sum_{j=1}^{F_{v}} \prod_{i=0}^{M} P\left(A_{i}^{j}\right), \tag{17}
\end{equation*}
$$

where the probabilities are determined by formulas (16). Considering that

$$
\mathrm{r}_{\mathrm{i}}= \begin{cases}1-\alpha, & \mathrm{i}=0 ; \\ \beta, & \mathrm{i} \in \overline{1, \mathrm{M}},\end{cases}
$$

(the probabilities $\alpha$ and $\beta$ are given by relations (8) and (9)), for the probability of the correct localization of the object, we obtain the final expression:

$$
\begin{equation*}
P_{c l}=\sum_{j=1}^{F_{v}} C_{\mathrm{F}_{v}}^{j}(1-\alpha)^{j} \alpha^{F_{v}-j}\left[\sum_{m=0}^{j-1} C_{\mathrm{F}_{\mathrm{v}}}^{\mathrm{m}} \beta^{\mathrm{m}}(1-\beta)^{\mathrm{F}_{\mathrm{v}}-\mathrm{m}}\right]^{\mathrm{M}} \tag{18}
\end{equation*}
$$

In order to ensure unambiguous decisions, it is necessary to develop an algorithm that performs an iterative process of processing with a varying threshold before obtaining a single solution.

One of the possible variants of such an algorithm is as follows. After calculating the average value estimate $\overline{\mathrm{S}}_{\mathrm{w}}$, the initial value of the threshold $l^{0}=\alpha \sigma$ is set (the algorithm tests showed that it is expedient to choose $\alpha \in 1.8 \ldots 2.2$ ), with respect to which the CI

$$
\mathbf{S}_{\mathrm{CI}}=\|\mathrm{S}(\mathrm{i}, \mathrm{j})\|
$$

is transformed into a binary image, which we denote by $\mathbf{H}^{0}$. By comparing this image with the RI in accordance with criterion (14), the matrix of the decision function $\left\|z_{i \mathrm{ij}}^{0}\right\|$ is calculated and the set is found as follows:

$$
\mathrm{M}^{0}=\left\{(\mathrm{k}, \mathrm{l}) \in \overline{1, \mathrm{~N}_{1}} \times\left.\overline{1, \mathrm{~N}_{2}}\right|_{\mathrm{kl}}=\max _{\mathrm{i}, \mathrm{j}} \mathrm{z}_{\mathrm{ij}}\right\} .
$$

Moreover, the maximum of the decision function $z_{\text {max }}^{0}$ is not necessarily equal to $F_{v}$, but it is possible that $z_{\text {max }}^{0}<F_{v}$. If the set $\mathbf{M}^{0}$ consists of one element, i. e. $\mathbf{M}^{0}=\{1(\mathrm{~m}, \mathrm{l})\}$, then the decision is made that the coordinates of the reference element of the object relative to the CI are m,l.

Otherwise, an iterative process is organized, which includes three stages. The first stage is associated with a change in the initial value of the threshold $l^{0}$ to $l^{1}$. In the second stage, a new binary image $\mathbf{H}^{1}$ is formed with respect to the threshold $\mathrm{l}^{1}$. The third stage is connected with the calculation of the matrix of the decision function $\left\|z_{i \mathrm{ij}}^{1}\right\|$ for the image $\mathbf{H}^{1}$ and the calculation of the set $\mathbf{M}^{1}$. To organize the iterative process, it is necessary to determine the magnitude of the iteration step and its sign. In the algorithm under consideration, in the threshold-adaptation mode, the solution is suggested to be assumed under the condition that the set $\mathbf{M}^{\mathbf{i}}$ consists of one element at some $i$-th iteration, and $z_{\text {max }}^{i}=F_{v}$. Consequently, if the maximum of the decision function is not unique and is equal to $F_{v}$, it is necessary to choose $\Delta l_{1}=1^{1}-1^{0}>0$. Otherwise, the iteration must have a negative sign.

The choice of the step of each iteration should be carried out in such a way that at the next i-th iteration the binary image of $\mathbf{H}^{\mathrm{i}}$ could slightly differ from $\mathbf{H}^{\mathrm{i}-1}$.

## 6. The solution of the problem of forming a unimodal decision function

Expression (18) for the probability of the correct localization of the object is suitable for evaluating the effectiveness of applying the CENS in the areas of the VS with the reference object by a uniquely determined system. In this case, the system forms a unimodal DF. If reference is made to a VS with several objects that are comparable in parameters to the RO, it becomes necessary to refine the result of the reference. For this, in the third stage, the search is performed for the largest DF value, corresponding to the complete coincidence of the CI with the RI.

The essence of the method is to form a set of matrices $\mathbf{G}_{i}$ of the DF with the subsequent determination of the largest number of units in the summary representation of the DF as $\sum_{\mathrm{i}=1}^{\mathrm{U}} \mathrm{G}_{\mathrm{i}}$.

The decision rule is that the fragment $\mathbf{G}_{\mathrm{j}} \subset \mathbf{G}$, for which

$$
\begin{equation*}
\mathrm{G}_{\mathrm{j}}=\sup _{\mathrm{i} \in \overline{0}, \mathrm{U}} G_{\mathrm{i}} \tag{19}
\end{equation*}
$$

is declared to coincide with the RI.
The index i assumes as many values as there are cuts $U$ of the fragments $\mathbf{G}_{j} \subset \mathbf{G}$ by the time of determining the cut with the greatest number of units.

As a criterion for localizing the RO, we choose an integral indicator of relative brightness, the values of which are formed as independent samples of $Q$ in the elements of the DF matrices $\mathbf{G}_{\mathrm{i}}$. All the resulting matrices $\mathbf{G}_{\mathrm{i}}$ are summed together elementwise. The resulting matrix $\mathbf{G}_{\mathrm{i}}$ contains in the elements $G_{j}(i, j)$ the values of the independent samples in the form of integrated luminance indices. The matrix $\mathbf{G}_{\mathbf{i}}$, with the largest number of units written as $\left(\sum_{i=1}^{\mathrm{U}} \mathrm{G}_{\mathrm{i}}=\max \right)$, is taken as the result of localizing the required reference object.

The cell $G_{j}(i, j)$, containing the maximum number of units, allows determining the number of stepwise comparison by rows and columns of the matrix at which the RO is localized. Thus, the coordinates of one of the selected angles of the DF matrix within the CI matrix are determined, which allows finding the coordinates of the RO in the field of the CI ma-
trix. As a result of implementing the three stages, a decision is made to localize the RO in the CI frame with several objects that are comparable in their parameters to the RO. The probability of the correct localization of the RO in accordance with the described algorithm is determined by the expression:

$$
\begin{align*}
& P_{c l}= \\
& =1-\left(1-\sum_{j=1}^{F_{v}} C_{F_{v}}^{j}(1-\alpha)^{j} \alpha^{F_{v}-j}\left[\sum_{m=0}^{j-1} C_{F_{v}}^{m} \beta^{m}(1-\beta)^{F_{v}-m}\right]^{Q}\right)^{U} \tag{20}
\end{align*}
$$

The results of estimating the probability of the correct localization and the DF formation for two values of the sig-nal-to-noise ratio are shown in Fig. 1-4.


Fig. 1. The result of estimating the probability of localizing the $R O$ in the Cl with a signal-to-noise ratio $\mathrm{q} \approx 10$


Fig. 2. The result of the DF formation with a signal-to-noise ratio $\mathrm{q} \approx 10$


Fig. 3. The result of estimating the probability of localizing the RO in the Cl with a signal-to-noise ratio $\mathrm{q} \approx 20$


Fig. 4. The result of the DF formation with a signal-to-noise ratio $\mathrm{q} \approx 20$

The analysis of the results of estimating the probability of the correct localization of the RO (Fig. 1, 3) and the formation of the DF (Fig. 2, 4) with the use of the VS with FOs has revealed that the application of the detection procedure
and the multi-threshold selection of the RO in the image allows ensuring the probability of the correct localization of the object that is close to 1 . At the same time, the presence of false objects in the image of the VS, comparable in parameters with the RO, does not affect the formation of the unimodal DF. Thus, the algorithm for implementing the developed method is distortion-proof, and it can be used in the CENS to ensure efficient functioning in conditions of a complex background-object situation.

## 7. Discussion of the results of developing the method of forming a unimodal decision function

The results of theoretical studies that are presented in the article on developing a method for forming a unimodal decision function are a continuation of the authors' development of the methods of secondary image processing in the CENS - in particular, methods for prompt and advance preparation of reference images. In addition, the authors developed a number of methods for the formation of a unimodal DF. These methods include a generalized method and an algorithm for linking the CENS to the distorted sections of a VS. A method was developed on an approach that is based on the formation of a field of correlation analysis of images, taking into account variations in brightness, contrast, and image structure. To bind the CENS to the VS with different object density, a method based on the formation of a field of fractal dimensions of images was proposed. However, the developed methods are not suitable for use in the CENS when linked to a VS of a high object density, when images comparable to informative parameters can occur along with the RO in the current image. As a result, there was a need to develop such a method.

The basis of the method developed in this study is the use of the CI representation in the form of an "object on the background" and the division of the task of forming the DF into three stages. The first stage consists in detecting the RO in the CI, being based on estimating the average background value and determining the quantization thresholds with respect to it. The choice of the magnitude of the quantization threshold determines the probability of occurrence of errors of the first and second kinds. These errors determine the minimum value of the signal-to-noise parameter at which the required probability of the correct localization of the RO is achieved. At the second stage, the multi-threshold selection of the RO is carried out. The step is an iterative process of processing images with a varying threshold until a single solution is obtained. The third stage is to refine the maximum DF value and to determine the coordinates of the RO in the field of the CI matrix.

The method can be suitable to navigate, for example, unmanned aircraft flying at low altitudes. This is due to the fact that a number of objects can become comparable with the size of the resolution element, with the need to take them into account in the CI. In addition, the method can be useful in solving the navigation problems of an aircraft equipped with a CENS, with the establishment of interference comparable in parameters with the RO. At the same time, the method may prove to be of little use for navigating high-speed aircraft with limitations on time of forming the DF due to the complication of the procedure for the formation of the command of control. This particularly applies to high-speed aircraft with steep flight paths.

Further studies will be aimed at improving the method of forming the DF, which will take into account distortions of the CI due to the geometry of the sight and the influence
of the fairing (for high-speed aircraft). In aggregate, the research results can become the basis for the development of a software package to form a unimodal DF in different conditions of applying the CENS.

## 8. Conclusion

1. Models for describing the current and reference images are proposed in this study. The CI model is represented by the brightness values of the corresponding objects and the VS backgrounds in the resolution elements. In the model, the RO has the greatest brightness. Objects that are close in brightness and commensurable with the RO are referred to false objects. Aircraft in the CI frame are randomly distributed and represented by a Poisson flow having the property of stationarity and ordinariness without any aftereffect.

The RI is given by the sign of the contrast and the geometric shape of the object, and it is binary. Elements of the object correspond to the value of 1 , whereas the background elements correspond to 0 .

The task was formulated to localize the RO of the CENS in an image with high object density of the "object on the background" type. The solution of the task of localizing the RO in the image is realized in three stages. The first stage is to detect the object, whereas the second is to select it preliminary against the background of the aircraft. The third stage is to clarify the maximum value of the DF.
2. A method for the detection and preliminary multithreshold selection of a RO in a CI with several bright objects has been developed. Its features are:

- the transformation of the CI into a binary one by estimating, during the phase of detecting the RO, the average background value and setting the image quantization threshold with respect to it, which in turn determines the probability values for the occurrence of errors of the first and second kinds;
- the assigning of objects in the VS and backgrounds to two classes: the RO and the background are determined by selecting the quantization threshold at the stage of preliminary multi-threshold selection of the RO.

An algorithm for localizing the RO in the image is developed by searching for a fragment of a binary CI with a maximum value of units that coincides with the reference image. Since a priori signal-to-noise value is not known, in order to improve the efficiency of the algorithm, a procedure for adapting the conversion threshold is proposed.
3. A method has been developed to refine the maximum value of the DF and to determine the coordinates of the RO in the field of the CI matrix. An analytical expression is obtained for determining the probability of localizing the RO with the procedure for searching for the maximum value of the DF and the choice of the threshold for the cuttings of the DF. The expression establishes the dependence of the probability of localizing the RO on the parameters that are determined in all stages of solving the problem of localizing the RO: detection, multi-threshold selection and search for the maximum DF value. By simulating the formation of the DF, numerical estimates of the probability of localizing the RO are obtained. It is established that for typical values of signal-to-noise ratios, the probability of localizing the RO in the presence of false objects in the image will be at least 0.9. This ensures the unimodality of the formed DF.

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