Побудовані узагальнені моделі одно-, двох- і трьохмасних вібромашин з прямолінійним поступальним рухом платформ і вібробудником у вигляді шарового, роликового або маятникового автобалансира. Виведені диференціальні рівняння руху. Рівняння приведені до вигляду, що не залежить від типу автобалансира. З узагальнених моделей можна отримувати частині, шляхом відкидання частини зовнішніх або внутрішніх пружно-в'язких опор

Ключові слова: інерційний віброзбудник, двочастотні вібрації, резонансна вібромашина, автобалансир, одномасна вібромашина, багатомасна вібромашина

Построены обобщенные модели одно-, двух- и трехмассных вибромашин с прямолинейным поступательным движением платформ и вибровозбудителем в виде шарового, роликового или маятникового автобалансира. Выведены дифференциальные уравнения движения. Уравнения приведены к виду, не зависящему от типа автобалансира. Из обобщенных моделей можно получать частные, путем отбрасывания части внешних или внутренних упруго-вязких опор

Ключевые слова: инерционный вибровозбудитель, двухчастотные вибрации, резонансная вибромашина, автобалансир, одномассная вибромашина, многомассная вибромашина

UDC 622.74: 621.928.235
DOI: 10.15587/1729-4061.2017.111216

EQUATIONS OF MOTION OF VIBRATION MACHINES WITH A TRANSLATIONAL MOTION OF PLATFORMS AND A VIBRATION EXCITER IN THE FORM OF A PASSIVE AUTO-BALANCER

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1. Introduction

Among such types of vibration machines as screens, vibration tables, vibration conveyors, vibration mills, etc., the promising ones are multi-frequency and resonant machines.

Multi-frequency vibration machines have larger performance efficiency [1] while resonant vibration machines are the most energy efficient [2]. That is why it is a relevant task to create vibration machines that combine the benefits of multi-frequency and resonant vibration machines [3].

The simplest and most reliable ways to excite the resonance oscillations of platforms and rotors are based on the Sommerfeld effect [4–16]. One of these techniques is based on using a ball, a roller, or a pendulum auto-balancer in the form of a vibration exciter [8, 11, 12]. The given technique makes it possible to excite the two-frequency resonant vibrations.

At present, the workability of the new technique for exciting two-frequency vibrations is not theoretically studied. A necessary stage of such a research is to build generalized models of new vibration machines and to derive their differential equations of motion.

2. Literature review and problem statement

In multi-frequency vibration machines, the basic technological process in the form of screening, dehydration, grinding, separation by size, etc. occurs more intensely [1]. Resonant vibration machines are more energy efficient because their vibration exciters of lower weight excite platform oscillations with a larger amplitude [2]. Multi-frequency vibration machines combine advantages of both types of vibration machines and, therefore, are the most promising [3].

Let us consider in detail techniques for the excitation of resonance oscillations of vibration machines based on the Sommerfeld effect [4]. Paper [5] describes a technique in which the pendulum is placed on the motor shaft. The rated rotational speed of the rotor of electric engine is slightly less than the resonant frequency of platform oscillations. Due to the Sommerfeld effect, the pendulum cannot accelerate to the nominal speed of the rotor's rotation and gets stuck on the resonance frequency of platform oscillations. The shortcoming of the method is that the electric engine is overloaded in this case. Article [6] describes a technique that employs a wind wheel, combined with the unbalance, instead of the electric engine. A stream of compressed air is fed to the wheel. The wheel gradually accelerates to the resonance frequency of the platform oscillations. The disadvantage of the given method is the low efficiency coefficient of the system «compressed air – wind wheel – platform». Paper [7] describes a technique in which pendulums are mounted on the low-power electric motors' shafts. The pendulums' motion is synchronized over time and they get stuck at the resonance frequency of platform oscillations. The drawback of the technique is the overload of electric motors.

In [8], it is proposed to excite two-frequency resonant vibrations of the platform using a vibration exciter in the form of a ball, a roller or a pendulum auto-balancer. A special mode of the pendulums' motion is exploited for this purpose [9], or of the balls or rollers [10], which occurs at small resistance forces to the motion of loads relative to the auto-balancer housing. Under this mode, the loads get together, cannot catch up with the shaft on which the auto-balancer is mounted, and get stuck on the resonance frequency of the platform oscillations. In [8], it is proposed to excite slow resonant oscillations of the platform by utilizing the loads that get stuck. It is also proposed to fix the unbalanced mass on the auto-balancer housing. The housing with the unbalanced mass rotate synchronously with the shaft. This excites rapid oscillations of the platform. Two-frequency vibrations parameters vary widely by changing speed of the rotor's rotation, the unbalanced mass on the auto-balancer housing, and the total mass of loads.

The new technique is based on the same Sommerfeld effect as the techniques in papers [5–7]. Only, instead of a low-power motor, one uses small forces of viscous resistance to the relative motion of loads. The unbalanced mass, combined with an auto-balancer housing, is used as the second vibration exciter.

It is assumed that a vibration exciter in the form of a passive auto-balancer is applicable for the one-, two-, three-mass vibration machines with a different kinematic of the platform motion. The workability of the new method was investigated for a screen with a rectilinear translational motion of the box by using a 3D simulation [11] and a field experiment [12].

Research into special mode of load motion in auto-balancers is addressed in papers [9, 10, 13–16].

In [9], by employing a computational experiment, authors discovered the phenomenon of the pendulums being stuck in a pendulum auto-balancer. In the examined model, the rotor rotation axis is horizontal. Pendulums are mounted on both sides of the rotor. The forces of gravity are taken into account. The rotor rotates at working angular velocity; the working angular velocity of pendulums' rotation is equal to one of the natural frequencies of the rotor's oscillations.

In [10], a phenomenon of the balls being stuck on the resonant velocity of the rotor's rotation in an auto-balancer was discovered experimentally, on a universal bench. The axis of rotor's rotation is vertical. There is one auto-balancer. The phenomenon was studied within the framework of a flat model of the rotor on isotropic supports with a multi-ball auto-balancer. It was established that when the balls get stuck, it is contributed by the following: small resistance forces to the balls' motion relative to the auto-balancer housing, an increase in the weight of balls, the presence of forces of viscous resistance in the rotor supports.

In [13], a phenomenon of the balls that get stuck in a ball or a roller auto-balancer was discovered analytically and confirmed in the course of a computational experiment. The rotor has two supports, it is positioned vertically. There is one auto-balancer.

In [14], in order to analytically study the effect of jamming, discovered in [9], authors applied the method of separating the movements. It was established that pendulums are stuck on one of the natural frequencies of rotor's oscillations. The authors found in the first approximation the law of motion of the rotor and pendulums under effect of getting stuck.

In [15], authors built in the plane of dimensionless parameters a region of the motion stability, in which two balls get stuck on the resonant rotation speed of the rotor. They used a flat model of the rotor with a two-ball auto-balancer.

In [16], the effect of pendulums getting stuck, found in [9], was explored at the experimental bench. Computational experiments were also performed. Results of the field and computational experiments almost coincided.

The above review demonstrates that it is a relevant task to theoretically investigate the workability of the technique for exciting the two-frequency vibrations by a ball, a roller, or a pendulum auto-balancer. For this purpose, it is necessary to build generalized models and derive differential equations of motion of one-, two-, and three-mass vibration machines with translational motion of vibration platforms and a vibration exciter in the form of a passive auto-balancer.

The scope of application of the generalized models and differential equations of motion: conducting theoretical research and computational experiments.

3. The aim and objectives of the study

The aim of present work is to build generalized mechanical-mathematical models of one-, two-, and three-mass vibration machines with vibration exciters in the form of passive auto-balancers.

To achieve the set aim, the following tasks have to be solved:

– to describe the generalized models of one-, two-, and three-mass vibration machines with a vibration exciter in the form of a ball, a roller, a pendulum auto-balancer;

– to derive differential equations of the motion of vibration machines.

4. Research methods

In order to build the mechanical-mathematical models of vibration machines, we apply elements of the theory of vibration machines [3] and elements of the theory of rotor machines with passive auto-balancers [10].

To derive the differential equations of motion, we employ basic theorems of dynamics and second-order Lagrange's equations [17].

5. Research results

5. 1. A single-mass vibration machine with a vertical translational motion of the platform

5. 1. 1. Description of the generalized model of a vibration machine

A vibration machine (Fig. 1) consists of the platform, mass M, and a vibration exciter in the form of a ball, a roller (Fig. 1, *b*) or a pendulum (Fig. 1, *c*) auto-balancer. The platform can move only rectilinearly translationally due to two stationary guides. The direction of platform motion forms angle α to the vertical. The platform is based on an elastic-viscous support with rigidity coefficient k and viscosity coefficient b. Position of the platform is determined by the y coordinate equal to zero in the static equilibrium state of the platform.

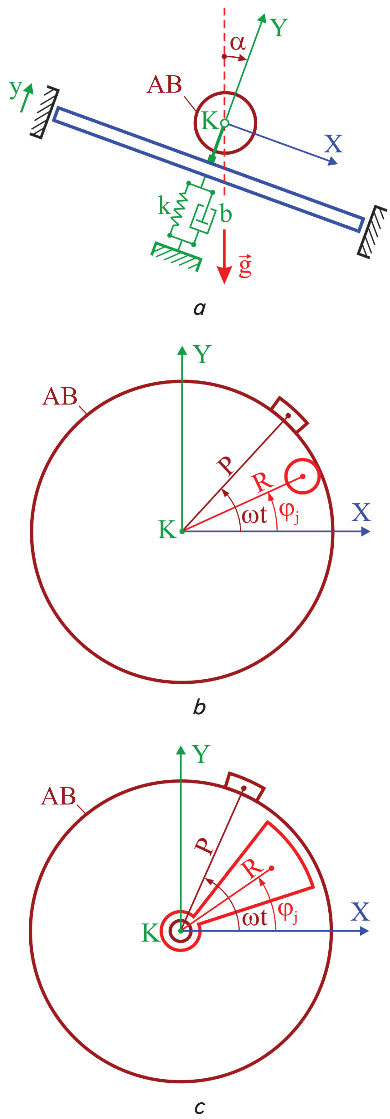

Fig. 1. Generalized model of a single-mass vibration machine: *a* – kinematic of the platform motion; *b* – kinematic of motion of the unbalanced mass, and a ball or a roller; *c* – kinematic of motion of the unbalanced mass and a pendulum

The auto-balancer housing revolves around a shaft – point K with a constant angular velocity ω.

The auto-balancer housing is rigidly coupled with a point unbalanced mass μ, located at a distance P from point K. Two mutually perpendicular axes X, Y originate at point K and form the right coordinate system. The X axis is parallel to the platforms, while the Y axis is parallel to the direction of platform motion. Position of the unbalanced mass relative to the housing is determined by angle ωt, where t is the time. The angle is measured from the X axis to the section that extends from point K and ends in the unbalanced mass.

The auto-balancer consists of N identical loads. The mass of one load is m. Mass center of the load can move in a circle with radius R centered at point K (Fig. 1, *b*, *c*). Position of load number j relative to the housing is determined by angle φ_j, $/ j=\overline{1,N} /$. The angle is measured from the X axis to the section that extends from point K and ends at the mass center of load number j. The motion of load relative to the auto-balancer housing is prevented by the force of viscous resistance with module

$$F_j = b_W v_j^{(r)} = b_W R|\varphi_j' - \omega|, \ / j=\overline{1,N}/, \tag{1}$$

where b_W is the coefficient of viscous resistance force, $v_j^{(r)} = R|\varphi_j' - \omega|$ is the module of motion speed of the mass center of load number j relative to the auto-balancer housing with a stroke at the magnitude denoting time derivative t.

5. 1. 2. Differential equations of motion of a single-mass vibration machine

Differential equations of motion of the platform.

Coordinate y_C of the center of masses of a mechanical system is derived from equality

$$M_\Sigma y_C = M y_K + \sum_{j=1}^{N} m(R\sin\varphi_j + y_K) + \mu(P\sin\omega t + y_K) =$$

$$= M_\Sigma y_K + mR\sum_{j=1}^{N}\sin\varphi_j + \mu P\sin\omega t, \tag{2}$$

where $M_\Sigma = M + Nm + \mu$ is the mass of the whole system, y_K is the coordinate of point K.

We shall introduce, accordingly, projections of the total unbalanced mass from loads onto the X, Y axes, and the unbalance from the unbalanced mass:

$$S_x = mR\sum_{j=1}^{N}\cos\varphi_j, \quad S_y = mR\sum_{j=1}^{N}\sin\varphi_j, \quad S_d = \mu P. \tag{3}$$

Then equality (2) takes the form

$$M_\Sigma y_C = M_\Sigma y_K + S_y + S_d \sin\omega t. \tag{4}$$

We find from (4)

$$M_\Sigma y_C' = M_\Sigma y' + S_y' + S_d \omega\cos\omega t,$$

$$M_\Sigma y_C'' = M_\Sigma y'' + S_y'' - S_d \omega^2 \sin\omega t, \tag{5}$$

where $y_K' = y'$, $y_K'' = y''$, since during translational motion the velocities and accelerations, respectively, of all points of the platform are the same.

The theorem of the motion of center of masses of a mechanical system yields the following differential equation of the platform motion:

$$M_\Sigma y_C'' = M_\Sigma y'' + S_y'' - S_d \omega^2 \sin\omega t = -b\dot{y} - ky.$$

Following the transforms

$$M_\Sigma y'' + by' + ky + S_y'' = S_d \omega^2 \sin\omega t. \tag{6}$$

Differential equations of motion of loads.

The kinetic energy of load number *j* is equal to the kinetic energy of translational motion with the center of masses and the kinetic energy of rotation around the center of masses:

$$T_j = T_j^{(tr)} + T_j^{(r)}. \qquad (7)$$

In its turn

$$T_j^{(tr)} = \frac{1}{2} m v_j^2, \; T_j^{(r)} = \frac{1}{2} J_C \omega_j^2, \qquad (8)$$

where v_j is the module of speed of the center of masses of the load; J_C is its main central axial moment of inertia; ω_j is the module of angular velocity of the load rotation around the center of masses.

Coordinates of the center of masses of load number *j*:

$$x_j = x_K + R\cos\varphi_j, \; y_j = y_K + R\sin\varphi_j \quad (x_K = \text{const}).$$

Projections of the speed of the center of masses of load number *j*:

$$v_{jx} = -R\varphi_j' \sin\varphi_j, \; v_{jy} = y' + R\varphi_j' \cos\varphi_j.$$

Square of the speed of the center of masses of load number *j*:

$$v_j^2 = v_{jx}^2 + v_{jy}^2 = = (-R\varphi_j' \sin\varphi_j)^2 + (y' + R\varphi_j' \cos\varphi_j)^2 = = y'^2 + R^2\varphi_j'^2 + 2R\varphi_j' y' \cos\varphi_j. \qquad (9)$$

Let the ball or a cylindrical roller number j roll on the auto-balancer housing without slipping. Then the absolute angular velocity of rotation of the ball or the roller

$$\omega_j = \omega(R+r)/r - \varphi_j' R/r = = \omega(R/r+1) - \varphi_j' R/r. \qquad (10)$$

The kinetic energy of the ball or roller number *j*:

$$T_j = T_j^{(tr)} + T_j^{(r)} = = \frac{1}{2} m(y'^2 + R^2\varphi_j'^2 + 2R\varphi_j' y' \cos\varphi_j) + + \frac{1}{2} J_C [\omega(R/r+1) - \varphi_j' R/r]^2 = = \frac{1}{2} m(y'^2 + 2R\varphi_j' y' \cos\varphi_j) + \frac{1}{2} mR^2\kappa\varphi_j'^2 + + \frac{1}{2} J_C [\omega^2 (R/r+1)^2 - 2\omega\varphi_j' (R/r+1) R/r], \qquad (11)$$

where

$$\kappa = 1 + J_C / mr^2 \qquad (12)$$

is the dimensionless coefficient. For ball $J_C = \frac{2}{5} mr^2$, roller $J_C = \frac{1}{2} mr^2$, and, respectively,

$$\kappa = 1 + \frac{2}{5} = \frac{7}{5}, \quad \kappa = 1 + \frac{1}{2} = \frac{3}{2}. \qquad (13)$$

For pendulum $\omega_j = \varphi_j'$. The kinetic energy of pendulum number *j*:

$$T_j = T_j^{(tr)} + T_j^{(r)} = = \frac{1}{2} m(y'^2 + R^2\varphi_j'^2 + 2R\varphi_j' y' \cos\varphi_j) + \frac{1}{2} J_C \varphi_j'^2 = = \frac{1}{2} m(y'^2 + 2R\varphi_j' y' \cos\varphi_j) + \frac{1}{2} mR^2\kappa\varphi_j'^2, \qquad (14)$$

where

$$\kappa = 1 + J_C / (mR^2). \qquad (15)$$

Note that for a mathematical pendulum, $J_C = 0$, $\kappa = 1$.

Differential equations of the load motion will be constructed using second-order Lagrange equations.

For ball or roller number *j*:

$$\frac{\partial T_j}{\partial \dot\varphi_j} = mR^2\kappa\varphi_j' + mRy' \cos\varphi_j - J_C \omega (R/r+1) R/r,$$

$$\frac{d}{dt}\frac{\partial T_j}{\partial \varphi_j'} = mR^2\kappa\varphi_j'' + mR(y'' \cos\varphi_j - \varphi_j' y' \sin\varphi_j),$$

$$\frac{\partial T_j}{\partial \varphi_j} = -mR\varphi_j' y' \sin\varphi_j;$$

left side of second-order Lagrange equations

$$\frac{d}{dt}\frac{\partial T_j}{\partial \varphi_j'} - \frac{\partial T_j}{\partial \varphi_j} = = mR^2\kappa\varphi_j'' + mR(y'' \cos\varphi_j - \varphi_j' y' \sin\varphi_j) + + mR\varphi_j' y' \sin\varphi_j = mR^2\kappa\varphi_j'' + mRy'' \cos\varphi_j. \qquad (16)$$

It is possible to verify that the left side of second-order Lagrange equations for pendulum number *j* will take similar form.

Dissipative Rayleigh function and potential energy (with accuracy up to a constant), corresponding to load number *j*:

$$R_j = \frac{1}{2} b_W (v_j^{(r)})^2 = \frac{1}{2} b_W R^2 (\varphi_j' - \omega)^2,$$

$$\Pi_j = mg[R\sin(\varphi_j - \alpha) + y_K \cos\alpha]. \qquad (17)$$

The right side of the differential equation of motion:

$$-\frac{\partial R_j}{\partial \varphi_j'} - \frac{\partial \Pi_j}{\partial \varphi_j} = -b_W R^2 (\varphi_j' - \omega) - mgR\cos(\varphi_j - \alpha). \qquad (18)$$

Equating left side (16) to the right side (18), we obtain:

$$\kappa mR^2\varphi_j'' + mRy'' \cos\varphi_j = = -b_W R^2 (\varphi_j' - \omega) - mgR\cos(\varphi_j - \alpha),$$

or, following the transform:

$$\kappa mR^2\varphi_j'' + b_W R^2 (\varphi_j' - \omega) + + mgR\cos(\varphi_j - \alpha) + mRy'' \cos\varphi_j = 0, \; / \, j = \overline{1,N} \, /. \qquad (19)$$

Note that the form of differential equations of the motion of system (6) and (19) is not dependent on the type of an auto-balancer.

5. 2. Two-mass vibration machine with a vertical translational motion of platforms

A generalized model of the two-mass vibration machine is shown in Fig. 2. It consists of two platforms of mass M_1 and M_2. Each platform is held by external elastic-viscous supports with a coefficient of rigidity k_i and viscosity coefficient b_i, $/i=1, 2/$. The platforms are connected by the inner elastic-viscous support with a coefficient of rigidity k_{12} and viscosity coefficient b_{12}.

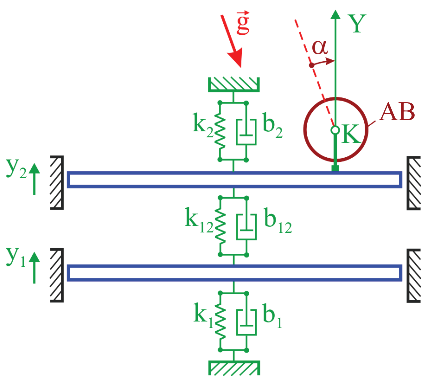

Fig. 2. Generalized model of a two-mass vibration machine (rotated at angle α)

Coordinates y_1, y_2 of the platforms are counted from the positions of static equilibrium of the platforms.

A passive auto-balancer – a ball, a roller, or a pendulum – is placed on the second platform.

The equations of motion of a two-mass vibration machine will be derived from the equations of motion of a single-mass vibration machine, by analogy.

Equation of motion of the first platform:

$$M_1 y_1'' + b_1 y_1' + k_1 y_1 + b_{12}(y_1' - y_2') + k_{12}(y_1 - y_2) = 0. \qquad (20)$$

Equation of motion of the second platform:

$$M_{2\Sigma} y_2'' + b_2 y_2' + k_2 y_2 + b_{12}(y_2' - y_1') + k_{12}(y_2 - y_1) + S_y'' = S_d \omega^2 \sin \omega t, \qquad (21)$$

where $M_{2\Sigma} = M_2 + Nm + \mu$.

Equation of motion of loads:

$$\kappa m R^2 \varphi_j'' + b_W R^2 (\varphi_j' - \omega) + mgR\cos(\varphi_j - \alpha) + mRy_2'' \cos\varphi_j = 0, \ /\ j = \overline{1, N}\ /. \qquad (22)$$

The models of particular two-mass vibration machines can be obtained from the generalized model by rejecting one of the external elastic-viscous supports.

5. 3. Three-mass vibration machines with a vertical translational motion of platforms

The generalized models of three-mass vibration machines are shown in Fig. 3. The structure of the models and conditional symbols are similar to those used in a two-mass vibration machine model.

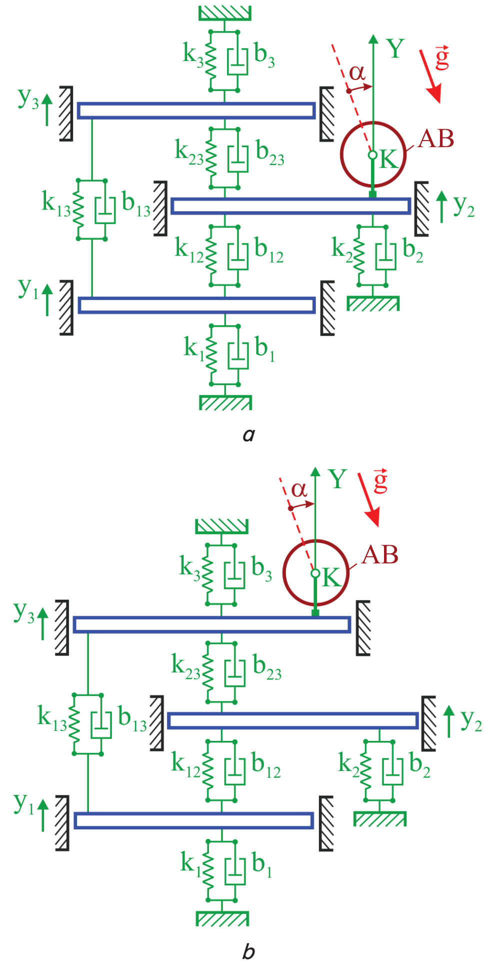

Fig. 3. Generalized models of three-mass vibration machines (rotated at angle α) where: *a* – the auto-balancer is mounted on the middle platform; *b* – on the upper platform

Coordinates y_1, y_2, y_3 of the platforms are counted from the positions of static equilibrium of the platforms.

Equations of motion of a three-mass vibration machine, in which the auto-balancer is mounted on the middle platform (Fig. 3, a).

Equation of motion of the first platform:

$$M_1 y_1'' + b_1 y_1' + k_1 y_1 + b_{12}(y_1' - y_2') + k_{12}(y_1 - y_2) + b_{13}(y_1' - y_3') + k_{13}(y_1 - y_3) = 0. \qquad (23)$$

Equation of motion of the second platform:

$$M_{2\Sigma} y_2'' + b_2 y_2' + k_2 y_2 + b_{12}(y_2' - y_1') + k_{12}(y_2 - y_1) + b_{23}(y_2' - y_3') + k_{23}(y_2 - y_3) + S_y'' = S_d \omega^2 \sin \omega t. \qquad (24)$$

Equation of motion of the third platform:

$$M_3 y_3'' + b_3 y_3' + k_3 y_3 + b_{13}(y_3' - y_1') + k_{13}(y_3 - y_1) + b_{23}(y_3' - y_2') + k_{23}(y_3 - y_2) = 0. \qquad (25)$$

Equation of motion of loads:

$$m\kappa R^2 \varphi_j'' + b_W R^2 (\varphi_j' - \omega) + mgR\cos(\varphi_j - \alpha) + mRy_2'' \cos\varphi_j = 0, \ /\ j = \overline{1, N}\ /. \qquad (26)$$

Equations of motion of a three-mass vibration machine, in which the auto-balancer is mounted on the upper platform (Fig. 3, b).

Equation of motion of the first platform:

$$M_1 y_1'' + b_1 y_1' + k_1 y_1 + b_{12}(y_1' - y_2') + \\ + k_{12}(y_1 - y_2) + b_{13}(y_1' - y_3') + k_{13}(y_1 - y_3) = 0. \quad (27)$$

Equation of motion of the second platform:

$$M_2 y_2'' + b_2 y_2' + k_2 y_2 + b_{12}(y_2' - y_1') + \\ + k_{12}(y_2 - y_1) + b_{23}(y_2' - y_3') + k_{23}(y_2 - y_3) = 0. \quad (28)$$

Equation of motion of the third platform:

$$M_\Sigma \ddot{y}_3 + b_3 y_3' + k_3 y_3 + b_{13}(y_3' - y_1') + k_{13}(y_3 - y_1) + \\ + b_{23}(y_3' - y_2') + k_{23}(y_3 - y_2) + S_y'' = S_d \omega^2 \sin \omega t. \quad (29)$$

Equation of motion of loads:

$$m\kappa R^2 \varphi_j'' + b_W R^2 (\varphi_j' - \omega) + \\ + mgR\cos(\varphi_j - \alpha) + mRy_3'' \cos\varphi_j = 0, \ / \ j = \overline{1, N} \ /. \quad (30)$$

The models of particular three-mass vibration machines can be obtained from the generalized model by rejecting:

– one or two external elastic-viscous supports;

– one of the three inner elastic-viscous supports;

– one or two external elastic-viscous supports and one of the three inner elastic-viscous supports.

6. Discussion of results of studying two-frequency vibration machines

The studies conducted allowed us to build generalized models of one-, two-, and three-mass vibration machines with a translational motion of platforms and a vibration exciter in the form of a ball, a roller, or a pendulum auto-balancer.

We obtained differential equations of motion of vibration machines and reduced them to the form independent of the type of an auto-balancer.

It is possible to obtain particular models from the generalized models of two- and three-mass vibration machines: by selecting the type of an auto-balancer, by removing part of the supports (external, inner). In order to reject a part of supports, it is required that the differential equations of motion of vibration platforms include zero rigidity and viscosity for the missing supports.

The differential equations of motion that we derived could be used both for analytical studies into dynamics of vibration machines and for performing computational experiments.

It should be noted that the differential equations of motion are not brought to the dimensionless form with the introduction of a small parameter not discussed either. It is planned to carry out in the further research.

In the future, it is also planned to search for the two-frequency motion modes of vibration machines, as well as to determine conditions for their existence and stability.

7. Conclusions

1. The generalized models have been built of one-, two- and three-mass vibration machines with a translational motion of platforms and a vibration exciter in the form of a ball, a roller, or a pendulum auto-balancer. In the generalized models, each platform is supported by an external elastic-viscous support while the platforms are interconnected by inner elastic-viscous supports.

2. We have derived differential equations of the motion of one-, two-, and three-mass vibration machines. Equations are reduced to the form independent of the type of an auto-balancer. The obtained equations are applicable both to the analytical studies into dynamics of appropriate vibration machines and for conducting computational experiments.

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