MATHEMATICS AND CYBERNETICS – APPLIED ASPECTS

Введено відстань Фреше між деревами та доведено, що ця відстань є метрикою. Розроблено метод і алгоритми визначення відстані між не опуклими областями. Спроектований і програмно реалізований модуль визначення відстані Фреше між скелетонами. Досліджено похибки результатів сегментації для метрик Хаусдорфа та Фреше між деревами на прикладі біомедичних зображень

Ключові слова: метрика Фреше, метрика Хаусдорфа, не опуклі області, біомедичні зображення, похибка сегментації

Введено расстояние Фреше между деревьями и доказано, что это расстояние является метрикой. Разработан метод и алгоритмы определения расстояния между не выпуклыми областями. Спроектирован и программно реализован модуль определения расстояния Фреше между скелетонами. Исследованы погрешности результатов сегментации для метрик Хаусдорфа и Фреше между деревьями на примере биомедицинских изображений

Ключевые слова: метрика Фреше, метрика Хаусдорфа, не выпуклые области, биомедицинские изображения, погрешности сегментации

DEVELOPMENT OF A METRIC AND THE METHODS FOR QUANTITATIVE ESTIMATION OF THE SEGMENTATION OF BIOMEDICAL IMAGES

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1. Introduction

Digital microscopy plays an important role in the analysis of biomedical images and in diagnosis. We define biomedical images as the images obtained using the technical tools employed for medical purposes in order to visualize processes. Of special importance is the quality of digital images in oncology. To diagnose a patient in oncology, images of individual cells (cytological images) and the images of a group of cells (histological) are used.

Modern digital microscopy has travelled a long way in its development from hand microscopes to modern robotized complexes.

To categorize digital microscopy, we shall introduce the following criteria: automation, software, the use of network technologies.

According to the first criterion, there are hand microscopes, which are independently operated by laboratory staff. Such microscopes are most common in the clinics of Ukraine. Automatics of the automated microscopes controls the motion and focusing of the preparation, change of filters, lenses, lighting [1]. Hardware of the system of automated microscopy (SAM) consists of a microscope, a video camera, and a computer. The basis of SAM software is the systems of image analysis (SIA). The latest modern group is the automated (ro-

botized) microscopes that enable automatic processing of images. Based on the automated microscopes, modern complexes of automatic microscopy are built (CAM) [2, 3].

In terms of software, there are software systems (SS) that perform systemic functions: control over a microscope, a video camera, etc. Another class of SS is the information systems, in which databases of images, databases of patients, incoming and outgoing reporting documents are implemented. The next class of SS is the systems for analysis of images, in which image processing algorithms are implemented at three levels – low, medium and high. A separate class of SS is the expert systems that can simulate reflections of a physician-expert when diagnosing a patient. The combination of expert systems and systems for analysis of images have spawned hybrid systems that combine technologies of knowledge bases and computer vision. This direction is promising at the present stage. Modern CAM include programs that enable the organization of telemedicine.

The level of using network technologies determines CAM that are implemented on separate workstations. Such use is limited by the local work of individual laboratory staff. Another level is the use of CAM in a local network of a clinic, hospital, which makes it possible for several specialists to work simultaneously. The use of CAM in the global Internet makes it possible to implement the technology of telemedicine.

We analyzed commercially available systems of digital microscopy that vary in price depending on configuration. For example, price of the system BioVision [4] is USD 1,850, VideoTesT-Morpho-5 [5] – USD 3,440, MEKOS-C2 [6] – USD 12,208.

The bulk of the price of such systems is the software.

The base of all SAM and CAM are SIA. SIA include all image processing levels: low, middle and high. Low level is used to improve the quality of images and contains algorithms for image filtering, histogram equalization, contrast improvement, etc. Middle level carries out segmentation, differentiation of image contours, calculation of contour and texture features. High level performs classification of micro objects based on attributes [7, 8].

The main operation at the middle level is segmentation. The accuracy of segmented objects affects subsequent calculations of quantitative attributes. At present, there is no universal image segmentation algorithm. Each algorithm is designed for a specific subject area. That is why assessment of the quality of segmentation results is very important. To assess the quality of segmentation, objective and subjective criteria are applied. Classification criteria for the evaluation of segmentation is given in paper [9].

Subjective estimate is given by a human and is based on qualitative indicators. An objective assessment is based on quantitative indicators. The advantage of employing quantitative indicators to evaluate segmentation is the absence of the human factor. The highest accuracy of segmentation evaluation is demonstrated by methods that are based on the use of metrics.

Therefore, it is important in terms of quantitative estimation of the quality of image segmentation to apply an approach that is based on using the Hausdorff and Fréchet metrics [10].

2. Literature review and problem statement

We shall analyze modern algorithms for calculating the Hausdorff and Fréchet distances. Algorithms to compute the Hausdorff distance are developed only for convex polygons. In paper [11], authors constructed an algorithm to reduce the number of vertices of a convex polygon for the set error ξ in the Hausdorff metric. The algorithm is used for convex polygons only. In article [12], authors calculated the Hausdorff distance between algebraic flat curves using the Voronoi diagrams. The algorithm is applied for particular cases of algebraic curves and has high computational complexity. The algorithm for finding the minimal Hausdorff distance in metrics L_i and L_∞ is reported in work [13]. The resulting computational complexity is $O(n^2\log^2 n)$. A search method for a given pattern of the image that has the smallest distance in the Hausdorff metric is given in paper [14]. In this case, the authors use a transmission of the assigned pattern to the desired image. The algorithm has high computational complexity. Article [15] reports a method of finding the minimum weighted tree based on the Hausdorff metric for a d-dimensional space. The problem of the approximation of such a tree is solved over polynomial time.

An algorithm to compute the discrete Fréchet distance for polygonal curves is given in article [16]. A given algorithm employs groups of conversion of rigid motions. Computational complexity is $O(m^2 n^2)$, where m and n are the number of segments along the first and second curve. In paper [17], authors show an algorithm for computing the Fréchet distance for surfaces, which are represented by simple polygons. The algorithm has polynomial complexity. In work [18], author developed an algorithm to calculate the Fréchet distance between two curves, which are assigned by a set of $m+n$ linearly approximated segments. Computational complexity is $O(m×n)$. Authors of [19] obtained, for closed polygonal curves, a computational complexity of $O(m×n)$ for the Fréchet metric. In [20], authors demonstrate an algorithm to compute a discrete Fréchet distance with inaccurately assigned vertices. For a d-dimensional space, they received computational complexity $O(d×m×n)$. An algorithm to calculate the Fréchet distance between non-flat surfaces is given in paper [21]. The authors reached the polynomial time in the L_∞ metric. In article [22], authors developed a fast algorithm for finding a similarity of polygonal trees in the Fréchet metric. The algorithm has polynomial complexity. In works [23, 24], finding the distance between parametrically set curves is carried out over polynomial time.

Thus, an analysis of the scientific literature reveals that modern algorithms for finding the Fréchet distance for flat closed curves have the least computational complexity $O(m×n)$. Algorithms for finding the Hausdorff distance between regions are computationally complex. There are no efficient algorithms for computing the Hausdorff distance for non-convex regions.

Therefore, it is necessary to develop a metric for finding a distance between non-convex regions. For this purpose, we shall use a description of the image region by skeletons [8]. Skeletons are the middle lines, which describe arbitrary regions and reduce dimensionality of a region description. Thus, skeletons are graphs without cycles, in other words, trees.

Therefore, further direction of present research is to develop a Fréchet metric between trees (skeletons), which would make it possible to find distances between arbitrary (convex and non-convex) geometrical objects. In addition, by employing the developed metric, it is necessary to construct a method and algorithms for computing the distance between non-convex regions.

3. The aim and objectives of the study

The aim of present study is to introduce a metric between trees to find the distance between non-convex regions of segmented images. This would make it possible to decrease the computational complexity of algorithms that compare non-convex regions.

To accomplish the aim, the following tasks have been set:
– to introduce and substantiate the Fréchet metric between trees;
– to devise a method for comparison of non-convex regions of images, based on the Fréchet metric between trees;
– to construct algorithms for computing distances between non-convex regions;
– to perform computer experiments to find distances between non-convex regions.

4. Computation of distance between non-convex regions based on the Fréchet metric

4. 1. The Fréchet metric between trees

A tree is a connected graph without cycles. A tree is called the root tree if it contains a selected point (root) [26].

We shall consider a topological tree in $R^n(R^2)$, that is, topological embedding of tree graphs. Thus, a topological tree is a triple (T, α, x_0), where $T \subseteq R^n$, $x_0 \in T$ and there is graph-tree T' and embedding f $f: T' \to R^n$, $\alpha(T')=T$, $f(x_0')=x_0$.

At each topological tree T there exists, therefore, two metrics. The first is induced from R^n (denoted as d) and the second is induced from a geodesic metric into T' by mapping α – denoted as ρ_T. How do we find $\rho_T(x, y)$? It is necessary to take prototypes of $\alpha_{-1}(x)$, $\alpha_{-1}(y)$ in T', and measure in graph T' the length of the segment connecting these points.

The mapping of graphs is called monotonous if it is monotonous along all segments that originate from the roots.

That is, the mapping $f: T \to S$ is monotonous if $f(x_0)=y_0$ (root in S) and, at distance x from x_0 along the segment (in metric ρ_T), point $f(x)$ is monotonously distanced from y_0 (in metric ρ_S).

We shall determine the Fréchet distance between topological trees T and S:

$$d_F(T,S)=$$
$$= \inf\left\{\sup\left\{d\left(f(x),g(x)\right)\middle|x \in R\right\}\middle|f:R \to T,g:R \to S\right\}, \quad (1)$$

here R is a tree and f, g are onto maps.

An example of trees mapping is shown in Fig. 1.
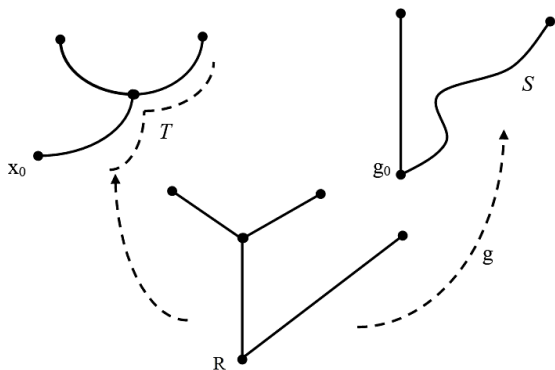
Fig. 1. Trees mapping: R, T, S — topological trees

Then f maps the left side R to T, and the right side is along the right side of T. Accordingly, g maps the right R to the right side S, and the left side R to the left side of S, making up the two ends of the fork.

Consider root trees in the plane R^2. If (T, t_0) is such a tree, then for each $t \in T$ through $\|t\|$ we denote $d(t, t_0)$. Here and hereafter, d denotes a geodesic distance inside the tree, that is, $d(t, t_0)$ indicates length of the arc inside T, which connects t and t_0.

Mapping of root trees.

We believe that $f:(S, s_0) \to (T, t_0)$ has the property that $f(s_0)=t_0$, and, for each geodesic segment J originating from point t_0, we obtain

$$s,s' \in J, \|s\| \leq \|s'\| \Rightarrow \|f(s)\| \leq \|f(s')\|. \quad (2)$$

To denote the Fréchet metric on the set of all embedded trees in R^2, we shall denote as ρ the Euclidean metric in R^2.

Assume

$$D(T,S)=$$
$$= \inf\left\{\sup\left\{\rho\left(f(r),g(r)\right)\middle|r \in R\right\}\middle|f:R \to T,g:R \to S\right\}. \quad (3)$$

First, it is necessary to make sure that the given definition is valid. Having T, S, we shall denote through R the bouquet of these two trees, $R=T \vee S$ (that is, the factor-space of combination $T \cup S$ relative to the equivalence relation, which identifies point t_0 and s_0) (Fig. 2).
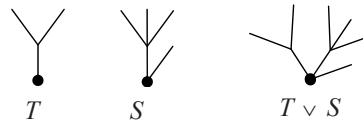

Fig. 2. A bunch of tree T and S

Then $f:R \to T$ we can accept the mapping that deforms S to a point, and $g:R \to S$, respectively, as the mapping that deforms T to a point.

Theorem. The function D is a metric.

Proof.

1) It is obvious that $D(T, S) \geqq 0$ for each T, S. It is also clear that $D(T, T)=0$ (it will suffice to accept the identical mapping as f and g).

Next, assume that $D(T, S)=0$. Hence, it is easy to derive $\rho_H=(T, S)=0$ (ρ_H denotes the Hausdorff metric on the plane). Thus, $T=S$.

2) The symmetry of function D directly follows from the definition.

3) Triangle inequality.

Let T, S, U be the trees and $\varepsilon>0$. There are trees P, R and mapping "onto" $f:P \to T$, $g:P \to S$ and $k:R \to S$, $h:R \to U$ so that

$$\sup\left\{\rho\left(f(p),g(p)\right)\middle|p \in P\right\} \leq D(T,S)+\varepsilon,$$

$$\sup\left\{\rho\left(k(r),h(r)\right)\middle|r \in R\right\} \leq T(S,U)+\varepsilon.$$

Consider the pullback of trees P and R, that is a tree

$$Q=\left\{(p,r) \in P \times R\middle|g(p)=k(r)\right\}.$$

If Q is not a tree, one may consider an onto map $Q' \to Q$, where Q is a tree.

Let $\alpha:Q \to P$, $\beta:Q(r)$ be the projections.

Then, for each $(p,r) \in Q$, we have (4):

$$\rho\left(f\alpha(p,r)\right),h\beta(p,r)=$$
$$= \rho\left(f(p),h(r)\right) \leq \rho\left(f(p),g\alpha(p,r)\right)+\rho\left(k\beta(p,r),h(r)\right)=$$
$$= \rho\left(f(p),g(p)\right)+\rho\left(k(r),h(r)\right) \leq D(T,S)+D(S,U)+2\varepsilon. \quad (4)$$

Thus, the theorem proved makes it possible to apply the Fréchet metric between trees to find a distance between non-convex regions.

4. 2. A method for estimating distance between skeletons of the non-convex regions

As a result of the skeletonization process, we obtain skeleton Sk of the image, which is a tree. Therefore, skeleton $Sk=G= =(V, E)$ is a non-directed graph where $V=\{v_0, v_1,..., v_k\}$ is the set of vertices, $E=\{u_0, u_1,..., u_m\}$ is the set of edges. For a non-directed graph, the edge is the set $\{u, v\}$, where $u, v \in V$, $u \neq v$.

For the assigned graph $G=(V, E)$, the path (route) of length k from vertex u to vertex u' is a sequence of vertices $\{v_0, v_1,...,v_k\}$, such that $u=u_0$, $u'=v_k$, $(v_{i-1},v_i) \in E$ for $i=1, 2,..., k$. The path includes vertices $v_0, v_1,..., v_k$ and edges $(v_0, v_1), (v_0, v_1),..., (v_{k-1}, v_k)$. If there is a path P from vertex u

to vertex u', then one can say that vertex u' is reachable from vertex u along path P, that is, $u\xrightarrow{P} u'$. A path is called simple if all the vertices of the path are different.

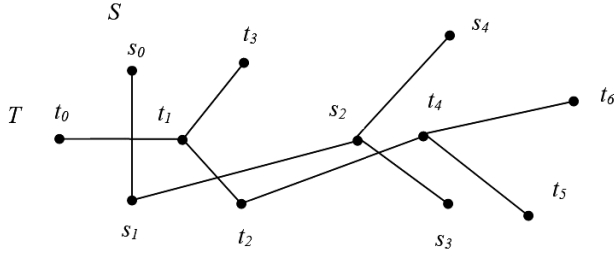Let two connected planar graphs without cycles be assigned (Fig. 3).

Fig. 3. Planar graphs T and S

The merging of graphs T and S is a set of C pairs of (t_i, s_j), which has the following properties:

1) $(t_0, s_0) \in C$;

2) if t_i is the end vertex in T, then there is such an end vertex s_j in S so that $(t_i, s_j) \in C$;

3) if s_k is the the end vertex in S, then there is such an end vertex t_i in T so that $(t_l, s_k) \in C$;

4) if $(t_i, s_j) \in C$ is a pair of end-vertices, then at the segment that connects (t_0, s_0) and (t_i, s_j) the set C assigns the connection.

We selected root t_0 in the graph and there are the end vertices. If we take one more similar graph with root $s0$, then the Fréchet distance between these graphs is computed in the following way:

$$D_F(T,S) = \min_C \left(\max_{(t_i,s_j)\in C} D_F\big([t_0,t_i],[s_0,s_j]\big) \right). \tag{5}$$

Thus, formula (5) for finding the Fréchet distance makes it possible to calculate distances between trees (skeletons) and to construct algorithms for finding a distance between non-convex regions.

4. 3. Algorithm for finding a distance between the non-convex regions based on the Hausdorff metric

Let the two non-convex polygons P and Q received after segmentation be assigned (Fig. 4).
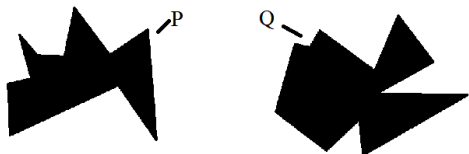
Fig. 4. Polygons P and Q

The algorithm for computing the Hausdorff distance between them is the following:

1) According to algorithm [27], we shall split polygons P and Q into sets of convex regions, that is:

$$P = O_{P_1} \cup ... \cup O_{P_i} \cup ... \cup O_{P_n},$$

where $i = \overline{1,n}$ is the quantity of convex regions of polygon P;

$$Q = O_{Q_1} \cup ... \cup O_{Q_j} \cup ... \cup O_{Q_m},$$

where $j = \overline{1,m}$ mis the quantity of convex regions of polygon Q;

2) Denote:

$$O_{P_1} \cup ... \cup O_{P_i} \cup ... \cup O_{P_n} = V,$$

$$O_{Q_1} \cup ... \cup O_{Q_i} \cup ... \cup O_{Q_m} = W;$$

3) Distance between polygons P and Q that is equal to the distance between inner regions of convex polygons P_i and Q_j will be represented as:

$$d(V,W) = \inf\left\{ \varepsilon > 0 \,|\, \forall i = \overline{1,n}, \exists j = \overline{1,m} \right.$$

and

$$d_H(O_i, O_j) \le \varepsilon \tag{6}$$

and vice versa

$$\forall_j = \overline{1,m}, \ \exists i = \overline{1,n} \ \text{and} \ d_H(O_i, O_j) \le \varepsilon,$$

where d_H is the Hausdorff distance;

4) the Hausdorff distance will be found based on the Attalah's algorithm [28]. Computational complexity of a given algorithm is $O(m, n)$, where m, n is the number of vertices of polygons.

4. 4. Algorithm of distance estimation between skeletons of the non-convex regions

Trees of the graph can exist without a selected root and with a selected root, that is, free trees. The following theorem holds for them.

Theorem [25]. Let $G=(V, E)$ be a non-directed graph. Then the following holds:

1. G is a free tree.

2. Any two vertices of G are connected through a single simple path.

3. G is a connected graph, but, when any edge is deleted from E, it ceases to be such a graph.

4. G is a connected graph, $|E| = |V| - 1$.

5. G is the acyclic graph, $|E| = |V| - 1$.

6. G is the acyclic graph, but, when adding any edge to E, we obtain a graph that includes a cycle.

We shall describe the main steps of the algorithm:

1. Following the skeletonization process over two images, we obtain skeletons $Sk_1 = G_1 = (V_1, E_1)$ and $Sk_2 = G_2 = (V_2, E_2)$, respectively. Let us represent the obtained trees $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ by applying adjacency matrices A_1 and A_2. The vertices should be numbered and arranged by numbers 1, 2, ..., $|V|$. Then the adjacency matrix $A = (\alpha_{ij})$, the size of $|V| \times |V|$, is such that

$$a_{ij} = \begin{cases} 1, \text{ if } (i,j) \in E, \\ 0, \text{ otherwise.} \end{cases}$$

Adjacency matrix A for a non-directed graph is equal to transposed matrix A^T. That is why we shall consider only those elements of the matrix, which are located along the main diagonal and above, that is, the submatrix A^*.

2. Form submatrices A^*_1 and A^*_2 based on incidence matrices A_1 and A_2 of graphs G_1 and G_2.

3. Based on them, create arrays of end-vertices $M_1 = \{m_1, m_2, ..., m_k\}$ and $M_2 = \{m_1, m_2, ..., m_p\}$.

4. By using submatrices A^*_1 and A^*_2 and arrays of end-vertices M_1 and $M2$, we shall find sets of paths $P_1 = \{p_1, p_2, ..., p_k\}$ and

$P_2=\{p_1, p_2,..., p_k\}$, each of which is the subset $p_i^1=\{v_0,v_r,...,v_i\}$ $i=\overline{1,k}$, and $p_i^2=\{v_0,v_r,...,v_i\}$ $i=\overline{1,r}$, where k and r is the number of end-vertices of trees G_1 and G_2, respectively.

5. Then the Fréchet distance will be computed from expression

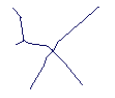$$D_F(P_1,P_2)=\min\left(\max_{(p_i^1,p_j^2)\in C} D_F\left(\left[p_0^1,p_i^1\right],\left[p_0^2,p_j^2\right]\right)\right).$$
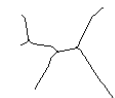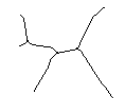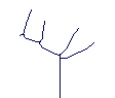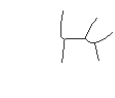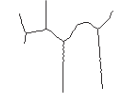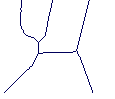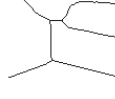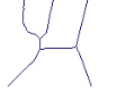
6. Computer experiments

The polygons that are examined in the present study were obtained following the segmentation of histologic and cytologic images from database [29]. As a result of the skeletonization process over polygons, we obtained skeletons of the examined micro objects.

Examples of polygons and their skeletons are shown in Table 1.

Table 1

Examples of polygons and their skeletons

No. of experi-ment	Polygon		Skeleton	
	Polygon of region 1	Polygon of region 2	Skeleton of region 1	Skeleton of region 2
1				
2				
3				
4				
5				
6				

Values of the Hausdorff distance between polygons are given in Table 2.

Values for the Fréchet distance for skeletons are given in Table 3.

Table 2

Values of the Hausdorff distance between polygons
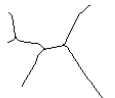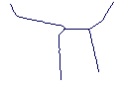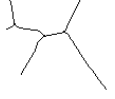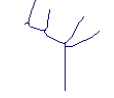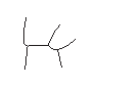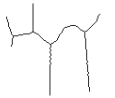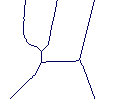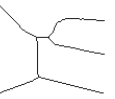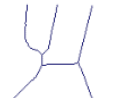
No.	Polygon of region 1	Polygon of region 2	The Hausdorff distance, pixels
1			34.48
2			40.01
3			36.12
4			53.93
5			93.19
6			71.58

Table 3

Values for the Fréchet distance for skeletons

No.	Skeleton of region 1	Skeleton of region 2	The Fréchet distance, pixels
1			34.48
2			41.0
3			36.87
4			54.20
5			94.0
6			72.42

Time for finding the distance between polygons and skeletons is shown in Fig. 5.
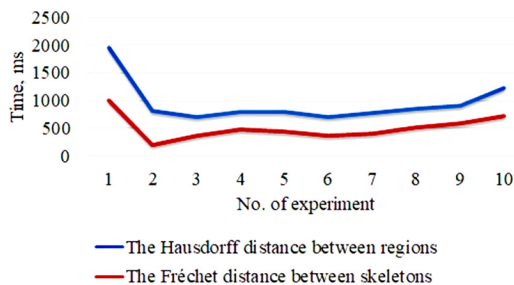
Fig. 5. Time for finding the Hausdorff distance between polygons and the Fréchet distance between skeletons

The values of relative error for the Hausdorff distance between polygons and the Fréchet distance between skeletons are shown in Fig. 6.
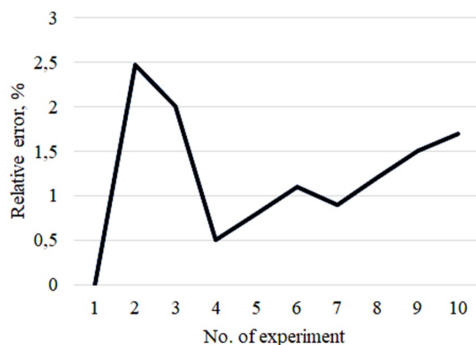
Fig. 6. Values of relative error for the Hausdorff distance between polygons and the Fréchet distance between skeletons

Computer experiments that we performed show that the deviation in the value of the Hausdorff distance between polygons and the Fréchet distance between skeletons is within 2–3 %.

7. Discussion of results of comparing the non-convex regions of images based on the Fr chet metric between trees

Modern algorithms for computing the Hausdorff distance between regions with low computational complexity exist only for the convex regions. Using the Hausdorff metric to calculate distances between the non-convex regions is computationally complicated. In this case, non-convex regions need to be converted into convex regions. This requires additional computational costs.

A characteristic of any region is the skeleton, which is a tree. That is why finding a distance between regions is replaced by finding a distance between skeletons. We proposed a new mathematical structure – the Fréchet metric between trees, which allowed us to solve the problem on computing distances between the non-convex regions.

The advantage of the Fréchet metric between trees is the possibility to calculate distances between the non-convex regions. This benefit is achieved by replacing the calculation of distances between regions with the computation of distances between skeletons of the regions. The method devised and the algorithms constructed have lower computational complexity compared to known algorithms for calculating the Hausdorff distance between the non-convex regions. Numerical simulation of the algorithms that we performed showed that the error in finding a distance based on the Hausdorff metric and on the Fréchet metric between trees lies within 2–3 %. In this case, however, the time for finding distances between regions when using the Fréchet metric between trees is significantly reduced, compared to the Hausdorff metric.

The first constraint for the application of the Fréchet metric between trees is the necessity to find skeletons of the regions. However, modern algorithms for finding skeletons possess low computational complexity. That is why finding the skeletons of regions is not a difficult task.

The second limitation is the need to separate the roots of skeletons. This limitation is caused by the fact that the introduced Fréchet metric between trees holds only to the root trees. This constraint can be eliminated by developing the Fréchet metric between the non-root trees. This is not a fundamental limitation. That is why the next steps in the research imply development of the Fréchet metric Fréchet for the non-root trees.

Results of the present study could be used not only for testing known and new segmentation algorithms, but also for image recognition, as well as image search in databases. This significantly expands the scope of application of the constructed metric.

8. Conclusions

1. It was found that the proposed Fréchet distance between trees is a metric. To establish it, we have proven axioms of identity, symmetry, and the triangle. The Fréchet metric between trees is based on the Fréchet metric and makes it possible to find distances between skeletons of images.

2. Based on the Fréchet metric between trees, we constructed a method for comparing the non-convex regions of images. The method is based on the algorithm of selection of image skeletons and on the algorithm for finding a distance between skeletons based on the Fréchet metric between trees. By employing the devised method, it has become possible to find distances between the non-convex regions.

3. Based on the method of comparison of the non-convex regions, we built algorithms for calculating a distance between the non-convex regions. These algorithms have lower computational complexity than the algorithms for computing the Hausdorff distance between the non-convex regions.

4. Computer experiments that we performed have shown that the error in calculating the distances, found by algorithms based on the Hausdorff metric and based on the Fréchet metric between trees, is within 2–3 %. In this case, the time for calculating the Fréchet distance between skeletons is on average 2 times less than the time for computing the Hausdorff distance between polygons.

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DEVELOPMENT OF THE METHOD FOR DYNAMIC REGULARIZATION OF SELECTED ESTIMATES IN THE CORRELATION MATRICES OF OBSERVATIONS

Розглянута задача формування вибіркових оцінок кореляційної матриці спостережень за критерієм «обчислювальна стійкість – спроможність» та проаналізована спроможність стійких оцінок при статичній регуляризації. Виявлена проблема задачі регуляризації оцінок, для вирішення якої запропоновано альтернативний метод динамічної регуляризації. Отримана оптимальна функція динамічної регуляризації вибіркових оцінок в умовах апріорної невизначеності та надані чисельні результати

Ключові слова: статична регуляризація, динамічна регуляризація, стійкість, збіжність, спроможність оцінок, кореляційна матриця

Рассмотрена задача формирования выборочных оценок корреляционной матрицы наблюдений по критерию «вычислительная устойчивость – состоятельность» и проанализирована состоятельность устойчивых оценок при статической регуляризации. Выявлена проблема задачи регуляризации оценок, для решения которой предложен альтернативный метод динамической регуляризации. Получена оптимальная функция динамической регуляризации выборочных оценок в условиях априорной неопределенности и представлены численные результаты

Ключевые слова: статическая регуляризация, динамическая регуляризация, устойчивость, сходимость, состоятельность оценок, корреляционная матрица

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1. Introduction

Inversion of the correlation matrix of observations belongs to the class of problems associated with the reversion of cause-effect relationships. This procedure is the basis for solving inverse statistical problems in applications of spectral analysis, space-time processing of multidimensional signals, control theory, identification, prediction and decision making [1–8].