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TECHNOLOGY AND SOFTWARE TO DETERMINE ADEQUATE NORMALIZED CORRELATION MATRICES IN THE SOLUTION OF IDENTIFICATION PROBLEMS

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Показано, що при класичному підході при нормуванні оцінок в діагональних елементах кореляційних матриць відсутні похибки від перешкод, а в інших же елементах ця похибка, навпаки, виникає. В результаті поліпшення обумовленості матриці від переходу до нормованих кореляційних матриць не спостерігається. Пропонуються технологія, софт для усунення цього недоліку і аналізу обчислювальних експериментів

Ключові слова: реальний сигнал, перешкода, кореляційна функція, нормована кореляційна матриця, вхідний – вихідний сигнал

Показано, что при классическом подходе при нормировании оценок в диагональных элементах корреляционных матриц отсутствуют погрешности от помех, а в остальных же элементах эта погрешность, наоборот, возникает. В результате улучшения обусловленности матрицы от перехода к нормированным корреляционным матрицам не наблюдается. Предлагается технология, софт для устранения этого недостатка и анализа вычислительных экспериментов

Ключевые слова: реальный сигнал, помеха, корреляционная функция, нормированная корреляционная матрица, входной – выходной сигнал

1. Introduction

It is known [1] that one of the main challenges in solving problems of automated control of industrial facilities is establishing the quantitative interrelations between input and output noisy signals characterizing the processes in those objects both in statics and dynamics. Such interrelations are called static and dynamic characteristics, respectively. These characteristics can be determined from differential equations of control objects. However, those differential equations are often unknown, which is why statistical methods are widely used – they make it possible to determine dynamic characteristics during normal operation of objects [1–3]. In practice, such dynamic characteristics as impulsive admittance and transfer functions of linear systems are determined by applying to their input artificial stimulation of a certain type (impulse, step function, sinusoids) and measuring the response. However, in that case, random uncontrollable disturbances are superimposed on these impacts. As a result, it proves impossible to precisely determine dynamic characteristics based on typical input signals [1–3].

2. Literature review and problem statement

The statistical correlation method for determining these dynamic characteristics is based on the solution of an integral equation that includes the correlation functions $R_{XX}(i\Delta t)$ and $R_{XY}(i\Delta t)$ of the input $X(i\Delta t)$ and output $Y(i\Delta t)$ signals. It allows us to obtain the dynamic characteristics of an object without disturbing its normal operation mode. Therefore, statistical methods are widely used for determining the dynamic characteristics of objects during their normal operation [1–3].

However, the application of statistical methods for building mathematical models of real-life industrial objects presents the following difficulty. Interferences and noises are imposed upon the useful signal (that has to be obtained with the least possible amount of distortion), thus hindering the calculation of the estimates of their static characteristics.

One should take into account that interferences and noises are also represented by random functions $\varepsilon(i\Delta t)$. The reasons behind the formation of interferences and noises can be very diverse [4]:

a) thermal noises;

b) noises caused by other machinery and equipment operating nearby;

c) noises caused by power supply sources;

d) noises caused by self-oscillations generated in feedback circuits, etc.

For instance, for deep-water offshore platforms, noises are caused by waves, wind, etc. Another example is the radio detector of an antenna under a wind load, which also represents a random time function.

In view of the above, many algorithms and technologies of filtration have been proposed with the aim of eliminating the effects of the noise on the result of identification of statistical models of the dynamics of control objects over a long period of time [4]. The ones that allow for eliminating the error of the noises caused by external factors have found a wide application. However, in many real-life objects, noises of technological processes are formed under the influence of various factors. Some of them reflect indirectly certain processes that cause defects in the objects under investigation. For this reason, the range of the noise spectrum frequently overlaps the spectrum of the useful signal. Besides, the spectra of the noise and the useful signal are not strictly stable. Therefore, filtration does not always yield the desired result. Sometimes, the spectrum of the useful signal is even distorted from the filtration.

Taking into account the above, the paper considers one possible option of creating alternative digital methods and technologies for eliminating the error induced by noise during the formation of correlation matrices in the process of identification of the dynamic model of industrial objects.

When applying the classical approach, it is possible to obtain more or less acceptable estimates of the characteristics of the investigated parameters only if the signal is modeled and complies with the classical conditions. In the case when the signal is received from real processes, the effect of noise, changing of the frequency range, differences in measurement units create a number of difficulties.

Because of this, the description of many analyzed processes through the classical mathematical apparatus of probabilistic methods proves to be inadequate, and when solving real problems, erroneous results are obtained. Precisely for these reasons, the correlation matrices, being formed from the correlation functions of real signals and having the most important applied importance, as well as underlying many engineering problems, are not adequately solved now. This is due to the noise-induced error in each element of the correlation matrix, which creates a perceptible error in the formation of the final normalized correlation matrix of noisy signals and the inaccuracy of the results in determining the statistical characteristics of the matrix as a whole. It should also be noted that the process of obtaining the output signal and determining its characteristics is accompanied by serious errors.

According to the existing literature [1], in order to eliminate the effects of the noise on the estimate of the correlation function for zero-time shift $\mu=0$, it is expedient to proceed to normalized estimates of the correlation functions. It is known that standardization or normalization reduces values of all transformed variables to a single range of values by expressing through the relation of these values to a certain value reflecting certain properties of a particular attribute [4–6]. At the same time, the normalization process really allows one to independently compare the cross-correlations of the absolute values of the data [7]. In addition, the normalization of the indicators is often required, and this is convenient in order to proceed to dimensionless variables [8]. Moreover, in most cases, random functions are approximated by a normalized correlation function [9]. It is common to proceed immediately to normalized correlation functions, as normalized correlation functions are considered to be convenient in that their values do not exceed unity [10]. It is also known that to estimate the degree of dependence of the cross sections of a random function, it is more convenient to use the normalization of the correlation function [11]. It is also common that the normalized correlation function is used to be able to compare processes with different values of variances [12].

Thus, the normalized cross-correlation matrices that consist of normalized correlation functions are of the utmost applied importance.

It is known that correlation functions mainly are calculated as elements of correlation matrices [13]. At the same time, correlation matrices are the main indicators of eigenvectors estimates [14]. Besides, solving tasks with large correlation matrices is one of the big spheres of computational mathematics and methods of optimization [15]. Correlation matrices are used for different engineering tasks as a tool of decision-making technologies. There is a great variety of tasks where correlation matrices are applied [16, 17].

It is known that the universality of the problem of improving the conditionality of correlation matrices is due to the fact that, in essence, all typical problems associated with the statistical analysis of the processes under investigation require the knowledge of correlation matrices composed of the estimates of auto- and cross-correlation functions. However, in practice, for real objects, based on the data of their normal operation, the estimates of the auto- and cross-correlation functions of input and output signals contain certain "micropulsations", due to unavoidable noise-induced errors [18]. Since the methods used to solve these problems are very sensitive to these "micropulsations", the correlation matrices turn out to be ill-conditioned, and the obtained solution is not adequate. Similar difficulties arise for any matrix equation, when ill-conditioned correlation matrices are used to solve it, each element of which contains a noise-induced error. It should also be noted that the process of obtaining the output signal and determining its characteristics is accompanied by serious errors.

To eliminate the difficulties caused by these reasons, many methods for improving the conditionality of the correlation matrices have been proposed. Despite the high academic level of these studies, the experience of their successful practical use in solving applied problems is not great [1–3]. Among these methods, a special place is occupied by the method of regularization and its modification [1–3], which are the most popular. The usefulness of regularization is substantiated by a large number of theoretical studies. However, for all its indisputable merits, this method has a significant drawback: the impossibility of practical selection of the optimal regularization parameter due to its dependence on unknown parameters. In view of this, there is also no complete guarantee that satisfactory results will be obtained after using the regularization method [4–10]. In this regard, there is an urgent need to develop new methods and algorithms aimed at eliminating the difficulties caused by the ill conditionality of correlation matrices. Thus, to solve the above – mentioned problems, it is necessary to develop alternative methods, algorithms and technologies that allow solving statistical identification problems even under ill conditionality of correlation matrices and in violation of the classical conditions. Thus, this paper is devoted to the technology of correct normalization of correlation matrices, the elements of which are normalized correlation functions.

3. The aim and objectives of the study

The aim of this technology is transforming initial (noisy matrices) matrices to the view which is almost equivalent to the view of matrices with elements which do not consist in any errors of the noise. It is done by eliminating the influence of the noise. It happens even in case of "bad" conditionality. This problem is solved by new technology for normalization of correlation matrices (cross- and auto-correlation matrices) (4), (5). This technology is for eliminating the influence of the noise/interference.

The comparative analysis should prove the difference between traditional equations and advantages of it. For solving this problem, the creation of new special software was suggested too. The software should be maximum informative (all necessary estimates of initial, noisy, corrected matrices) should be calculated.

It should be obvious thanks to easy interface. The interface computes both variants: traditional and suggested one. Thus, the purpose was:

– to suggest new equations for normalized correlation functions and normalized correlation matrices;

– to suggest a new technology which easily helps one to eliminate the errors of noise in correlation functions which are elements of correlation matrices;

– to create the special new software for applying the new technology;

– to compare the results of correlation matrices of useful signals (without any noise), classical normalized correlation matrices of noisy signals and normalized correlation matrices counted by the suggested technology;

– to open the possibility of easy comparison of their statistical estimates.

4. Technology for determining equivalent normalized correlation matrices for solving identification problems.

The technology for obtaining correct values and characteristics of normalized correlation matrices in solving identification problems in the case of ill conditionality is as follows. In the following paragraphs, we propose a technology for forming the corrected values of normalized correlation matrices, which provides improved accuracy of the obtained estimates even under ill conditionality in solving identification problems.

1. For each noisy input signal $g_1(t), g_2(t), g_3(t)$ and output signal $\eta(t)$, the estimates of the auto- and cross-correlation functions are calculated [19]:

$$R_{\overset{\circ}{g}_i \overset{\circ}{g}_j}(\mu) = \frac{1}{N}\sum_{k=1}^{N} \overset{\circ}{g}_i(k\Delta t)\overset{\circ}{g}_j\big((k+\mu)\Delta t\big),$$

$$R_{\overset{\circ}{g}_i \overset{\circ}{\eta}}(\mu) = \frac{1}{N}\sum_{k=1}^{N} \overset{\circ}{g}_i(k\Delta t)\overset{\circ}{\eta}\big((k+\mu)\Delta t\big),\ i,j=\overline{1,n}. \tag{1}$$

2. For each noisy input signal $\overset{\circ}{g}_i(t)$ and output signal $\overset{\circ}{\eta}(t)$, the noise variances $D^*(\varepsilon_i)$, $D^*(\phi)$ are calculated [7–10]:

$$D^*(\varepsilon_i) = \\ = \frac{1}{N}\sum_{k=1}^{N}\Big[\overset{\circ}{g}_i(k\Delta t)\overset{\circ}{g}_i(k\Delta t) - 2\overset{\circ}{g}_i(k\Delta t)\overset{\circ}{g}_i\big((k+1)\Delta t\big) + \overset{\circ}{g}_i(k\Delta t)\overset{\circ}{g}_i\big((k+2)\Delta t\big)\Big], \tag{2}$$

$$D^*(\phi) = \\ = \frac{1}{N}\sum_{k=1}^{N}\Big[\overset{\circ}{\eta}(k\Delta t)\overset{\circ}{\eta}(k\Delta t) - 2\overset{\circ}{\eta}(k\Delta t)\overset{\circ}{\eta}\big((k+1)\Delta t\big) + \overset{\circ}{\eta}(k\Delta t)\overset{\circ}{\eta}\big((k+2)\Delta t\big)\Big]. \tag{3}$$

3. The normalized correlation matrices of the useful signals, noisy signals, and corrected normalized correlation matrices $\vec{r}^{\,k}_{\overset{\circ}{X}\overset{\circ}{X}}(0)$, $\vec{r}^{\,k}_{\overset{\circ}{X}\overset{\circ}{Y}}(0)$ are formed:

$$\vec{r}^{\,k}_{\overset{\circ}{X}\overset{\circ}{X}}(0) = \\ = \begin{bmatrix} 1 & \frac{R_{\overset{\circ}{g}_1\overset{\circ}{g}_2}(0)}{\sqrt{\big(D(g_1)-D^*(\varepsilon_1)\big)\cdot\big(D(g_2)-D^*(\varepsilon_2)\big)}} & \cdots & \frac{R_{\overset{\circ}{g}_1\overset{\circ}{g}_n}(0)}{\sqrt{\big(D(g_1)-D^*(\varepsilon_1)\big)\cdot\big(D(g_n)-D^*(\varepsilon_n)\big)}} \\ \frac{R_{\overset{\circ}{g}_2\overset{\circ}{g}_1}(0)}{\sqrt{\big(D(g_2)-D^*(\varepsilon_2)\big)\cdot\big(D(g_1)-D^*(\varepsilon_1)\big)}} & 1 & \cdots & \frac{R_{\overset{\circ}{g}_2\overset{\circ}{g}_n}(0)}{\sqrt{\big(D(g_2)-D^*(\varepsilon_2)\big)\cdot\big(D(g_n)-D^*(\varepsilon_n)\big)}} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{R_{\overset{\circ}{g}_n\overset{\circ}{g}_1}(0)}{\sqrt{\big(D(g_n)-D^*(\varepsilon_n)\big)\cdot\big(D(g_1)-D^*(\varepsilon_1)\big)}} & \frac{R_{\overset{\circ}{g}_n\overset{\circ}{g}_2}(0)}{\sqrt{\big(D(g_n)-D^*(\varepsilon_n)\big)\cdot\big(D(g_2)-D^*(\varepsilon_2)\big)}} & \cdots & 1 \end{bmatrix}. \tag{4}$$

$$\vec{r}^{\,k}_{\overset{\circ}{X}\overset{\circ}{Y}}(0) = \\ = \left[\frac{R_{\overset{\circ}{g}_1\overset{\circ}{\eta}}(0)}{\sqrt{\big(D(g_1)-D^*(\varepsilon_1)\big)\cdot\big(D(\eta)-D^*(\phi)\big)}} \quad \frac{R_{\overset{\circ}{g}_2\overset{\circ}{\eta}}(0)}{\sqrt{\big(D(g_2)-D^*(\varepsilon_2)\big)\cdot\big(D(\eta)-D^*(\phi)\big)}} \quad \cdots \quad \frac{R_{\overset{\circ}{g}_n\overset{\circ}{\eta}}(0)}{\sqrt{\big(D(g_n)-D^*(\varepsilon_n)\big)\cdot\big(D(\eta)-D^*(\phi)\big)}}\right]. \tag{5}$$

It should also be noted that $D^*(\varepsilon_i)$ was calculated by two methods: by the classical method and by the T. A. Aliev's method. The calculations using the classical formula were performed solely for comparison with the values of the noise variances calculated from the Aliev's formula. This was possible, because the noisy signal was obtained by adding the useful signal and the noise, i. e. $g(t)=x(t)+\varepsilon(t)$, with different distribution laws, and consequently, the values of the noise variances are known in advance and can be compared.

Thus, this technology for improving the formation and evaluation of normalized correlation matrices, even with a high degree of ill conditionality, allows one, by eliminating the effects of the noise characteristics, to transform the original matrices into a form almost equivalent to the form of the matrix whose elements do not contain noise-induced errors.

5. Software tools for determining equivalent normalized correlation matrices in solving identification problems

To test the effectiveness of the technology for forming the robust normalized correlation matrices and obtaining adequate statistical characteristics of noisy signals, namely several input and output signals, numerous computational experiments were conducted. To ensure the effectiveness of the experiments, we have developed independent software called signals_v1.

For this purpose, absolutely independent software was created in JavaFX using Java JDK v1.8: Compiled on Intel i7 x64 Windows 10. This was done for several important reasons. First, Matlab is extremely expensive and its installation takes several hours, and the libraries required for signal processing cause additional difficulties. In addition, Matlab takes up several Gb of memory. Second, Matlab is designed for computers that meet high standards and parameters. The advantages of our software are as follows.

1. It "weighs" only 2 Mb, which makes it very "light".
2. The installation is easy and takes a split second.
3. The program interface is designed solely to solve this problem and can easily be used by an ordinary operator.
4. It does not require ɨ any additional programs and does not impose special requirements for the system.

We will still specify the requirements for the PC: OS Windows Vista/7/10, JRE v1.8, or any Linux platform with JRE v.1.8 installed.

Three useful signals X(kΔt) were generated with the step of pi/100 up to 24*pi and 3 noises ε(kΔt) by the random unit generator with different distribution laws. Thus, 2,400 points (estimates) were obtained for each of the 3 input noisy signals and noises. Then noisy signals of the $g(k\Delta t)=X(k\Delta t)+\varepsilon(k\Delta t)$ form were generated. The output signal of the $y(k\Delta t)=k0+k1*s1(i\Delta t)+k2*s2(i\Delta t)+k3*s3(i\Delta t)$ form was also generated, where k is the coefficients. Then a fourth noise was generated to form a noisy output signal. Thus, the output signal also consisted of 2,400 samples.

Then all the signals were centered. After that, the normalized correlation functions of the input useful signals were calculated for forming the correlation matrices of the useful signals, i. e.

M(1,1)=*rns*1*s*1(1); *M*(1,2)=*rns*1*s*2(1);

M(1,3)=*rns*1*s*3(1);

M(2,1)=*rns*2*s*1(1); *M*(2,2)=*rns*2*s*2(1);

M(2,3)=*rns*2*s*3(1);

M(3,1)=*rns*3*s*1(1); *M*(3,2)=*rns*3*s*2(1);

M(3,3)=*rns*3*s*3(1).

Further, the normalized correlation functions of the noisy signals were calculated for forming the correlation matrices of the noisy signals calculated from the classical formulas [1–3].

M(1,1)=*rng*1*g*1(1); *M*(1,2)=*rng*1*g*2(1);

M(1,3)=*rng*1*g*3(1);

M(2,1)=*rng*2*g*1(1); *M*(2,2)=*rng*2*g*2(1);

M(2,3)=*rng*2*g*3(1);

M(3,1)=*rng*3*g*1(1); *M*(3,2)=*rng*3*g*2(1);

M(3,3)=*rng*3*g*3(1).

Next, the values of the noise variances are calculated using the T. A. Aliev's formula [4].

Then the corrected normalized correlation functions of the noisy signals were calculated to form the corrected normalized correlation matrices of the noisy signals calculated from the new formulas.

M(1,1)=*rkn*11(1); *M*(1,2)=*rkn*12(1);

M(1,3)=*rkn*13(1);

M(2,1)=*rkn*12(1); *M*(2,2)=*rkn*22(1);

M(2,3)=*rkn*23(1);

M(3,1)=*rkn*13(1); *M*(3,2)=*rkn*23(1);

M(3,3)=*rkn*33(1).

In turn, the normalized output signal was generated (useful, noisy, corrected, respectively):

Y=[*rnx*4*x*1(1);*rnx*4*x*2(1);*rnx*4*x*3(1)];

*Y*_za6um=[*rng*4*g*1(1);*rng*4*g*2(1);*rng*1*g*3(1)];

*Y*_korrektir=[*rkn*41(1);*rkn*42(1);*rkn*43(1)].

In addition, taking into account the possibility of finding the noise variance, the values of the coefficients *k*0, *k*1, *k*2, *k*3 were calculated. Further, the errors of the corrected estimates obtained from the values of the useful signal estimates were calculated.

After the calculations, the determinants $\Delta_{\overset{\circ}{X}\overset{\circ}{X}}(\mu)$, $\Delta_{\overset{\circ}{g}\overset{\circ}{g}}(\mu)$, $\Delta^{k}_{\overset{\circ}{X}\overset{\circ}{X}}(\mu)$, and the conditioning numbers $H\left(\vec{r}_{\overset{\circ}{X}\overset{\circ}{X}}(\mu)\right)$, $H\left(\vec{r}_{\overset{\circ}{g}\overset{\circ}{g}}(\mu)\right)$, $H\left(\vec{r}^{k}_{\overset{\circ}{X}\overset{\circ}{X}}(\mu)\right)$ of the normalized correlation matrices $\vec{r}_{\overset{\circ}{X}\overset{\circ}{X}}(\mu)$, $\vec{r}_{\overset{\circ}{g}\overset{\circ}{g}}(\mu)$ of the useful and noisy signals and the corrected matrix $\Delta\vec{r}^{k}_{\overset{\circ}{X}\overset{\circ}{X}}(\mu)$ were calculated for the operative and visual comparative analysis. Also, the values of the noise variance, both those specified and those calculated by Aliev's formula, the coefficients for all 3 cases, namely the useful signal, the normalized noisy signal and the corrected normalized noisy signal were shown.

Further, the errors of the corrected estimates obtained from the values of the useful signal estimates were calculated.

Types of computational experiments conducted.

Experiment N1. Three useful input signals

$$X_1(i\Delta t) = 40\sin(i\Delta t) + 282,$$

$$X_2(i\Delta t) = 50\sin(i\Delta t - 0.4) + 125,$$

$$X_3(i\Delta t) = 73\sin(i\Delta t + 0.39) + 155$$

and the output signal

$$Y(i\Delta t) = 100 + 3x_1(i\Delta t) + 8x_2(i\Delta t) - 5x_3(i\Delta t)$$

are generated. The noises $\varepsilon_1(t)$, $\varepsilon_2(t)$, $\varepsilon_3(t)$, $\phi(t)$ obey the normal distribution law with the mathematical expectations $m_{\varepsilon_1} \approx m_{\varepsilon_2} \approx m_{\varepsilon_3} \approx m_y \approx 0$ and mean – square deviations $\sigma_{\varepsilon_1} \approx 12$, $\sigma_{\varepsilon_2} \approx 15$, $\sigma_{\varepsilon_3} \approx 13$, $\sigma_y \approx 18$. The classical conditions are fulfilled for the useful signals and the noise.

Experiment N2. Three useful input signals

$$X_1(i\Delta t) = 40\sin(i\Delta t) + 101,$$

$$X_2(i\Delta t) = 50\sin(i\Delta t - 0.49) + 24\cos(0.78 \cdot i\Delta t) + 119,$$

$$X_3(i\Delta t) = 71\sin(i\Delta t + 0.28) - 44\cos(0.37 \cdot i\Delta t) + 177$$

and the output signal

$$Y(i\Delta t) = 101 + 11x_1(i\Delta t) - 7x_2(i\Delta t) + 6x_3(i\Delta t)$$

are generated. The noises obey the normal distribution law with $m_{\varepsilon_1} \approx m_{\varepsilon_2} \approx m_{\varepsilon_3} \approx m_y \approx 0$ and $\sigma_{\varepsilon_1} \approx 9$, $\sigma_{\varepsilon_2} \approx 14$, $\sigma_{\varepsilon_3} \approx 23$, $\sigma_y \approx 100$. For the second and third useful signals, the condition of the consistence of the mathematical expectation is violated.

6. Data processing (Discussion): comparative analysis of computational experiments

1) The elements of the normalized matrix of the noisy signals are very different from the elements of the matrix of the useful signals. It is evident from the example which was done with special software (Fig. 1, *b*, Fig. 2, *b*):

$$r_{\overset{\circ}{g}_i\overset{\circ}{g}_j}(\mu) \neq r_{\overset{\circ}{X}_i\overset{\circ}{X}_j}(0),$$

$$r_{\overset{\circ}{g}_i\eta}(0) \neq r_{\overset{\circ}{X}_i\overset{\circ}{Y}}(0), \quad i,j = \overline{1,n}.$$

However, the elements of the corrected normalized matrix of noisy signals are commensurable with the elements of the matrix of useful signals. It is evident from the example which was done with special software (Fig. 1, *b*, Fig. 2, *b*):

$$r^{k}_{\overset{\circ}{X}_i\overset{\circ}{X}_j}(\mu) \approx r_{\overset{\circ}{X}_i\overset{\circ}{X}_j}(\mu),$$

$$r^{k}_{\overset{\circ}{X}_i\overset{\circ}{Y}}(\mu) \approx r_{\overset{\circ}{X}_i\overset{\circ}{Y}}(\mu), \quad i,j = \overline{1,n}.$$

2) The elements of the matrix $\Delta\vec{r}_{\overset{\circ}{g}\overset{\circ}{g}}(\mu)$ of the relative errors of the noisy input signals range from 0 % to 14.12 %, and the elements of the column vector $\Delta\vec{r}_{\overset{\circ}{g}\eta}(0)$ of the relative errors from 7.5 % to 23.9 % (Table 1, rows 2, 4, column 2). The elements of the matrix $\Delta\vec{r}^{k}_{\overset{\circ}{X}\overset{\circ}{X}}(0)$ of the relative errors in the matrix $\vec{r}^{k}_{\overset{\circ}{X}\overset{\circ}{X}}(0)$ of the input signals range only from 0 % to 5.28 %, and the elements of the column vector $\Delta\vec{r}^{k}_{\overset{\circ}{X}\overset{\circ}{Y}}(0)$ range from 1.5 % to 5.3 %.

3) Despite the fact that the value of the conditioning number of the matrix of the noisy signals differs significantly from the value of that of the matrix of the useful signals, i. e.

$$H\left(\vec{r}_{\overset{\circ}{g}\overset{\circ}{g}}(0)\right) \neq H\left(\vec{r}_{\overset{\circ}{X}\overset{\circ}{X}}(0)\right)$$

and the value of the conditioning number of the corrected normalized matrix is not the same as that of the matrix of the useful signals, i. e.

$$H\left(\vec{r}^{R}_{\overset{\circ}{X}\overset{\circ}{X}}(0)\right) \neq H\left(\vec{r}_{\overset{\circ}{X}\overset{\circ}{X}}(0)\right),$$

the noisy output signal and its coefficients are close to the values of the useful output signal and its coefficients.

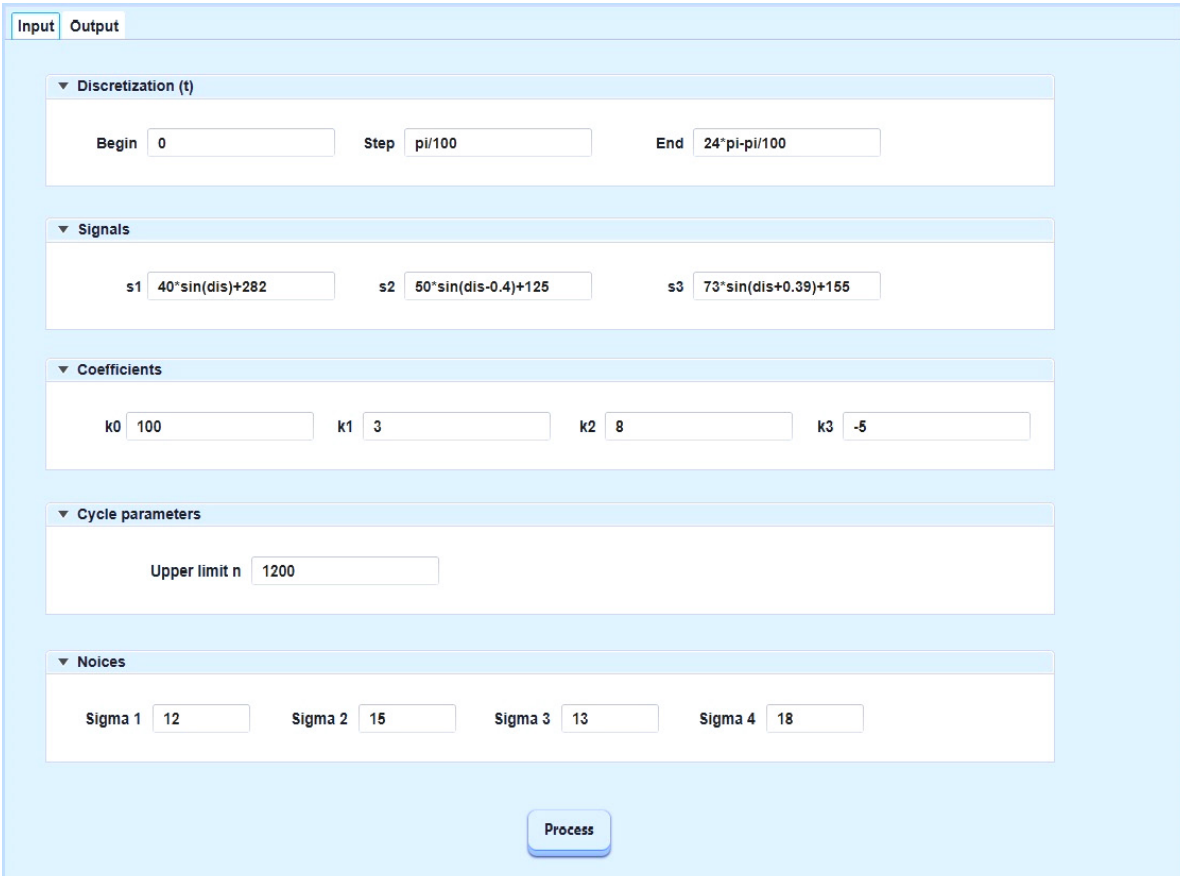
4) The found noise variance (calculated from the T. A. Aliev's formula) practically matches the given noise variance (Fig. 1, *b*, Fig. 2, *b*).

Thus, the use of the developed technology makes it possible to obtain values of normalized correlation matrices of noisy signals that are practically equivalent to those of correlation matrices of useful signals, i. e. to eliminate the effects of the noise.

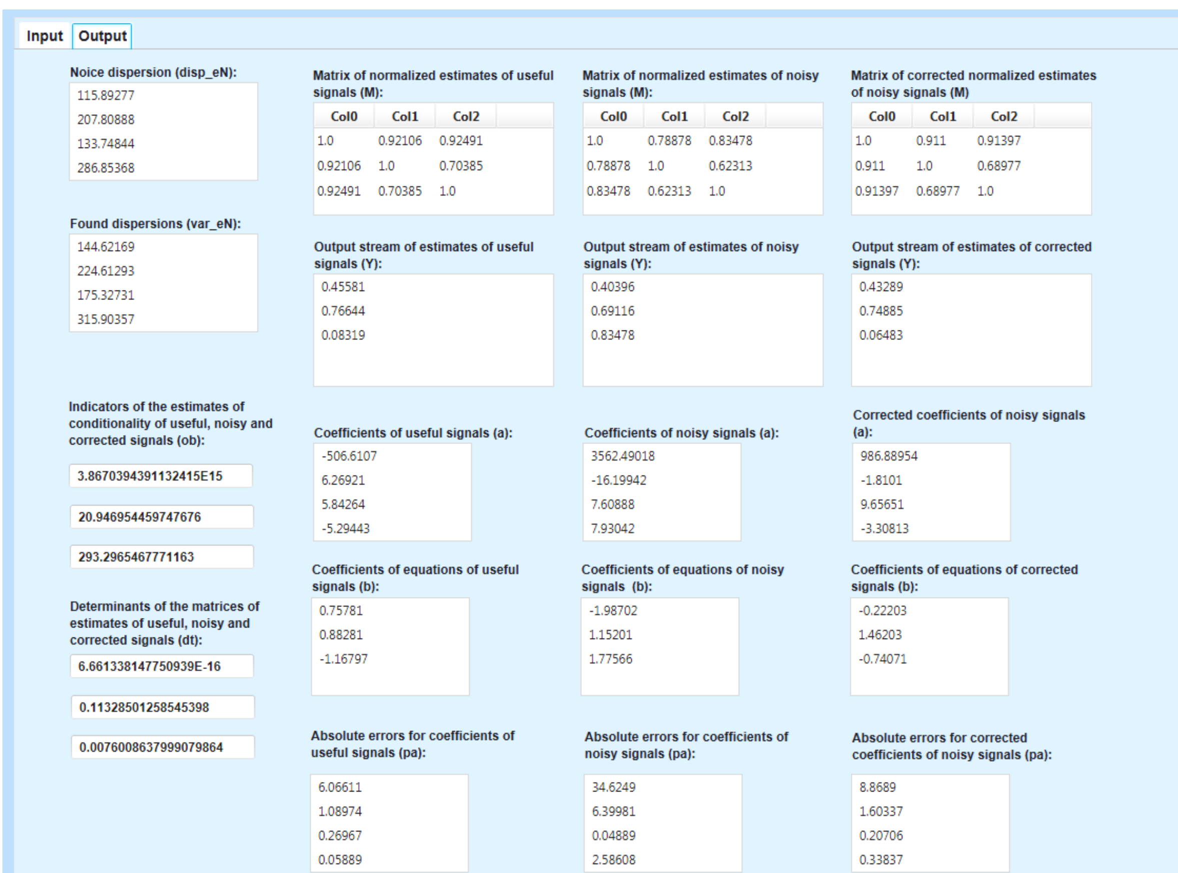
The main advantage of this technology is that it helps one to eliminate the influence of the noise and correctly compute the normalized correlation matrices. Thanks to different input parameters (with different units of count) after the procedure of normalization one can receive more exact output results or better equation which describe the output signal.

The disadvantage of this technology is that in the case when the analyzed object is not an industrial object, but a construction object (bridge or any other strategic object) and does not have different input parameters, one should put at least 3 sensors in different points. This is quite problematic and expensive.

For developing these experiments, one must have the natural experiments on real objects parallel with computational ones.

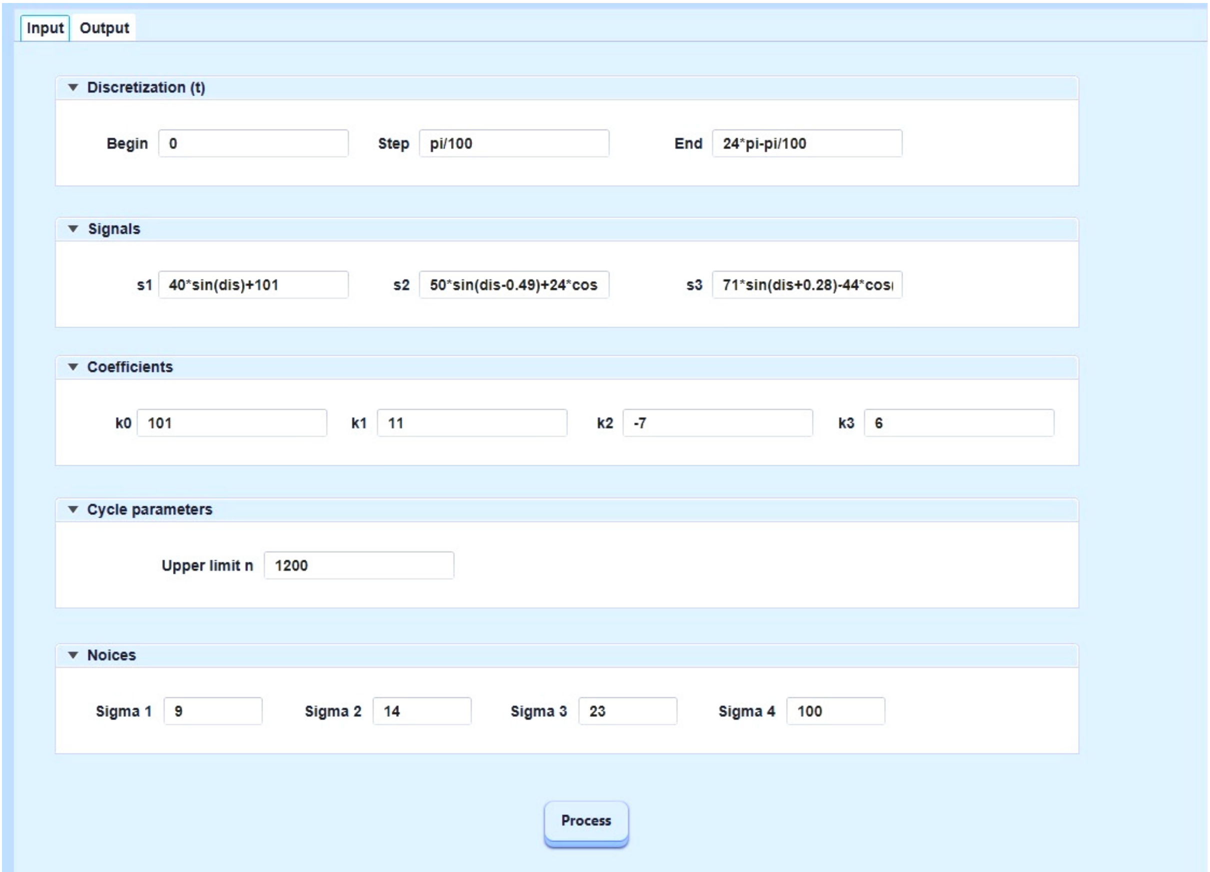


a

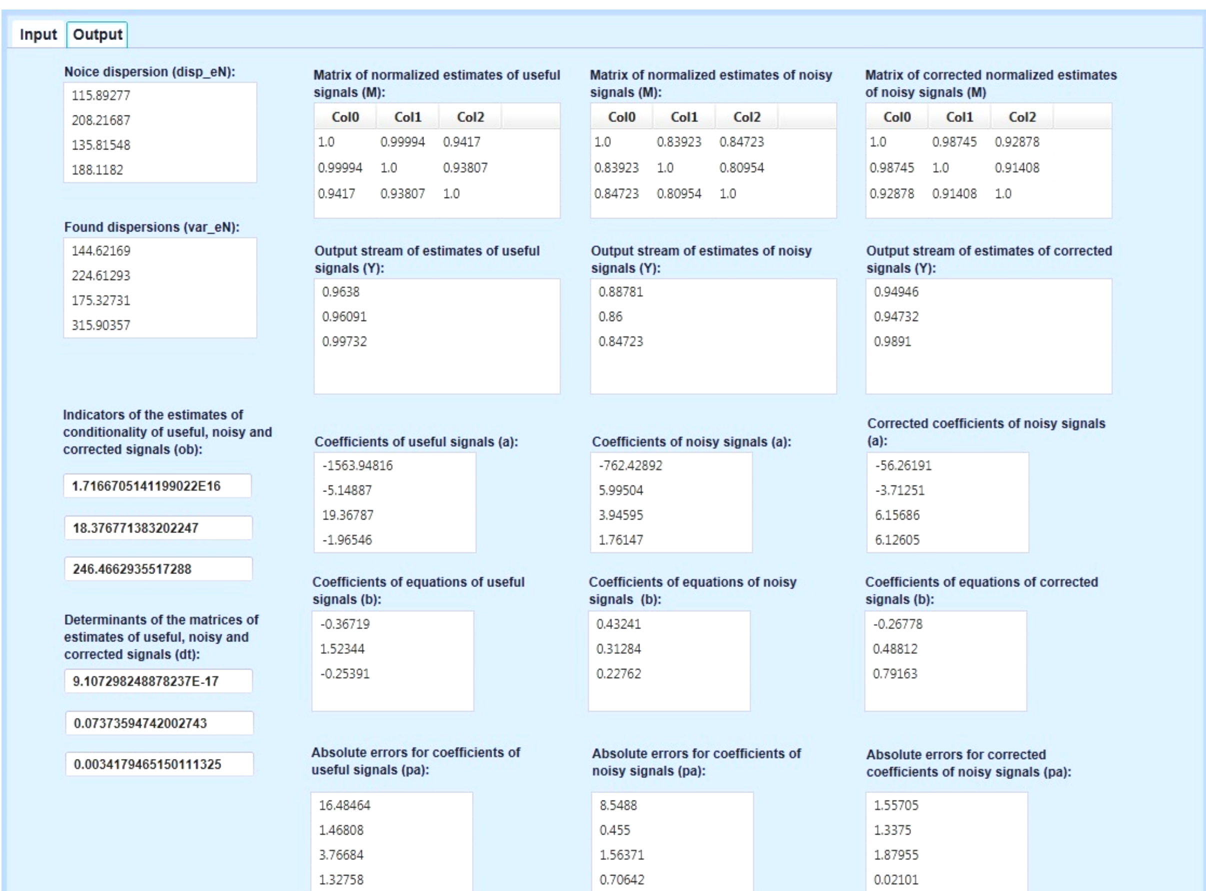


b

Fig. 1. Software interface: *a* — initial data; *b* — results window



a



b

Fig. 2. Software interface: *a* – initial data; *b* – results window

7. Conclusions

After the analysis of numerous computational experiments, the following conclusions are drawn:

1. The proposed new equations for normalized correlation functions and normalized correlation matrices ignore the impact of noise even in the case when traditional conditions [1–3] are not satisfied and matrices are with "bad" conditionality. It is clear from Fig. 1, *a*, Fig. 2 as the conditionality of all three matrices is very high.

2. A new technology which easily helps one to eliminate the errors of noise in correlation functions which are elements of correlation matrices is proposed. Thus, corrected elements of normalized correlation matrices of noisy signals are almost equal to the elements of the normalized correlation matrix of useful signals. As it is clear from Fig 1, *b*, Fig 2, *b*, correlation matrices of corrected normalized estimates (4) of noisy signals are practically equal to correlation matrices of useful signals.

3. The software which is suggested for formalizing corrected normalized matrices is helpful for showing that new technology gives positive results (similarity of normalized correlation matrices of noisy signals to normalized correlation matrices of useful signals). At the same time, this is proven by comparing the variances (dispersion) which are founded by 2 expressions: classical one [3] and suggested one (2). As the noisy signal was modeled as $g(t)=x(t)+e(t)$, the estimate of variance was evident. That is why, in the program there are 2 values for each experiment – calculated by the standard function and determined by the Aliev's expression (2) Fig. 1, *b*, Fig. 2, *b*. I need to notice that it was counted for all 4 noises (3 for input signals and one for output signal). From values which are shown in the software's window (Fig. 1, *b*, Fig. 2, *b*), it is clear that variances which are calculated from (2) are closer to real estimates of variance.

4. At the same time, the software helps one to easily compare (both traditional and suggested technology) ~~of~~ other statistical estimates of corrected normalized correlation matrices of noisy signals with normalized correlation functions of useful signals. The software is helpful to determine and compare the determinant of all 3 matrices (matrix of useful signals, normalized matrix of noisy signals and corrected normalized matrix of noisy signals) Fig. 1, *b*, Fig. 2, *b*. It is evident that the estimate of the determinant of the corrected matrix is closer to the determinant of the useful signal matrix. Thus, the influence of the noise is less. Besides, the errors of the input signal's coefficients $y(k\Delta t)=k0+k1\cdot s1(i\Delta t)+k2\cdot s2(i\Delta t)+k3\cdot s3(i\Delta t)$ are minimalized too Fig. 1, *a*, *b*, Fig. 2 *a*, *b*.

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