Розглянуто двовимірну задачу теорії пружності для двох спаяних різнорідних півплощин, що містять тріщини. Побудовано системи сингулярних інтегральних рівнянь першого роду на контурах тріщин. Числовий розв'язок інтегральних рівнянь одержано методом механічних квадратур для випадків довільно орієнтованої, а також дволанкової ламаної тріщини. Визначено коефіцієнти інтенсивності напружень у вершинах тріщини в залежності від кута нахилу для різних пружних характеристик півплощин

Ключові слова: коефіцієнт інтенсивності напружень, сингулярне інтегральне рівняння, рівномірно розподілений тиск, спаяні різнорідні півплощини

Рассмотрена двумерная задача теории упругости для двух спаянных разнородных полуплоскостей, содержащих трещины. Построены системы сингулярных интегральных уравнений первого рода по контурам трещин. Числовое решение интегральных уравнений получено методом механических квадратур для случаев произвольно ориентированной, а также двухзвенной ломаной трещины. Определены коэффициенты интенсивности напряжений в вершинах трещины в зависимости от угла наклона для различных упругих характеристик полуплоскостей

Ключевые слова: коэффициент интенсивности напряжений, сингулярное интегральное уравнение, равномерно распределенное давление, спаянные разнородные полуплоскости

UDC 539.3

DOI: 10.15587/1729-4061.2017.114359

EXAMINING ELASTIC INTERACTION BETWEEN A CRACK AND THE LINE OF JUNCTION OF DISSIMILAR SEMI-INFINITE PLATES

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1. Introduction

In real solid bodies that are the elements of engineering structures, there is always a certain amount of micro defects whose growth under the influence of the applied power loads leads to the emergence of cracks resulting in local or total destruction of the body. Practice shows that such a phenomenon is characteristic of high-strength and low-plastic materials. Therefore, it is important theoretically and practically to study stress distribution in the vicinity of stress concentrators of the crack type. In this case, the intensity of stresses at the top of the cracks is expressed by stress intensity coefficients (SIC). These parameters make it possible to determine threshold value of power load at which a crack starts to grow with the body being locally destroyed.

The method of singular integral equations (SIE) was employed to study the intensity of stresses in the vicinity of tops of an arbitrarily oriented crack, as well as a broken crack, which crosses the line of junction of two dissimilar half-planes. In this case, uniformly distributed normal pressure is set on the shores of the crack. Such a theoretical model reflects to some extent a mechanism of destruction of engineering structures with cracks when the water contained in them freezes to ice. The pressure on the shores of the crack created in this way may cause the growth of the crack.

Therefore, such studies are important for estimating the strength in terms of the mechanics of destruction. In particular, in the case of piecewise homogeneous bodies with a crack, it is possible to reduce stress intensity coefficients through appropriate selection of mechanical characteristics of the composite components.

2. Literature review and problem statement

Research into elastic state in the welded dissimilar half-planes with cracks is addressed in a number of publications. In particular, a crack was studied, located in parallel to the lines of junction of dissimilar half-planes when stretching stresses are assigned on the infinity of the formed plane [1]. Termoelastic problem for such a region with uniform distribution of temperature over entire piecewise-homogeneous plane with a crack was examined in [2, 3].

In paper [4], authors in a two-dimensional model considered a problem on bending a plate weakened by coaxial crack and slit. The resulting solution makes it possible to analyze the effect of the interaction of variable-type defects on the stressed state near tops.

Authors of [5] analyzed elastic interaction between two spherical cracks, located along the outer surface of hollow parts, placed in a heterogeneous environment during action of an uniaxial stretching load.

Authors of [6] obtained analytical solution to a two-dimensional problem of elasticity theory of screw dislocation near the surface crack of mode III, a shear crack during action of anti-flat deformation. They determined effect of the dislocation on a stress intensity coefficient.

In paper [7], a problem of interaction between a crack and an elastic inclusion was reduced to solving singular integral equations of the Cauchy type. Based on this result, the authors analyzed singular behavior of the solution for a crack with branches.

Based on the numerical solution to the bound three-dimensional elastic-dynamic problem, the influence of massive inclusion of the hard disk on the adjacent slit-like crack was examined in [8].

A problem on the circular, absolutely rigid, inclusion of arbitrary shape, which is located in the transversally isotropic half-space under conditions of smooth contact with the second half-space, was reduced to a system of two-dimensional singular integral equations. Authors of [9] investigated the asymptotics of stresses in the vicinity of an inclusion and defined directions of the largest and the lowest concentration of stresses.

In article [10], a problem of elasticity theory for a half-plane with many cracks was reduced to a singular integral equation using the modified comprehensive potential under condition of free stretching. The authors obtained a system of singular integral equations with a distributed dislocation function.

Assume that a boundless body (the plane) consists of two elastic isotropic dissimilar bodies (half-spaces) s^+ i S^- with a junction line L_0. The body is weakened by N. rectilinear cracks $L_n(n=\overline{1,N})$. We consider that all contours $L_n(n=\overline{1,N})$ do not have common points and each of them is associated with the local $x_nO_ny_n$ coordinate system whose axis O_nx_n forms an angle α_n with the Ox axis, which coincides with contour L_0. Points O_n define in the xOy coordinate system complex coordinates of N). Then the relationship between coordinates of the points of the plane in the local and main coordinate system is assigned by dependences:

$$z = z_n e^{i\alpha_n} + z_n^0,\quad \mu_k(t_k),\ k=\overline{1,N},\quad z_n = x_n + iy_n.$$

Assume that a perfect mechanical contact is assigned on contour L_0

$$[N(t_0)+iT(t_0)]^+ = [N(t_0)+iT(t_0)]^-;$$

$$(u_0+iv_0)^+ - (u_0+iv_0)^- = 0,\quad t_0 \in L_0, \tag{1}$$

the shores of the cracks do not contact in the process of deformation and a self-equilibrium load is set on them

$$[N(t_n)+iT(t_n)]^\pm = p_n^*(t_n),\quad n=\overline{M+1,N}, \tag{2}$$

t_n are the complex coordinates of the point on contour L_n in the local $x_nO_ny_n$ coordinate system.

An analysis of major scientific literary sources revealed that still unexplored and undeveloped are the mathematical models, which are applied to study the interaction between a crack and the line that connects two dissimilar half-planes, as well as the intersection of a crack with the line of junction of half-planes in piecewise homogeneous bodies with cracks. Given this, there is a necessity to construct mathematical models for determining such mechanical loads at which a crack starts to grow while the body undergoes local destruction. Exploring such models will make it possible to propose one of the approaches, for example by selecting the components of half-planes, welded together, with appropriate mechanical characteristics, to prevent the growth of a crack.

3. The aim and objectives of the study

The aim of present work is to determine the two-dimensional elastic state in welded dissimilar half-endless plates containing a rectilinear randomly-oriented or a two-link irregular crack under conditions of power load on the shores of the crack. This will make it possible to determine critical values of mechanical load on the shores of a crack in order to prevent crack growth, which will not allow the local destruction of the body.

To achieve the set aim, the following tasks had to be solved:

– to obtain two-dimensional mathematical model in the form of singular integral equations on the contours of cracks in order to determine perturbed power stresses due to the presence of cracks;

– to find numerical solutions to singular integral equations of the problem of elasticity theory for a specified region under the action of normally distributed pressure on the shores of the crack;

– to identify and explore stress intensity coefficients at the tops of a crack and to detect the effects of mechanical character.

4. Main results of research into stressed state in the welded dissimilar half-planes, weakened by a crack

4. 1. System of integral equations of the problem of elasticity theory for welded dissimilar half-planes, weakened by cracks

Complex potentials $\Phi(z)$, $\Psi(z)$ will be selected in the form [1]

$$\Phi(z)=\Phi_1(z)+\Phi_2(z),$$

$$\Psi(z)=\Psi_1(z)+\Psi_2(z), \tag{3}$$

where

$$\Phi_1(z) = \frac{1}{2\pi}\sum_{k=1}^{N}\int_{L_k}\frac{g'_k(t_k)e^{i\alpha_k}dt_k}{\zeta_k - z};\quad \zeta_k = t_k e^{i\alpha_k} + z_k^0; \tag{4}$$

$$\Psi_1(z) = \frac{1}{2\pi}\sum_{k=1}^{N}\int_{L_k}\left[\frac{\overline{g'_k(t_k)}e^{-i\alpha_k}\overline{dt_k}}{\zeta_k - z} - \frac{\bar{\zeta}_k g'_k(t_k)e^{i\alpha_k}dt_k}{(\zeta_k - z)^2}\right];$$

$$\Phi_2(z)=\frac{1-\Gamma_0}{2\pi(1+\chi_-\Gamma_0)}\times$$
$$\times\sum_{k=1}^{N}\int_{L_k}\left[\frac{g'_k(t_k)dt_k}{z-\overline{\zeta_k}}+\frac{(\zeta_k-\overline{\zeta_k})\overline{g'_k(t_k)dt_k}}{(\overline{\zeta_k}-z)^2}\right];$$

$$\Psi_2(z)=\frac{1-\Gamma_0}{2\pi(1+\chi_-\Gamma_0)}\sum_{k=1}^{N}\int_{L_k}\left\{\frac{\overline{\zeta_k}g'_k(t_k)dt_k}{(\overline{\zeta_k}-z)^2}+\right.$$
$$\left.+\left[\frac{(\overline{\zeta_k}-\zeta_k)(\overline{\zeta_k}+z)}{(\overline{\zeta_k}-z)^3}-\frac{1}{\overline{\zeta_k}-z}\right]\overline{g'_k(t_k)dt_k}\right\};$$

$\Gamma_0=G_+/G_-$; $G_+(G_-)$ is the module of shear, $\mu^+(\mu^-)$ is the Poisson's ratio of the upper (lower) half-plane, respectively, $\chi=(3-\mu)/(1+\mu)$ is for the generalized flat stressed state, $g'_k(t_k)$ are the unknown derivatives from a jump of displacements for crossing a line of cracks $L_n(n=\overline{1.N})$. Functions $g'_k(t_k)$ should possess integrated features at the ends of the crack.

We shall note that the choice of complex potentials in the form (3), (4) provides exact satisfaction of the second equality of boundary condition (1) on contour L_0. As a consequence, the order of the system of integral equations, which we obtain after fulfillment of the remaining boundary conditions is reduced by unity.

By satisfying, employing integral representations (3), (4), the boundary conditions on cracks contours (2), we shall obtain a system of N unknown singular equations of the first kind relative to the N unknown function on cracks contours L_k, $k=\overline{1,N}$, which do not contain any unknown function on contour L_0

$$\frac{1}{2\pi}\sum_{k=1}^{N}\int_{L_n}\left[R_{nk}(t_k,\tau_n)g'_k(t_k)dt_k+S_{nk}(t_k,\tau_n)\overline{g'_k(t_k)}\ \overline{dt_k}\right]=$$
$$=P_n(\tau_n), \qquad (5)$$

where

$$R_{nk}(t_k,\tau_n)=R^1_{nk}(t_k,\tau_n)-\frac{1-\Gamma_0}{1+\chi_-\Gamma_0}e^{i\alpha_k}\left\{\frac{1}{\overline{T_{nk}}}-\frac{(\overline{\zeta_k}-\zeta_k)}{T^2_{nk}}-\right.$$
$$\left.-e^{-2i\alpha_n}{}_n\cdot\frac{(2\eta_n-\zeta_k-\overline{\eta_n})(\overline{\zeta_k}-\zeta_k)}{T^3_{nk}}-\frac{1}{T_{nk}}\right\};$$

$$S_{nk}(t_k,\tau_n)=S^1_{nk}(t_n,\tau_n)+$$
$$+\frac{1-\Gamma_0}{1+\chi_-\Gamma_0}e^{-i\alpha_k}\left[\frac{(\zeta_k-\overline{\zeta_k})}{\overline{T^2_{nk}}}-\frac{1}{T_{nk}}+e^{-2i\alpha_n}\cdot\frac{H_{kn}}{\overline{T^2_{nk}}}\right];$$

$$R^1_{nk}(t_k,\tau_n)=e^{i\alpha_k}\left[\frac{1}{H_{nk}}+\frac{e^{-2i\alpha_n}}{\overline{H_{nk}}}\right];$$

$$S^1_{nk}(t_k,\tau_n)=e^{-i\alpha_k}\left[\frac{1}{\overline{H_{nk}}}-\frac{e^{-2i\alpha_k}H_{nk}}{\overline{H^2_{nk}}}\right];$$

$$H_{kn}=\zeta_k-\eta_n;\ \ T_{nk}=\zeta_k-\overline{\eta_n},\ \ \eta_n=\tau_n e^{i\alpha_n}+z^0_n.$$

A system of integral equations (5), in the case of internal cracks, has, for arbitrary right side, the only solution in the class of functions $g'_k(t_k)\in H^*$, $k=\overline{1,N}$ provided the following conditions are satisfied

$$\int_{L_k}g'_k(t_k)dt_k=0,\ \ k=\overline{1,N}, \qquad (6)$$

which ensure unambiguous displacements when traversing the contours of cracks.

4. 2. Crossing a two-link crack with a line of the half-planes junction

We shall consider an irregular crack, formed by two slits with a common point at the line of junction of dissimilar half-planes. Assume that the bottom half-plane contains an incision L_1 of length $2l_1$ perpendicular to the edge of junction L_0. The upper end of incision L_1 at an angle α to the Ox axis is the origin of lateral incision L_2 of length $2l_2$ directed to the upper half-plane (Fig. 1). Normally distributed pressure p is assigned on the shores of an irregular crack. In this case, we shall obtain from the system of integral equations (5) two integral equations of the first kind on contours of L_1 and L_2 relative to unknown functions $g'_1(t_1)$ and $g'_2(t_2)$

$$\frac{1}{2\pi}\int_{L_1}\left[R_{11}(t_1,\tau_1)g'_1(t_1)dt_1+S_{11}(t_1,\tau_1)\overline{g'_1(t_1)}\ \overline{dt_1}\right]+$$
$$+\frac{1}{2\pi}\int_{L_2}\left[R_{12}(t_2,\tau_1)g'_2(t_2)dt_2+S_{12}(t_2,\tau_1)\overline{g'_2(t_2)}\ \overline{dt_2}\right]=p^*_1(\tau_1),$$
$$\tau_1\in L_1; \qquad (7)$$
$$\frac{1}{2\pi}\int_{L_1}\left[R_{21}(t_1,\tau_2)g_1(t_1)\mathrm{d}t_1+S_{21}(t_1,\tau_2)\overline{g'_1(t_1)}\ \overline{dt_1}\right]+$$
$$+\frac{1}{2\pi}\int_{L_2}\left[R_{22}(t_2,\tau_2)g'_2(t_2)\mathrm{d}t_2+S_{22}(t_2,\tau_2)\overline{g'_2(t_2)}\ \overline{dt_2}\right]=p^*_2(\tau_2),$$
$$\tau_2\in L_2.$$

The condition of uniqueness of displacements after bypassing a contour of the irregular crack takes the form

$$\frac{2G_-}{1+\chi_-}e^{i\alpha_1}\int_{L_1}g'_1(t_1)\mathrm{d}t_1+\frac{2G_+}{1+\chi_+}e^{i\alpha_2}\int_{L_2}g'_2(t_2)\mathrm{d}t_2=0. \qquad (8)$$

Considering

$$\alpha_1=\pi/2,\ \ z^0_1=-il_1,\ \ \alpha_2=\alpha,\ \ z^0_2=l_2e^{i\alpha},\ \ \varepsilon=l_2/l_1$$

and by substituting

$$t_1=l_1\xi,\ \tau_1=l_1\eta,\ t_2=l_2\xi,\ \tau_2=l_2\eta$$

we record a system of two integral equations in the parameterized form

$$\int_{-1}^{1}\left[R^*_{11}(\xi,\eta)\psi_1(\xi)+S^*_{11}(\xi,\eta)\overline{\psi_1(\xi)}\right]d\xi+$$
$$+\int_{-1}^{1}\left[R^*_{12}(\xi,\eta)\psi_2(\xi)+S^*_{12}(\xi,\eta)\overline{\psi_2(\xi)}\right]d\xi=2\pi R_1(\eta),\ |\eta|<1; \qquad (9)$$

$$\int_{-1}^{1}\left[R^*_{21}(\xi,\eta)\psi_1(\xi)+S^*_{21}(\xi,\eta)\overline{\psi_1(\xi)}\right]d\xi+$$
$$+\int_{-1}^{1}\left[R^*_{22}(\xi,\eta)\psi_2(\xi)+S^*_{22}(\xi,\eta)\overline{\psi_2(\xi)}\right]d\xi=2\pi R_2(\eta),\ |\eta|<1;$$

where

$$\psi_1(\xi)=g'_1(l_1\xi);$$

$$\psi_2(\xi)=g'_2(l_1\xi);$$

$$R_1(\eta)=p_1^*(\eta_1);$$

$$R_2(\eta)=p_2^*(\eta_2);$$

$$R_{nk}^*(\xi,\eta)=l_kR_{nk}(l_k\xi,l_n\eta);$$

$$S_{nk}^*(\xi,\eta)=l_kS_{nk}(l_k\xi,l_n\eta),$$

(k=1, 2; n=1, 2),

$$\eta_1=l_1\eta e^{-i\pi/2}-il_1;$$

$$\eta_2=l_2\eta e^{-i\alpha}-l_2e^{i\alpha}.$$

Condition (8) will take the form

$$\frac{2G_-}{1+\chi_-}\int_{-1}^{1}\psi_1(\xi)d\xi+\frac{2\varepsilon G_+}{1+\chi_+}e^{i\alpha}\int_{-1}^{1}\psi_2(\xi)d\xi=0. \qquad (10)$$

For an irregular crack in the homogeneous plate, the integral equation was examined in [11], and in particular it is shown that a feature at the point of breaking a crack is always less than at the ends. Because it is important in this problem to determine stress intensity coefficients at the tops of a crack, then we shall apply the approach suggested in [11] where it is shown that nuclei $R_{12}^*(\xi,\eta)$, $S_{12}^*(\xi,\eta)$, $R_{21}^*(\xi,\eta)$, $S_{21}^*(\xi,\eta)$ have unmovable features. Then functions $\psi_1(\eta)$ and $\psi_2(\eta)$ at, respectively, points η=1 and η=–1 have a feature that differs from the root. But, as it is known from [11], the order of features of functions $g_1'(t_1)$ and $g_2'(t_2)$ in the angular points is always lower than at the ends of the incision. Thus, functions $\Psi_1(\eta)$ and $\Psi_2(\eta)$ can be represented in the form

$$\psi_1(\eta)=u_1(\eta)/\sqrt{1-\eta^2},$$

$$\psi_2(\eta)=u_2(\eta)/\sqrt{1-\eta^2},$$

considering the conditions are satisfied

$$u_1(1)=0;\ u_2(-1)=0. \qquad (11)$$

By applying to integral equations (9) and condition (10) the quadrature formulae of Gaussian-Chebyshev, we shall obtain a system of $2n-1$ algebraic equations for determining $2n$ unknown functions $u_1(\xi_k)$ and $u_2(\xi_k)$, $k=1,2...n$.

$$\frac{1}{n}\sum_{k=1}^{n}\left[R_{11}^*(\xi_k,\eta_m)u_1(\xi_k)+S_{11}^*(\xi_k,\eta_m)u_1(\xi_k)\right]+$$
$$+\frac{1}{n}\sum_{k=1}^{n}\left[R_{12}^*(\xi_k,\eta_m)u_2(\xi_k)+S_{12}^*(\xi_k,\eta_m)u_2(\xi_k)\right]=$$
$$=2\pi R_1(\eta_m), \qquad (12)$$

$$m=1,2,....,n-1;$$

$$\frac{1}{n}\sum_{k=1}^{n}\left[R_{21}^*(\xi_k,\eta_m)u_1(\xi_k)+S_{21}^*(\xi_k,\eta_m)u_1(\xi_k)\right]+$$
$$+\frac{1}{n}\sum_{k=1}^{n}\left[R_{22}^*(\xi_k,\eta_m)u_2(\xi_k)+S_{22}^*(\xi_k,\eta_m)u_2(\xi_k)\right]=2\pi R_2(\eta_m),$$

$$m=1,2,....,n-1;$$

$$\frac{2iG_-}{1+\chi_-}\sum_{k=1}^{n}u_1(\xi_k)+\frac{2\varepsilon G_+}{1+\chi_+}e^{i\alpha}\sum_{k=1}^{n}u_1(\xi_k)=0; \qquad (13)$$

$$\xi_k=\cos(\pi(2k-1)/2k),$$

$$\eta_m=\cos(\pi m/n).$$

In order to obtain a closed system of equations, we add to system (12), (13) one more of the equations

$$\sum_{k=1}^{n}(-1)^k u_1(\xi_k)ctg\frac{2k-1}{4n}\pi=0,$$

$$\sum_{k=1}^{n}(-1)^{k+n} u_2(\xi_k)tg\frac{2k-1}{4n}\pi=0, \qquad (14)$$

which are obtained based on equalities (11).

Calculations demonstrate that the numerical solution practically does not depend on the choice of the first or of the second equality (14).

Stress intensity factors for the lower (–) and the upper (+) tops the knee cracks We have expressions [11] for stress intensity coefficients in the lower (–) and upper (+) tops of irregular crack

$$K_I^+-iK_{II}^+=\frac{\sqrt{\pi l_2}}{n}\sum_{k=1}^{n}(-1)^{k+1}u_2(\xi_k)ctg\frac{2k-1}{4n}\pi;$$

$$K_I^--iK_{II}^-=\frac{\sqrt{\pi l_1}}{n}\sum_{k=1}^{n}(-1)^{k+n}u_1(\xi_k)tg\frac{2k-1}{4n}\pi.$$

In these formulas, SIC $K_I^\pm$, $K_{II}^\pm$ are the valid magnitudes, that characterize the stressed-strained state in the vicinity of the tops of a crack.

4. 3. Two welded half-planes with a randomly-oriented crack

We shall consider two welded dissimilar half-planes with a junction line L_0, along which there is a perfect mechanical contact (equality of stresses and displacements). The lower half-plane is weakened by crack L_1 of length $2l_1$ with a center point (0; –ih). The crack forms angle α with the Ox axis with an evenly distributed normal pressure of intensity p assigned on the shores (Fig. 2, *a*). In this case, we shall obtain form the system of equations (11) one integral equation on contour L_1 in which the right side equals to

$$p_1^*(\tau_1)=-p.$$

$$\frac{1}{2\pi}\int_{L_1}\left[R_{11}(t_1,\tau_1)g'_1(t_1)\mathrm{d}t_1+S_{11}(t_1,\tau_1)\overline{g'_1(t_1)}\,\overline{dt_1}\right]=$$
$$=p_1^*(\tau_1),\ \tau_1\in L_1 \qquad (15)$$

and that has the only solution provided the condition is satisfied.

$$\int_{L_1}g_1'(t_1)\mathrm{d}t_1=0, \qquad (16)$$

which ensures the uniqueness of displacements when bypassing the contour of the crack.

5. Analysis of the obtained numerical results

Given that evenly distributed normal pressure p is assigned on the shores of the irregular crack, the right sides in equations (7) will take the form

$$p_1^*(\tau_1) = p_2^*(\tau_2) = -p.$$

Graphs for the dimensionless stress intensity coefficients K_I/K_0 and K_{II}/K_0 ($K_0 = p\sqrt{l}$) are shown in Fig. 1, *a, b*.

Dash curves correspond to the values of coefficients of intensity at the upper top of a crack (vertex A), solid curves – at the bottom (vertex B). Curves 1 correspond to the value of l_2/l_1=1, curves 2 – l_2/l_1=0.5. The numerical solution to algebraic equations (12)–(14) was obtained by the method of mechanical quadratures [11] at $\chi_+=\chi_-=2$, G_+/G_-=0.2.

Graphs for the dimensionless stress intensity coefficients K_I/K_0 and K_{II}/K_0 ($K_0 = p\sqrt{l}$), in the case of an arbitrarily oriented crack, are shown in Fig. 2, 3.

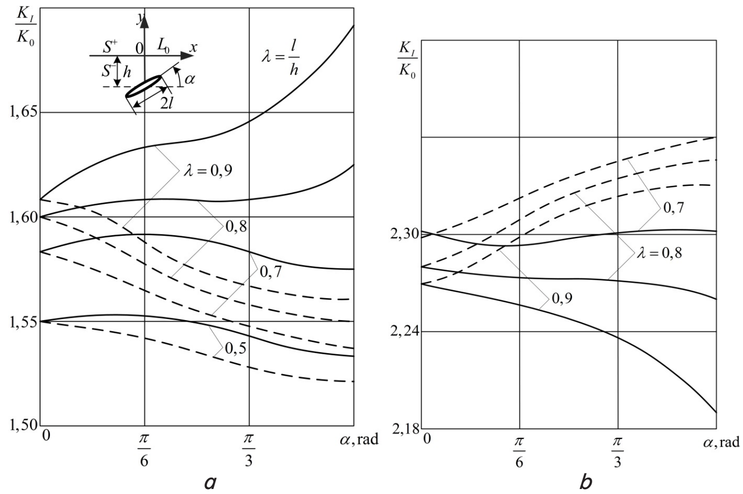

Fig. 2. Dependence of dimensionless SIC K_I/K_0 on the angle of inclination of crack α: *a* – G_+/G_-=0.5; *b* – G_+/G_-=2

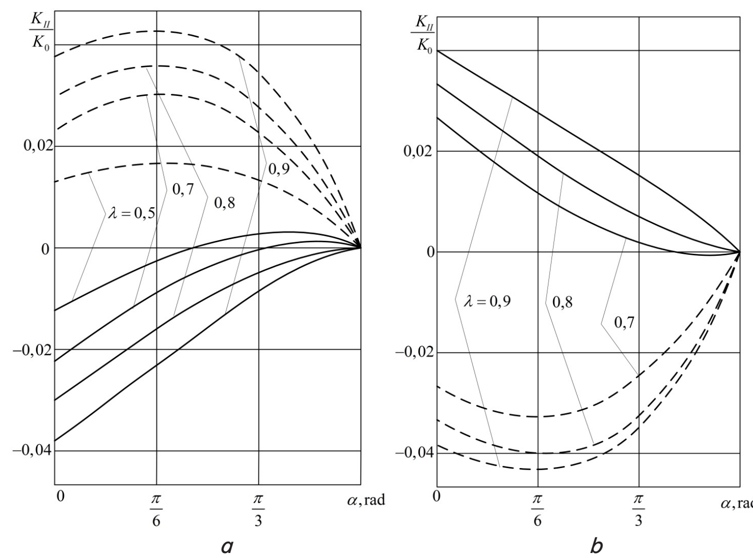

Fig. 3. Dependence of dimensionless SIC K_{II}/K_0 on the angle of inclination of crack α: *a* – G_+/G_-=0.5; *b* – G_+/G_-=2

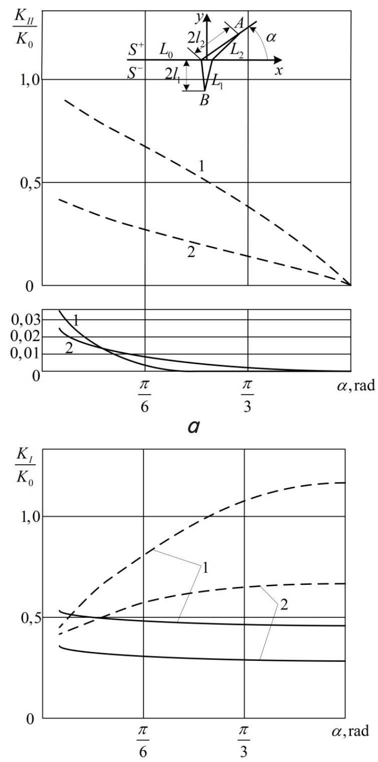

Fig. 1. Dependence of dimensionless SIC on the angle of inclination α of the upper link of an irregular crack: *a* – K_{II}/K_0; *b* – K_I/K_0

We constructed dependences of stress intensity coefficients on the crack inclination angle α for different values of parameter λ=1/*h* when G_+/G_-=0.5 (Fig. 2, *a,* Fig. 3, *a*), and G_+/G_-=2 (Fig. 2, *b*, Fig. 3, *b*). Solid curves correspond to the coefficients of intensity at the right top of the crack (closest to the line of separation of half-planes), dashed curves – at the left. The numerical solution to integral (15), (16) equations was obtained by the method of mechanical quadratures [11] at $\chi_+=\chi_-=2$.

6. Discussion of results of research into interaction between a crack and a junction line of dissimilar half-planes

If the side link of a crack is in the less rigid half-plane ($G_+<G_-$), then SIC K_I/K_0 at the top of the bottom link (vertex B) does not increase significantly when the upper link approaches the junction border L_0. SIC K_I/K_0 of the upper link (vertex A) reaches a maximum value when an irregular crack becomes a straight crack, perpendicular to the line of half-planes junction (Fig. 1, *b*). SIC K_{II}/K_0 for both tops of the irregular crack reach a maximum simultaneously to the upper lateral link approaching the line of half-planes junction (Fig. 1, *a*). In this case, the values of SIC K_{II}/K_0 in the lower vertex B is an order of magnitude lower than those at the upper vertex A.

Stress intensity coefficient K_I/K_0 is always greater (smaller) for that top of the crack that is closer to the upper softer (more rigid) half-plane (Fig. 2, *a, b*). At a significant distance from the crack from the upper, less stiff $G_+<G_-$), half-plane, coefficient K_I/K_0 accepts maximum values at α=0

(a crack is parallel to the line of separation of L_0). With the crack approaching the boundary of division, the maximum of K_I/K_0 shifts to angle $\alpha=\pi/2$ (Fig. 2, *a*). If the upper half-plane is more rigid ($G_-<G_+$), then the maximum of K_I/K_0 is always achieved at $\alpha=\pi/2$ for a more remote top of the crack (Fig. 2, *b*). Intensity coefficient K_{II}/K_0 is always greater for the top of the crack, which is closer to the line of separation of half-planes, regardless of the rigidity of the upper half-plane. In this case, stress intensity coefficient K_{II}/K_0 accepts a maximum value for the angles of inclination of a crack close to $\alpha=\pi/6$ (Fig. 3, *a, b*).

In the problem on two welded half-planes with an arbitrarily oriented straight crack the shores of the crack do not touch. Then, according to σ_θ – criterion (crack original growth hypothesis), it is possible to derive from the equations of boundary equilibrium [12]

$$\cos^3\frac{\theta_*^\pm}{2}\left(K_{\mathrm{I}}^\pm-3K_{\mathrm{II}}^\pm\,\mathrm{tg}\frac{\theta_*^\pm}{2}\right)=\frac{K_{1C}}{\sqrt{\pi}}$$

critical values for normal pressure of intensity p_{cv}, when the body starts breaking down locally, from formula

$$p_{\text{кр}}=\frac{1}{\sqrt{\pi l}}\cdot\frac{K_{1C}}{\cos^3\frac{\theta_*^\pm}{2}\left(k_1^\pm-3k_2^\pm\,\mathrm{tg}\frac{\theta_*^\pm}{2}\right)},$$

where $k_1^\pm=K_I^\pm/K_0,\ k_2^\pm=K_{II}^\pm/K_0,\ K_0=p\sqrt{l},\ K_{1C}$ is the constant that characterizes resistance of a material to destruction and which is determined experimentally;

$$\theta_*^\pm=2\,\mathrm{arctg}\frac{k_1^\pm-\sqrt{(k_1^\pm)^2+8(k_2^\pm)^2}}{4k_2^\pm}$$

$\theta_*^\pm$ – angles of original crack growth from tops $l^\pm$.

Table 1

Relative values of normal pressure intensity on the shores of crack $\tilde{p}/p_{cv}$, for different angles of inclination of the crack α_i of parameter $\lambda=1/h=0.9$

α	$G_+/G_-=5$		$G_+/G_-=0.2$	
	l^-	l^+	l^-	l^+
0	0.632	0.632	0.438	0.438
π/6	0.634	0.624	0.436	0.443
π/3	0.638	0.612	0.432	0.449
π/2	0.640	0.595	0.430	0.456

It follows from the numerical results in Table 1 that with a crack's top approaching junction line with a more rigid environment ($G_-<G_+$), the critical value of the intensity of normal pressure p_{cv} grows, while with a less rigid environment ($G_+<G_-$) – reduces.

The resulting simplified numerical procedure for solving integral equations (12)–(14) is effective in the case when it is necessary to determine the distribution of stresses only in the vicinity of the tops of the crack. If it is required to further explore the intensity of stresses in the vicinity of the point of breaking down a crack, then the solution is to be found using the Gauss-Jacobi quadrature formulae. These formulas correctly represent special features of solution at an angular point.

Practical value of the present work lies in the possibility of a more complete accounting of actual stressed-strained state in the piecewise-homogeneous elements of a structure with cracks that work under conditions of different mechanical loads. The results of specific studies that are given in the form of graphs could be used when designing rational operational modes of structural elements. In this case, the possibility is obtained for preventing the growth of a crack through the appropriate selection of composite's components with the corresponding mechanical characteristics.

The present study is continuation of previously examined problems on the piecewise-homogeneous bodies of similar geometry under the action of heat load and is extension to be applied for mechanical stresses.

7. Conclusions

1. We constructed a 2-dimensional mathematical model for the problem of elasticity theory for two welded dissimilar half-planes with cracks in the form of singular integral equations (SIE) of the first kind on the contours of cracks. Such an approach makes it possible to obtain a numerical solution to SIE by the application of the high precision method of mechanical quadratures. This method implies representation of SIE in the form of an appropriate finite system of linear algebraic equations, based on the solutions to which one funds an approximate solution to integral equations with a preset accuracy.

2. We obtained numerical solutions to SIE (employing the method of mechanical quadratures) in particular cases of two welded dissimilar half-planes with one randomly-oriented crack, as well as a two-link irregular crack, which crosses the line of junction when uniformly distributed normal pressure acts on the shores of the crack. This makes it possible to determine stress intensity coefficients (SIC) at the tops of the crack, which are subsequently used to determine critical values of the normal pressure on the shores of the crack at which a crack starts to grow.

3. We constructed graphic dependences of SIC, which characterize the distribution of intensity of stresses at the tops of a crack, on the angle of crack inclination and elastic characteristics of half-planes. We determined relative critical values of normal pressure on the shores of the crack from the equations of equilibrium for different angles of crack inclination. These results make it possible to determine the limit of permissible values of normal pressure on the shores of the crack and could be used when designing rational operational modes of structures' elements in terms of preventing the growth of cracks.

References

1. Zeleniak, V. Napruzhennia v spaianykh riznoridnykh pivploshchynakh z vkliuchenniam i trishchynoiu za diy roztiahu [Text] / V. Zeleniak, R. Martyniak, B. Slobodian // Visnyk Natsionalnoho universytetu «Lvivska politekhnika». – 2008. – Issue 625. – P. 54–58.
2. Savruk, M. P. Plane problem of thermal conductivity and thermal elasticity for two joined dissimilar half-planes with curved inclusions and cracks [Text] / M. P. Savruk, V. M. Zelenyak // Soviet Materials Science. – 1988. – Vol. 24, Issue 2. – P. 124–129. doi: 10.1007/bf00736348

3. Zeleniak, V. Modeliuvannia termopruzhnoho dvovymirnoho stanu dvokh spaianykh riznoridnykh pivploshchyn z vkliuchenniamy i trishchynamy [Text] / V. Zeleniak, B. Slobodian // Fizyko-matematychne modeliuvannia ta informatsiyni tekhnolohiy. – 2010. – Issue 12. – P. 94–101.
4. Shatskyi, I. P. Vzaiemodiya trishchyny z kolinearnoiu shchilynoiu za zghynu plastyny [Text] / I. P. Shatskyi, T. M. Daliak // Visnyk Zaporizkoho natsionalnoho universytetu. Fizyko-matematychni nauky. – 2015. – Issue 1. – P. 211–218.
5. Tagliavia, G. Elastic interaction of interfacial spherical-cap cracks in hollow particle filled composites [Text] / G. Tagliavia, M. Porfiri, N. Gupta // International Journal of Solids and Structures. – 2011. – Vol. 48, Issue 7-8. – P. 1141–1153. doi: 10.1016/j.ijsolstr.2010.12.017
6. Chu, S. N. G. Elastic interaction between a screw dislocation and surface crack [Text] / S. N. G. Chu // Journal of Applied Physics. – 1982. – Vol. 53, Issue 12. – P. 8678–8685. doi: 10.1063/1.330465
7. Ming-huan, Z. Interaction between crack and elastic inclusion [Text] / Z. Ming-huan, T. Ren-ji // Applied Mathematics and Mechanics. – 1995. – Vol. 16, Issue 4. – P. 307–318. doi: 10.1007/bf02456943
8. Mykhas'kiv, V. V. Interaction between rigid-disc inclusion and penny-shaped crack under elastic time-harmonic wave incidence [Text] / V. V. Mykhas'kiv, O. M. Khay // International Journal of Solids and Structures. – 2009. – Vol. 46, Issue 3-4. – P. 602–616. doi: 10.1016/j.ijsolstr.2008.09.005
9. Kryvyy, O. F. Interface circular inclusion under mixed conditions of interaction with a piecewise homogeneous transversally isotropic space [Text] / O. F. Kryvyy // Journal of Mathematical Sciences. – 2012. – Vol. 184, Issue 1. – P. 101–119. doi: 10.1007/s10958-012-0856-6
10. Elfakhakhre, N. R. F. Stress intensity factor for multiple cracks in half plane elasticity [Text] / N. R. F. Elfakhakhre, N. M. A. Nik long, Z. K. Eshkuvatov // AIP Conference Proceedings. – 2017. – P. 020010-1–020010-8. doi: 10.1063/1.4972154
11. Savruk, M. P. Dvumernye zadachi uprugosti dlya tel s treshchinami [Text] / M. P. Savruk. – Kyiv: Naukova dumka, 1981. – 324 p.
12. Panasyuk, V. V. Raspredelenie napryazheniy okolo treshchin v plastinah i obolochkah [Text] / V. V. Panasyuk, M. P. Savruk, A. P. Datsyshin. – Kyiv: Naukova dumka, 1976. – 444 p.