*Проведено моделювання сходу колеса залізничного екіпажу з рейок на підставі спільного розгляду фрикційного взаємодії з рейками набігаючого і збігаючого коліс колісної пари. Запропоновано оновлений критерій безпеки від сходу з рейок, на 10–50 % нижчий за класичний критерій стійкості Надаля, що робить запропонований критерій більш надійним при оцінці безпеки від сходу залізничних екіпажів з рейок*

*Ключові слова: залізничний екіпаж, сход з рейок, критерій безпеки, кут набігання, гребеневий контакт*

*Проведено моделирование схода колеса железнодорожного экипажа с рельсов на основании совместного рассмотрения фрикционного взаимодействия с рельсами набегающего и сбегающего колес колесной пары. Предложен обновленный критерий безопасности от схода с рельсов, который на 10–50 % ниже классического критерия устойчивости Надаля, что делает предложенный критерий более надежным при оценке безопасности от схода железнодорожных экипажей с рельсов*

*Ключевые слова: железнодорожный экипаж, сход с рельсов, критерий безопасности, угол набегания, гребневой контакт*

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# RESEARCH ON THE SAFETY FACTOR AGAINST DERAILMENT OF RAILWAY VEHICLES

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## 1. Introduction

The most frequent (up to 25 %) among the main causes of disasters, accidents and serious incidents in the railway transport is derailing. The first derailment in the history of railways occurred in the United States in 1833. It happened with the passenger train Camden & Amboy at a speed of 40 km per hour. About 100 passengers perished in the accident. Since then, there have been thousands of derailments of rolling stocks, which resulted in tens of thousands of victims. In recent decades, the number of rolling stock derailments has decreased noticeably; unfortunately, however, they happen with quite a frequent regularity [1–5].

The railway derailment mechanism is a complex process that is the subject of intensive research around the world. The main criterion for safe movement of the rail stock is the stability factor against derailment in the form of a correlation between the transverse force and the vertical load of the wheel on the rail [6]. The safety criterion against derailment in the Nadal theory is greatly simplified; it involves a large number of assumptions and disregards many features of the wheel-rail contact [7, 8]. In particular, Nadal's formula does not take into account the effect produced on the safety parameters by the angle of the wheel running onto the rail and the mode of motion (traction, coasting or braking) [9, 10]. The correct definition of the safety criterion against derailment is of great social and economic importance. Therefore, its clarification is aimed at improving the safety of rail transport, so it is important and relevant.

## 2. Literature review and problem statement

The safety factor against the derailment of rail vehicles or the coefficient of stability is determined by the following formula [7]:

$$k_y = \left[\frac{Y}{P}\right] \div \frac{Y}{P} \geq \left[k_y\right], \qquad (1)$$

where $P$ is the vertical load of the wheel on the rail; $Y$ is the lateral force in the flange contact; $k_y$ is the minimum allowable safety factor against derailment; $\left[\frac{Y}{P}\right]$ is the ratio of the lateral force to the vertical load in the flange contact at which the surface of the rolling wheel comes off the rail.

According to Nadal [6],

$$\left[\frac{Y}{P}\right] = \frac{\mathrm{tg}\beta - \mu}{1 + \mu \cdot \mathrm{tg}\beta}, \qquad (2)$$

where $\beta$ is the flange inclination angle; $\mu$ is the friction coefficient in the wheel-rail contact.

Formula (1) is valid only for cases when the angle of the wheel approaching equals zero ($\psi = 0$).

Many attempts have been made to improve Nadal's formula to increase the safety of rail transport even at the stage of design development.

A refinement of the Nadal formula is the Wagner formula [11], which takes into account the deviation of the normal wheel load on the rail from the vertical direction:

$$\left[\frac{Y}{P}\right]=\frac{\mathrm{tg}\beta-\mu\cdot\cos\gamma}{(1+\mu\cdot\mathrm{tg}\beta)\cdot\cos\gamma}, \qquad (3)$$

where $\gamma$ is the angle of deviation of the normal load of the wheel on the rail from the vertical position.

Based on Nadal's formula, in [12], an analysis is given of the role of spin in the flange contact. A conclusion is made that the obtained data are more realistic than those obtained by Nadal's formula. A new safety criterion is proposed, which produces less conservative values than the Nadal equation.

An attempt to improve Nadal's formula is also made in [13]. The refinement is based on taking into account the influence of the longitudinal contact force of creep and the angle of the wheel approaching the rail. The obtained results, however, differ little from the Nadal criterion, since they do not take into account the slip in the main contact and the regime of motion of the wheel pair.

In [14], the study is focused on the main types of derailment of railway vehicles. Three of them are related to the characteristics of the tracks, and two of them are associated with the shape of the wheel and the rail profile. It is proven that the inclination angle of the wheel flange is the main geometric parameter that affects the probability of derailment. Paper [15] provides statistics and analyzes the causes of large-scale accidents on the railway. Research on the flange contact as the main factor of safety is emphasized as highly essential.

In [16], an equation is formulated for estimating the wheel load with allowance for centrifugal forces, deviation of the trajectory geometry and deformation of the secondary suspension of railway vehicles. A method is proposed for calculating the critical safety factor, taking into account the influence of the wheel-on-rail angle and the equivalent coefficient of friction.

In study [17], the features of contact between the wheel and the rail are analyzed as to the following:

– the three-dimensional distribution of reactions in the contact of the wheel with the rail;

– the features of the kinematics of a two-point contact;

– the redistribution of force and kinematic parameters between contacts during a re-contact;

– the dependence of the direction of force in the flange contact on the angle of approach and the mode of movement of the wheel pairs of locomotives (traction, coasting or braking);

– the probability of a vertical component of the frictional force in the flange contact.

The results of the tests in [18] are based on the data of a numerical simulation of the motion of a railway vehicle along its track with lateral and vertical irregularities. The irregularities were random. The degree of safety was assessed through the ratio of the transverse and vertical forces at the point of contact of the wheel with the rail. The level of comfort was assessed by the level of lateral and vertical acceleration in accordance with ISO 2631-1:1997.

There is a general consensus that the dangerous angle of the flange of the railway wheel is the main cause of train crashes. The authors of [19] estimate the influence of the coefficient of friction in the contact of the flange with the rail on the probability of an accident. It is noted that the coefficient of friction varies greatly depending on various factors: the state of the surfaces of the wheel and the rail, the atmospheric conditions, and the speed of the rolling stock. The danger of derailment was estimated by the Nadal criterion for different friction coefficients. As a result, it was found that measures to reduce the coefficient of friction are effective even in cases where the coefficient of derailment exceeds Nadal's limit criterion.

In study [20], a method of protection against derailment is proposed based on the coefficient of a wheel pair collapse and the wheel unloading ratio. This method allows assessing the safety of derailment, whether the load is within the boundary limit of the safety zone. Dynamic tests were carried out using an indirect method of measuring the wheel-rail load for the subway. It was found that the safety indicators for the vehicle wagging exceed the permissible limit, whereas in stable conditions they are within safety limits.

In [21], the dynamic characteristics of three different types of trolleys are compared. Tests are performed on the railway car main dynamic indicators such as the vertical and horizontal dynamics factors, as well as the safety factor against derailment.

Study [22] considers the process of derailment of railway vehicles at low speeds in curved sections of the track. The mechanism for lifting the flange to quantify the factors causing the vehicles to derail is studied. The results of the tests on passenger trains with a real derailment are analyzed. Also, tests are carried out on a roller stand. A method is proposed for assessing the level of safety against derailment in curved sections of the track.

Paper [23] describes the mechanism of derailment and the method of continuous control of the force between the wheel and the rail. New criteria for assessing the risk of crash are proposed.

Study [24] develops a three-dimensional nonlinear dynamic model of the system of wheel pairs and their suspension. The influence of the coefficient of friction and the speed of motion on derailment is investigated. In addition, various methods for lubricating rails and their effect on the risk of derailment are studied. Also, the features of a two-point wheel-rail contact are considered, and recommendations are given to improve the safety of derailment.

A brief review of some problems of the dynamics of railway vehicles is presented in [25], including an analysis of the derailment mechanism. The authors of the paper argue that none of the existing safety criteria completely determines the probability of derailment. Simultaneous use of several safety criteria is suggested.

Study [26] presents the results of analyzing the mechanism of derailment. The role of the approach angle in the process of derailing the wheel is shown. However, the authors confine themselves to describing the mechanism of derailment and do not present calculated dependencies.

The procedure for calculating the longitudinal location of the flange contact, in particular the eccentricity, is presented in [27], using the Wang method and a three-dimensional grid. The results of this simulation are compared with the results in which the location of the contact was calculated by the classical method. It is concluded that a more accurate contact location by the Wang method gives a smaller angle of the flange inclination. In addition, it is specified that the influence of the longitudinal location of the point of contact on the modeling of the vehicle's motion is insignificant for the flange inclination angle below 40 degrees and the approach angle below 1 degree.

In work [28], structural features of the wheelsets of railway vehicles, especially their ability to self-alignment, are analyzed.

In [29, 30], the characteristics of derailment of vehicles on the basis of mathematical models of the spatial configuration of a wheel and a rail are investigated. It is noted that a detailed description of the geometry of contact between wheels and rails is crucial for the accuracy of the dynamic analysis of railway vehicles. The following issues are considered:

– studying the geometry of the contact to determine the location of the main and flange contacts;

– profiling surfaces of contacting bodies;

– calculating relative velocities at the points of contact;

– determining contact forces.

Papers [31, 32] present requirements for a modern rolling stock with a detailed consideration of the corresponding characteristics for freight railway cars. However, the work does not indicate the potential and ways to improve the dynamic characteristics of the rolling stock and especially the safety performance in various states of the cars.

Study [33] is devoted to improving the carrier systems of a rolling stock by refining the methods of researching their strength properties in statics. At the same time, the dynamics issues, taking into account the traffic safety factors, are not considered.

Paper [34] examines the existing methods for assessing the stability of railway rolling stocks against derailment. It is established that the factor of stability against the rolling of the wheel flange onto the railhead is an integral indicator of traffic safety. This figure is determined taking into account the vertical load, lateral force, frictional forces in the contact of the flange of the wheel with the rail, and the geometrical parameters of a wheel pair.

Despite numerous studies [1–34] of safety criteria against derailment, they do not take into account a number of dynamic factors. In particular, the joint influence of the wheel-on-rail angle on the rail and the effect of the slip forces on the process of the wheel pair derailment have not been adequately investigated. Both from a practical and theoretical point of view, the estimation of the safety factor for the lateral force in the flange contact is inconvenient and inaccurate. The difficulty lies in analyzing the horizontal flange force, which is largely influenced by the friction force in the contact of the second wheel of a wheel pair.

Given the importance of the issue of traffic safety in railway transport, it seems appropriate to redefine the safety criterion based on a study of factors that have not been taken into account yet.

## 3. The aim and objectives of the study

The aim of the study is to clarify the safety criterion against the derailment of railway stocks on the basis of a full pattern of the frictional interaction between the wheel pair and the track. The improvement is based on a more precise consideration of the kinematics of the flange contacting of wheels and rails.

To achieve this aim, the following objectives were set:

– to model the derailment of the wheel of a railway stock on the basis of joint consideration of the frictional interaction with the rails of the approaching and running-off wheels of the wheel pair;

– to improve Nadal's classical stability criterion on the basis of taking into account parameters that have not been considered yet: in particular, the effects of friction in the contact of the running-off wheel with the rail on the horizontal lateral load on the flange of the approaching wheel.

## 4. Methods for studying the coefficient of stability against derailment

The modeling of contact interaction between a wheel and a rail is made by an example of a standard wheel profile (DSTU 11018-2000). The equilibrium equations are formulated with the use of simplifications and idealizations that are typical of classical mechanics. Friction contact forces are determined on the basis of the Amonton-Coulomb law. The two-point flange contact of the wheel and the rail is conditionally reduced to points in the centers of the contact spots in accordance with the Hertz theory.

## 5. The results of studying the safety criterion against derailment

### 5. 1. The kinematics of wheel pairs' contacting with rails

The motion of a wheel pair is simulated in the skew position on the rail track, with the angle of approaching $\psi$. Fig. 1 shows the speed plans (blue vectors) and forces (red vectors) in the contacts of the wheel pair with the rails. The oncoming wheel (2) has a two-point contact with the rail at points $K_{21}$ (main contact) and $K_{22}$ (flange contact). The running-off wheel (1) has a one-point contact at $K_{11}$.

The case is considered when the approaching wheel is in a state of derailment. As a condition for the derailment of the wheel, the load in the main contact of the approaching wheel is assumed to be zero.

When simulating the kinematic characteristics of the wheel pair contact with the rails, the following parameters and their designations are considered:

$V$ is the velocity of the center of the wheel pair along the track axis;

$V_{\varphi 11}$ is the circumferential speed of the wheel at the contact point $K_{11}$, associated with the rotation of the wheel pair relative to its own axis;

$V_{11}$ is the speed of the wheel sliding on the rail in the contact point $K_{11}$;

$V_{11x}$ and $V_{11y}$ are the projections of the speed $V_{11}$ on the axis Ox and Oy, respectively;

$V_{\varphi 22xy}$ is the projection of the peripheral speed of the wheel at the contact point $K_{22}$, which is associated with the rotation of the wheel pair relative to its own axis, to the horizontal plane Oxy;

$V_{22xy}$ is the projection of the sliding speed of the wheel relative to the rail at the contact point $K_{22}$ to the horizontal plane Oxy;

$V_{22x}$ and $V_{22y}$ are the projections of the speed $V_{22}$ on the axis Ox and Oy, respectively;

$V_{22xz}$ is the projection of speed $V_{22}$ on the longitudinal vertical plane Oxz;

$Y_f$ is the axial reaction in the axle unit, acting on the wheel pair as the frame force;

$S_{11y}$ and $S_{22y}$ are the projections of the frictional forces $S_{22}$ in the contacts $K_{11}$ and $K_{22}$ on the Oy axis;

$S_{22xz}$, $S_{22x}$, and $S_{22z}$ are the projections of the frictional force $S_{22}$ in the contact $K_{22}$ on the longitudinal vertical plane Oxz, the axis Ox, and the axis Oz, respectively;

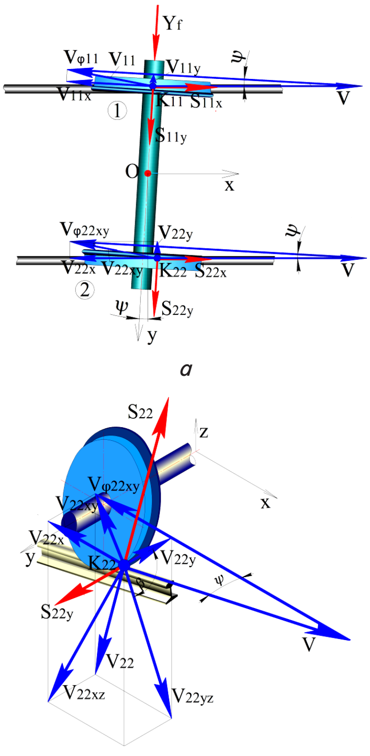
$\beta$ is the angle of the flange inclination.

The vector equations of slip velocities in the contacts points $K_{11}$ and $K_{22}$ are obtained on the basis of the superposition principle as components of the relative displacement of rolling surfaces of the wheels and the rails:

$$\bar{V}_{11}=\bar{V}+\bar{V}_{\varphi 11};\quad \bar{V}_{22xy}=\bar{V}+\bar{V}_{\varphi 22xy}. \tag{4}$$

$$\bar{V}_{22}=\bar{V}_{22xy}+\bar{V}_{22xz}+\bar{V}_{22yz}. \tag{5}$$

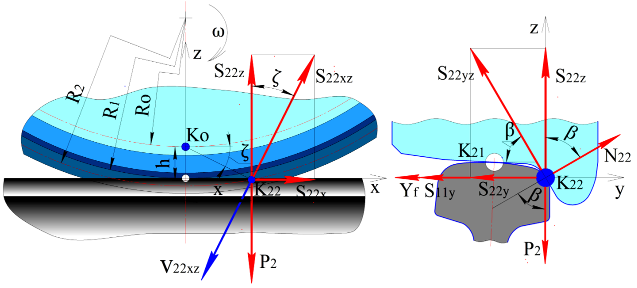
Fig. 2 shows a diagram of the contact forces in the flange contact in the projections onto the longitudinal vertical plane Oxz and the transverse vertical plane Oyz.



*a*

*b*

Fig. 1. The system of velocity vectors (blue color) and forces (red color) in the contacts of the wheel pair with the rails: *a* is the projection of the velocities and forces in the contacts of the wheel pair with the rails on the horizontal plane Oxy; *b* is the spatial picture of the velocities and forces in the flange contact of the approaching wheel



*a* *b*

Fig. 2. The system of the projections of forces in the flange contact: *a* shows the projections onto the longitudinal vertical plane Oxz; *b* shows the projections onto the transverse vertical plane Oyz

In Fig. 2, the following designations are accepted:

$\zeta$ is the angle determining the position of the vector $S_{22xz}$ with respect to the vertical axis;

$\omega$ is the angular velocity of the wheel pair rotation about its own axis;

$N_{22}$ is the normal load in the flange contact $K_{22}$;

$K_0$ is the instantaneous center of the wheel rotation;

$R_0$ is the distance from the center of the wheel to the instantaneous center $K_0$ of the wheel rotation;

$R_1$ and $R_2$ are the wheel radii in the main and flange contacts, respectively;

$P_2$ is the vertical load in the flange contact $K_{22}$.

### 5. 2. Force interaction of the wheel and the rail

The total horizontal lateral load in the flange contact $Y$ is equal to the sum of the frame force $Y_f$ and the slip forces $S_{11y}$ and $S_{22y}$ in the contacts $K_{11}$ and $K_{22}$:

$$Y=Y_f+S_{11y}+S_{22y}. \tag{6}$$

The frictional contact forces $S_{11y}$ and $S_{22y}$ in the contacts $K_{11}$ and $K_{22}$ in the theory of controlling wheeled vehicles are called slip forces. The slip forces $S_{11y}$ and $S_{22y}$ are friction forces that are directed opposite to the corresponding slip velocity vectors $V_{11y}$ and $V_{22y}$. The forces $S_{11y}$ and $S_{22y}$ can be approximated by the formulae of the Coulomb law. In this case, the most dangerous case with regard to derailing is the case when the angle is $\zeta=0$.

$$S_{11y}=P_1\cdot\mu;\quad S_{22y}=N_{22}\cdot\mu\cdot\cos\beta, \tag{7}$$

where $P_1$ is the vertical load in the contact $K_{11}$; $\mu$ is the coefficient of sliding friction in the contact between the wheels and the rails.

Then

$$Y=\mu\left(P_1+N_{22}\cdot\cos\beta\right)+Y_f. \tag{8}$$

The equations of equilibrium of the contact forces are represented as the sum of the projections on the Oy and Oz axes (Fig. 2, *b*):

$$\begin{cases}\sum F_y=0: \\ \sum F_z=0:\end{cases}\quad \begin{cases}Y-N_{22}\cdot\sin\beta=0; \\ N_{22}\cdot\mu\cdot\sin\beta+N_{22}\cdot\cos\beta-P_2=0.\end{cases} \tag{9}$$

From the second equation of (9),

$$N_{22}=\frac{P_2}{\mu\cdot\sin\beta+\cos\beta}. \tag{10}$$

Given that the first wheel has no flange contact and the main contact of the second wheel is completely unloaded, we can assume that $P_1=P_2=P$. Then from the first equation of (9), we obtain

$$\mu\cdot P\cdot\left(1+\frac{\cos\beta}{\mu\cdot\sin\beta+\cos\beta}\right)+Y_f-P\frac{\sin\beta}{\mu\cdot\sin\beta+\cos\beta}=0. \tag{11}$$

### 5. 3. Safety factor against derailment

The critical relation of the frame force to the vertical load at which the main contact is completely unloaded can be found by the formula

$$\left[\frac{Y_f}{P}\right]=\frac{tg\beta\left(1-\mu^2\right)-2\mu}{\mu\cdot tg\beta+1}. \tag{12}$$

Fig. 3 shows the comparative dependence of the critical ratio $[Y_f/P]$ on the inclination angle and the friction coefficient μ, calculated by the classical formula (2) and by the refined formula (12).

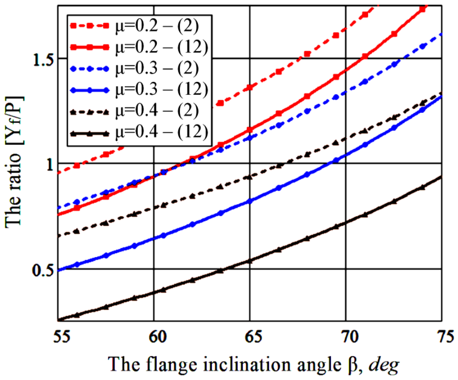


Fig. 3. The comparative dependences of the critical ratio $[Y_f/P]$ on the inclination angle β and the friction coefficient μ, calculated by the classical formula (2) and by the refined formula (12)

The values of the critical ratio $[Y_f/P]$ according to the classical Nadal formula and the refined formula (12) have significant differences. In this case, the refined formula (12) produces smaller values of the ratio $[Y_f/P]$.

## 6. Discussion of the results of studying the safety factor against derailment

For many decades, Nadal's formula has remained the main criterion for assessing the stability of rail vehicles as to derailment [6]. Many researchers [7–10] find it primitive and unable to take into account many factors affecting the safety of railway vehicles. Numerous attempts have been made to correct Nadal's formula with the aim of clarifying it [11–24]. However, derailments of locomotives and railway cars continue to occur quite regularly [25]. This study is another attempt to refine the safety criterion against derailment based on Nadal's theory.

The idea of the study is based on a hypothesis that includes the following starting positions for the wheel pair contacting the rails:

– sliding in the flange contact of the oncoming wheel can create a friction force the vertical component of which reduces the vertical load on the rail, and the horizontal component increases the lateral load in the flange contact;

– sliding in the main contact of the running-off wheel creates frictional forces that increase the total horizontal lateral load on the flange of the approaching wheel.

The combination of these factors and their effect on the safety factor against derailment was not considered in the known studies [1–34].

The specified formula for the safety factor against derailing of the wheels is as follows:

$$k_y = \frac{P}{Y_f} \div \frac{\mu \cdot tg\beta + 1}{tg\beta\left(1-\mu^2\right)-2\mu} \geq [k_y], \tag{13}$$

where $[k_y]$ is the minimum permissible standard safety factor of the stability against the derailment of wheels.

As can be seen from Fig. 3, the values of the ratio $[Y_f/P]$, obtained by the refined formula (12), depending on the values of the inclination angle β and the friction coefficient μ, are 10–50 % lower than those calculated according to the classical Nadal formula (2).

Nadal's safety criterion improvement is suggested on the basis of the features of the frictional interaction of a wheel pair with rails, which have been previously disregarded in similar studies. Thus, account is taken of the following:

– the dependence of the vertical component of the frictional force in the flange contact of the approaching wheel on the angle of the approach and the mode of the wheel pair motion;

– the influence of the frictional force in the flange contact of the approaching wheel pair on the increase in the lateral load on the flange;

– the influence of the frictional force in the contact of the running-off wheel with the rail, creating frictional forces to increase the lateral load on the flange of the approaching wheel.

In addition, unlike in the traditional approach, the safety factor of the wheel steadiness against derailment is assessed taking the frame force, rather than the lateral load on the flange, as the main factor of safety.

The cumulative effect of these factors on the derailment conditions has not been considered before this study.

Dependence (15) increases the accuracy of determining the safety factor against derailing, which helps increase the safety level of vehicles at the stage of their design and operation.

At the same time, we must admit possible inaccuracies in determining the safety criterion according to the formula proposed in the study. This is due to the fact that the problem was considered in a static formulation, without taking into account the dynamic processes in the contact of the wheel with the rail. First of all, these are frictional self-oscillations, typical of the flange contact.

## 6. Conclusion

1. The study has specified the safety factor – the Nadal criterion – against the derailment of the wheels of a railway stock. The improvement is based considering the complete model of the wheel pair contacting, taking into account the slip forces at the contact points of the approaching and running-off wheels.

2. The modeling of the frictional contact interaction between the wheel pair and the rails included the following previously overlooked factors:

– the dependence of the vertical component of the frictional force in the flange contact of the approaching wheel on the angle of the wheel running onto the rail;

– the influence of the frictional force in the flange contact of the approaching wheel on the increase in the horizontal lateral load on the flange;

– the influence of the frictional force in the contact of the running-off wheel on the increase in the lateral load on the flange of the approaching wheel.

3. In contrast to the traditional approach, when evaluating the stability of the wheel against derailment, it was suggested to use the frame force as the main safety factor instead of lateral load on the flange. This approach to determining the stability criterion gives more reliable results, since the frame force is more accessible for experimental and theoretical analysis.

4. The proposed safety factor against derailment of a railway stock, for different values of the flange inclination angle and the friction coefficient, is 10–50 % lower than Nadal's classical stability criterion. This makes the proposed criterion more reliable in assessing safety of a railway rolling stock derailment.

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