ADAPTIVE CONTROL OVER NONLINEAR OBJECTS USING THE ROBUST NEURAL NETWORK FCMAC
(p. 4-14)

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The paper explores issues related to the application of artificial neural networks (ANN) when solving the problems on identification and control of nonlinear dynamic systems. We have investigated characteristics of the network, which is a result of the application of the apparatus of fuzzy logic in a classical CMAC neural network, which is titled FCMAC – Fuzzy Cerebral Model Arithmetic Computer. We studied influence of the form of receptive fields of associative neurons on the accuracy of identification and control; various information hashing algorithms that make it possible to reduce the amount of memory required for the implementation of a network; robust learning algorithms are proposed allowing the use of a network in systems with strong perturbations. It is shown that the FCMAC network, when selecting appropriate membership functions, can be applied in order to synthesize indirect control systems with and without a reference model; it is more efficient to use it in control systems with the reference model. This sharply reduces the quantity of training pairs and simplifies the coding due to the narrower range of the applied values of input signals. The results obtained are confirmed by simulation modeling of the processes of identification of and control over nonlinear dynamical systems.

Keywords: artificial neural network, fuzzy-CMAC, identification, modeling, indirect adaptive control, hashing.

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THE METHOD FOR EVALUATION OF EDUCATIONAL ENVIRONMENT SUBJECTS' PERFORMANCE BASED ON THE CALCULATION OF VOLUMES OF M-SIMPLEXES (p. 15-25)

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We propose the method for comprehensive performance evaluation of subjects of educational environments, specifically higher educational institutions, based on calculation of generalized volume of the m-simplex. The vertices of the m-simplex are found based on performance scores of subjects of educational environments by different categories. To find a comprehensive performance score of subjects of educational environments, it is proposed to calculate generalized volume of the constructed m-simplex, based on calculation of the Cayley-Menger determinant. The numerical methods for calculation of this determinant for different cases of location of vertices of m-simplex were considered.

A list of five major categories of evaluation of higher education institutions was compiled and selection of indicators for these categories was performed. The method of comprehensive performance evaluation of the subjects of educational environments based on calculation of generalized volume of m-simplex was verified in the developed information-analytical system. This method was compared with the ideal point method and the weighed scores method. The feature of the proposed method is its self-sufficiency, because the method does not require solution of ancillary problems in calculation of a comprehensive score, such as selection of weight coefficients and the ideal point, involvement of experts, etc. It was shown that the proportional changes in a comprehensive score, calculated by the proposed method, correspond to small changes of certain categories. The method of setting a tendency of activity development of subjects of educational environments by calculating the derivative of a comprehensive score in time was presented.

The methods for performance evaluation of subjects of educational environments can be used in scientific and educational institutions, as well as in private companies that are engaged in creation of high-tech applied information technologies.

Keywords: m-simplex, rating of higher educational institution, evaluation of subject of educational environment, Cayley-Menger determinant.

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DEVELOPMENT OF THE METHOD FOR MODELING OPERATIONAL PROCESSES FOR TASKS RELATED TO DECISION MAKING (p. 26-32)

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A solution to any problem is valuable only if the results of the solution are reliable. This axiomatic statement is trivial. However, despite the fact that this statement is undeniable, it is violated regularly in problems of operations research. This is due to the fact that the proposed evaluation criteria are not verified and possibility of using indirect methods of effectiveness evaluation of operations is not substantiated.

The problem point of modeling was shown using the example of comparison of operational processes, based on equally effective operations with multiple duration. This point is related to the fact that it is necessary to take into account inter-operational losses while modeling the process, based on the use of shorter operations in relation to an alternative operational process. Ignoring these losses can lead to making a mistaken decision or to errors in verification of estimation indicators.

In a number of cases the problem can be solved using specially conducted experimental studies. However, if there is a set of unveri-

fied estimation expressions or in verification problems, a formal approach can be used. Under this approach, uncertainty, arising in the process of modeling, can be removed by using the capabilities of a verified indicator itself.

The developed method for modeling allows us to determine the scope of constraints on parameters of modeled operations. The probability of an error in results of modeling in problems, related to decision making, is excluded on condition of taking into account this region.

Keywords: operations research, modeling of operational processes, method of modeling, verification of effectiveness formula.

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ACCELERATION ANALYSIS OF THE QUADRATIC SIEVE METHOD BASED ON THE ONLINE MATRIX SOLVING (p. 33-38)

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The algorithm for the online matrix solving is proposed. The rate of acceleration of the basic quadratic sieve method based on the online matrix solving is investigated. Acceleration of the quadratic sieve method will reduce the runtime, the complexity of the algorithm and expand the set of numbers, where this algorithm is the best.

It is shown that the modified algorithm has increased the number of successful decompositions. That is, the number of cases where the basic quadratic sieve (standard sieving interval and size of the factor base) failed to form a matrix to obtain a solution was reduced. This became possible due to the fact that in the modified algorithm there is no need to obtain all L^a+2 B-smooth numbers prior to diagonalization of the matrix, as in the case of the basic method. Among other important characteristics of this method, it should be noted that when used, the same operations as in the basic quadratic sieve method are performed, only their order is changed. The computing complexity decreases if the set of B-smooth numbers, for which the power matrix vectors form a linearly dependent system, are found quickly.

According to the data obtained, the modified QS method, based on the online matrix solving, provides an acceleration of about 5.45 percent for numbers of 10^{130} in size. It is shown that improvements associated with solving the matrix cannot lead to a significant increase in the sieving interval. After all, the rate of acceleration decreases with increasing number N. Further improvement to the quadratic sieve method should be related to methods aimed at a significant reduction of the sieving interval and the size of the factor base, which in relative terms should be the greater, the higher N.

Keywords: factorization, quadratic sieve, B-smooth, online matrix solving, factor base.

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DEVELOPMENT OF GRAPHICANALYTICAL MODELS FOR THE SOFTWARE SECURITY TESTING ALGORITHM (p. 39-46)

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An analysis of one of the main types of software testing, namely security testing has been made. It was established that there are a number of specific features associated with the possibility of negative manipulation with software products. A graphic-analytical model of the algorithm of testing software security was developed. The model based on the theory of semi-Markov processes provides an adequate structural description of the actual testing process. However, accuracy of this model essentially depends on accuracy of reproduction of

densities of distribution of duration of the system residence in each of the possible states. An alternative model that uses the method of probability-time graphs is less demanding. For its implementation, it is sufficient to know the mean values of duration of residence in each of the states and the probability of transitions from one state to another. Correlations were obtained for calculating statistical characteristics and density of distribution of the mean time of execution of the software security testing algorithm. The model can be used to study basic stages of software security testing. Application of this model will reduce software vulnerability and improve security of the IT project as a whole. Also, the model is applicable when developing new methods, algorithms, and procedures for managing the IT projects.

Keywords: security testing, graphic-analytical model, semi-Markov process, Laplace transform, generating function.

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APPLICATION OF PIECEWISE-CUBIC FUNCTIONS FOR CONSTRUCTING A BEZIER TYPE CURVE OF C^1 SMOOTHNESS (p. 46-52)

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We have proposed and implemented a new method for constructing a spline curve of third degree, which possesses the properties of both a cubic spline and the Bezier curve. Similar to building the Bezier curves, control points are assigned, whose position affects the shape of the curve. In the proposed approach, the sections of the straight line that connect control points are tangent to the curve that is constructed. The location of touch points can be different, allowing the construction of different curves for one set of control points. A special feature of the proposed method is assigning, in abscissa of control points, some unknown spline values that are found from conditions for the continuity of the first derivatives of the curve at these points. Finding the coefficients of polynomials that make up the curve comes down to solving a system of linear equations with a three-diagonal matrix. The built curve is a piecewise-cubic function, continuous along with its first derivative throughout the entire interval. Conditions were found for any set of control points in the form of inequalities, which parameters of the curve must meet, at which the curve does exists and it is unique. These conditions follow from the requirement for a diagonal advantage of the matrix of the system for determining coefficients of the curve. A series of computational experiments were performed, which showed that the curve effectively inherits the shape assigned by control points. Similar to the Bezier curves, the proposed curve could be used in computer graphics systems and computer systems for technical design, specifically for the creation of fonts, drawings of parts, elements of transportation vehicles' bodies, etc.

Keywords: spline curve of third degree, Bezier curve, shape parameters of curve.

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GENERALIZATION OF ONE ALGORITHM FOR CONSTRUCTING RECURRENT SPLINES (p. 53-62)

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We have analyzed two algorithms, close in composition, for constructing a smoothing spline, which imply a change only in the last link of the spline when new experimental data arrive. The main feature of the N. D. Dicoussar algorithm is the form of a polynomial representation in order to describe a link of the spline. It is shown that a given polynomial is one of the hierarchical form of the Hermitian polynomial.

We have proposed a modification to the D. A. Silaev algorithm for constructing a smoothing spline with different orders of smoothness: from zero to the second, aimed at enhancing the stability of this algorithm. To this end, we substantiated recommendations related to the form of polynomials representation, which describe the links of splines of the specified form. For this purpose, we estimated conditionality of matrices used in the algorithm. For the spline of zero-order smoothness, the most advisable is to apply a polynomial in the N. D. Dicoussar form, and for splines with higher orders of smoothness of joining the links, it is appropriate to use different forms of the Hermitian polynomials.

Based on computational examples, a possibility was demonstrated to generalize the D. A. Silaev algorithm to construct a spline with links of various lengths, which is determined by the rate of change in the examined parameter. That makes it possible to reduce the volume of information that contains a description of the spline itself, and to prevent such a widespread shortcoming of approximation when using polynomials as parasitic oscillations. It was shown as well that in the presence of significant measurement errors in experimental data there may occur a need to decrease the length of the spline's link (compared to that derived by the D. A. Silaev rule) in order to provide the spline with a property of robustness.

Keywords: smoothing spline, a time series, algorithm stability, matrix conditionality, algorithm resource-intensity.

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