*Обчислювальними експериментами досліджена стійкість двочастотних режимів руху одномасної вібромашини з прямолінійним поступальним рухом платформи і віброзбудником у вигляді двокульового автобалансира. Оцінена точність закону руху системи, раніше знайденого методами малого параметра. Знайдена функція, яка наближує критичну швидкість обертання ротора, при переході через яку зникає стійкість двохчастотного режиму руху*

*Ключові слова: інерційний віброзбудник, двочастотні вібрації, автобалансир, одномасна вібромашина, ефект Зомерфельда, стійкість руху*

*Вычислительными экспериментами исследована устойчивость двухчастотных режимов движения одномассной вибромашины с прямолинейным поступательным движением платформы и вибровозбудителем в виде двухшарового автобалансира. Оценена точность закона движения системы, ранее найденного методами малого параметра. Найдена функция, приближающая критическую скорость вращения ротора, при переходе через которую исчезает устойчивость двухчастотного режима движения*

*Ключевые слова: инерционный вибровозбудитель, двухчастотные вибрации, автобалансир, одномассная вибромашина, эффект Зоммерфельда, устойчивость движения*

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# ON STABILITY OF THE DUAL-FREQUENCY MOTION MODES OF A SINGLE-MASS VIBRATORY MACHINE WITH A VIBRATION EXCITER IN THE FORM OF A PASSIVE AUTO-BALANCER

**V. Yatsun**
PhD, Associate Professor
Department of Road Cars and Building**
E-mail: yvkr@i.ua

**G. Filimonikhin**
Doctor of Technical Sciences, Professor, Head of Department*
E-mail: filimonikhin@ukr.net

**A. Haleeva**
PhD, Associate Professor
Department of tractors and agricultural machinery, operating and maintenance
Mykolayiv National Agrarian University
Georgiya Gongadze str., 9, Mykolayiv, Ukraine, 54020
E-mail: galeevaantonina@gmail.com

**A. Nevdakha**
PhD*
E-mail: aunevdaha@ukr.net

*Department of Machine Parts and Applied Mechanics**
**Central Ukrainian National Technical University
Universytetskyi ave., 8, Kropivnitskiy, Ukraine, 25006

## 1. Introduction

Among such vibratory machines as screeners, vibratory tables, vibratory conveyers, vibratory mills, etc., the promising ones are the multi-frequency-resonance machines [1]. They combine high performance of multi-frequency vibratory machines [2], and the largest efficiency of resonance vibratory machines [3].

In [4], it was proposed to excite dual-frequency resonance vibrations by passive auto-balancers. In order to design vibratory machines with the new vibration exciter, it is required to examine their dynamics. Theoretical research into dynamics of vibratory machines includes such stages as a description of the model and construction of differential equations of the vibratory machine, the search for various possible steady motion modes and study into their stability.

At present, such a procedure is relevant in order to investigate the stability of dual-frequency motion modes of the single-mass vibratory machine with translational rectilinear motion of the platform and a vibration exciter in the form of a passive auto-balancer.

## 2. Literature review and problem statement

Authors of [4] proposed to apply passive auto-balancers (a ball, a roller, a pendulum) as the dual-frequency exciters of vibrations. To do this, a special motion regime of pendulums [5], balls, or rollers, is employed [6]. Under this mode, loads are tightly pressed against each; cannot accelerate to the rotation speed of the shaft, onto which an auto-balancer is mounted; and get stuck at the resonance frequency of platform oscillations. This induces slow resonant oscillations of the platform. The unbalanced mass is placed on the auto-balancer's casing. This excites rapid oscillations of the platform at the frequency of shaft rotation. Vibration parameters are changed by altering: a shaft rotation speed, the unbalanced mass, a total load mass. The new vibration excitation technique employs the Sommerfeld effect [7].

Authors of [8] devised generalized models of single-, dual-, and three-mass vibratory machines with translational motion of vibratory platforms and a vibration exciter in the form of a ball, a roller, or a pendulum auto-balancer. They derived differential equations for the motion of vibratory machines. In [9], by using a small parameter method, possible frequencies at which loads get stuck and corresponding dual-frequency motion modes for a single-mass vibratory machine were analytically found. It was established that depending on the system's parameters and the rotor speed, there are one or three possible frequencies at which loads get stuck. In the case of three frequencies at which loads get stuck, two frequencies are close to the natural frequency of platform oscillations, and one frequency is close to the frequency of rotor rotation. Results of study [9] (applied in this paper) are described below in more detail.

In practice, of all possible motion modes reported in [9], only the steady motion will be implemented. It is therefore important to investigate stability of the dual-frequency modes of motion. It is also important to estimate the accuracy of approximated formulae intended to search for the potential frequencies at which loads get stuck and describe the dual-frequency modes of motion.

Analytically, the stability of motions of rotor machines with auto-balancers is explored by Lyapunov in a small, that is, at very small deviations of the perturbed motion from the unperturbed one.

In [10], authors studied stability of different steady motions of the isolated system consisting of a rotating body and two pendulums, mounted onto its longitudinal axis. Elements of the theory of bifurcations of motions were employed [11]. In a coordinate system that rotates synchronously with a carrying body, the equations of motion and steady motions are stationary. The authors applied, as a bifurcation parameter, the distance from the center of mass of the carrying body to the plane of pendulums. All possible steady motions (positions of relative equilibrium) were found as functions of the bifurcation parameter. To study the stability of various steady motions, the authors relied on that the stability of motions may change to instability (and vice versa) only when the bifurcation parameter acquires its critical values [11]. A given method makes it possible to explore the stability of steady motions of the system without additional assumptions about the ratios of smallness among parameters of the system. However, the method is effective only when one knows all possible established motions and all critical values of the bifurcation parameter. It should be noted that authors of [9] accepted as the bifurcation parameter the rotor speed. They found characteristic velocities (special values for the bifurcation parameter), exceeding which leads to the occurrence or merging different modes at which loads get stuck. In this case, not all possible (alternative to getting stuck) motion modes have been found. It is not possible to solve the problem on stability of the established modes of getting stuck using methods of the motion bifurcation theory.

In [6], the stability of the auto-balancing regime was studied in the framework of a flat model of rotor on isotropic supports, balanced by a multi-ball (multi-roller or multi-mass) autobalancer. In a coordinate system that rotates synchronously with the rotor, the equations of motion and steady motion, corresponding to balancing), are stationary. A characteristic equation was constructed for a system of differential equations that describe the process of auto-balancing. This is the eighth-degree polynomial. It is not possible to find stability conditions in a general case by using known criteria or by searching for (exact) polynomial roots. Therefore, to study the stability, roots were searched for approximately, by decomposing in a truncated series for powers of a small parameter [12]. The method holds only for the case of isotropic supports. To study stability in the wider area of change in the system's parameters, it is necessary to repeatedly decompose the roots of a polynomial at different ratios of smallness among parameters [12].

In [13–19], motion stability, corresponding to the pendulums or the balls getting stuck, is investigated analytically by Lyapunov (in a small) in combination with methods of the small parameter: in [13–17] – using the method of synchronization of dynamical systems [13]; in [18, 19] – applying the method of separation of motions [10].

Authors of [13–15] examined the effect of pendulums getting stuck in vibratory machines. For a pendulum, mounted on the electric motor shaft, placed on the platform of a vibratory machine, studies were conducted for the following cases: a low-power electric motor [13]; an electric motor whose rated rotation frequency slightly exceeds resonance frequency of oscillations of the platform [14]. In [15], a vibratory machine was studied, in which two low-power electric motors are placed on the platform and the shaft of each electric motor holds a pendulum.

The effect of balls getting stuck in the auto-balancer was investigated within the spatial model of the rotor, statically balanced by a two-ball auto-balancer [16]; the flat rotor model, statically balanced by a two-ball auto-balancer [17]. The effect of pendulums getting stuck in the auto-balancer was examined within the framework of a spatial model of the rotor balanced by: statically, by a two-pendulum auto-balancer [18]; dynamically, by two two-pendulum auto-balancers [19].

The studies reported in [13–19] established that the balls or pendulums get stuck at one of the natural frequencies of oscillations of the rotor or the platform. Comparison of these results with the results of paper [9] reveals that the combination of Lyapunov method with methods of the small parameter failed to detect such phenomena as:

– splitting one frequency at which loads get stuck into two in the vicinity of the natural oscillation frequency of the platform (rotor);

– dependence of the magnitude and quantity of possible frequencies at which loads get stuck on the rotor speed and other system's parameters.

This is because these phenomena are not discoverable based on the solutions found at zero (and sometimes first) approximation for a small parameter. In this case, the search for solutions in higher approximations is a complex and labor-intensive mathematical problem.

The vibratory machine, considered in present work, has an asymmetry in supports and its dynamics is affected by a large number of dimensionless parameters. Given this, its stability is investigated numerically. Research results are interpreted applying the theory of motion bifurcation. In addition, by employing a computational experiment, one searches for a function of dimensionless parameters of the system, bringing closer the critical speed. When a rotor exceeds this speed, dual-frequency motion modes of the vibratory machine become unstable.

## 3. The aim and tasks of the study

The aim of present study is to examine stability of dual-frequency modes of motion of the platform of a single-mass vibratory machine with translational rectilinear motion of the platform and a vibration exciter in the form of a passive auto-balancer. The results to be obtained would make it possible to design such vibratory machines with a steady dual-frequency motion mode of the platform.

To accomplish the aim, the following tasks have been set:

– to estimate the ranges of change in the dimensionless parameters influencing the dynamics of a vibratory machine;

– to investigate stability, by a computational experiment, using the methods, previously defined, for a small parameter of dual-frequency motion modes of the platform;

– to estimate accuracy of the approximated laws that govern the motion of a vibratory platform;

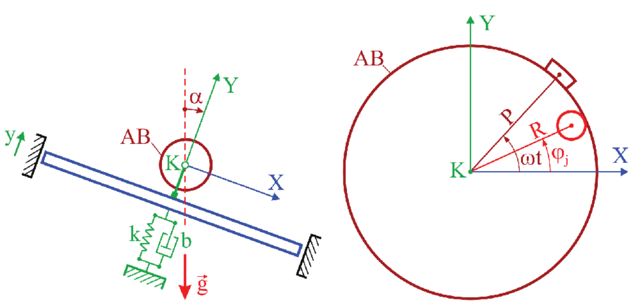
– to estimate, employing a computational experiment, the effect of dimensionless parameters on critical speed, above which a dual-frequency mode loses stability;

– to derive, applying a computational experiment, an analytical function for the approximate calculation of this critical speed.

## 4. Research methods

### 4. 1. Description of the generalized model of a vibratory machine

A vibratory machine (Fig. 1) is composed of the platform, mass $M$, and a vibration exciter in the form of a ball, a roller, or a pendulum auto-balancer [8]. The platform can move only translationally rectilinearly (Fig. 1, *a*). Direction of the platform motion creates angle $\alpha$ to the vertical. The platform rests against an elastic-viscous support with a rigidity coefficient $k$ and a viscosity coefficient $b$. Position of the platform is defined by the $y$ coordinate, equal to zero in the state of static equilibrium of the platform.




*a* *b*

Fig. 1. Model of a single-mass vibratory machine with translational rectilinear motion of the platform [8]: *a* — kinematics of the platform motion; *b* — kinematics of the motion of the unbalanced mass and a load

The casing of an auto-balancer revolves around a shaft – point $K$ with a constant angular velocity $\omega$ (Fig. 1, *b*). The unbalanced mass $\mu$ is rigidly coupled to the casing of the auto-balancer. It is located at distance $P$ from point $K$. Position of the unbalanced mass is defined by angle $\omega t$, where $t$ is the time.

The auto-balancer consists of $N$ identical loads. Mass of one load is $m$. Center of the masses of load can move in a circle with radius $R$ centered at point $K$ (Fig. 1, *b*). Position of load No. $j$ is defined by angle $\varphi_j$, $/j=\overline{1,N}/$. Motion of the load relative to the casing of the auto-balancer is prevented by the force of viscous resistance. Its modulus:

$$F_j = b_W v_j^{(r)} = b_W R\,|\varphi_j' - \omega|, \quad /j=\overline{1,N}/,$$

where $b_W$ is the coefficient of the viscous resistance force, $v_j^{(r)} = R\,|\varphi_j' - \omega|$ is the modulus of motion speed of the center of massed of load No. $j$ relative to the casing of the auto-balancer; bar after the magnitude denotes time derivative $t$.

### 4. 2. Differential equations of motion in a dimensionless form

Differential equations of motion in the dimensionless form [8]:

$$\ddot{v} + 2h\dot{v} + v + \ddot{s}_y = \delta n^2 \sin n\tau,$$
$$\ddot{\varphi}_j + \varepsilon\beta(\dot{\varphi}_j - n) + \sigma\cos(\varphi_j - \alpha) + \varepsilon\ddot{v}\cos\varphi_j = 0,$$
$$/j=\overline{1,N}/, \qquad (1)$$

where:

– dimensionless variable and time:

$$v = y/\tilde{y}, \quad \tau = \tilde{\omega} t; \qquad (2)$$

– dimensionless parameters:

$$h = \frac{b}{2M_\Sigma\tilde{\omega}}, \quad \delta = \frac{\mu P}{NmR}, \quad n = \frac{\omega}{\tilde{\omega}},$$
$$\varepsilon = \frac{Nm}{\kappa M_\Sigma}, \quad \beta = \frac{b_W M_\Sigma}{Nm^2\tilde{\omega}}, \quad \sigma = \frac{g}{\kappa R\tilde{\omega}^2}; \qquad (3)$$

– dimensionless projection of the imbalance created by corrective loads:

$$s_x = \frac{1}{N}\sum_{j=1}^{N}\cos\varphi_j, \quad s_y = \frac{1}{N}\sum_{j=1}^{N}\sin\varphi_j. \qquad (4)$$

In turn, in formulae (2), (3):

– characteristic scales:

$$\tilde{y} = \frac{NmR}{M_\Sigma}, \quad \tilde{\omega} = \sqrt{\frac{k}{M_\Sigma}}; \qquad (5)$$

– mass of the entire system:

$$M_\Sigma = M + Nm + \mu; \qquad (6)$$

– for a ball, a roll, a pendulum, respectively:

$$\kappa = \frac{7}{5}, \quad \kappa = \frac{3}{2}, \quad \kappa = 1 + J_C/(mR^2), \qquad (7)$$

where $J_C$ is the principal central axial pendulum's moment of inertia; $g = 9{,}81$ m/s$^2$ is the free fall acceleration. Note that $\tilde{\omega}$ is the natural (resonance) oscillation frequency of the platform.

### 4. 3. Dual-frequency modes of motion

A dual-frequency motion mode of the platform derived in the zero approximation ($\varepsilon = 0$) takes the form [9]:

$$v_0=\frac{A\Omega^2}{(1-\Omega^2)^2+4h^2\Omega^2}\begin{bmatrix}(1-\Omega^2)\sin(\Omega\tau+\gamma_0)-\\-2h\Omega\cos(\Omega\tau+\gamma_0)\end{bmatrix}+$$
$$+\frac{\delta n^2}{(1-n^2)^2+4h^2n^2}\left[(1-n^2)\sin(n\tau)-2hn\cos(n\tau)\right], \quad (8)$$

The values for constant parameters $A$, $\gamma_0$ are not defined, and the frequency at which corrective loads get stuck $\Omega$ is the root of polynomial:

$$P(\Omega)=\chi\Omega^5-(n-\Omega)[(1-\Omega^2)^2+4h^2\Omega^2]=$$
$$=a_0\Omega^5+a_1\Omega^4+a_2\Omega^3+a_3\Omega^2+a_4\Omega+a_5=0, \quad (9)$$

where

$$\chi=Ah/\beta,\ \ a_0=1+\chi,\ \ a_1=-n,\ \ a_2=-2(1-2h^2),$$
$$a_3=2n(1-2h^2),\ \ a_4=1,\ \ a_5=-n. \quad (10)$$

Corrective loads move in line with laws:

$$\varphi_j=\Omega\tau+\psi_j,\ \ /j=\overline{1,N}/, \quad (11)$$

where $\psi_j$ are the undefined constants, in this case:

$$A\cos\gamma_0=\frac{1}{N}\sum_{j=1}^{N}\cos\psi_j,\ \ A\sin\gamma_0=\frac{1}{N}\sum_{j=1}^{N}\sin\psi_j,$$
$$A^2=\frac{1}{N^2}\left[\left(\sum_{j=1}^{N}\cos\psi_j\right)^2+\left(\sum_{j=1}^{N}\sin\psi_j\right)^2\right],$$
$$\mathrm{tg}\,\gamma_0=\sum_{j=1}^{N}\sin\psi_j\Big/\sum_{j=1}^{N}\cos\psi_j. \quad (12)$$

Dual-frequency vibratory machines will be the more energy efficient the smaller the forces of viscous resistance are (in supports, and forces that prevent the motion of balls relative to the casing of the auto-balancer). Based on the results of paper [9], in this case, there are three characteristic rotor speeds:

$$\tilde{n}_1\approx1+\frac{3}{4}\sqrt[3]{4\chi},\ \ \tilde{n}_2=1+\frac{\chi}{4h^2}=1+\frac{A}{4\beta h},$$
$$\tilde{n}_3\approx\frac{\chi}{4h^2}+1+\frac{9}{16}\chi+\frac{3}{2}\left(1+\frac{27}{32}\chi\right)h^2. \quad (13)$$

When they passed, the number or properties of possible frequencies at which loads get stuck are change.

In this case, $1<\tilde{n}_1\ll\tilde{n}_2<\tilde{n}_3\ll n$ and at rotor speeds:

– lower than $\tilde{n}_1$ $(0<n<\tilde{n}_1)$, there is only one frequency at which loads get stuck $\Omega_1$, and $0<\Omega_1<1$;

– exceeding $\tilde{n}_1$, but lower than $\tilde{n}_2$ $(\tilde{n}_1<n<\tilde{n}_2)$, there are three frequencies at which loads get stuck $\Omega_{1,2,3}$, so that $0<\Omega_1<1<\Omega_2<\Omega_3<n$;

– exceeding $\tilde{n}_2$, but lower than $\tilde{n}_3$ $(\tilde{n}_2<n<\tilde{n}_3)$, there are three frequencies at which loads get stuck $\Omega_{1,2,3}$, so that $1<\Omega_1<\Omega_2\ll\Omega_3<n$;

– exceeding $\tilde{n}_3$ $(n>\tilde{n}_3)$, there is only one frequency at which loads get stuck $\Omega_1$, so that $1\ll\Omega_1<n$.

**4. 4. Procedure for determining the onset of a dual-frequency motion mode and estimating the accuracy of the approximated formulas**

***Procedure for determining the onset of a dual-frequency motion mode.***

The onset of a dual-frequency motion mode is determined by the angular speeds of rotation of the centers of masses of loads. Under a dual-frequency motion mode these speeds should be nearly identical magnitudes that are lower than the rotor speed.

In addition, a dual-frequency mode is determined from the characteristic shape of the chart for a vibratory acceleration of the platform.

***Procedure for estimating the accuracy of approximated formulae.***

The «precise» laws of motion of loads and the platform are understood here as the laws, derived by integrating the differential equations of motion (1). Precise and approximated laws of motion are compared at a certain time interval. In this interval, the system is in steady motion with the platform performing several (3)–(5) slow oscillations.

***Procedure for estimating stability of the dual-frequency modes of motion.***

Dimensionless parameters of the system are fixed. We search, with an accuracy to 0.05, for the largest value of the dimensionless rotor speed $n_{cr}$, at which a dual-frequency motion mode is still steady. If it exceeds 0.05, a given mode loses stability.

***Procedure for searching for a function that brings closer the critical speed.***

The form of a function is determined based on the results of examining the influence of various parameters on critical speed. Coefficients in the function are derived by the least squares method.

## 5. Research results

**5. 1. Reducing the equations of motion to a normal form**

We shall introduce new variables:

$$z_0=v,\ \ z_1=\dot{v}=\dot{z}_0,$$
$$z_2=\varphi_1,\ \ z_3=\dot{\varphi}_1=\dot{z}_2,\ldots,z_{2j}=\varphi_j,$$
$$z_{2j+1}=\dot{\varphi}_j=\dot{z}_{2j},\ldots,z_{2N}=\varphi_N,\ \ z_{2N+1}=\dot{\varphi}_N=\dot{z}_{2N}. \quad (14)$$

Introduce a matrix and a vector:

$$A=\begin{pmatrix}1 & \eta\cos z_2 & \eta\cos z_4 & \cdots & \eta\cos z_{2N-2} & \eta\cos z_{2N}\\ \varepsilon\cos z_2 & 1 & 0 & \cdots & 0 & 0\\ \varepsilon\cos z_4 & 0 & 1 & \cdots & 0 & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\ \varepsilon\cos z_{2N-2} & 0 & 0 & \cdots & 1 & 0\\ \varepsilon\cos z_{2N} & 0 & 0 & 0 & 0 & 1\end{pmatrix},$$

$$B=\begin{pmatrix}-2hz_1-z_0+\eta\sum_{j=1}^{N}z_{2j+1}^2\cos z_{2j}+\delta n^2\cos n\tau\\ -\varepsilon\beta(z_3-n)\\ \vdots\\ -\varepsilon\beta(z_{2N+1}-n)\end{pmatrix}. \quad (15)$$

Then a system of equations (1) in the normal form will take the following form:

$$\dot{z}_0 = z_1,\quad \dot{z}_{2j} = z_{2j+1},\quad /j=\overline{1,N}/,$$

$$(\dot{z}_1, \dot{z}_3, \ldots, \dot{z}_{2N+1})^{\mathrm{T}} = A^{-1}B. \qquad (16)$$

A system of equations (16) with coefficients from (3), (5)–(7) will be employed in order to conduct computational experiments.

Computational experiments will be conducted for the case of 2 loads in the absence of the force of gravity ($\sigma=0$).

**5. 2. Estimation of the magnitudes of dimensionless parameters**

A vibratory table is calculated.

***Data on the platform and its supports:***

$M_0$=36 kg – mass of the platform and the parts connected to it;

$M$=90 kg – mass of the loaded platform and the parts connected to it;

$k$=145.083·10$^3$ N/m – a support rigidity coefficient;

$b$=150 N·s/m – a support viscosity coefficient.

***Data on the auto-balancer and the unbalanced mass***:

$N$=2, $\kappa$=7/5 – a two-ball auto-balancer;

$D_b$=0.043 m – a ball diameter;

$$m=\frac{4}{3}\pi\rho\left(\frac{D_b}{2}\right)^3=\frac{4}{3}\cdot 3.142\cdot 7800\cdot\left(\frac{43\cdot 10^{-3}}{2}\right)^3=0.325\text{ kg}$$ – mass of a steel ball;

$D_r$=0.19 m – diameter of the ball running track;

$R=0.5\cdot(D_r-D_b)=0.5\cdot(0.19-0.043)=0.0735$ m – distance from the longitudinal axis of the rotor to the center of masses of the ball;

$b_w$=0.03 N·s/m – coefficient of forces of viscous resistance to the motion of the ball;

$\mu$=0.1 kg – unbalanced mass;

$P$=0.105 m – distance from the longitudinal axis of the rotor to the unbalanced mass.

***Dimensional parameters and magnitudes***:

$M_{\Sigma 0}=M_0+N\cdot m+\mu$=36.75 kg – mass of the system without a load;

$\tilde{y}=1.3\cdot 10^{-3}$ m – characteristic scale for the displacement of the system without a load;

$M_\Sigma=M+N\cdot m+\mu$=90.75 kg – mass of the loaded system;

$\tilde{y}=0.5264\cdot 10^{-3}$ m – characteristic scale for the displacement of a loaded system;

$n_{0\max}=\frac{1}{2\pi}\sqrt{k/M_{\Sigma 0}}=10$ Hz – resonance frequency of platform oscillations;

$n_{0\min}=\frac{1}{2\pi}\sqrt{k/M_{\Sigma}}=6.36$ Hz – resonance frequency of the loaded platform oscillations.

***Dimensionless parameters***:

$\varepsilon$=0.0051÷0.0126; $\beta$=0.138÷0.537;

$\delta$=0.22; $h$=0.021÷0.032.

***Hereafter we accepted the following intervals of change in the dimensionless parameters*:**

$\varepsilon$=0.005÷0.05; $\beta$=0.1÷0.6;

$\delta$=0÷1; $h$=0.01÷0.07.

These intervals correspond to the required specifications of the designed machine. On the other hand, they make it possible to change characteristics of dual-frequency vibrations of the platform in a wide range.

**5. 3. Stability of dual-frequency modes of motion of the system and the estimation of accuracy of the approximated laws of motion**

196 computational experiments were conducted at different values of the dimensionless parameters from the examined range of their change. These experiments were performed to study both the stability of dual-frequency modes of motion and the influence of system's parameters on critical speed.

***Results of a typical experiment*** conducted for the following values of dimensionless parameters: $\varepsilon$=0.01; $\beta$=0.4; $\delta$=0.25; $h$=0.03.

For these values, the characteristic critical speeds:

$$\tilde{n}_1\approx 1.808,\quad \tilde{n}_2\approx 21.833,\quad \tilde{n}_3\approx 21.877.$$

Fig. 2 shows charts of frequencies at which balls may get stuck (the real roots of a polynomial (9)).

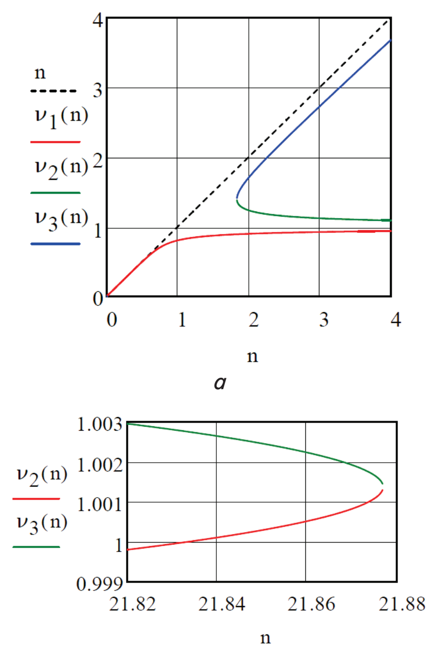



Fig. 2. Frequencies (Hz) at which balls may get stuck: *a* – emergence of second and third frequencies; *b* – merger of first and second frequencies

In the range of dimensionless rotor speeds (0, $n_{cr}$], where $n_{cr}\approx$7.0, only the dual-frequency mode of motion is stable, at which balls get stuck at a pre-resonant frequency $\Omega_1$.

The balls under this mode:

– create the greatest imbalance;

– rotate synchronously ($\varphi_1(\tau)=\varphi_2(\tau)$) as a whole.

At the rotor speeds that exceed critical speed $n_{cr}$, there may occur, depending on the magnitudes of dimensionless parameters:

– auto-balancing;

– a stable dual-frequency motion mode with a frequency at which loads get stuck $\Omega_3$;

– a chaotic motion of the system.

These modes of motion are not explored in detail, since operation of the designed single-mass vibratory machines is based on using only the first mode of motion.

The charts that describe a dual-frequency motion mode of the system at $n$=5 are shown in Fig. 3.

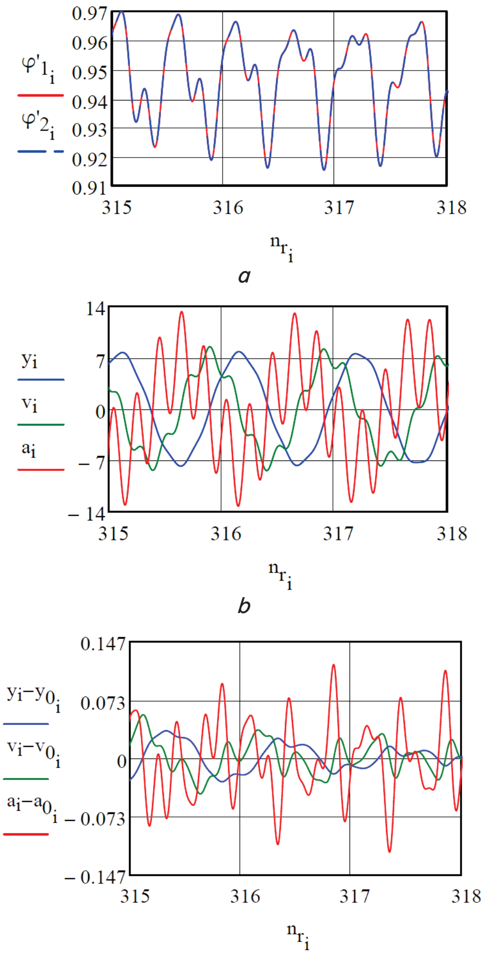



Fig. 3. Charts of change in the dimensionless: *a* – rotation speeds of the centers of balls; *b* – displacement ($y_i$), velocity ($v_i$), and acceleration ($a_i$) of the platform; *c* – differences between the approximated and actual displacements, velocities, and accelerations of the platform

The charts demonstrate the number of slow oscillations of the platform on the horizontal axis. Possible frequencies at which balls get stuck (the roots of equation (9)) are $\Omega_1\approx 0.9465$, $\Omega_2\approx 1.0732$, $\Omega_3\approx 4.61875$. The frequency at which a ball gets stuck, derived from averaging the angular velocity of rotation of the ball center:

$$\bar{\Omega}_1 = \frac{1}{\Delta\tau}\int_{\tau_{max}-\Delta\tau}^{\tau_{max}} \dot{\varphi}_j(\tau)d\tau = 0.9469, \quad /j=1,2/, \qquad (17)$$

where $\tau_{max}$ is the experiment duration, $\Delta\tau$ is the time interval at which the averaging is performed. Difference between $\Omega_1$ and $\bar{\Omega}_1$ is 0.04 %. The differences between the approximated and the actual motions, velocities and accelerations of the platform do not exceed 1.105 %.

Similar results were obtained when performing other computational experiments. The experiments have proved that for any values of dimensionless parameters of the system in the examined range there is a critical rotor speed $n_{cr}$, such that:

– in the range of rotor speeds $(0, n_{cr}]$, only the first mode at which balls get stuck is stable;

– upon exceeding this critical speed, the first mode loses stability.

**5. 4. Estimation of influence of various parameters on critical speed**

**5. 4. 1. Influence of the small parameter ε on critical speed**

Fig. 4 shows dependence charts of critical speed on parameter ε at different δ and β=0.4, *h*=0.03.

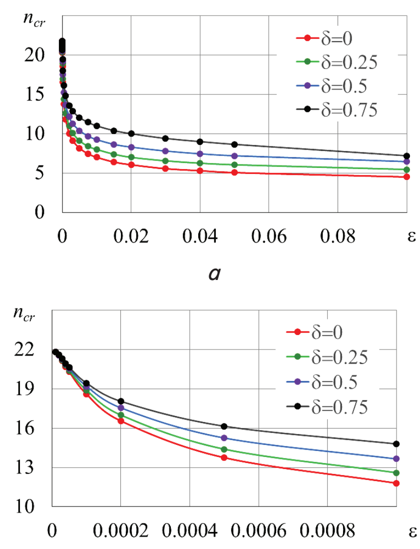



Fig. 4. Dependence of critical speed on parameter ε at different δ and β=0.4, *h*=0.03: *a* – throughout the interval (0, 0.1]; *b* – in the vicinity of ε=0

Fig. 4 shows that:

– at a decrease in parameter ε critical speed approaches a characteristic speed $\tilde{n}_2$ at *A*=1;

– at a zero approximation for ε, critical speed coincides with $\tilde{n}_2$;

– $\tilde{n}_2$ cannot be used to calculate the critical speed $n_{cr}$ because even at a slight increase in ε critical speed rapidly deviates from the characteristic speed.

**5. 4. 2. Influence of parameters β, δ, *h* on critical speed**

Fig. 5, 6 show dependences of critical speed on parameters *h*, β, δ.

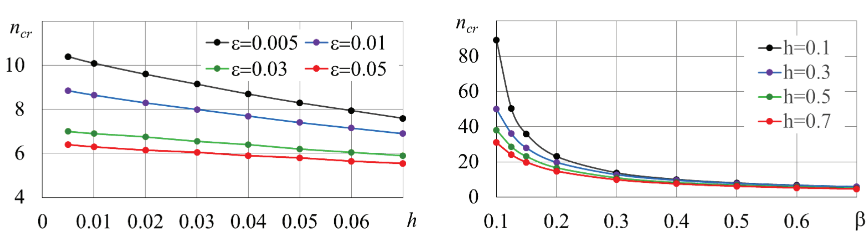



Fig. 5. Dependence of critical speed on parameter: *a* – *h* (at different ε and β=0.4, δ=0.25); *b* – β (at different *h* and ε=0.005, δ=0.25)

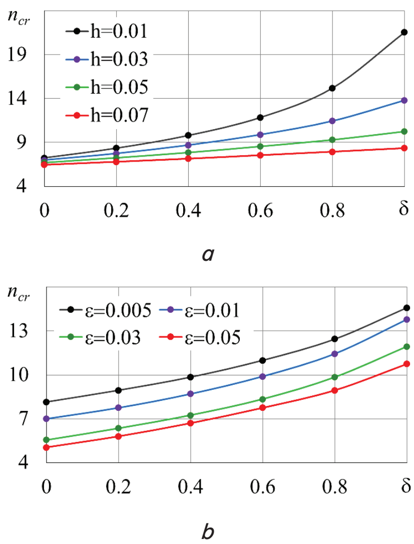



Fig. 6. Dependence of critical speed on parameter δ: *a* – at different *h* and ε=0.01, β=0.4; *b* – at different ε and β=0.4, *h*=0.03

Charts in Fig. 5, 6 show that in the examined range of change in the parameters critical speed considerably depends on dimensionless parameters of the system; in this case:

– $n_{cr}$ monotonically increases for parameter δ;

– $n_{cr}$ monotonically decreases for parameters e, β, *h*;

– dependence on parameter *h* is close to linear;

– dependences on parameters ε, β, δ are essentially non-linear.

**5. 5. Approximation of critical speed using an analytic function**

Based on the results from chapters 5. 3 and 5. 4, critical speed depends on all dimensionless parameters of the system: $n_{cr}=n_{cr}(h,\beta,\varepsilon,\delta)$.

The search for this critical speed by a combination of Lyapunov method and the methods of a small parameter requires consideration of higher approximations for a small parameter. Therefore, it is a complex and labor-intensive mathematical problem.

On the other hand, at ε=0:

$$n_{cr}(h,\beta,0,\delta)=\tilde{n}_2\big|_{A=1}=1+\frac{1}{4\beta h}.$$

Hence, it follows that critical speed is appropriate to be searched for in the form:

$$n_{cr}(h,\beta,\varepsilon,\delta)= =1+\frac{1+f_1(\varepsilon)\cdot g_1(h,\beta,\delta)}{4[\beta+f_2(\varepsilon)\cdot q_2(h,\beta,\delta)][h+f_3(\varepsilon)\cdot g_3(h,\beta,\delta)]}, \quad (18)$$

where $f_i$, $g_i$, $/i=1,2,3/$ are the analytical functions; in this case, $f_i(\varepsilon)\xrightarrow[\varepsilon\to 0]{}0$.

We tested different powers from ε as functions $f_i$. We tested as functions $g_i$: linear and quadratic forms; fractions of linear forms; exponents of linear and quadratic forms. The best result was demonstrated by the function of form (18), in which:

$$f_i(\varepsilon)=\varepsilon,\quad /i=1,2,3/,$$

$$g_1(h,\beta,\delta)=\frac{1+a_1\delta+a_2h+a_3\beta}{b_0+b_1\delta+b_2h+b_3\beta},$$

$$g_2(h,\beta,\delta)=\frac{1+c_1\delta+c_2h+c_3\beta}{d_0+d_1\delta+d_2h+d_3\beta},$$

$$g_3(h,\beta,\delta)=\frac{1+e_1\delta+e_2h+e_3\beta}{p_0+p_1\delta+p_2h+p_3\beta}. \quad (19)$$

The following parameter values were derived by the least-squares method:

$$a_1=-26.5235,\quad a_2=-16.7424,\quad a_3=-30.1288,$$

$$b_0=-0.2537,\quad b_1=-0.8080,$$

$$b_2=-3.9439,\quad b_3=0.6984,$$

$$c_1=1.7134,\quad c_2=8.2055,\quad c_3=-28.7751,$$

$$d_0=-0.3455,\quad d_1=-0.5673,$$

$$d_2=4.3034,\quad d_3=8.2679,$$

$$e_1=-9.2714,\quad e_2=229.7664,\quad e_3=43.5133,$$

$$f_0=-0.0992,\quad f_1=2.4911,$$

$$f_2=35.3149,\quad f_3=-0.1019. \quad (20)$$

Coefficients (20) were determined based on the results of all 196 conducted computational experiments.

Function (20) produces an error not exceeding 6 %. The function makes it possible to calculate approximately the critical speed, assuming that dimensionless parameters of the system lie in the considered region. The function was derived formally; its form cannot be used for the interpretation of physical processes.

The accuracy of calculation of critical speed can be improved by the introduction of new components and coefficients to function (18). In this case, the application of the function in practice will be complicated.

We shall find, at ε~0, critical rotor speed in a dimensional form. At ε~0: $n_{cr}\approx\tilde{n}_2$. On the other hand, $n_{cr}=\omega_{cr}/\tilde{\omega}$ and, therefore, at ε~0:

$$\frac{\omega_{cr}}{\tilde{\omega}}\approx\tilde{n}_2=1+\frac{1}{4\beta h}=1+\frac{1}{4\cdot\dfrac{b_W M_\Sigma}{Nm^2\tilde{\omega}}\cdot\dfrac{b}{2M_\Sigma\tilde{\omega}}}=1+\frac{Nm^2\tilde{\omega}^2}{2b_W b}.$$

Hence, we find a dimensional critical speed at ε~0:

$$\omega_{cr}\approx\tilde{\omega}\left(1+\frac{Nm^2\tilde{\omega}^2}{2b_W b}\right). \quad (21)$$

It follows from (21) that in order to increase the region of stability for a dual-frequency mode of motion ($\omega_{cr}$), it is required to:

– increase the mass of load (*m*) or the total mass of loads (*Nm*);

– reduce the force of viscous resistance that prevents the motion of load ($b_w$);

– reduce the force of viscous resistance that prevents the motion of platform ($b$);

– increase the frequency of natural oscillations of the platform ($\tilde{\omega}$).

## 6. Discussion of results of studying the stability of dual-frequency modes of motion of the single-mass vibratory machine

The examined dual-frequency vibratory machines will be the more energy-efficient the smaller the forces of viscous resistance (in supports, and forces that prevent the motion of balls relative to the casing of the auto-balancer). In addition, in actual machines, the ratio of the mass of loads to the mass of the entire machine is small. These conditions make it possible to estimate the region of change in the dimensionless parameters in actual energy efficient machines.

In accordance with the computational experiments, when accelerating the rotor, a dual-frequency motion mode sets in eventually at which balls get stuck at a pre-resonant frequency $\Omega_1$. This mode is stable at speeds lower than a certain critical speed $n_{cr}$, lesser than $\tilde{n}_2$.

Despite the strong asymmetry of supports, an auto-balancer excites almost perfect dual-frequency vibrations. Deviations of the precise solution (derived by integration) from the approximated solution (previously found by the method of the small parameter) are equivalent to the ratio of the mass of balls to the mass of the entire machine. Thus, for actual machines, it does not exceed 2 %. Hence, it follows that when calculating the considered vibratory machines, the law of dual-frequency vibrations of the platform, established previously in [9], is applicable, as well as the corresponding frequency at which loads get stuck $\Omega_1$.

Critical speed $n_{cr}$ is a function of all dimensionless parameters of the system $n_{cr}=n_{cr}(h,\beta,\varepsilon,\delta)$. In the examined region of change in the dimensionless parameters, critical speed decreases monotonically for parameters $\varepsilon$, $\beta$, $h$, and monotonically increased for parameter $\delta$. At a decrease in $\varepsilon$ (a ratio of the mass of balls to the mass of the entire system), $n_{cr}$ strives to $\tilde{n}_2$. However, characteristic speed $\tilde{n}_2$ cannot be used for the approximate calculation of critical speed $n_{cr}$ due to rapidly growing error arising at an increase in $\varepsilon$.

To increase the region of stability of the dual-frequency motion mode (dimensional critical speed $\omega_{cr}$), it is required to:

– increase the mass of load ($m$) or the total mass of loads ($Nm$);

– reduce the force of viscous resistance that prevents the motion of load ($b_w$);

– reduce the force of viscous resistance that prevents the motion of platform ($b$);

– increase the frequency of natural oscillations of the platform ($\tilde{\omega}$).

The search for a function for the approximate calculation of critical speed, suitable over the entire range of change in dimensionless parameters, using the methods of the small parameter, is a complicated mathematical problem. More effective is the method of regression analysis. The method makes it possible to find the appropriate function. However, this function is not an approximation for a small parameter. Therefore, it cannot be used to interpret physical processes occurring in the system.

In the future, we plan to design, fabricate, and test a vibratory table with translational rectilinear motion of the platform and a vibration exciter in the form of a ball auto-balancer.

## 7. Conclusions

1. In actual energy-efficient vibratory machines, the forces of external and internal resistance should be small, with the mass of loads much less than the mass of the platform. Under these conditions, dimensionless parameters that determine the dynamics of a vibratory machine are in the intervals: $\varepsilon=0.005\div0.05$; $\beta=0.1\div0.6$; $\delta=0\div1$; $h=0.01\div0.07$. In this case, a vibratory machine has three characteristic rotor speeds. These speeds exceed the resonance frequency of oscillations of the platform. In this case, at the rotor speeds:

– lower than the first characteristic speed, there is only one possible frequency at which loads get stuck, it is less than the resonance frequency of oscillations of the platform;

– positioned between the first and second characteristic speeds, there are three possible frequencies at which loads get stuck, among which only one is a pre-resonant frequency;

– positioned between the second and third characteristic speeds, there are three possible frequencies at which loads get stuck; all of them are over-resonant frequencies;

– exceeding the third characteristic speed, there is only one possible frequency at which loads get stuck; it is an over-resonant frequency and it is close to the rotor speed.

2. Under a stable dual-frequency motion mode, the loads: create the greatest imbalance, rotate synchronously as a whole, get stuck at the lowest possible frequency of getting stuck; only if it is a pre-resonant frequency.

There is a critical speed above which a dual-frequency motion mode loses stability. This speed is less than the second characteristic speed.

3. Despite the strong asymmetry in supports, an auto-balancer excites almost perfect dual-frequency vibrations. Deviations of the precise solution (derived by integration) from the approximates solution (established previously by the method of the small parameter) are equivalent to the ratio of the mass of loads to the mass of the entire machine. That is why, for actual machines, deviations do not exceed 2 %.

4. The critical speed significantly depends on all dimensionless parameters of the system. In the examined region of change in the dimensionless parameters, critical speed changes monotonically for each parameter.

At a decrease in the ratio of the mass of balls to the mass of the entire system ($\varepsilon$), critical speed tends to the second characteristic speed.

However, the second characteristic speed cannot be used to calculate the critical speed. This is because even at a slight increase in $\varepsilon$ the critical speed rapidly deviates from the characteristic speed.

To extend the region of stability of a dual-frequency motion mode, it is required to:

– increase the mass of load or the total mass of loads;

– reduce the force of viscous resistance that prevents the motion of load;

– reduce the force of viscous resistance that prevents the motion of platform;

– increase the frequency of natural oscillations of the platform.

5. The second characteristic speed is to be effectively used to search for the form of a function that brings closer the critical speed. In this function, coefficients are determined based on the results of a computational experiment applying a method of least squares. In the examined region of change in the parameters the derived function produces an error not exceeding 6 %.

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