UDC 656.615 : 658.589
DOI: 10.15587/1729-4061.2018.133690

ELABORATION OF THE EQUIPMENT REPLACEMENT TERMS TAKING INTO ACCOUNT WEAR AND TEAR AND OBSOLESCENCE

I. Lapkina
Doctor of Economic Sciences, Professor,
Head of Department*
E-mail: lapkina@ukr.net
M. Malaksiano
PhD, Associate Professor*
E-mail: malax@ukr.net
*Department of management of logistics
systems and projects
Odessa National Maritime University
Mechnikova str., 34, Odessa, Ukraine, 65029

Розв’язано задачу обґрунтування термінів заміни обладнання, що схильне до фізичного і морального зносу, на більш досконале обладнання з аналогічним рівнем продуктивності. Для цього запропоновано методику багатокритеріальної оптимізації значень показників при переході на обладнання нового типу. Досліджено значення EAC (Equivalent Annual Cost), які розраховано як для декількох циклів замін обладнання, так і для нескінченної кількості циклів.

Отримано оцінки ступеня розсіювання можливих значень EAC в залежності від термінів служби обладнання старого і нового типу в умовах, коли динаміка операційних витрат схильна до випадкових коливань. Для цього використовувалися коваріаційні функції випадкових процесів, що описують динаміку операційних витрат на обладнання старого і нового типу. На підставі коваріаційних функцій були отримані оцінки функцій середньоквадратичних відхилень випадкових значень EAC.

З використанням отриманої оцінки ступеня розсіювання можливих значень EAC була досліджена задача багатокритеріальної оптимізації. Такий підхід має велике практичне значення, тому що для багатьох підприємств є важливим не тільки середній очікуваний рівень показників функціонування обладнання, але й ступень розсіювання можливих значень цих показників. В результаті дослідження розроблено методику планування термінів заміни обладнання. Запропонована методика дозволяє обґрунтовувати строки замін обладнання старого типу на нове обладнання, беручи до уваги як середні очікувані показники EAC, так і їх рівень коливань. Дослідження показали, що за рахунок вибору часу оновлення обладнання можна помітно зменшити ступень розсіювання можливих значень показників експлуатації обладнання, при цьому лише незначно поступившись його середнім очікуваним значенням

Ключові слова: заміна обладнання, оптимізація значень показників використання обладнання, equivalent annual cost, багатокритеріальне оцінювання

1. Introduction

Efficient use of equipment is a key factor of successful operation of many enterprises. Therefore, the issues of justifying the optimum terms of equipment renewal are of great practical interest. The advisability of equipment replacement arises for several reasons. First, this is due to wear and tear. Wear and tear can be caused by abrasion of parts, fatigue of materials, oxidation and other reasons. The increase in wear and tear is manifested in the increase in the number of breakdowns and growth of operating costs of equipment. Secondly, the advisability of equipment replacement is associated with obsolescence, caused by the emergence of more efficient or cheaper analogs of equipment in the market. In many cases, the advisability of equipment renewal is determined not by one type of wear, but by the combined effect of wear and tear and obsolescence on equipment.

Planning of terms of replacement of aging equipment with new equipment often should take into account the impact of various random factors. Equipment performance depends on external factors such as equipment utilization rate, the nature of the work performed, weather conditions, etc., which are subject to accidental changes. Even for similar machines used under identical conditions, the dynamics of wear and tear can be significantly different. And this should be taken into account when planning the terms of equipment renewal.

Currently, many enterprises in Ukraine, including seaports, need upgrading of equipment. This causes a significant practical interest in the development of scientifically based methods for determining the optimum terms of switching to a new type of equipment. The relevance of work in this direction is also explained by the need to develop control methods aimed at reducing the degree of dispersion of possible values of equipment performance indicators.

2. Literature review and problem statement

In [1] and [2], the optimum terms of equipment replacement under the conditions when operating costs of equipment are subject to random fluctuations have been investigated. Also in [1], the issues of the influence of tax rates on the determination of optimum terms of equipment utilization have been studied. In [2], the influence of random fluctuations in the equipment utilization rate on the optimum service life has been examined. However, in [1, 2] the possibility of capital repairs of equipment and the obsolescence factor have not been taken into account.

In [3, 4], the issues of determining the terms of replacement of equipment parts have been largely solved. In [5] and [6], the issues of determining the optimum service life of equipment, taking into account the possibility of restoration have been investigated. Thus, in [5], the possibility of a single equipment repair with a subsequent replacement for a coal-fired power plant has been considered. In [6], handling equipment has been discussed and the possibility of multiple major repairs has been allowed. Both in [5] and [6], the random factor when changing the operating costs of equipment has been taken into account. However, these works have not addressed the issues related to justifying the service life of aging equipment in connection with the emergence of the corresponding new types.

In [7], the issues of the influence of the number of technological operations on reliability and wear and tear of equipment have been considered. In [8], the problem of finding the optimum terms of equipment replacement taking into account taxation and the depreciation policy has been investigated. For this, the mathematical model based on a partial differential equation has been proposed. For the modeling of random fluctuations, the Brownian process has been used. Failures of complex multi-component technical systems have been studied in [9]. In this work, the influence of the technical system configuration, quality of elements and interaction between them on failures of the entire system and changes in associated costs has been analyzed. Models of cost estimation for systems with various configurations of elements have been presented and a sensitivity analysis has been carried out. At the same time, the issues related to obsolescence of equipment remained beyond the scope of these studies.

In [10] and [11], the problems of stable functioning of transport systems under conditions of uneven cargo traffic and the rationale for the choice of the optimum structure of the equipment fleet have been investigated. However, in [10, 11], attention hasn't been paid to the justification of equipment replacement strategies, taking into account technological progress.

The work [12] is devoted to planning the terms of introduction of new technologies. The models and conclusions proposed in it are based on certain assumptions regarding the rates of technological progress and emergence of new technologies.

In [13], the issues of early and delayed replacement of equipment in the deterministic model with the account for technological progress have been studied. It has been assumed that the level of income and expenses associated with equipment varies over time. The optimum time of deterministic replacement of equipment has been determined using the dynamic programming model.

In [14], the issues of determining the optimum time of the introduction of advanced equipment in view of uncertainty of the time of emergence of new technologies and their efficiency have been also studied. Three strategies of introduction of technological innovations have been considered. The substantiation of the choice of a strategy at various levels of uncertainty, efficiency, and also the pace of technological innovation has been carried out.

In order to justify the time of equipment replacement taking into account available spare parts stocks and considering the technical progress, it has been proposed in [15] to use the model of the Markov decision process. It has been assumed that due to technical progress in the market, new types of equipment may appear over time. But when replacing old equipment with new equipment, spare parts for old equipment become useless.

However, in [12–15], the influence of wear and tear of equipment on the determination of optimum replacement terms has not been investigated.

In a number of works, including [1–11], when determining the optimum terms of equipment replacement, only the factor of wear and tear or only the factor of emergence of new technologies has been taken into account. Also, many authors emphasize the need to study the influence of the random factor in justifying the equipment replacement terms. In [1–15], only some of the above factors have been in a varying degree considered. At the same time, in a number of cases, it is necessary to fully take into account the aggregate of all these factors when making decisions on equipment replacement. Therefore, the issues related to justifying the equipment replacement terms on the basis of a comprehensive assessment of both wear and tear and obsolescence, as well as accounting for the random factor, are of great importance.

3. The aim and objectives of the study

The aim of the study is to develop methods of planning the optimum terms of replacement of wearing equipment with more advanced equipment of new type, taking into account the degree of dispersion of possible values of equipment performance indicators.

To achieve the aim, the following objectives are accomplished:

– to justify the choice of equipment performance assessment criteria when switching to a new type of equipment;

– to carry out a quantitative estimation of the degree of uncertainty of performance indicators, depending on the choice of the service life of old and new equipment;

– to determine a multi-criteria estimate of average values and degree of dispersion of possible values of equipment performance indicators, depending on the service life of old and new equipment.

4. Materials and methods of the study of equipment performance indicators

4. 1. Justification of optimum terms of equipment replacement

The situation when the operating mode and utilization rate of equipment remain constant throughout the life, and all technical and operational characteristics are governed by the relevant regulations is considered. If technical and operational characteristics cease to meet specified standards of performance, reliability or safety over time, then the use of such equipment is considered unacceptable, and it must be repaired or replaced. For example, this applies fully to seagoing vessels: their usage modes are constant, and the technical condition is strictly controlled by the maritime register. It is economically irrational to use a seagoing vessel not with full intensity because it has rather high fixed costs, which can only be covered by intensive use. The situation is similar with railway rolling stock. The same situation is with complex handling equipment of sea ports. So, reducing the operating time of an aging harbor crane or reducing the speed of the harbor crane entails the irrational use of the berth, warehouse and dockers, increases the costly unproductive time of vessel berthing in the port. For such equipment, operating costs associated with the growth in the number of maintenance and repair works usually increase over time, while performance and reliability must remain at a certain level.

The main criterion in making decisions on replacement of such equipment, as a rule, is the change in the economic performance of its operation. It is often difficult to estimate the profit share of the entire enterprise, which falls on a particular machine, if this machine is only a part of a complex production chain. Therefore, it is natural to consider the unit cost of equipment operating time (or unit cost) or indicators that are derivatives of the unit cost as an indicator of equipment performance under such conditions. The *EAC* indicator reflects the redistributed share of total costs of equipment per unit operating time. At the same time, *EAC* considers cash flow discounting, which is a mandatory requirement in case of complex equipment having a long service life [16]. The use of the *EAC* indicator does not require reduction to a single planning horizon when comparing the results of utilization of equipment with different service lives. With such problem statement, *NPV*, *IRR* and many other widely used indicators for equipment performance assessment are not suitable. Therefore, the *EAC* indicator was chosen in the paper as the main criterion for estimating the efficiency of complex port equipment.

In some cases, it may take a long time from the moment of making a decision to purchase complex equipment prior to commissioning. This is due to the fact that, as a rule, complex equipment, including complex seaport handling equipment, is made to order. The manufacture, transportation and installation of equipment, as well as previous stages associated with the preparation and approval of feasibility studies to obtain permits and attract investments for purchase, often take years. Therefore, the issues related to replacement of such equipment cannot be solved promptly, based only on the current condition of equipment or short-term forecasts. In many cases, replacement of such equipment should be planned long before its technical condition reaches an unsatisfactory level.

Thus, the delay between the control action (in this case, the decision to replace equipment) and the response of the control object (commissioning of new equipment) is too great. At the same time, the values of equipment performance indicators are subject to random fluctuations. Moreover, the values of performance indicators of equipment at the time of replacement often correlate little with the values of performance indicators at the time of making a replacement decision (for example, graphs of covariance functions of operating costs for container cranes are shown in Fig. 6, 7). In [17], the developed apparatus for the synthesis and stability analysis of adaptive control systems is presented. However, due to the above-mentioned features, it cannot be successfully used in some cases to solve the problem of finding the optimum terms for replacing complex equipment. In this paper, we propose a different approach to the study of this problem.

In most cases, equipment repair and maintenance payments are discrete and aggregated monthly, quarterly or annually. Therefore, when evaluating the equipment effectiveness post factum, discrete models are often used. However, continuous regression curves of possible changes in indicators and continuous optimization methods are more convenient for forecasting. Therefore, attention should be given to the development of an approach based on the use of continuous models.

In [18], the definition of efficiency is given and its main function as an optimization criterion is defined. However, as noted in [18], the constraint on the use of such indicator in optimization problems is the imperfection of architectural solutions of automatic productions.

The rationale for the choice of the *EAC* indicator as an optimization criterion is presented in [16, 19].

Consider the situation where old equipment is replaced with more advanced equipment of comparable performance. We introduce the following notation:

A_o – the cost of purchase and installation of old equipment, USD;

A_n – the cost of purchase and installation of new equipment, USD;

$c_o(t)$ – the average rate of operating costs of old equipment after operation for t years, USD/year;

$c_n(t)$ – the average rate of operating costs of new equipment after operation for t years, USD/year;

$S_o(t)$ – the cost of dismantling and sale of old equipment after operation for t years, USD;

$S_n(t)$ – the cost of dismantling and sale of new equipment after operation for t years, USD;

T_o – the time during which it is planned to use old equipment, years;

T_n – the time during which it is planned to use new equipment, years.

Since the service life of complex equipment is typically several years, discounting should be used to assess the utilization efficiency throughout the life cycle. Let r be the annual continuously compounded interest rate. Using the known continuous compounding formula (for example, [19]), the current operating costs of old equipment when used for T_o years can be found:

$$PV(C_{o,oper}(T_o)) = \int_0^{T_o} c_o(\tau)\cdot e^{-r\cdot\tau}\mathrm{d}\tau. \tag{1}$$

The current capital costs of old equipment, when used for T_o years, are found by the formula

$$PV(C_{o,cap}(T_o)) = A_o + S_o(T_o)\cdot e^{-r\cdot T_o}. \tag{2}$$

Thus, the current total costs of old equipment when used for T_o are

$$PV\left(C_{o,total}(T_o)\right) = PV\left(C_{o,cap}(T_o)\right) + PV\left(C_{o,oper}(T_o)\right) =$$
$$= A_o + S_o(T_o)\cdot e^{-r\cdot T_o} + \int_0^{T_o} c_o(\tau)\cdot e^{-r\cdot\tau}\mathrm{d}\tau. \tag{3}$$

To substantiate the optimum service life of equipment, it is possible to use the current total costs of equipment in cyclic replacements with the same type of equipment during an infinite planning horizon $PV\left(C^{\infty}_{o,total}(T_o)\right)$. Summing up the terms of the geometric progression, it is easy to show that

$$PV\left(C^{\infty}_{o,total}(T_o)\right) = PV\left(C_{o,total}(T_o)\right)\cdot\frac{1}{1-e^{-r\cdot T_o}}. \tag{4}$$

To compare the equipment performance on time intervals of different lengths, the *EAC* (Equivalent Annual Cost) indicator is often used (for example, [16]). *EAC* equals the current value of equipment costs multiplied by the $CRF(T_o, r)$ value (Capital Recovery Factor), where

$$CRF(T_o, r) = \frac{e^r - 1}{1-e^{-r\cdot T_o}}. \tag{5}$$

In this case, the *EAC* indicator, calculated on the basis of one complete utilization cycle of old equipment, is given by the formula

$$EAC_o(T_o)=PV\left(C_{o,total}(T_o)\right)\cdot\frac{e^r-1}{1-e^{-r\cdot T_o}}. \qquad (6)$$

The equipment operation term T_o^* in which the expression (4) or (6) will take a minimum value can be considered optimum. Although the formulas (4) and (6) reflect different performance indicators of equipment, it is obvious that they reach minimum values at the same value of $T_o=T_o^*$.

Similarly, it can be found that the optimum service life T_n^* of new equipment, provided it is replaced by equipment of the same type, is the minimum point of the expression

$$EAC_n(T_n)=PV\left(C_{n,total}(T_n)\right)\cdot\frac{e^r-1}{1-e^{-r\cdot T_n}}, \qquad (7)$$

where

$$PV\left(C_{n,total}(T_n)\right)=A_n+S_n(T_n)\cdot e^{-r\cdot T_n}+\int_0^{T_n}c_n(\tau)\cdot e^{-r\cdot\tau}\mathrm{d}\tau. \qquad (8)$$

If at the same level of performance there is the inequality $EAC_n(T_n^*)<EAC_o(T_o^*)$, this gives grounds to consider that new equipment, when used in the given conditions, is economically more expedient than old equipment.

Let us investigate the issue of finding the optimum terms of equipment replacement when switching from old equipment to new equipment, which is comparable in performance to old, but is more economically feasible. To this end, consider two reasoning patterns: the first pattern, based on the analysis of a finite time interval consisting of two complete cycles of equipment replacement, and the second – based on an infinite time interval.

We consider the first pattern, based on the analysis of the time interval, consisting of two complete utilization cycles of equipment. Within this pattern, it is planned to use old equipment during the first cycle of T_o years. Then it is planned to use new equipment during the second cycle of T_n years. It is easy to see that the current total costs during these two cycles can be calculated as

$$\begin{aligned}&EAC_{on}(T_o,T_n)=\\&=\left[PV\left(C_{o,total}(T_o)\right)+PV\left(C_{n,total}(T_n)\right)\cdot e^{-r\cdot T_o}\right]\cdot\frac{e^r-1}{1-e^{-r\cdot(T_o+T_n)}}.\end{aligned} \qquad (9)$$

The *EAC* value for two complete utilization cycles of old and new equipment is a function of the two variables T_o and T_n. The values of $T_o=T_o^{**}$ and $T_n=T_n^{**}$, at which the expression (9) reaches the minimum can be considered optimum service lives of equipment when switching from old equipment to new equipment.

Let us find the *EAC* value for the second reasoning pattern. Within this pattern, we will analyze the infinite planning horizon. And, during the first full cycle of work, it is planned to use old equipment. Then, during all subsequent cycles of the same duration T_n, new equipment will be used. In this case, obviously,

$$\begin{aligned}&EAC_{on}^{\infty}(T_o,T_n)=\\&=\left[PV\left(C_{o,total}(T_o)\right)+e^{-r\cdot T_o}\cdot\sum_{k=0}^{\infty}\left(e^{-r\cdot T_n\cdot k}\cdot PV\left(C_{n,total}(T_n)\right)\right)\right]\cdot\left(e^r-1\right)=\\&=\left[PV\left(C_{o,total}(T_o)\right)+PV\left(C_{n,total}(T_n)\right)\cdot\frac{e^{-r\cdot T_o}}{1-e^{-r\cdot T_n}}\right]\cdot\left(e^r-1\right).\end{aligned} \qquad (10)$$

We determine the values of $T_o=T_o^{***}$ and $T_n=T_n^{***}$, at which the expression (10) reaches the minimum. It is easy to check that $T_n^{***}=T_n^*$.

4. 2. Assessment of the influence of equipment replacement terms on possible fluctuations in EAC values

Due to the influence of various random factors, as equipment ages, significant fluctuations in the operating costs of equipment are possible. Therefore, it makes sense to describe the dynamics of changes in the rates of the operating costs of equipment using random processes $c_o(t,\omega)$ and $c_n(t,\omega)$, where $\omega\in\Omega$, (Ω, A, P) is the probability space. Moreover, the mathematical expectations of these random processes are $c_o(t)=E(c_o(t,\omega))$ and $c_n(t)=E(c_n(t,\omega))$, respectively. In this case, the current operating costs of equipment will be described by the corresponding random processes

$$PV(C_{o,oper}(T_o,\omega))=\int_0^{T_o}c_o(\tau,\omega)\cdot e^{-r\cdot\tau}\mathrm{d}\tau, \qquad (11)$$

$$PV(C_{n,oper}(T_n,\omega))=\int_0^{T_n}c_n(\tau,\omega)\cdot e^{-r\cdot\tau}\mathrm{d}\tau. \qquad (12)$$

Thus, the *EAC* value will also be a random variable.

Let us study the influence of the choice of the service life of equipment on the dispersion of values of the random variable *EAC* for the chain of successively replaced machines of old and new type, namely – on the value of the standard deviation $\sigma(EAC_{on}(T_o, T_n, \omega))$. To this end, consider the covariance functions of the random processes $c_o(t,\omega)$ and $c_n(t,\omega)$, respectively:

$$\begin{aligned}&K_o(t_1,t_2)=\\&=E\left((c_o(t_1,\omega)-c_o(t_1))(c_o(t_2,\omega)-c_o(t_2))\right),\end{aligned} \qquad (13)$$

$$\begin{aligned}&K_n(t_1,t_2)=\\&=E\left((c_n(t_1,\omega)-c_n(t_1))(c_n(t_2,\omega)-c_n(t_2))\right).\end{aligned} \qquad (14)$$

It is natural to assume that the dynamics of the operating costs of each subsequent machine does not depend on the costs of previous machines. Therefore, using (9) and the properties of covariance functions of random processes (for example, [20]), we obtain

$$\begin{aligned}&\sigma^2\left(EAC_{on}(T_o,T_n,\omega)\right)=\\&=\left(\frac{e^r-1}{1-e^{-r\cdot(T_o+T_n)}}\right)^2\times\\&\times\left[\sigma^2\left(PV\left(C_{o,total}(T_o,\omega)\right)\right)+e^{-2\cdot r\cdot T_o}\cdot\sigma^2\left(PV\left(C_{n,total}(T_n,\omega)\right)\right)\right]=\\&=\left(\frac{e^r-1}{1-e^{-r\cdot(T_o+T_n)}}\right)^2\times\\&\times\left[\int_0^{T_o}\int_0^{T_o}K_o(t_1,t_2)\cdot e^{-r\cdot(t_1+t_2)}\mathrm{d}t_1\mathrm{d}t_2+\right.\\&\left.+e^{-2\cdot r\cdot T_o}\cdot\int_0^{T_n}\int_0^{T_n}K_n(t_1,t_2)\cdot e^{-r\cdot(t_1+t_2)}\mathrm{d}t_1\mathrm{d}t_2\right].\end{aligned} \qquad (15)$$

Similarly, from (10) for the second reasoning pattern, we obtain

$$\sigma^2\left(EAC_{on}^{\infty}(T_o,T_n,\omega)\right)=$$

$$=\left(e^r-1\right)^2\cdot\left[\begin{array}{l}\sigma^2\left(PV\left(C_{o,total}(T_o,\omega)\right)\right)+e^{-2\cdot r\cdot T_o}\times\\ \times\sigma^2\left(\sum_{k=0}^{\infty}\left(e^{-r\cdot T_n\cdot k}\cdot PV\left(C_{n,total}(T_n,\omega)\right)\right)\right)\end{array}\right]=$$

$$=\left(e^r-1\right)^2\cdot\left[\begin{array}{l}\sigma^2\left(PV\left(C_{o,total}(T_o,\omega)\right)\right)+\\ +\dfrac{e^{-2\cdot r\cdot T_o}}{1-e^{-2\cdot r\cdot T_n}}\cdot\sigma^2\left(PV\left(C_{n,total}(T_n,\omega)\right)\right)\end{array}\right]=$$

$$=\left(e^r-1\right)^2\cdot\left[\int_0^{T_o}\int_0^{T_o}K_o(t_1,t_2)\cdot e^{-r\cdot(t_1+t_2)}\mathrm{d}t_1\mathrm{d}t_2+\right.$$

$$\left.+\frac{e^{-2\cdot r\cdot T_o}}{1-e^{-2\cdot r\cdot T_n}}\cdot\int_0^{T_n}\int_0^{T_n}K_n(t_1,t_2)\cdot e^{-r\cdot(t_1+t_2)}\mathrm{d}t_1\mathrm{d}t_2\right]. \qquad (16)$$

In practice, in addition to optimizing the values of equipment performance indicators, the degree of dispersion of values of these indicators is also important. Based on the above methods of estimating the average expected indicators and the level of *EAC* fluctuations, we can consider the two-criteria minimization problem

$$\min_{T_o,T_n}\left(E(EAC_{on}(T_o,T_n,\omega)),\sigma(EAC_{on}(T_o,T_n,\omega))\right). \qquad (17)$$

In (17), the search of equipment replacement terms T_o and T_n is carried out, at which the minimum values of $E(EAC_{on}(T_o, T_n, \omega))$ and $\sigma(EAC_{on}(T_o, T_n, \omega))$ are attained. It is similarly possible to investigate the problem

$$\min_{T_o,T_n}\left(E(EAC_{on}^{\infty}(T_o,T_n,\omega)),\sigma(EAC_{on}^{\infty}(T_o,T_n,\omega))\right) \qquad (18)$$

of finding the equipment replacement terms T_o and T_n, at which balanced minimum values of $E(EAC_{on}^{\infty}(T_o,T_n,\omega))$ and $\sigma(EAC_{on}^{\infty}(T_o,T_n,\omega))$. are achieved.

5. Results of the study of optimum terms of equipment replacement

We investigate optimum replacement terms for harbor container cranes. The costs of equipment of the old and new type are A_o=240 thousand USD and A_n=195 thousand USD, respectively. The average dynamics of operating costs for container cranes is described by the functions $c_o(t)=75.07+0.21\cdot t^{2.11}$, thousand USD/year and $c_n(t)=$ $=60.52+0.22\cdot t^{2.52}$, thousand USD/year. These functions were obtained on the basis of the regression analysis of changes in operating costs for a sample of 27 old-type container cranes. Fig. 1 shows the box plot, which presents the dynamics of changes in operating costs of old-type container cranes according to this sample. In Fig. 2, the solid line shows the curve of changes in average operating costs of old-type container cranes, and the dashed lines – the borders of the band, within which, with a 0.9 probability, the values of operating costs are found.

The way the form of the law of distribution of the actual values of the operating costs of equipment changes is seen from the histograms in Fig. 3, 4, reflecting distributions of values of operating costs at specified timepoints. In these figures, red lines represent the densities of normal distributions, the mathematical expectations and standard deviations of which correspond to sample indicators, as well as the results of the Kolmogorov-Smirnov and Shapiro-Wilk normality tests. As can be seen, at the beginning of equipment service life, the distribution of average costs per unit time of equipment operation almost does not differ from normal (Fig. 3, 4), but the deviation from the normal distribution law becomes noticeable over time (Fig. 4). A similar trend is observed for new equipment.

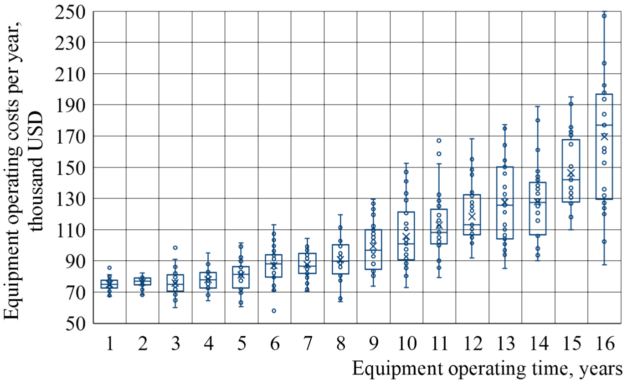

Fig. 1. Statistical data on changes in operating costs of old container cranes

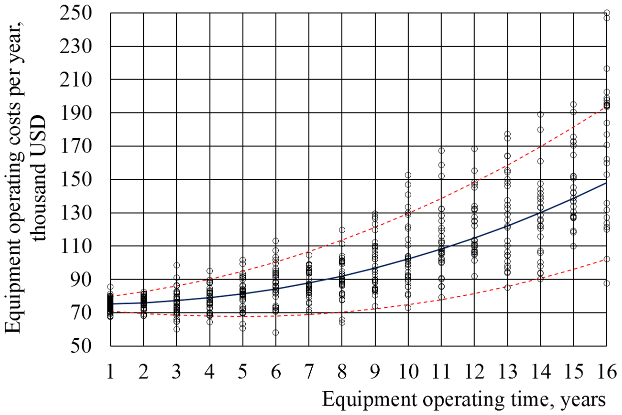

Fig. 2. Changes in operating costs for old container cranes

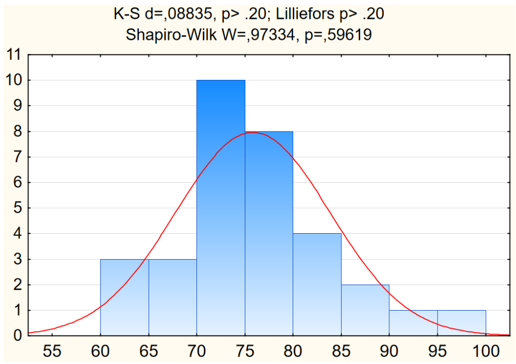

Fig. 3. Histogram of distribution of operating cost values for old container cranes three years after the start of operation

The cost of dismantling and sale of equipment depending on service life is described by the function $S_o(t)=-5.15-$ $-140.01\cdot(1.14\cdot t+1)^{-1.32}$ thousand USD for old equipment and $S_n(t)=-1.50-159.99\cdot(0.51\cdot t+1)^{-2.02}$ thousand USD – for new. These functions were also obtained on the basis of the statistical analysis of actual data. The annual interest rate given in shares is assumed to be r=0.1.

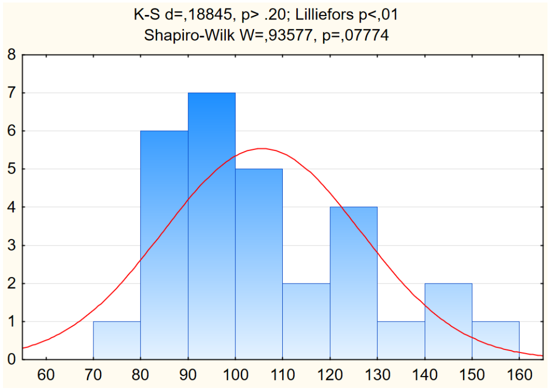

Fig. 4. Histogram of distribution of operating cost values for old container cranes ten years after the start of operation

All numerical computations by the formulas (1)–(18), necessary for plotting the graphs and finding the optimum values given in the paper, were implemented in the Maple mathematical computing environment. The graphs of changes in the values of $EAC_o(T)$ and $EAC_n(T)$, calculated by the formulas (1)–(7), are shown in Fig. 1. The expression (6) reaches a minimum with $T_o^* = 12.28$, and the expression (7) – with $T_n^* = 8.05$.

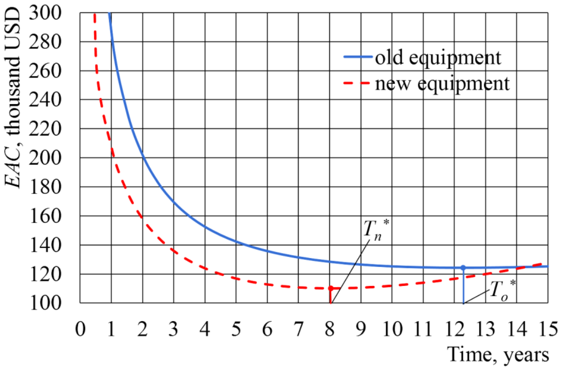

Fig. 5. Graph of *EAC* change for old and new equipment depending on service life

Fig. 6, 7 show the surfaces of changes in the *EAC* values calculated by the formulas (9), (10), with two replacement cycles and with an infinite number of equipment replacement cycles, depending on the time of utilization of old and new equipment.

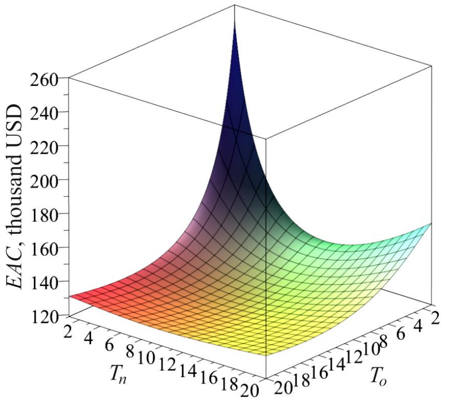

Fig. 6. Changes in the $EAC_{on}(T_o, T_n)$ values with two equipment replacement cycles

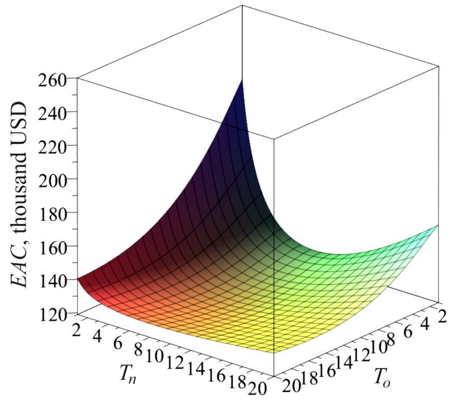

Fig. 7. Changes in the $EAC_{on}^{\infty}(T_o, T_n)$ values with an infinite number of equipment replacement cycles

The minimum value of $EAC_{on}(T_o, T_n)$ is reached with $(T_o^{**}, T_n^{**}) = (10.93,\ 8.83)$ and is 121.29 thousand USD. And the minimum value of $EAC_{on}^{\infty}(T_o, T_n)$ is reached with $(T_o^{***}, T_n^{***}) = (10.15,\ 8.05)$ and is 119.60 thousand USD.

Fig. 8 shows the graphs of changes in the *EAC* values for each individual piece of equipment when replacing old equipment with new equipment. Fig. 9 shows the graphs of *EAC* changes for each individual piece of equipment with one operation cycle of old equipment and an infinite number of subsequent operation cycles of new equipment. In Fig. 8 and Fig. 9, the blue solid line represents the curve of changes in the *EAC* values for old equipment, and the red dotted line shows the *EAC* curves for new equipment.

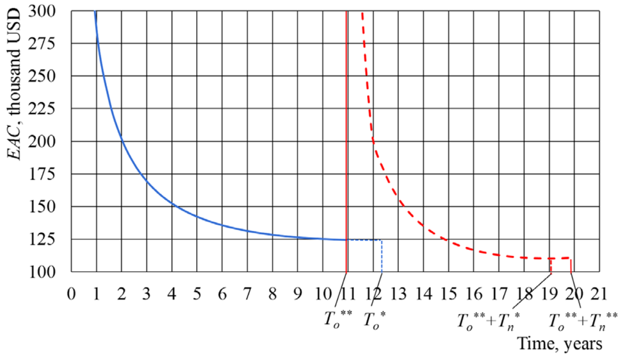

Fig. 8. Graph of *EAC* change with two equipment replacement cycles

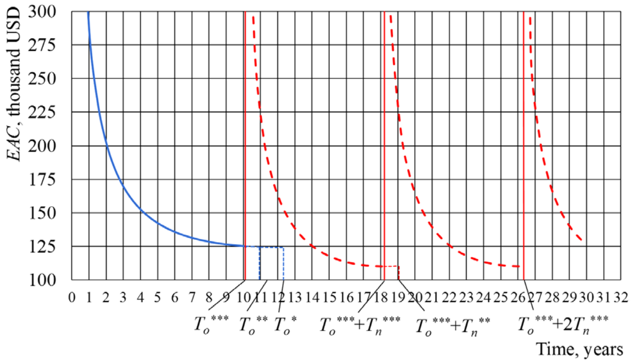

Fig. 9. Graph of *EAC* change with an infinite number of equipment replacement cycles

Calculations showed that the value of T_o^* significantly exceeds the value of T_o^{**}, and T_n^{**} exceeds T_n^*. It is obvious that the more efficient the new equipment is, the greater the difference between T_o^* and T_o^{**}.

We investigate the degree of dispersion of *EAC* values depending on the choice of equipment replacement terms with possible random changes in operating costs. Estimation of the operating cost function $c_o(t)$ and the covariance function $K_o(t_1, t_2)$ for old equipment is usually not very difficult, since in most cases there is enough statistical data for old equipment. But for new equipment, there is sometimes not enough information for reliable statistical estimation of equipment utilization under given specific conditions. In this case, the estimate of the dynamics of the average level of operating costs and the covariance function $K_n(t_1, t_2)$ can be obtained on the basis of technical documentation and expert assessments. Introduction and first experience of using innovative equipment are often associated with increased risks. And this risk should be taken into account when justifying the values of the covariance function $K_n(t_1, t_2)$.

Further, when describing the covariance functions for old and new equipment, we will use the following expression

$$K(t_1,t_2) = (a_1 \cdot (t_1+t_2)^{a_2} + a_3) \cdot \exp\left(\frac{-(t_1-t_2)^2}{a_4 \cdot (t_1+t_2)^{a_5} + a_6} \right), \quad (19)$$

where the set of constant coefficients $a=(a_1;...; a_6)$ is determined by regression analysis methods, separate for each type of equipment. Such choice of a type of covariance function makes it easy to interpret the effect of each of the constant coefficients on the properties of the studied random process of changes in the operating costs of equipment. Thus, the coefficients a_1, a_2 and a_3 determine the growth rate, the curve shape and the initial value of the dispersion function, respectively, for the random process under study. The coefficients a_4, a_5 and a_6 reflect the way in which the degree of interrelation between the values of the operating costs of equipment is changed at different timepoints. The use of covariance functions of the type (19) made it possible to describe quite accurately the process of changes in the operating costs for container cranes. However, this form of covariance functions is not universal. The form of the function $K(t_1, t_2)$ should be selected and justified individually for different types of equipment and different operating conditions.

For the considered old type container cranes, a set of values of constant coefficients a_o=(0.3501; 2.21; 35.21; 0.002; 2.50; 15.40) in the function (19) was determined based on the regression analysis. Accordingly, for container cranes of new type, the values of coefficients a_n=(0.0075; 3.50; 105.21; 0.015; 3.01; 5.61) were obtained.

Fig. 10, 11 present the graphs of the covariance functions $K_o(t_1, t_2)$ and $K_n(t_1, t_2)$ for container cranes of the old and new type determined by the function of the form (19) with the above values of the coefficients. From Fig. 10, 11, it can be seen that dispersion of the operating cost values for new equipment is much higher than for old equipment. Moreover, for new equipment, there is also a greater correlation between the values of operating costs at different timepoints than for old.

Verification of the indicators was carried out on the basis of actual operation data of harbor container cranes. The values of all input data for calculations by the formulas (1)–(18) given in the paper, including statistical estimates of the covariance functions $K_o(t_1, t_2)$ and $K_n(t_1, t_2)$ are obtained using the statistical analysis of actual data.

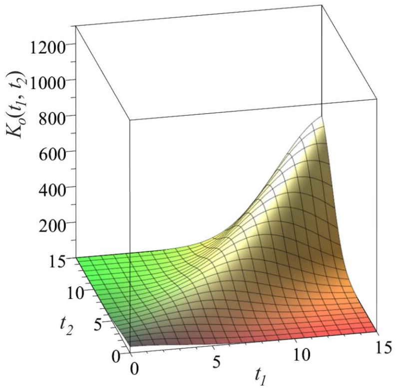

Fig. 10. Graph of the covariance function $K_o(t_1, t_2)$

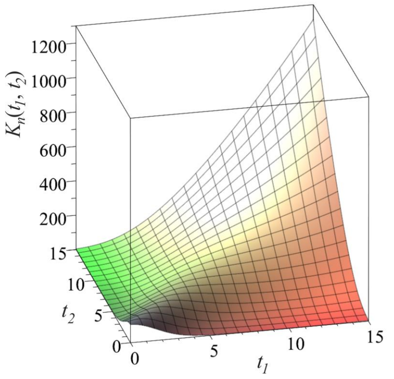

Fig. 11. Graph of the covariance function $K_n(t_1, t_2)$

Fig. 12 presents the graph of changes in the standard deviation of the *EAC* values calculated by the formula (15), depending on equipment replacement terms with two replacement cycles. A similar graph of changes in the standard deviation of the *EAC* values calculated by the formula (16) with an infinite number of replacement cycles is shown in Fig. 13.

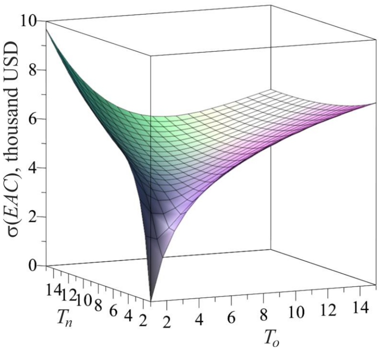

Fig. 12. Graph of the function $\sigma(EAC_{on}(T_o, T_n, \omega))$

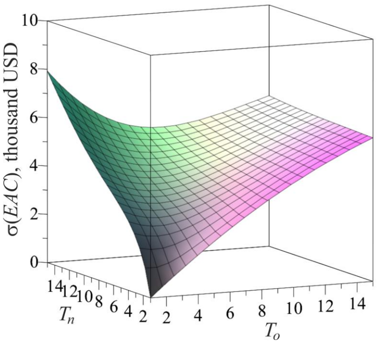

Fig. 13. Graph of the function $\sigma(EAC_{on}^{\infty}(T_o, T_n, \omega))$

Comparing Fig. 6, 12, and also Fig. 7, 13, we can see that the values of T_o and T_n, optimum from the point of view of the average *EAC*, will not correspond to the minimum values of standard deviations of *EAC*. Therefore, in order to justify such a choice of equipment replacement terms, at which the balance between the minimum and the dispersion degree of *EAC* values is achieved, it is necessary to investigate the multi-criteria optimization problems (17) and (18). Let us analyze the relationship between the values of $E(EAC_{on}(T_o, T_n, \omega))$ and $\sigma(EAC_{on}(T_o, T_n, \omega))$. Fig. 14 shows the set of points, the coordinates of which are respectively equal to $E(EAC_{on}(T_o, T_n, \omega))$ and $\sigma(EAC_{on}(T_o, T_n, \omega))$ and are determined by the choice of the values of T_o and T_n. The unimprovable points lying on the Pareto frontier of the multi-criteria optimization problem are given in red (17). The coordinates of some unimprovable solutions, as well as the corresponding values of equipment replacement terms T_o and T_n, are shown in Table 1.

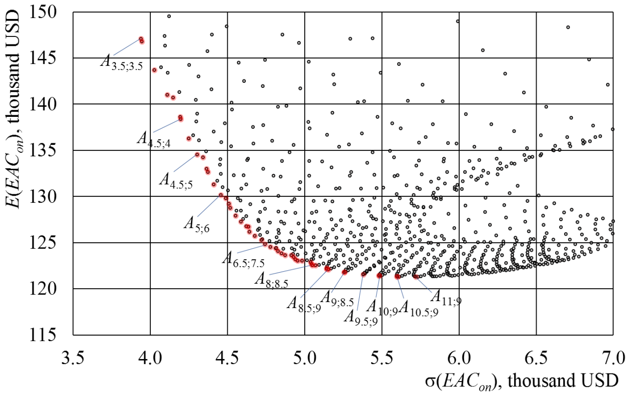

Fig. 14. Relationship between the values of $E(EAC_{on}(T_o, T_n, \omega))$ and $\sigma(EAC_{on}(T_o, T_n, \omega))$ for different values of T_o and T_n

Table 1

Some unimprovable solutions of the two-criteria optimization problem (17)

Point designation on the Pareto frontier, $A_{T_o;T_n}$	$E(EAC_{on}(T_o, T_n, \omega))$	$\sigma(EAC_{on}(T_o, T_n, \omega))$
$A_{3.5;3.5}$	147.10	3.94
$A_{4.5;4}$	138.37	4.20
$A_{4.5;5}$	134.52	4.31
$A_{5;6}$	130.16	4.46
$A_{6.5;7.5}$	124.84	4.74
$A_{8;8.5}$	122.56	5.05
$A_{8.5;9}$	122.12	5.16
$A_{9;8.5}$	121.82	5.25
$A_{9.5;9}$	121.57	5.38
$A_{10;9}$	121.40	5.48
$A_{10.5;9}$	121.31	5.60
$A_{11;9}$	121.29	5.72

The points lying on the Pareto frontier of the multi-criteria optimization problem (18) are given in red in Fig. 15. The coordinates of some of these points are shown in Table 2.

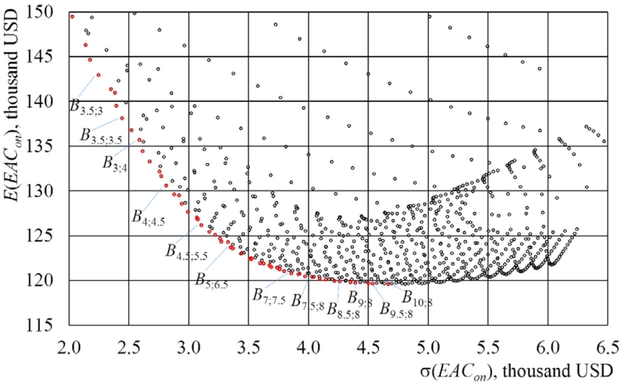

Fig. 15. Relationship between the values of $E(EAC_{on}^{\infty}(T_o, T_n, \omega))$ and $\sigma(EAC_{on}^{\infty}(T_o, T_n, \omega))$ for different values of T_o and T_n

Table 2

Some unimprovable solutions of the two-criteria optimization problem (18)

Point designation on the Pareto frontier, $B_{T_o;T_n}$	$E(EAC_{on}^{\infty}(T_o, T_n, \omega))$	$\sigma(EAC_{on}^{\infty}(T_o, T_n, \omega))$
$B_{3;4}$	135.69	2.59
$B_{3.5;3}$	142.98	2.25
$B_{3.5;3.5}$	138.14	2.44
$B_{4;4.5}$	130.62	2.81
$B_{4.5;5.5}$	126.19	3.11
$B_{5;6.5}$	123.61	3.37
$B_{7;7.5}$	120.82	3.86
$B_{7.5;8}$	120.40	4.03
$B_{8.5;8}$	119.90	4.26
$B_{9;8}$	119.74	4.39
$B_{9.5;8}$	119.64	4.54
$B_{10;8}$	119.60	4.67

It is obvious that the points that did not fall on the Pareto frontier are of no practical interest. To find the optimum service life of equipment, it is advisable to consider only unimprovable solutions of the problems (17), (18).

This paper does not assume the use of a single integrated criterion for solving the problems (17) and (18). On the contrary, the reduction of the problems (17) and (18) to the study of any integrated indicators or convolutions of criteria would make the analysis of the problem of choosing the optimum equipment replacement terms more formal and would cut off much of the information that could help the DM (decision-maker) to make a more reasonable choice. Not the packing of all indicators into a single integrated indicator, but the construction and analysis of the entire Pareto frontier, makes it possible to analyze the problem as deeply as possible (for example, [21]).

6. Discussion of the results of the study of optimum equipment replacement terms

The shape of the surfaces $\sigma(EAC_{on}(T_o, T_n, \omega))$ and $\sigma(EAC_{on}^{\infty}(T_o, T_n, \omega))$ is due to the influence of several factors. Comparing Fig. 12, 13, it can be seen that the values of standard deviations for the indicator $EAC_{on}(T_o, T_n, \omega)$ are greater than for $EAC_{on}^{\infty}(T_o, T_n, \omega)$. This difference is especially pronounced in cases when T_n significantly exceeds T_o. This is explained by the effect of overlapping and mutual compensation of random multidirectional fluctuations of independent random variables that make up an infinite sum in the evaluation of $\sigma(EAC_{on}^{\infty}(T_o, T_n, \omega))$. Thus, the values of the indicators of chains of successively replaced machines on the long planning horizon are more stable. It should also be borne in mind that the level of fluctuations in the operating costs of machines that are the first in the chain of successively replaced equipment have a greater contribution to the values of the indicators $\sigma(EAC_{on}(T_o, T_n, \omega))$ and $\sigma(EAC_{on}^{\infty}(T_o, T_n, \omega))$ due to discounting.

Comparing the Pareto frontiers presented in Fig. 14, 15 and Table 1, 2, it should be noted that the unimprovable solutions of the multicriteria optimization problem (17) correspond to longer terms of equipment utilization than those that correspond to unimprovable solutions of the problem (18).

The studies have shown that the minimum value of $E(EAC_{on}(T_o, T_n, \omega))$ equals 121.29 thousand USD and is achieved in the case when the service lives of old and new equipment are respectively 11 and 9 years. In this case, the value of $\sigma(EAC_{on}(T_o, T_n, \omega))$ is 5.72 thousand USD. In Fig. 14, this solution corresponds to the point $A_{11;9}$. Let us compare the points $A_{11;9}$. and $A_{9;8.5}$ in Fig. 14. As can be seen, by reducing the service lives of old and new equipment to 9 and 8.5 years, respectively, it is possible to significantly decrease the value of $\sigma(EAC_{on}(T_o, T_n, \omega))$, while slightly worsening $E(EAC_{on}(T_o, T_n, \omega))$. A similar effect can be observed in Fig. 15, analyzing the Pareto frontier of solutions of the problem (18).

Since $T_o^{***} < T_o^{**} < T_o^{*}$ and $T_n^{**} > T_n^{***} = T_n^{*}$, it is necessary to specify in what situations it is expedient to use each of these values. The values T_o^{*} and T_n^{*} determine the optimum terms of equipment replacement, due only to wear and tear. The optimum terms of equipment replacement, taking into account both wear and tear and obsolescence, are determined by the values T_o^{**}, T_n^{**}, T_o^{***} and T_n^{***}. In this case, if accounting of risks is not in question, the choice of the values T_o^{***} and T_n^{***} as optimum service lives of equipment can be considered the most reasonable, since the maximum planning horizon is taken into account. However, on the maximum planning horizon, estimates of equipment performance fluctuations may be blurred and significantly underestimated. Therefore, when studying the dispersion degree of the values of equipment performance indicators, it may be more appropriate in some cases to consider the planning horizon consisting of two equipment replacement cycles.

7. Conclusions

1. The paper proposes the methodology for multicriteria assessment of equipment performance when switching to a new type of equipment. For equipment performance assessment, taking into account both wear and tear and obsolescence at different time intervals, it is proposed to use the *EAC* indicator. The calculations used the *EAC* values calculated for several equipment replacement cycles (9) and for an infinite number of cycles (10). The studies have shown that when planning the equipment renewal terms, it is advisable to shorten the life of old equipment in comparison with those terms that would be optimum when replaced with old equipment.

2. The quantitative estimate of the dispersion degree of the values of the *EAC* indicator depending on the choice of the service lives of old and new equipment under conditions when the dynamics of operating costs is subject to random fluctuations is obtained. To do this, covariance functions for random processes that describe the dynamics of the operating costs of old and new equipment were used. Based on these covariance functions, estimates of functions of standard deviations of the *EAC* values were obtained.

3. Based on the obtained estimate of the dispersion degree of the *EAC* values, the technique for planning the equipment replacement terms was proposed. This technique allows justifying the terms of replacement of old equipment with new equipment, taking into account both the average expected *EAC* indicators and the level of possible *EAC* fluctuations. The studies have shown that due to the choice of equipment renewal terms (Fig. 14, 15), the degree of dispersion of the *EAC* values can be significantly reduced, slightly sacrificing the average expected value. Thus, the optimum (from the point of view of the minimum of average values and dispersion degree of *EAC*) replacement terms of equipment subject to wear and tear and obsolescence with new equipment were justified in the work.

References

1. Adkins R., Paxson D. Replacement decisions with multiple stochastic values and depreciation // European Journal of Operational Research. 2017. Vol. 257, Issue 1. P. 174–184. doi: 10.1016/j.ejor.2016.07.006
2. Lapkina I., Malaksiano M. Estimation of fluctuations in the performance indicators of equipment that operates under conditions of unstable loading // Eastern-European Journal of Enterprise Technologies. 2018. Vol. 1, Issue 3 (91). P. 22–29. doi: 10.15587/1729-4061.2018.123367
3. Nahorny V. V. Automated control cutting based prediction of individual life of the cutting tool // Radio Electronics, Computer Science, Control. 2017. Issue 3. P. 183–191. doi: 10.15588/1607-3274-2017-3-20
4. Zaloga V. A., Nagorniy V. M., Nagorniy V. V. Kontrol' dinamicheskogo sostoyaniya metalloobrabatyvayushchey tekhnologicheskoy sistemy i prognozirovanie ee resursa. Sumy: Sumskiy gosudarstvenniy universitet, 2016. 242 p.
5. Optimal replacement policies for an uncertain rejuvenated asset / Stutzman S., Weiland B., Preckel P., Wetzstein M. // International Journal of Production Economics. 2017. Vol. 185. P. 21–23. doi: 10.1016/j.ijpe.2016.12.018
6. Malaksiano N. A. On the stability of economic indicators of complex port equipment usage // Actual Problems of Economics. 2012. Issue 12. P. 226–233.
7. Mihaylov V. V. Nadezhnost' elektrosnabzheniya promyshlennyh predpriyatiy. Moscow: Energiya, 1973. 168 p.

8. Zambujal-Oliveira J., Duque J. Operational asset replacement strategy: A real options approach // European Journal of Operational Research. 2011. Vol. 210, Issue 2. P. 318–325. doi: 10.1016/j.ejor.2010.09.011
9. Liu B., Wu J., Xie M. Cost analysis for multi-component system with failure interaction under renewing free-replacement warranty // European Journal of Operational Research. 2015. Vol. 243, Issue 3. P. 874–882. doi: 10.1016/j.ejor.2015.01.030
10. Lapkina I. O., Malaksiano M. O., Malaksiano M. O. Optimization of the structure of sea port equipment fleet under unbalanced load // Actual Problems of Economics. 2016. Issue 9. P. 364–371.
11. Lapkina I. O., Malaksiano M. O. Modelling and optimization of perishable cargo delivery system through Odesa port // Actual Problems of Economics. 2016. Issue 3. P. 353–365.
12. Hagspiel V., Huisman K. J. M., Nunes C. Optimal technology adoption when the arrival rate of new technologies changes // European Journal of Operational Research. 2015. Vol. 243, Issue 3. P. 897–911. doi: 10.1016/j.ejor.2014.12.024
13. Adkins R., Paxson D. Deterministic models for premature and postponed replacement // Omega. 2013. Vol. 41, Issue 6. P. 1008–1019. doi: 10.1016/j.omega.2013.01.002
14. Chronopoulos M., Siddiqui A. When is it Better to Wait for a New Version? Optimal Replacement of an Emerging Technology under Uncertainty // Norwegian School of Economics, Department of Business and Management Science. 2014. URL: https://brage.bibsys.no/xmlui/bitstream/handle/11250/217638/1/DiscussionPaper.pdf
15. Nguyen T. P. K., Yeung T. G., Castanier B. Optimal maintenance and replacement decisions under technological change with consideration of spare parts inventories // International Journal of Production Economics. 2013. Vol. 143, Issue 2. P. 472–477. doi: 10.1016/j.ijpe.2012.12.003
16. Jones T. W., Smith D. An historical perspective of net present value and equivalent annual cost // Accounting Historians Journal. 1982. Vol. 9, Issue 1. P. 103–110. doi: 10.2308/0148-4184.9.1.103
17. Stability of adaptive systems: passivity and averaging analysis / Anderson B. D. O., Bitmead R. R., Johnson C. R., Kokotovic Jr., P. Y., Kosut R. L., Mareels I. M. V., Praly L., Riedle B. D. MIT Press, 1986. 326 p.
18. Lutsenko I. Definition of efficiency indicator and study of its main function as an optimization criterion // Eastern-European Journal of Enterprise Technologies. 2016. Vol. 6, Issue 2 (84). P. 24–32. doi: 10.15587/1729-4061.2016.85453
19. Jardine A., Tsang A. Maintenance, Replacement, and Reliability. Theory and Applications. CRC Press, 2013. 364 p. doi: 10.1201/b14937
20. Volkov I. K., Zuyev S. M., Tsvetkova G. M. Sluchaynye protsessy. Moscow: Izd-vo MGTU, 1999. 448 p.
21. Podinovskiy V. V., Nogin V. D. Pareto-optimal'nye resheniya mnogokriterial'nyh zadach. Moscow: Nauka, 1982. 256 p.