Розглянуто побудову уточненої математичної моделі поздовжньо-поперечних коливань водовіддільної колони в площині дії векторів швидкості потоку рідини, що омиває колону. Дана модель дає змогу вивчати напружено-деформований стан водовіддільної колони із одночасним впливом на неї з боку моря і зміною зусилля натягу її верхнього кінця. Окрім цього в моделі уточнено силовий вплив, що чиниться на водовіддільну колону з боку промивальної рідини, яка протікає по ній.

На основі розробленої математичної моделі створено імітаційну модель роботи системи «бурове судно – натяжна система водовіддільної колони канатного типу – водовіддільна колона» в мові моделювання modelica і проведено серію чисельних експериментів при різній висоті нерегулярного хвилювання моря. Отримані результати показують, що запропонована модель дає на 22–40 % більші розрахункові значення амплітуди поперечних коливань і на 10–25 % більші розрахункові значення згинаючих моментів в критичних січеннях в порівнянні із результатами класичної моделі поперечних коливань. Найбільше розходження результатів моделювання спостерігається при помірному хвилюванні моря і з ростом висоти хвилювання розбіжність двох моделей зменшується. Виходячи із отриманих результатів в прикладних задачах по дослідженню роботи водовіддільної колони при помірному хвилюванні моря не рекомендується нехтувати впливом зміни в часі сил натягу водовіддільної колони

Ключові слова: водовіддільна колона, напружено-деформований стан, імітаційна модель, поздовжньо-поперечні коливання, нерегулярне хвилювання моря, modelica

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STUDYING THE COUPLED AXIAL AND LATERAL OSCILLATIONS OF THE DRILLING RISER UNDER CONDITIONS OF IRREGULAR SEAWAYS

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1. Introduction

The complexity of well-drilling under deep sea conditions is primarily connected to overcoming powerful and highly mobile water masses. This is predetermined by sea currents and seaways that cause drifting and pitching of a floating drilling rig and deform marine risers.

Therefore, when constructing and operating the risers as one of the most loaded elements of the drilling system, it is necessary to study a change in the stressed-strained state in operation to avoid the emergence of negative and accidental phenomena.

For a long time, construction and study into theoretical models has remained the main method for studying the dynamics of operation of risers. According to experts, this is mainly because of lower study costs in comparison with field observations and the impossibility of modeling the important factors acting on the risers in the studies with scale models [1].

At present, a large number of mathematical models have been developed to study dynamics of the risers [1–3]. However, complexity of the object under study leads to the fact that the studies of operation of risers are usually carried out under conditions of idealized regular seaways. In addition, when constructing a mathematical model, the influence of a number of secondary force factors acting on the riser at operation is usually neglected or represented by the simplified equivalent models. It hinders complete description of the dynamic processes taking place in the riser and presents an obstacle for a further improvement of structures and expansion of the application of risers.

2. Literature review and problem statement

Typically, when analyzing operation of the riser, it is represented as a long sectioned hollow bar of a circular cross section with axially distributed parameters modeled as a Bernoulli-Euler beam. Proceeding from this, the model of lateral oscillations in the plane of action of the velocity vectors of the flows passing over the riser has found the widest application in calculations of the stressed-strained state and operation dynamics of the riser. It is described by the following equation:

$$E_R I_R \frac{\partial^4 w(z,t)}{\partial z^4} - \frac{\partial}{\partial z}\left(T_e(z)\frac{\partial w(z,t)}{\partial z}\right) + \rho_R \frac{\partial^2 w(z,t)}{\partial t^2} =$$
$$= C_M \frac{\rho_w \pi D_e^2}{4}\frac{\partial w_w(z,t)}{\partial t} - C_A \frac{\rho_w g p D_e^2}{4}\frac{\partial^2 w(z,t)}{\partial t^2} +$$
$$+ C_D \frac{\rho_w D_e}{2}\left(w_w(z,t) - \frac{\partial w(z,t)}{\partial t}\right)\left|w_w(z,t) - \frac{\partial w(z,t)}{\partial t}\right|, \quad (1)$$

where E_R is the modulus of elasticity of the riser material; I_R is the axial moment of inertia of the riser cross section; $T_e(z)$ is the "effective" tension force in riser sections [13]; ρ_R is the weight of a unit of the riser length; $w(z, t)$ is the value of lateral displacement of riser sections; C_M is a coefficient of inertial component in Morison's equation; C_D is the resistance factor in Morrison's equation; ρ_w is sea water density; D_e is the outside diameter of the riser; $w_w(z ,t)$ is the velocity of the point of the stream passing over the riser.

This model was used in [4, 5] to study operation of the riser under irregular seaways and the action of sea currents.

However, model (1) enables taking into account the effect of only main force factors on dynamics of the riser: hydrodynamic forces from the sea, forces of inertia and weight which incompletely reflects peculiarities of operation of the riser. Currently, there is an interest in constructing new mathematical models that would clarify the impact of existing force factors and take into account new factors acting on the riser during its operation. Authors of work [6] proposed to replace the nonlinear term of the Morison's equation in equation (1) with an equivalent combined model of Duffing and Van der Pol and carried out an experimental verification of the proposed approach for irregular seaways. This approach is also used in [7] when investigating the restoring forces that occur in the riser. In the study of influence of a multiphase flow on dynamics of the riser [8], a modified equation (1) was considered with an additional accounting of influence of inertial forces from the transient, centripetal and Coriolis accelerations of the fluid flow in the riser. Construction of a simplified alternative model of lateral oscillation of a riser described by a differential equation of the second order is considered in [9]. Disadvantage of the above-mentioned works is that when studying dynamics of the riser, it is considered that the tension force is unchanged over time, which cannot be ensured under real conditions. The study of influence of the dynamic component of the tension force on dynamics and stability of the riser was considered in papers [10, 11]. However, the simplified representation of variation of the tensile force in the riser along its length does not enable an accurate characterization of their influence on the dynamic processes in the riser.

A more detailed study of influence of the tension force in the riser on its operation dynamics is possible by considering its combined axial and lateral oscillations. An idealized model of axial and lateral oscillations in a simply supported pipe with a fluid flowing in it and a stationary flow passing over it is considered in paper [12]. Axial and lateral oscillations in the riser were studied in more detail in papers [13–17]. Combined axial and lateral oscillations in a sloping non-rigid riser is considered in [13]. The study of nonlinear resonance was performed in [14] by analyzing a model of axial and lateral oscillations in the riser taking into account inertial forces brought about by passage of a flushing fluid and vertical displacement of the upper end of the riser. Papers [15, 16] study reaction of the riser to variation of the force tensioning its upper end by construction of a finite-element model of axial and lateral oscillations of the riser.

In existing works, only some issues of dynamics of the riser were considered. In problem statement, this has allowed us to study operation of the riser in ideal conditions neglecting a number of important factors. Complexity of studying operation of the riser consists in that in the process of operation, it is influenced by two main factors that have a decisive effect on the oscillatory processes in the riser and have the same nature of origin. They are hydrodynamic forces acting in the vertical and horizontal directions caused by the fluid passing over the riser and variation of the force tensioning it. Given the fact that the mathematical model of axial and lateral oscillations of the riser has a pronounced nonlinear character, this makes it impossible to use the principle of superposition in the study of the stressed-strained state.

Given this, it is important to study operation of the riser simultaneously taking into consideration the force influence of the sea and variation of the force that tensions its upper end because of the tensioning system features.

3. The aim and objectives of the study

This study objective was to elucidate the dynamics of operation and the stresses-strained state of the riser under an interconnected effect of irregular seaways and the tensioning system of the riser.

To attain this objective, the following tasks were formulated:

– to work out a refined mathematical model of axial and lateral oscillations of the riser;

– to obtain a simulation model of the system ''floating drilling rig – tension system of the riser – riser'' in the Modelica modeling language;

– to conduct simulation of the riser operation at various levels of irregular seaways;

– to analyze the results of simulation modeling by comparing the results obtained with the results of the classical model (1).

4. Physical model of the system under study

A drilling system was taken as the object under study. Its calculation scheme is given in Fig. 1. Drilling ship 1 conducts well drilling under sea conditions at depth d. The wellhead A is connected to drill ship 1 by riser 3, the lower end of which is fixed by spherical joint 4 to the wellhead equipment and the upper end is held by the tensioning system.

The rope-type tensioning system of the riser consists of eight tensioning devices $T_{1\ldots8}$ mounted on deck 7 of the drilling ship.

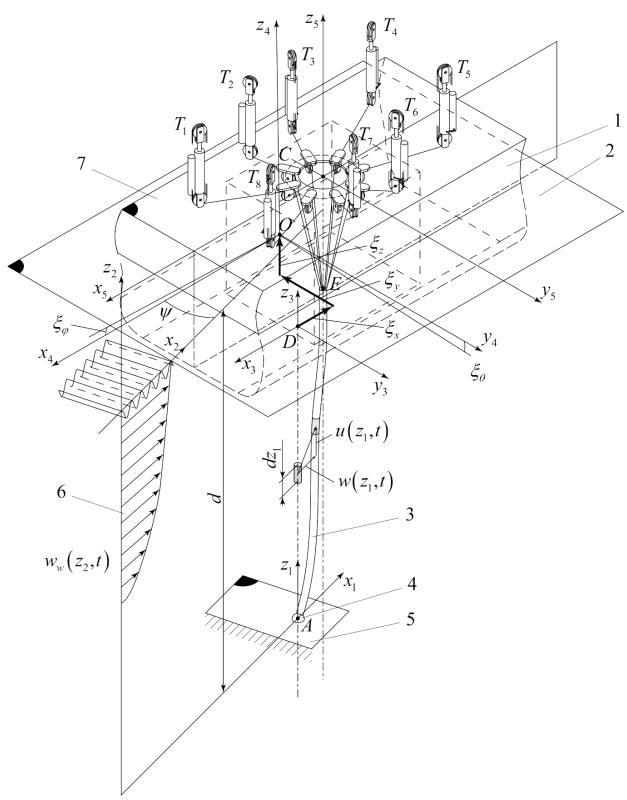

Fig. 1. Design diagram of the system under study

In operation, the system is subjected to effects of seaways 6 specified in a plane determined by the angle ψ to the plane of symmetry of drilling ship 1. The sea state is described by function $\eta(t)$ and variation of horizontal and vertical components of the fluid flow velocity caused by it is described by functions $w_w(z_2, t)$ and $u_w(z_2, t)$ respectively. Seaways causes the ship rolling described by three vectors ξ_x, ξ_y, ξ_z and two angles ξ_φ, ξ_θ. As a result, the center of inertia of the ship moves from point D, which is the projection of the well head on surface 2 of the quiet sea to an arbitrary point O. The water passing over the riser causes its deformation described in the axial direction by function $u(z_1, t)$ and in lateral direction by function $w(z_1, t)$. For convenience of calculations, take the coordinate systems: x_1z_1 in which oscillations of the riser will be studied; x_2z_2 in which velocity and acceleration of the fluid passing over the riser will be determined; $x_3y_3z_3$, $x_4y_4z_4$ and $x_5y_5z_5$ related to the geometry of the drilling ship and necessary for assigning drilling ship rolling and the study of operation of the riser tensioning system [18].

5. Construction of a mathematical model for the axial and lateral oscillations of the riser

We represent riser 3 shown in Fig. 1 as a sectioned hollow rod of a round cross section with constant parameters along its length in which the flushing fluid of density ρ_f flows with constant velocity v_f. Describe the riser by external and internal diameters D_e and D_i, respectively, cross-sectional area A_R, weight ρ_R of a unit of length of the riser, a Young's modulus E_R of the riser wall material and moment of inertia I_R of its cross section.

To deduce the mathematical model of axial and lateral oscillations of the riser, cut an element of length dz_1 from it together with the flushing fluid in it (Fig. 2).

Apply internal force factors to its edges: axial force T_O, bending moment M_O and bending force Q_O. Apply external and inertial forces to the element that are acting on it during its operation. The external forces include weight f_{gR}, force f_b exerted by the float sections, friction force f_{fe} between the riser walls and the flushing fluid flowing therein and the hydrodynamic forces f_{Mn} and $f_{M\tau}$ in the horizontal and vertical directions exerted by the flow of the fluid passing over the riser. Inertial forces include forces f_{iu}, and f_{iw} from axial and lateral displacements of the riser element and forces f_{fi}, f_{fn}, and f_{fc} brought about by relative, axipetal and Coriolis accelerations of the flushing fluid resulting from its flow through the deformed portion of the riser.

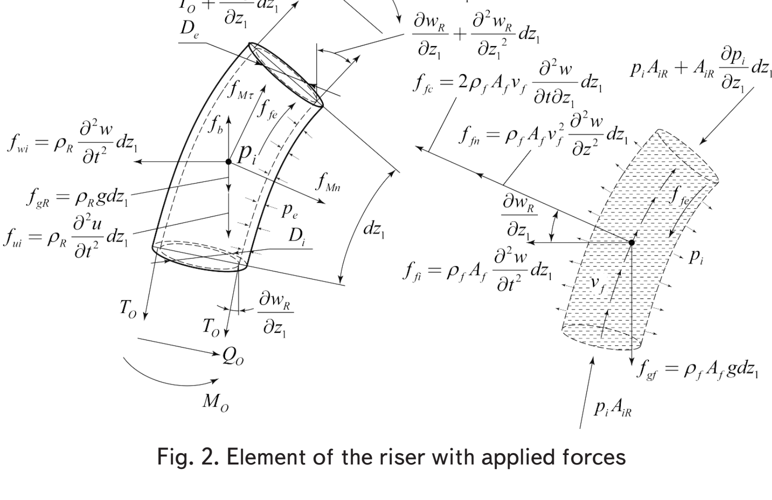

Fig. 2. Element of the riser with applied forces

Assume that:

$$\sin\frac{\partial w}{\partial t} = \frac{\partial w}{\partial t}, \quad \cos\frac{\partial w}{\partial t} = 1,$$

the magnitude of stress according to the Euler-Bernoulli theory is equal to:

$$\varepsilon = \frac{\partial u}{\partial z_1} + \frac{1}{2}\left(\frac{\partial w}{\partial z_1}\right)^2.$$

Consequently, the axial force is $T_O = E_R A_R \varepsilon$ and the value of the bending moment:

$$M_O = E_R I_R \frac{\partial^2 w}{\partial z_1^2}.$$

Apply the principle of D'Almerbert, the theory of "effective" tension and make transition $dz_1 \to 0$ to obtain the following system of equations describing axial and lateral oscillations of the riser section:

$$-\rho_R \frac{\partial^2 u}{\partial t^2} - \rho_R k_b g + E_R A_R \frac{\partial}{\partial z_1}\left(\frac{\partial u}{\partial z_1} + \frac{1}{2}\left(\frac{\partial w}{\partial z_1}\right)^2\right) - \lambda \frac{\partial u}{\partial t} + \frac{\partial}{\partial z_1}\left(p_e A_e - p_i A_i\right) + \eta S_i + f_{M\tau} = 0;$$

$$-\rho_R \frac{\partial^2 w}{\partial t^2} + E_R A_R \frac{\partial}{\partial z_1}\left(\frac{\partial u}{\partial z_1} + \frac{1}{2}\left(\frac{\partial w}{\partial z_1}\right)^2\right)\frac{\partial w}{\partial z} + E_R A_R\left(\frac{\partial u}{\partial z_1} + \frac{1}{2}\left(\frac{\partial w}{\partial z_1}\right)^2\right)\frac{\partial^2 w}{\partial z^2} - \frac{\partial^2 M}{\partial z^2} - \rho_f A_i\left(\frac{\partial^2 w}{\partial t^2} + v_f^2 \frac{\partial^2 w}{\partial z_1^2} + 2 v_f \frac{\partial^2 w}{\partial z \partial t}\right) + f_{Mn} = 0, \quad (2)$$

where k_b is a coefficient of the riser weight compensation with float sections; S_i is the perimeter of the riser channel; λ is a coefficient of the riser damping; A_i is the cross-sectional area of the riser channel; p_e and p_i are the external and internal pressures acting on the riser from the side of the sea and the mud, which are calculated from the following formulae:

$$p_e = \rho_w (d - z_1) g;$$

$$p_i = \rho_f (d - z_1) g + \rho_f \frac{v_f^2}{2} + \rho_f \frac{f}{D_i}\frac{v_f^2}{2}(d - z), \quad (3)$$

f_{Mn} and $f_{M\tau}$ are the hydrodynamic forces that arise as a result of washing of the riser by a flow of fluid which is calculated using the Morison equations:

$$f_{M\tau} = C_{M\tau} \frac{\rho_w \pi D_{ei}^2}{4} \frac{\partial u_w}{\partial t} + C_{D\tau} \frac{\rho_w D_{ed}}{2}\left(u_w - \frac{\partial u}{\partial t}\right)\left|u_w - \frac{\partial u}{\partial t}\right|;$$

$$f_{Mn} = C_{Mn} \frac{\rho_w \pi D_{ei}^2}{4} \frac{\partial w_w}{\partial t} - C_A \frac{\rho_w g \pi D_e^2}{4} \frac{\partial^2 w}{\partial t^2} + +C_{Dn} \frac{\rho_w D_{ed}}{2} \left(w_w - \frac{\partial w}{\partial t} \right) \left| w_w - \frac{\partial w}{\partial t} \right|, \quad (4)$$

where D_{ei}, D_{ed} are the reduced diameters of inertia and resistance of the riser; C_{Mn} and $C_{M\tau}$ are normal and tangential coefficients of inertia; C_{Dn} and $C_{D\tau}$ are normal and tangential coefficients of resistance; C_A is the coefficient of connected weight of water; u_w and w_w are the vertical and horizontal components of velocity of the fluid flow that passes over the riser determined in accordance with the spectral theory of seaways by decomposition of the JONSWAP energy spectrum:

$$S_\eta(\omega) = \frac{ag^2}{\omega^5} \exp\left(-\beta \left(\frac{\omega_p}{\omega} \right) \right) \gamma^\alpha;$$

$$\alpha = \exp\left(-\frac{(\omega - \omega_p)^2}{2\sigma^2 \omega_p^2} \right), \quad \sigma = \begin{cases} 0.07 \ \omega \le \omega_p; \\ 0.09 \ \omega > \omega_p; \end{cases}$$

$$\mu(t) = \sum_{n=1}^{400} \zeta_n \sin(\omega_n t + \varepsilon_n), \quad \varepsilon_n \in \{0..2\pi\}, \quad \Delta\omega = \frac{5\omega_p}{400};$$

$$\omega_n \in \{\Delta\omega(n-1)..\Delta\omega \cdot n\}, \quad k_n = \frac{\omega_n^2}{g}, \quad \zeta_n = \sqrt{2 \cdot S_{\zeta\zeta}(\omega_n)\Delta\omega};$$

$$w_w(z_2, t) = \sum_{n=1}^{400} \zeta_n \omega_n \exp(k_n z_2) \cos(\omega_n t + \varepsilon_n);$$

$$u_w(z_2, t) = \sum_{n=1}^{400} \zeta_n \omega_n \exp(k_n z_2) \sin(\omega_n t + \varepsilon_n), \quad (5)$$

where $S_\eta(\omega)$ is the energy spectrum; ω_p is the peak frequency of the spectrum; γ is the peak gain factor; k_n is the wave number; ζ_i, ω_i, ε_i are the amplitude, frequency and phase of the i-th harmonic into which the energy spectrum of seaways is decomposed.

The sections of the riser are connected to each other through the following boundary conditions:

$$u_i\big|_{z_1=l_i} = u_{i+1}\big|_{z_1=l_i}, \quad w_i\big|_{z_1=l_i} = w_{i+1}\big|_{z_1=l_i}, \quad \frac{\partial w_i}{\partial z_1}\bigg|_{z_1=l_i} = \frac{\partial w_{i+1}}{\partial z_1}\bigg|_{z_1=l_i};$$

$$E_R I_R \frac{\partial^2 w_i}{\partial z_1^2}\bigg|_{z_1=l_i} + E_R I_R \frac{\partial^2 w_{i+1}}{\partial z_1^2}\bigg|_{z_1=l_i} = 0,$$

$$E_R I_R \frac{\partial^3 w_i}{\partial z_1^3}\bigg|_{z_1=l_i} + E_R I_R \frac{\partial^3 w_{i+1}}{\partial z_1^3}\bigg|_{z_1=l_i} = 0;$$

$$E_R A_R \left(\frac{\partial u_i}{\partial z_1} + \frac{1}{2} \left(\frac{\partial w_i}{\partial z_1} \right)^2 \right)\Bigg|_{z_1=l_i} + +E_R A_R \left(\frac{\partial u_{i+1}}{\partial z_1} + \frac{1}{2} \left(\frac{\partial w_{i+1}}{\partial z_1} \right)^2 \right)\Bigg|_{z_1=l_i} = 0, \quad (6)$$

where l_i is the coordinate of the i-th connection and (i+1)-th sections of the riser.

Present the boundary condition of fastening the lower end of the riser in a form of an ideal hinge in the following form:

$$u\big|_{z_1=0} = 0, \quad w\big|_{z_1=0} = 0, \quad \frac{\partial^2 w}{\partial z_1^2}\bigg|_{z_1=0} = 0. \quad (7)$$

The boundary condition of fastening of the upper end of the riser is as follows:

$$\frac{\partial^2 w}{\partial z_1^2}\bigg|_{z_1=L} = 0, \quad E_R I_R \frac{\partial^3 w}{\partial z_1^3}\bigg|_{z_1=L} = F_x(t),$$

$$E_R A_R \left(\frac{\partial u}{\partial z_1} + \frac{1}{2} \left(\frac{\partial w}{\partial z_1} \right)^2 \right)\Bigg|_{z_1=L} = F_z(t), \quad (8)$$

where $F_x(t)$ and $F_z(t)$ are the horizontal and vertical components of the force influence from the riser tensioning system on the upper end of the riser. They are determined by simulation of the riser tensioning system.

6. Conducting simulation of operation of the system under study

Based on the proposed mathematical model of axial and lateral oscillations of the riser (3)–(8) and the simulation model of the tensioning system described by the author in [18], an imitation model of the studied system was constructed in the environment of Modelica multispectric simulation (hereinafter, the proposed model). A simulation model of lateral oscillations of the riser was also constructed. It is described by equation (1). Its results were used for comparative analysis (hereinafter the standard model). In view of impossibility of describing simulation models in the Modelica language using differential equations with partial derivatives for constructing models of riser sections described by equations (1), (3)–(8), the numerical method of lines (NMOL) [19] was used. This method implies the discretization of all differential equations with partial derivatives except for one using known numerical methods with subsequent representation of the equation in a form of a system of ordinary differential equations. In construction of the simulation model, discretization of equations (1), (3)–(8) was conducted by dimension z_1 in steps of ≈10 m using the central finite-difference scheme of the second order.

The riser operation was studied at various levels of irregular seaways. Parameters of its energy spectrum are shown in Table 1. Operation of the 21-inch diameter riser consisting of two riser sections was studied: a 274.3 m section without floats and a 365.8 m section with floats. Parameters of the sections of which the riser was built are summarized in Table 2 and other parameters of the system under study are given in Table 3.

Simulation modeling was carried out in a time interval of 0–500 s with an iteration step of 0.01 s using a CVODES solver with an accuracy of 10^{-6} to solve linear equations and 10^{-12} to solve nonlinear equations. The simulation has resulted in obtaining of a set of discrete time rows of change of all parameters of the system under study. Fig. 3 shows graphical representation of the magnitude of change of tension of the upper end of the riser in time for various seaways values obtained in simulation and Fig. 4 shows a change in the angle of deflection of the lower spherical hinge.

Table 1

Parameters of the JONSWAP spectrum depending on the significant wave height [20]

No.	Significant wave height $H_{1/3}$, m	Peak frequency ω_p, rad/s	Coefficient of peak amplification γ
1	3.0	0.79	3.3
2	4.0	0.7	3.3
3	5.0	0.63	3.3
4	6.0	0.57	3.3

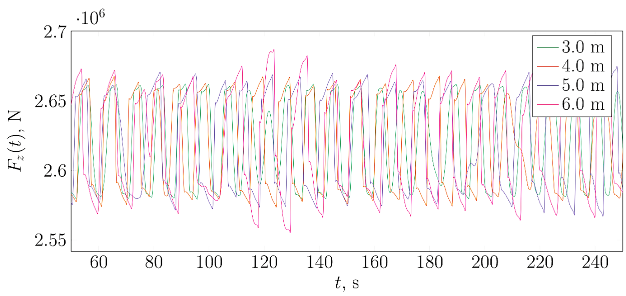

Fig. 3. Variation of the force of tensioning of the upper end of the riser $F_z(t)$ in time t at various significant wave height $H_{1/3}$

Table 2

Basic geometric and mechanical parameters of the riser sections

No.	Parameters	21-inch section	
		With floats	With no floats
1	Weight of the section without floats, N	158,552	158,552
2	Section length, m	22.86	22.86
3	Weight of floats for a section, N	98,216	0
4	Weight of the immersed section with floats, N	2,869.1	137,783
5	Weight compensation by floats	97.92 %	–
6	Reduced diameter of resistance D_{ed}, mm	1,410	1,049
7	Coefficient of resistance C_{Dn}	1.00	1.00
8	Coefficient of resistance $C_{d\tau}$	0.03	0.03
9	Reduced diameter of inertia D_{ei}, mm	1,410	952.5
10	Coefficient of inertia C_{Mn}	1.9	1.9

The obtained numerical data of variation of the basic system parameters were later used for calculating indicators of operation dynamics of risers and determining their critical cross-sections and maximum stresses.

Table 3

Basic parameters of the system under study

No.	Parameter	Unit
	Tensioning system of the riser	
1	Number of the tensioning devices	8
2	Type of the tensioning devices	Single
3	Diameter of the hydraulic cylinder, mm	470
4	Diameter of the hydraulic cylinder shaft, mm	400
5	Volume of the gas tank of low pressure, m^3	0.5
6	Volume of the gas tank of high pressure, m^3	3.2
7	Diameter of a pulley of the pulley block, mm	1,200
8	Diameter of the pulley block rope, mm	44,5
9	Hydraulic fluid	Eriffon 818
10	Working gas	Nitrogen
11	Friction forces in the hydraulic cylinders from nominal force	1.5 %
	Other system parameters	
12	Sea water density ρ_w, kg/m^3	1,025
13	Mud density ρ_f, kg/m^3	1,200
14	Velocity of the mud flow v_f, m/s	1.1

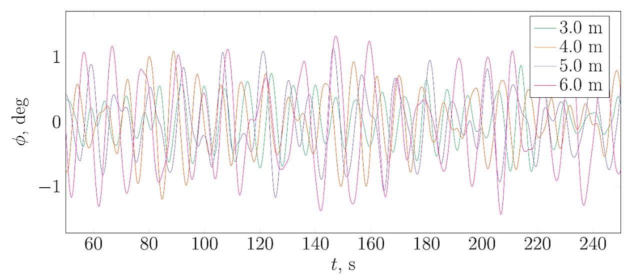

Fig. 4. Variation of the deflection angle φ of the lower spherical hinge in time t at various significant wave height $H_{1/3}$

7. Discussion of the results obtained in the simulation modeling of operation of the riser under conditions of unregular seaways

Analysis of the results obtained in the simulation modeling of the system under study has shown that because of peculiarities of functioning of the tensioning system, there was a pronounced nonlinear variation of force of tensioning of the upper end of the riser. Its maximum value was 5.8 % of the static value at significant wave height $H_{1/3}$=3.0 m and increased to 7.6 % at significant wave height $H_{1/3}$=6.0 m. This indicates that the tensioning system is one of the main factors influencing dynamics of the object under study. To estimate the effect of variation of the force of tensioning in the sections of the riser and the additional force factors caused by the flow of the mud in the riser on its lateral oscillations, consider Fig. 5. It shows comparison of distribution of maximum, minimum and mean square values of lateral oscillations along the riser length. These graphs were obtained by analyzing results obtained in simulation of the riser operation at various seaways levels for the proposed (3)–(8) and standard (1) models.

The proposed model has demonstrated higher calculated values of lateral oscillations in cross-sections of the riser. For example, magnitude of the mean square deviation for a critical section 55.86 m at significant wave height $H_{1/3}$=6.0 m calculated by the proposed model was 22 % larger than for the standard model and for a critical section 39.9 m at $H_{1/3}$=3.0 m, it was respectively larger by 30.3 %. Similar results were observed for other critical sections of the riser. The average difference between the magnitude of the mean square deviation for the two models along the length of the riser was from 47.2 % at $H_{1/3}$=4.0 m to 37.3 % at $H_{1/3}$=6.0 m. The obtained data show that a growth of the seaways level leads to that the difference between the two models tends to decrease.

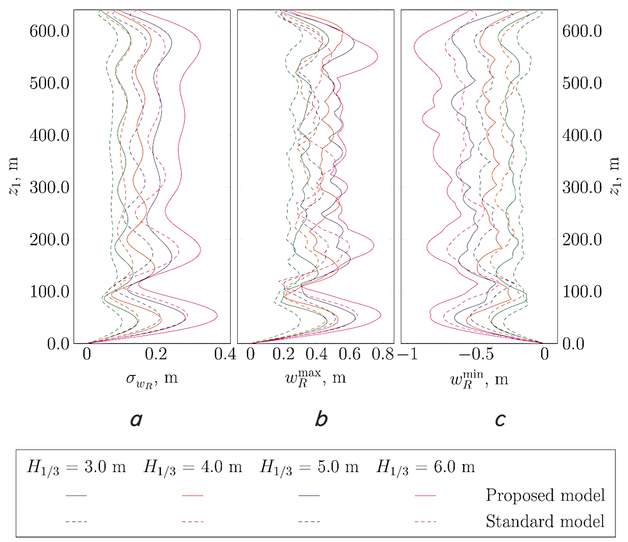

Fig. 5. Distribution of values of lateral displacements of the riser sections along the riser length z_1 at various significant wave height $H_{1/3}$: rms (*a*); maximum (*b*); minimum (*c*)

Comparison of the obtained graphs shows that the standard model gives a much larger relative difference between minimum and maximum values along the length of the riser. In the graphs Δw_R^{max} and Δw_R^{min}, there is appearance of new local extrema and a much larger estimate of difference of oscillations in noncritical sections of the riser. For example, local extrema at significant wave height $H_{1/3}$=3.0 m was observed in the sections of 44.89 m, 134.66 m, 324.41 m, 484.76 and 584.98 m where the values of the maximum difference of lateral oscillations of the riser were 0.848 m, 0.763 m, 0.584 m, 0.658 m, and 0.811 m, respectively. At significant wave height $H_{1/3}$=6.0 m, extrema were observed in the following sections of the riser: 54.86 m, 184.53 m. 429.64 m and 569.95 m. The values of maximum difference between lateral oscillations in these sections were 1,625 m, 1,604 m, 1,438 m, and 1,719 m, respectively.

The obtained results show that when seaways level increases, a decrease in the difference between the results of the standard and proposed model of the riser operation dynamics is observed. This indicates that the influence of the dynamic component of tensioning of the riser in verification calculations of the riser in extreme operation conditions can be neglected. However, in the studies of operation of the riser in usual drilling conditions, in particular the study of the effect of deformation of the riser on operation of the drill riser, this should not be neglected.

Another important parameter affecting operation of the riser is the magnitude of bending moments that arise in its cross-sections. Comparison of the results of imitation modeling according to the standard and proposed models (Fig. 6) shows that the difference between the values of bending moments obtained by the proposed model is greater compared to the results obtained by the standard model.

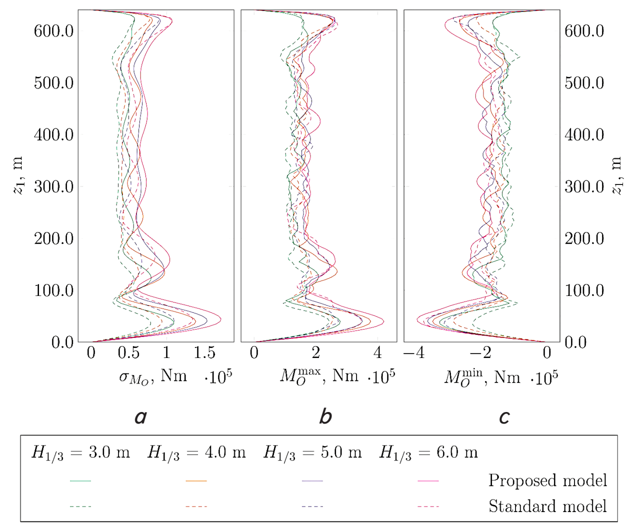

Fig. 6. Distribution of bending moments arising in cross-sections of the riser along its length z_1 at various levels of significant wave height $H_{1/3}$: rms (*a*); maximum (*b*); minimum (*c*)

For example, in a critical section of 39.9 m, the bending moment is 0.6 MNm at a seaways level $H_{1/3}$=3.0 m, 0.72 MNm at $H_{1/3}$=4.0 m, 0.71 MNm at $H_{1/3}$=5.0 m and 0.81 MNm at $H_{1/3}$=6.0 m. These values are 22.7 %, 25.7 %, 18.2 %, and 11.2 % higher compared to the values obtained by the standard model. In the upper critical section, there is a less pronounced difference between the bending moment values calculated for two models. For example, the bending moment value calculated by the proposed model is 9.1 % higher at $H_{1/3}$= 4.0 m and 6.2 % higher at $H_{1/3}$=6.0 m.

Comparison of the bending moment values arising in the upper and lower critical sections shows that the value of the bending moments in the lower critical section is 30–35 % higher than the value in the upper critical section. In general, difference between the two models along the riser length is from 37 % at significant wave height $H_{1/3}$=4.0 m to 24.8 % at $H_{1/3}$=6.0 m.

The obtained results show that when seaways are moderate, the strained state of the riser depends to a large extent not only on the hydrodynamic forces brought about by washing of the riser by the fluid flow but also on the variation of the force of tensioning of its upper end.

One of the further lines of further development of this study is obtaining of a model of spatial oscillations of the riser taking into account the force factors brought about by surge vortices of the flow passing over the riser.

9. Conclusions

1. A refined mathematical model of axial and lateral oscillations of the riser was proposed. It additionally takes into account the influence of power factors caused by the flow of fluid in the riser. Calculation of the pressure gradient of the mud along the length of the riser was refined and moving of the upper end of the riser was taken into consideration.

2. A simulation model of the system "drilling ship - tensioning system of the risers – riser" in Modelica language was created. It enables studying the stationary and transient processes in the riser taking into account the simultaneous influence of interconnected force factors: hydrodynamic forces caused by conditions of irregular seaways and change of tension force by the tensioning system of the riser.

3. The simulation results show that the proposed model produces a 22–40 % higher calculated lateral oscillation amplitude and the 10–25 % higher calculated bending moments in the critical sections of the riser.

4. The obtained results indicate that in the studies of dynamics of the riser in conditions of slight sea, it is not appropriate to ignore the influence of change in time of the tensile forces in the sections of the riser.

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