Розглянуто теоретичні аспекти теорії катастроф та описано можливості застосування методів теорії катастроф для оцінювання фактичних даних геолого-технічного контролю процесу поглиблення свердловин. Досліджено метод розпізнавання динамічного режиму поглиблення свердловини з метою управління цим процесом. Доведено доцільність використання не тільки елементарної катастрофи Р. Тома типу «збірка», але й феноменологічної моделі Vapor Pressure.

В результаті досліджень визначено таку ознаку як «час буріння 1 м породи» для розпізнавання динамічної стійкості процесу, який схильний до стрибкоподібних змін.

Визначено основні аргументи для застосування теорії катастроф щодо моделювання динамічної стійкості процесу поглиблення свердловин.

Показано, що поведінку системи в катастрофічному стані можна описати не лише канонічним кубічним рівнянням, розв'язання якого виконується за формулами Кардано, але і феноменологічною моделлю, побудованою на засадах холістичного підходу.

Використовуючи холістичний підхід до розв'язання задач моделювання динамічної стійкості процесу поглиблення свердловин, запропоновано структуру базової феноменологічної математичної моделі динаміки розвитку катастроф.

Доведено, що на початкових етепах розвитку катастроф, коли спостерігається збільшення часу, що витрачається на буріння одного метра порід по глибині свердловини, за допомогою запропонованої феноменологічної математичної моделі можна змоделювати динаміку розвитку катастроф. Під час подальшого поглиблення свердловини, коли спостерігається зменшення часу, що витрачається на буріння одного метра порід, динаміку розвитку катастрофи можна описати цим самим законом, але коефіцієнти моделі і їх знаки будуть іншими. Вона має змогу ідентифікувати параметри моделі за єдиним алгоритмом, а також прогнозувати появу катастрофи на етапі її розвитку. Це сприяє запобіганню ускладнень і аварій в процесі поглиблення свердловини. За результатами імітаційного моделювання підтверджено теоретичні висновки щодо вибору типу моделі як оптимальної для опису катастроф в процесі поглиблення свердловин та встановлено, що запропонована феноменологічна модель є адекватною реальним процесам.

Водночас слід дотримуватись базових принципів теорії катастроф, що дозволяє забезпечити ефективне прогнозування і виявлення передаварійних ситуацій і ускладнень, які виникають в процесі поглиблення свердловин.

Отримані дані корисні і важливі тому, що дозволяють удосконалити математичне і програмне забезпечення системи автоматизованого управління процесом поглиблення свердловин і зменшити аварійність в бурінні

Ключові слова: теорія катастроф, процес поглиблення свердловин, динамічна стійкість, моделювання, холістичний підхід, феноменологічна математична модель

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MODELING OF DYNAMIC STABILITY OF THE WELL DEEPENING PROCESS BASED ON THE CATASTROPHE THEORY

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1. Introduction

The well deepening process is a non-linear, non-reproducible, stochastically chaotic process. It is realized in conditions of a priori and current uncertainty regarding its parameters and structure. Besides, the process is influenced by various types of additive and multiplicative obstacles and develops over time. All this causes a necessity of developing methods for recognizing current dynamic conditions of well deepening and optimal control of this process. Application of mathematical models based on Renee Thoma catastrophe theory may be an effective tool for solving this problem [1, 2].

The catastrophe theory enters the synergy methodology together with fractal geometry, probability theories, algorithms, cellular automata, as well as categories and topos [3, 4]. Such objects and phenomena as attractors, bifurcation, self-realization, chaos and deterministic chaos, fractals and dissipative processes are consistently associated with synergetics. Note that a man-machine system, that is, a complex control system the constituent elements of which is a human operator cannot be in an equilibrium for a long time. In order to maintain its stable functioning and development, it is necessary to understand clearly the laws according to which it changes. Particularly it is necessary to understand its behavior under the influence of external destabilizing disturbing factors. Hence, creation of a universal approach to understanding events in the well deepening process when there are abrupt changes in the process indicators is extremely important. To solve such problems, it is expedient to use the catastrophe theory which is an effective analytical tool used to study and predict instability of systems of various classes [5–7].

The essence of the catastrophe theory consists in that it analyzes points where not only the first derivative of a function is zero but derivatives of higher order are also zero. Dynamics of development of such points can be studied by a Taylor decomposition of potential function at small changes of input parameters.

Random changes in geological and technological conditions of well drilling lead to pre-emergency situations. Therefore, the problem of drilling automation [8] is not only automated control of the drilling process but also prevention of emergencies. The cause-effect relations are determined for detection of the drilling process instability [9]. Effectiveness of using such an approach is confirmed by the results of operation of a drilling automation system [10]. Particular attention is paid to identification of pre-emergency situations caused by the drill string vibration [11] as well as the methods of real-time analysis [12, 13]. Peculiarities of detecting vibration deviations from normative values in drilling deep vertical wells as well as study of the bifurcation phenomenon [14] which is a sign of emergency are considered. However, such approaches can only be used to solve the problem of emergency identification. From the point of view of general problems, the problems of all these studies consist in the lack of possibility of using the proposed methods for modeling dynamic stability of the well drilling process. The unsolved part of the problem is the lack of tools for linking recurrent algorithms of cumulative sums to the automation system.

The catastrophe theory is used to study and predict time spent on drilling one meter of well [14]. Synergetic principles of studying quality of evaluation of dynamic conditions of well deepening and recognition of operating conditions have been used in [15]. However, since the studied dynamic process is chaotic, it is not always possible to accurately describe and predict conditions of its operation. Since new time, spatial or functional structures may appear in it, practical interest in the study of chaos structure and dynamic stability in the well deepening process is of practical interest. This part of the problem is unresolved.

It should be noted that scientists of the Society of Petroleum Engineers (SPE) and the International Association of Drilling Contractors (IADC) [16] pay due attention to the issues of preventing drilling emergencies, namely in the subsystem of well irrigation. The IADC/SPE international conferences are deeply concerned with the issues of automation of drilling control and management [17] as well as the issue of emergency-free drilling based on innovative technologies [18]. The problems of all these studies consist in that the drilling process is non-reproducible and the models formed for a particular well cannot be used for the other well. This causes a need for development of phenomenological models of dynamic conditions of well deepening based on a holistic approach for the process control problems. It should be borne in mind that the process has three control actions: axial bit load, frequency of bit rotation and the irrigation fluid flow.

According to the approach proposed by Rene Thom [2, 3], in the course of studying the effect of five or less active parameters on the final drilling results, there are only seven generalized structures of description of bifurcation paths which are given in Table 1.

Table 1

Elemental catastrophes by Rene Thom

Catastrophe type	Formula
Potential functions with one variable	
Fold	$F(x,a)=\frac{1}{3}x^3+ax$
Gather	$F(x,a,b)=\frac{1}{4}x^4+\frac{1}{2}x^2+bx$
«Dovetail»	$F(x,a,b,c)=\frac{1}{5}x^5+\frac{1}{3}ax^3+\frac{1}{2}bx^2+cx$
«Butterfly»	$F(x,a,b,c,d)=x^6+ax^4+bx^3+cx^2+dx$
Potential functions with two variables	
Hyperbolic umbilics	$F(a,b,c)=x^3+y^3+axy+bx+cy$
Elliptic umbilics	$F(a,b,c)=\frac{x^3}{3}-xy^2+a(x^2+y^2)+bx+cy$
Parabolic umbilics	$F(a,b,c,d)=yx^2+y^4+ax^2+by^2+cx+dy$

The catastrophe theory allows one to predict situational changes concerning the investigated object and behavior of the whole system. This theory is particularly suitable for studying the well drilling processes characterized by various changes in behavior and invisible transitions. This provides a possibility of predicting non-quantitative course of various processes that accompany the well deepening process.

At the same time, as the study of the outlined problem has shown, at least in Ukrainian scientific environment, there is still no general approach to the methods of modeling dynamic stability of the well deepening process based on the catastrophe theory at a level of SCADA-systems.

2. Literature review and problem statement

The most influential results of studies of the use of the catastrophe theory in the technology of deepening wells are presented in papers [13, 15].

Actual materials on drilling wells in shelf fields of Vietnam, in particular, in the White Tiger field, were used in [15]. Wells were constructed at large depths (3.700÷4.500 m), in rocks of 7÷10 hardness groups and the fifth grade of abrasivity at formation pressure of 0.83÷0.87 of hydrostatic pressure and drilling mud absorption.

The high hardness and abrasiveness of rocks as well as longitudinal and transverse oscillations of the drill column were the causes of spatial well curving and missing the target coordinates. For development of the method of recognition of dynamic well deepening conditions and control of this process, data of the Geoservice station of geological and technological control of the drilling process were used with recorded changes during drilling 1 m of the well, parameters of drilling conditions and other indicators. Qualitative estimation of dynamic conditions was carried out based on the R. Thoma elemental catastrophe of the «fold» type:

$$\frac{dT}{dx} = AT^2 + BT + C, \qquad (1)$$

where dT/dx is the intensity of change of drilling time with change in depth, x, of the well; $T = \sum t$ is the time of well drilling as it is deepened; A, B, C are the coefficients characterizing mining and geological conditions and technological parameters of drilling. They were determined by the method of least squares according to experimental data of the Geoservice complex.

Proceeding from the fact that if behavior of the system can be described by differential equation (1), the set of emergencies is determined from the condition:

$$\frac{dT}{dx} = \frac{d^2T}{dx^2}$$

and takes the following form:

$$B^2 - 4AC = 0. \qquad (2)$$

If $B^2-4AC > 0$, then the dynamic conditions of well deepening were considered to be stable.

If $B^2-4AC < 0$, then the conditions are unstable, chaotic, which can lead to the well curvature and deflection of the well face from the target point.

To solve this problem, point values of intensity of change of the time taken for drilling 1 m of the well were determined first and then the values of A, B, C coefficients and B^2-4AC discriminant were determined using the least squares method for 3 or 4 points.

The calculation results were used in plotting graphs of dependence of the time spent in drilling 1 m of rocks and B^2-4AC values on the well depth.

To identify the chaos control capabilities, the values of fractal, d, and spatial, n, dimensionalities of the technological process were calculated. If $d \in (1{,}0 \div 2{,}0)$, than such chaos was considered controlled. If $d > 2$, then chaotic unpredictable oscillations were observed in the system.

The value of spatial dimensionality, n, equals a number of factors involved in realization of the process of well deepening ($n = 1, 2, 3, 4, \ldots$).

The analysis carried out shows that:

– the formula of elemental catastrophe of the fold type (1) used by the authors does not correspond to the formula of *R.* Thom given in Table 1. Equation (1) as well as the equation of fold, in essence, is a parabola but with a vertex at the point:

$$\left(\frac{4AC - B^2}{4A} - \frac{B}{2A}\right),$$

while the vertex of *R.* Thom's parabola lies in the origin;

– it was not known what was the step of discretization used in determining the values of dT/dx in the well depth. If the step of discretization does not meet the requirements of the Shannon-Kotelnikov theorem, then the results obtained have a large error;

– calculations are of retrospective nature and the results obtained cannot be used to control the well deepening process in on-line mode.

To solve the problem of dividing rocks into homogeneous packets as a criterion, authors of [13, 21] have proposed parameter L of functional dependence:

$$\nu_t = \nu_0 \phi(L,t),$$

where ν_0 is the initial value of the mechanical speed of drilling; $\phi(L,t)$ is the function of wear of the bit tools; ν_t is the mechanical speed of drilling.

It characterizes total physical and mechanical properties of rocks.

It was shown that when the bit passes from one formation to another, L_i indicator expands its value stepwise and the process of calculating the value of L_i generates a sequence of discrete quantities:

$$L_i = \dot{L} + \nu_L,$$

where $\dot{L}$ is the mathematical expectation of the value of L_i; ν_L is the additive obstacle caused by the instrumental errors of measurements on the one hand and by the change in the rock properties within the formation on the other hand.

Since the value of $\dot{L}$ is distorted by the obstacle and the jump of $\dot{L}$, that corresponds to the moment of the bit transition through formation will be masked by this obstacle.

It was assumed that by the time of formation change, statistical characteristics of the obstacle, ν_L, are known, and after transition of the bit to another layer, stationarity of obstacles did not change. The following problem was considered: determine the moment of the bit transition to another formation based on observations of the L_i magnitude at constant parameters of conditions. The following method of determining the rock boundaries was proposed for this problem solution. Based on observations of the random discrete L_i process, the function is formed:

$$g_i = \frac{1}{\sigma_L^2}\left(L_i - \hat{L}\right)^2,$$

where

$$\sigma_L^2 = M\left[\left(L_i - \hat{L}\right)^2\right]$$

is dispersion of the additive obstacle, ν_L.

By the time of formation change, $\sigma_L = \sigma_L^{(1)}, L_i = L_i^{(1)}, \hat{L} = \hat{L}^{(1)}$ and the g_i, sequence at each step of observations coincides with the sequence

$$\left\{\frac{1}{\sigma_L^{(1)2}} v_{L_i}^{(1)}\right\},$$

where $v_L^{(1)} = L_i^{(1)} - \hat{L}^{(1)}$, hence, $M[g_i] = 1$.

If transition of the bit from one formation to another has occured at a discrete moment of time, i_0, then the equation $L_i = \dot{L}^{(2)} + v_L{}^{(2)}$, will take the following form taking into account that $L_i = \dot{L}^{(2)} + v_L{}^{(2)}$:

$$g_i = \frac{1}{\sigma_L^{(2)2}}\left(\Delta\hat{L} + v_{L_i}^{(2)}\right)^2,$$

where $\Delta\hat{L} = \hat{L}_i^{(2)} - \hat{L}^{(1)}$, $\sigma_L^{(2)2} = M\left(L_i - \hat{L}^{(2)}\right)^2$.

Estimation of mathematical expectation of random sequence (4) will be:

$$M[g_i]=(\Delta\widehat{L}^2+\sigma_L^{(2)2})/\sigma_L^{(1)2},\ \text{at } i>i_0.$$

Centering of g_i sequence has resulted in $M[g_i-1]=0$ at $<i_0$ and when $i>i_0$, the estimate of mathematical expectation of the g_i sequence was determined by formula (5).

At each step of observations, together with the discrete g_i sequence, it was proposed to calculate the function:

$$G_i=\sum_{R=1}^{i}\frac{g_R-1}{\sqrt{2i}}.$$

Since:

$$M[G_i]=\frac{1}{\sqrt{2i}}\sum_{R=1}^{i}M[g_R-1]=0,$$

then, until the moment of discrete time while the bit has not moved to another formation, mathematical expectation of the G_i function is zero. It was shown that after the bit has passed to the next layer,

$$M[G_i]=\frac{S_m(i-i_0-1)}{\sqrt{2i}},$$

where

$$S_m=\frac{\Delta\widehat{L}^2}{\sigma_L^{(1)2}}+\frac{\sigma_L^{(2)2}}{\sigma_L^{(1)2}}-1.$$

Hence, until the moment when the bit crosses fornation boundaries, the value of the G_i function varies near the mean value. After the bit has passed to another fornation ($i>i_0$), the $|G_i|$ sequence on average grows over time. Given this property of the G_i function, to determine the time moment i_0 of the bit transition to another formation, the procedure of comparing the G_i value at each time point with a certain threshold Δg was used. For estimation of the i_0 value for which the condition $|G_i|\geq\Delta g$ (where $\Delta g=2.5$) is fulfilled is taken.

It was shown that in order to reduce the error of estimating the time moment when the bit crosses boundary between two layers, it is expedient to jointly use the G_i function and the so-called Z-algorithm the main parameters of which are numbers N, α and c which must satisfy the following conditions: $N\geq N_0=[1/\alpha]$, $0<\alpha<0.5$, $0.5<c<1$ where $[1/\alpha]$ is the integer part of the number 1/2. At each step of observation of the random variable L_i, it was proposed to calculate the function:

$$Z(m,N)=\left|\frac{1}{m}\sum_{i=1}^{m}L_i-\frac{1}{N-m}\sum_{i=m+1}^{N}L_i\right|,$$

for which the rule of detecting formation boundaries is as follows:

$$d_N(i_0)=\begin{cases}1, \text{ if } \ \widehat{m}/N>c,\\ 0, \text{ if } \ \widehat{m}/N\leq c,\end{cases}$$

where $\dot{m}=i_0$ is the estimate of the moment when the bit crosses the border of two formations.

A procedure for determining the i_0 moment by Z-algorithm was proposed.

The minimum number of observations in which the algorithm operates is 4. Absolute error based on the results of the experiments conducted by the author at a depth of 1.500–2.865 m was $\Delta=1.57\div1.78$ m.

Disadvantage of this method is that the mean error value increases with growth of dispersion of the L_i sequence and the well depth. In addition, the weight of new information decreases with increase in the number of observations which reduces the rate of growth of the G_i function and accuracy of the algorithm.

This aalysis shows that the method of G_i function and Z-algorithm are suitable for detecting beginning of emergency but they do not allow one to create a model of dynamic stability of the well deepening process in conditions of a priori and current uncertainty the parameters and structure of the control object.

Consequently, the analysis has established that no results of mathematical modeling of dynamic stability of the well drilling process were found to date. At the same time, authors of papers [13, 15, 21] have shown that the main and final periods of the bit functioning in the well bottom are characterized by such indicators of the well deepening process as the time of drilling 1 m of rocks (h/m) or mechanical speed of drilling (m/h). This makes it possible to detect the moment of logical completion of the bit path on the basis of emergency of the «fold» type and G_i–Z-algorithm, the bit transition to a formation with other physical, mechanical and abrasive properties (productive formations, zones with abnormally high and abnormally low formation pressures, sinks, etc.) However, the scientific and practical problem of modeling dynamic stability of the well deepening process was not solved till now.

It was caused by significant limitations including inability of obtaining current information for automated control system and information on coordinate violations in on-line mode. This is explained by the fact that the well deepening process is a non-reproducible non-linear stochastically chaotic process that develops over time and influenced by various types of additive and multiplicative interferences and functions in conditions of prior and current uncertainty as for its parameters and structure.

In this regard, solution of the problem of constructing effective models of dynamic stability of the well deepening process should be based on a single holistic approach and such mathematical models should be phenomenological.

3. The aim and objectives of the study

The study objective was to develop a phenomenological mathematical model of emergency development dynamics in the well drilling process enabling identification of emergency beginning and ensuring safe operation of the system in general.

To achieve this objective, the following tasks were formed:

– to establish the main arguments for application of the catastrophe theories to modeling dynamic stability of the well deepening process;

– to study dynamic stability of the well deepening process on the basis of experimental data on change of the time taken for drilling 1 m rocks in deepening the well;

– to propose a structure of the basic phenomenological mathematical model of the emergency development dynamics by means of applying a holistic approach to solution of the problems of modeling dynamic stability of the well deepening process.

4. Materials and methods to study the dynamic stability of the well deepening process

The following methods, approaches and techniques for studying complex control objects form the methodological basis of this work:

– fundamentals of the catastrophe theory as a synergy methodology for evaluation and recognition of dynamic conditions of well deepening;

– holistic approach to analysis of the complex character of etiology of emergencies in the well deepening process and synthesis of the phenomenological model.

Methodological apparatus: a systematic approach and methods of mathematical analysis on the basis of which simulation of dynamic stability of the well deepening process was performed.

The set of methods and techniques used:

– Cardano formula: for solving canonical cubic equations;

– Correlation analysis: for choosing a phenomenological model of emergency development in the well deepening process;

– Graphical analysis: for visual presentation of theoretical and practical material.

5. Essence of the signs of emergencies that arise in the well deepening process as in a complex polyergotic system

Dynamic stability of the well deepening process is influenced by various factors: geological, technological and driller (human operator) errors.

Regarding the well deepening process, all emergencies (accidents and complications) can be divided into emergencies caused by:

– geological reasons;

– technological reasons;

– driller errors.

The first group can include such emergencies as seizure of the bit tools or collapse of the well walls. This group also includes complications: loss of flushing fluid in the well, narrowing of the well, gas escape, crossing of aquifers with high pressure, sinks, zones with abnormal formation pressures.

The second group of emergencies includes breakdown of the drill column, catastropheic wear of the bit tools or supports, and others.

As for the third group of factors, it was assumed [6] to identify two types of human errors: active errors and latent (hidden) malfunctions.

Since people design, produce and operate such complex technical systems as drilling rigs and manage them in conditions of uncertainty, their decisions and actions largely determine causes of emergencies.

Active errors, i. e. dangerous actions of the driller, include errors, delays in decision making, negligent interaction with mechanisms and procedural violations. Note that the effects of active errors are usually manifested instantaneously.

However, latent malfunctions which are always present in complex systems remain for a long time and are not harmless untill certain conditions appear. The latent malfunctions include bad design characteristics of the system, unidentified defects of some elements of the system, insufficient control of technical condition, inefficient automation functioning, etc. [6].

Situational factors may trigger activation of latent malfunctions. Besides, unpredictable nature of latent malfunctions may aggravate likelihood of active driller errors. It is known [5, 7] that the catastrophe theory enables prediction of situational changes that affect both the drilling process and behavior of entire system. At the same time, it is possible to predict non-quantitative occurrence of various processes: collapse of the well walls, catastrophic bit wear, zones with abnormal formation pressures. Since a catastrophe in drilling can be presented as a jump from one state to another, its main feature is that duration of this jump is very small compared with duration of the stable state, i. e. the well deepening process. For example, according to the data given in [15], the time spent for drilling 1 m of rock at a depth of 4.365–4.389 m varies within 4.13–9.51 minutes and duration of drilling of this interval is 152.98 min. That is, the mean mechanical drilling speed at this interval is 9.41 m/h and duration of emergencies ranges from 0.08 to 0.25 hours (4.8–15 min.).

Consider that the well drilling control system is multilevel. Any uncertainty or incident in the input parameters at a lower level leads to changes in the output parameters of the subsystems of higher levels and the system in general.

Most often, the R. Thom’s emergency takes place in practice, which was called the gather emergency [7].

Note that the most important issue in solving practical problems in the well drilling system is applicability of a particular mathematical apparatus to a particular problem under study.

However, under actual conditions of drilling deep wells, catastrophes occur and therefore it is important to be able to recognize them. Since there are only experimental data during consideration of emergencies and there is no commonly accepted theory for their interpretation, there is no ready-made potential function that could be studied. Moreover, even the form of the equation that describes it is unknown and such an equation cannot be derived. Consequently, presence of malfunctions or emergencies in the well drilling system being a complex polyergotic control system can only be determined by the presence of its attributes. Such an attribute in drilling is the time spent on drilling 1 m rocks in the well.

6. Modeling of dynamic stability of the well deepening process on the basis of the R. Thom’s catastrophe

Dynamic stability of the well deepening process is a changeable category prone to potential jump-like changes.

The main arguments for applying the catastrophe theory to modeling dynamics of the well drilling process are as the follows [9, 16–18]:

– the well deepening system is dynamic, nonlinear;

– the system tends to maintain its steady state for as long as possible;

– the current state of the system depends on initial conditions;

– the system trajectories are irreversible; the deepening process is unreproducible and develops in time;

– the system functions under conditions of priori and current uncertainty regarding the object parameters and structure;

– the well deepening process proceeds under the influence of various types of obstacles in conditions of deficiency of a priori and current information.

A catastrophe is a sharp abrupt change of the system behavior nature at a gradual change of its operation parameters which leads to the system degradation.

Consequently, in such rendition, emergencies can be considered as the periods of operation of the well drilling system at a catastrophic wear of the bit or its transition to the rocks with different properties. Catastrophes are also the collapse of the well walls, grip of the drill tool, the bit entry to a zone with abnormal formation pressure, exposure of productive levels, etc. Sharp deterioration of technical and economic indicators of drilling can also be identified as a catastrophe.

In order to predict types of emergencies in the future, it is necessary to determine the type of functional dependence using empirical data which can serve as a basis for prediction of changes in the well drilling control system outside the study period with a high degree of reliability.

To develop the method of recognition of dynamic conditions of the well deepening process and control this process, we shall use experimental data on the change of such important indicator of the well deepening process as the time T spent for drilling 1 m rocks in the well. Time T which is spent for drilling each meter of a well is one of the factors influencing the cost of drilling one meter of well [20]:

$$B_s = B_h T + \left(B_h t_{sp} + B_d\right) h_d^{-1}, \text{ conv.un./m},$$

where B_h is the cost of one hour of operation of the drilling unit, conv. un., $T = t_b/\Delta h$ is the time spent for drilling one meter of rock in the well, h/m, t_{sp} is the time spent on the descent-lifting operations connected with replacement of the bit and extension of the drill column as well as auxiliary operations connected with the bit travel, h, t_d is the drilling time, h., B_d is price of the drill, conv.un., h_d is the bit footage, m.

It is seen from the formula that the first component of cost $B_h \cdot T$ of one meter of drilling in the process of drilling isotropic rocks is linearly increasing while the second component $\left(B_h t_{sp} + B_d\right) h_d^{-1}$ decreases according to the hyperbole law. These regularities do not change with decrease or increase in the cost of one hour of the drilling rig operation, B_h, or the bit cost, B_d, and the time of descent-lifting operations. However, the first component depends essentially on the T parameter which in turn depends on both the control actions and the properties of the rocks that are drilled.

A prerequisite for changing control actions is not only the desire to provide a process with minimum costs but also a jump-like change of the T indicator.

The data of more than 20 wells drilled in the Trans-Carpathian region have been analyzed.

The graph of change of the time taken to drill 1 m of rock on one of these wells is shown in Fig. 1. The presence of such sign of drilling emergency as a catastrophic jump of the T indicator was recorded by researchers long ago [9, 14, 15].

If we consider the hypothesis of existence of emergency in the well deepening process, diversity of the catastrophe may only take the form of «gather» [6] since any features represent a set of individual «gathers» and «folds». Note that among the elementary emergencies described by R. Thoma, only the «gather» has two guiding parameters which meet conditions of functioning of the well deepening process.

Therefore, for a qualitative assessment of dynamic conditions of well deepening, we shall use the elementary catastrophe of the «gather» type (Table 1) by equating the first, second and third derivatives of $F(T_\Sigma, a, b)$ to zero:

$$F(T_\Sigma, a, b) = \frac{1}{4}T_\Sigma^4 + \frac{1}{2}aT_\Sigma^2 + bT_\Sigma, \tag{3}$$

$$T_\Sigma^3 + aT_\Sigma + b = 0, \tag{4}$$

$$3T_\Sigma^2 + a = 0, \tag{5}$$

$$6T_\Sigma = 0, \tag{6}$$

where a, b are the coefficients characterizing mining and geological conditions and technological parameters of drilling; they can be determined by the method of least squares according to empirical data.

The ability of recognition of dynamic conditions of the well deepening process progress is based on the following prerequisites.

If the system behavior can be described by a differential equation (4), then the set of jump-like changes of conditions is determined from the condition:

$$\frac{dT_\Sigma}{dh} = \frac{d^2T_\Sigma}{dh^2} = 0, \tag{7}$$

$$\frac{d^2T_\Sigma}{dh^2} = 3T_\Sigma^2 + a = 0, \tag{8}$$

where dT_Σ / dh is the intensity of change of the time taken for drilling 1 m of rocks in the well, h.

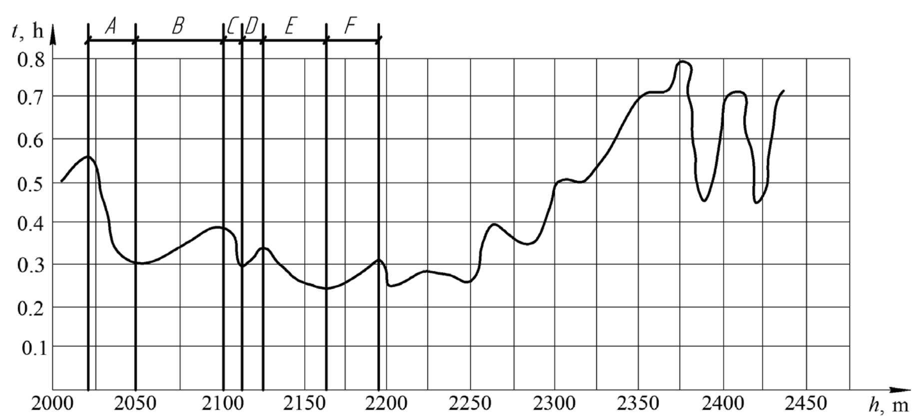


Fig. 1. Graph of change in the time of drilling 1 m rocks in the well t, h: $\sum t = T_\Sigma$ is total time of drilling the well as it is deepened

Condition (7) is satisfied at critical points, and the condition (8) – in doubly degenerated critical points.

Positions of critical points are found by solving equation (4) which defines a two-dimensional diversity in a three-dimensional space with coordinate axes $T_\Sigma - a - b$.

The cubic equation roots can be determined by the Cardano formulas [19].

According to the Cardano formula, the cubic equation roots (4) in the canonical form:

$$T_{\Sigma_1} = \alpha + \beta, \tag{9}$$

$$T_{\Sigma_{2,3}} = -\frac{\alpha+\beta}{2} \pm \frac{\alpha-\beta}{2}\sqrt{3}, \tag{10}$$

where

$$\alpha = \sqrt[3]{-\frac{b}{2} + \sqrt{Q}}; \tag{11}$$

$$\beta = \sqrt[3]{-\frac{b}{2} - \sqrt{Q}}; \tag{12}$$

$$\alpha\beta = -\frac{a}{3}; \tag{13}$$

$$Q = \left(\frac{a}{3}\right)^3 + \left(\frac{b}{2}\right)^2. \tag{14}$$

The values of α and β are auxiliary roots, so they have three values. From all possible pairs of α and β, one should choose those that satisfy equation (13).

The discriminator of a polynomial $T_\Sigma^3 + aT_\Sigma + b$ is equal in this case to $\Delta = -108Q$.

When applying formulas (9)÷(14) to each of the three values of α, it is necessary to take such β that the condition $\alpha\beta = -a/3$ is fulfilled. Such value of β always exists.

By using the Q values, one can identify stable conditions of well deepening and unstable chaotic conditions which can bring about the well curvinge and deflection of the well bottom from the target point.

Indeed, if all coefficients of the cubic equation (4) are valid, then Q is valid and the type of roots can be defined by its sign:

– $Q>0$: one real root and two dual, complex roots; dynamic conditions of well deepening are stable;

– $Q=0$: one single real root and one double roots or if $a=b$, then one triple real root; the system is at the stability limit, approaching the unstable state that indicates complexity of the well deepening conditions;

– $Q<0$: three real roots, deepening conditions are unstable, chaotic which can lead to the well curving.

Thus, the algorithm of solution of equation (4) is as follows:

– first, determine any value $\sqrt{Q}$;

– calculate three values of the cubic root, α;

– calculate β value for each value of α by means of formula (12):

$$\beta = \frac{a}{3\alpha}.$$

As a result, three pairs of α and β quantities are obtained.

Values of equation (4) roots are found for each pair of quantities α and β by the formula $T_\Sigma = \alpha + \beta$.

Condition $\alpha\beta = -a/3$ must be fulfilled for each pair of roots. To solve the canonical cubic equation (4), the following Cardano formula must be applied [19]:

$$T_\Sigma = \sqrt[3]{-\frac{b}{2} - \sqrt{\frac{b^2}{4} + \frac{a^3}{27}}} + \sqrt[3]{-\frac{b}{2} + \sqrt{\frac{b^2}{4} + \frac{a^3}{27}}}, \tag{15}$$

where

$$\frac{b^2}{4} + \frac{a^3}{27} = D$$

is a discriminant of the cubic equation.

Solution of equation (4) is given in the form: $T = \alpha + \beta$. In complex numbers, the cubic root has three different meanings. To obtain solutions, it is necessary to select such pairs of values of the cubic root that $\alpha\beta = -a/3$. Such pairs are necessarily found to be exactly 3. For a substantiated choice of the model, it is necessary to compare models of R. Thom's elemental emergencies with other models.

7. Comparison of models for description of emergencies in the well deepening process

To compare the R. Thom's model of elemental catastrophes of the «gather» type with other models, consider the largest of the catastrophes shown in Fig. 1. For the study, take the following ranges: 2,389÷2,408 and 2,408÷2,420 m.

The results of comparative analysis of the R. Thom's mathematical model of «gather» type with other models are given in Tables 2, 3.

From the set of models given in Tables 2, 3, the Vapor Pressure model was selected. It has provided the highest correlation coefficient and minimum mean square error of the test emergency approximation:

$$y = e^{a + \frac{b}{x} + c\ln x}, \tag{16}$$

where y is the generalizing synergetic factor, i.e. the time of drilling 1 m of rocks; a, b, c are the model parameters; x=1, 2, 3..., k is the well depth.

To compare the R. Thom's model of elementary catastrophe of the «gather» type with the Vapor Pressure model, consider six catastrophic sections of the process, $t(h)$ shown in Fig. 1 (A: 2,020÷2,050 m; B: 2,050÷2,100 m; C: 2,104÷2,116 m; D: 2,116÷2,124 m; E: 2,125÷2,165 m; F: 2,165÷2,195 m).

For each studied section of the process, $t(h)$, parameters of the models and all indicators characterizing quality of approximation of the dependence $t(h)$ were determined for R. Thom's model of «gather» type and the Vapor Pressure model.

The results of approximation of experimental data obtained at different depths of the well are given in Table 4.

Analysis of the data shown in Table 4 shows that the mathematical model (16) adequately and more accurately reflects relationships with the correlation coefficients:

$$R - \text{square} = 0{,}8877 \div 0{,}996.$$

This enables effective solution of the problem of identifying in real time parameters of the mathematical model of emergencies.

Table 2

The results of comparative analysis of the R. Tom's mathematical model of the «gather» type with other models in the range 2,389÷2,408 m

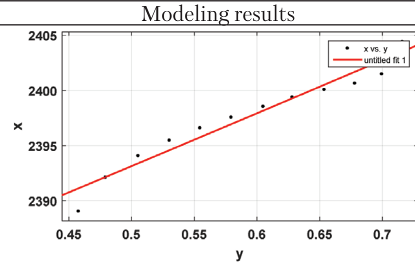
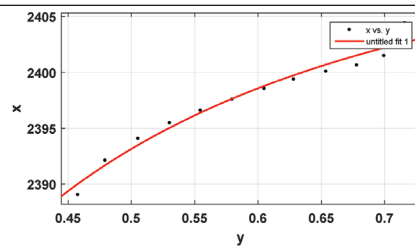
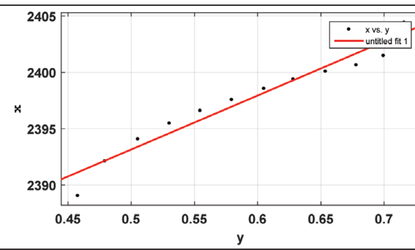
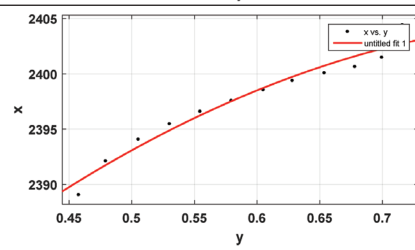
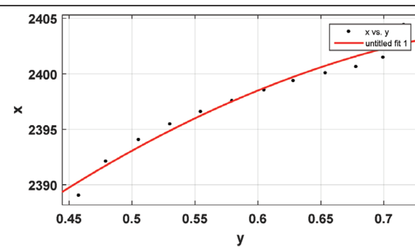
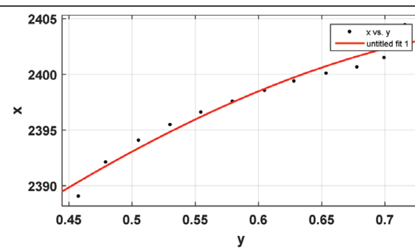
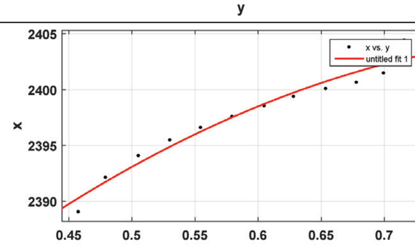
No.	Model type	Modeling results
1	**R. Thom** General model: f(x)=x^3+a*x+b Coefficients (with 95% confidence bounds): a=46.95 (39.05, 54.85); b=2370 (2365, 2374) Goodness of fit: SSE: 10.46 R-square: 0.9482 Adjusted R-square: 0.9431 RMSE: 1.023	
2	**Vapor Pressure** General model: f(x) = exp(a+b/x+c*log(x)) Coefficients (with 95% confidence bounds): a=7.796 (7.784, 7.807); b=-0.00911 (-0.02406, 0.00584) c=-0.004233 (-0.03041, 0.02194) Goodness of fit: SSE: 6.54 R-square: 0.9676 Adjusted R-square: 0.9604 RMSE: 0.8525	
3	**Exponential** General model Exp1: f(x)=a*exp(b*x) Coefficients (with 95% confidence bounds): a=2369 (2365, 2374); b=0.02002 (0.01674, 0.0233) Goodness of fit: SSE: 10.37 R-square: 0.9487 Adjusted R-square: 0.9435 RMSE: 1.018	
4	**Fourier** General model Fourier1: f(x)=a0+a1*cos(x*w)+b1*sin(x*w) Coefficients (with 95% confidence bounds): a0=-4.983e+07 (-3.582e+15, 3.582e+15); a1=4.984e+07 (-3.582e+15, 3.582e+15) b1=7.966e+04 (-2.862e+12, 2.862e+12); w=0.001816 (-6.526e+04, 6.526e+04) Goodness of fit: SSE: 7.515 R-square: 0.9628 Adjusted R-square: 0.9488 RMSE: 0.9692	
5	**Gaussian** General model Gauss1: f(x)=a1*exp(-((x-b1)/c1)^2) Coefficients (with 95% confidence bounds): a1=2405 (2397, 2413); b1=0.879 (0.5198, 1.238) c1=5.39 (2.086, 8.694) Goodness of fit: SSE: 7.526 R-square: 0.9627 Adjusted R-square: 0.9545 RMSE: 0.9145	
6	**Linear Fitting** Linear model: f(x)=a*(sin(x-pi)) + b*((x-10)^2) + c Coefficients (with 95% confidence bounds): a=-322.2 (-765.5, 121.2); b=11.66 (-7.9, 31.22) c=1186 (-791.8, 3164) Goodness of fit: SSE: 7.82 R-square: 0.9613 Adjusted R-square: 0.9527 RMSE: 0.9321	
7	**Polynomial** Linear model Poly2: f(x)=p1*x^2 + p2*x + p3 Coefficients (with 95% confidence bounds): p1=-82.19 (-183.3, 18.89); p2=144.7 (25.57, 263.8) p3=2341 (2307, 2376) Goodness of fit: SSE: 7.515 R-square: 0.9628 Adjusted R-square: 0.9545 RMSE: 0.9138	

Table 3

The results of comparative analysis of the R. Thom's mathematical model of the «gather» type with other models in the range 2,408÷2,420 m

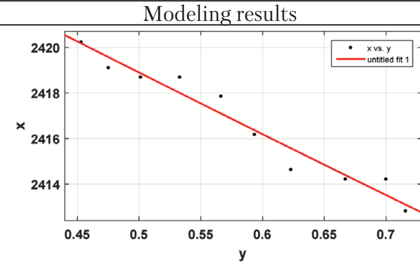
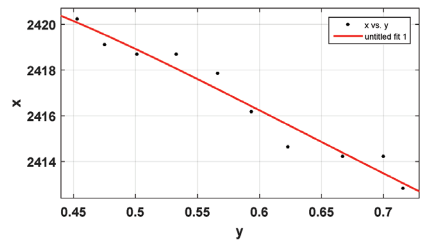
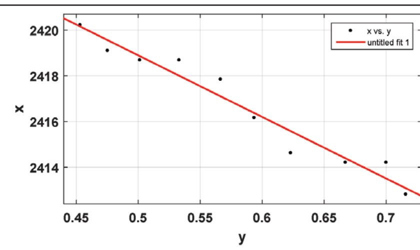
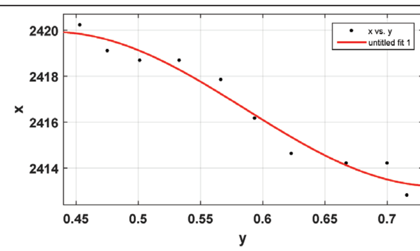
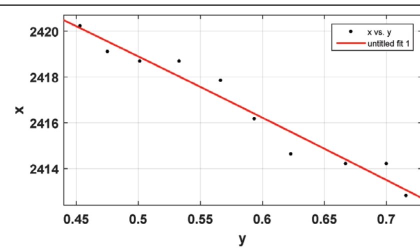
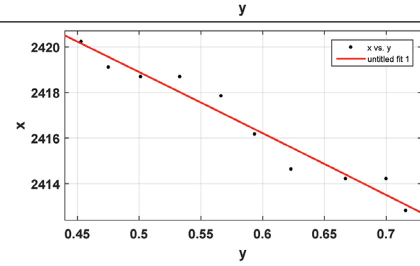
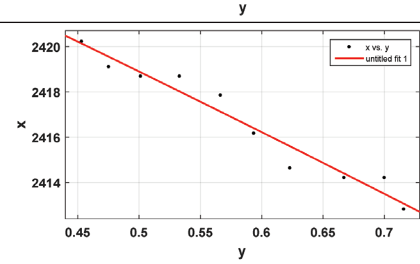
No.	Model type	Modeling results
1	**R. Thom** General model: f(x)=x^3+a*x+b Coefficients (with 95% confidence bounds): a=-27.97 (-32.85, -23.08); b=2433 (2430, 2436) Goodness of fit: SSE: 2.799 R-square: 0.9529 Adjusted R-square: 0.947 RMSE: 0.5915	
2	**Vapor Pressure** General model: f(x)=exp(a+b/x+c*log(x)) Coefficients (with 95% confidence bounds): a=7.79 (7.781, 7.799); b=-0.004638 (-0.01597, 0.006693) c=-0.01457 (-0.03454, 0.005399) Goodness of fit: SSE: 2.743 R-square: 0.9538 Adjusted R-square: 0.9406 RMSE: 0.626	
3	**Exponential** General model Exp1: f(x)=a*exp(b*x) Coefficients (with 95% confidence bounds): a=2432 (2430, 2435); b=-0.01114 (-0.01316, -0.009122) Goodness of fit: SSE: 2.796 R-square: 0.9529 Adjusted R-square: 0.947 RMSE: 0.5912	
4	**Fourier** General model Fourier1: f(x)=a0+a1*cos(x*w) + b1*sin(x*w) Coefficients (with 95% confidence bounds): a0=2417 (2415, 2418); a1=-1.276 (-21.09, 18.54) b1=-3.112 (-9.225, 3.001); w=10.06 (-0.4398, 20.55) Goodness of fit: SSE: 2.399 R-square: 0.9596 Adjusted R-square: 0.9394 RMSE: 0.6323	
5	**Gaussian** General model Gauss1: f(x)=a1*exp(-((x-b1)/c1)^2) Coefficients (with 95% confidence bounds): a1=2487 (482.7, 4491); b1=-4.55 (-149.3, 140.2) c1=30.37 (-397.4, 458.1) Goodness of fit: SSE: 2.793 R-square: 0.953 Adjusted R-square: 0.9395 RMSE: 0.6317	
6	**Linear Fitting** Linear model: f(x)=a*(sin(x-pi))+b*((x-10)^2)+c Coefficients (with 95% confidence bounds): a=-11.73 (-331, 307.6); b=1.948 (-12.15, 16.05) c=2237 (811.7, 3663) Goodness of fit: SSE: 2.796 R-square: 0.9529 Adjusted R-square: 0.9395 RMSE: 0.632	
7	**Polynomial** Linear model Poly2: f(x)=p1*x^2 + p2*x + p3 Coefficients (with 95% confidence bounds): p1=-2.416 (-76.26, 71.43); p2=-24.09 (-110.8, 62.62) p3=2432 (2407, 2457) Goodness of fit: SSE: 2.793 R-square: 0.953 Adjusted R-square: 0.9395 RMSE: 0.6317	

Table 4

The results of comparative analysis of mathematical models: R. Thom's «gather» type model and the Vapor Pressure model

Well depth	Model type	Model parameters			Coefficient of correlation	RMS error
		a	b	c		
A	$f(x)=x^3+ax+b$	–79.12	2,067	–	0.906	2.328
	$f(x)=e^{a+\frac{b}{x}+c\ln x}$	7.6	0.010	0.008425	0.937	1.992
B	$f(x)=x^3+ax+b$	468.8	1,913	–	0.985	1.916
	$f(x)=e^{a+\frac{b}{x}+c\ln x}$	7.721	–0.004	0.068	0.986	1.933
C	$f(x)=x^3+ax+b$	–75.43	2,135	–	0.842	1.139
	$f(x)=e^{a+\frac{b}{x}+c\ln x}$	7.635	–0.029	–0.098	0.879	1.094
D	$f(x)=x^3+ax+b$	228.4	2,046	–	0.985	0.487
	$f(x)=e^{a+\frac{b}{x}+c\ln x}$	7.689	–0.026	–0.044	0.987	0.548
E	$f(x)=x^3+ax+b$	–3337.6	2,240	–	0.928	3.138
	$f(x)=e^{a+\frac{b}{x}+c\ln x}$	7.665	0.061	0.169	0.989	1.32
F	$f(x)=x^3+ax+b$	390.3	2,073	–	0.987	1.114
	$f(x)=e^{a+\frac{b}{x}+c\ln x}$	7.725	–0.028	–0.050	0.997	0.616

At the initial stages of the emergency development when there is an increase in time of drilling 1 m of rocks, it is possible to simulate the emergency dynamics by means of its model (16).

In this case, the constant a will be positive and the constants b and c negative. During further deepening of the well, when there is a decrease in time of drilling 1 m of rocks the emergency dynamics can be described by the same law but constant a will be negative and the constants b and c become positive.

The model (16) differs fundamentally from other models in the following: in a case of its use at every step, h, (for example, $h=0.2$ m) in real time, all coefficients of the model are adjusted according to the well drilling conditions when new information comes.

That is, the coefficients a, b, c change with the change of the main indicators of the well deepening process. Parameter a is responsible for the general amplitude of the simulated curve. Parameter b indicates the moment of «emergency « (in our case, this is the extremum of the curve $y=f(x)$). Parameter c indicates logarithmic trend of the curve.

Consequently, vector of the model (16) parameters, $(a,b,c)=\bar{G}$, defines essence of participation of each component of the deepening process in the emergency formula. Vector $\bar{G}$ combines all external factors that influence the emergency dynamics: physical, mechanical and abrasive properties of the rocks, technical state of the bit, parameters of the irrigation liquid, formation pressure, temperature in the well, etc. These factors depend on the well depth.

Thus, use of the model (16) makes it possible to more accurately simulate dynamics of emergencies in the well deepening process and effectively solve the problem of identifying parameters of the model by a single algorithm.

8. Discussion of results obtained in the simulation of dynamic stability of the well deepening process

The merits of the study include establishment of the fact that the catastrophe theory makes it possible to assess current state of the well deepening process from the positions of local or global stability. The possibility of using mathematical apparatus of catastrophe theories to describe dynamic stability of the well deepening process, in particular, the R. Thom's elemental catastrophe of the «gather» type and the Vapor Pressure model has been proved. The content of the «time of drilling 1 m of rock» was revealed as a diagnostic criterion of potentially hazardous drilling situations.

The main disadvantage of the proposed method of modeling dynamic stability of the «gather» type is the necessity of determining the type of functional dependence and discriminants of the cubic equation with the help of empirical data and a retrospective analysis since this procedure takes some time to be minimized.

Therefore, by the use of a holistic approach to solving the problems of modeling dynamic stability of the well deepening process, phenomenological mathematical model of dynamics of solution of the problems of emergencies such as Vapor Pressure was substantiated and chosen. The obtained results are useful because it is possible to recognize conditions of well deepening based on the phenomenological model. They can be useful in prediction of loss of stability of the system with a high degree of reliability.

Further studies should be conducted in the direction of study of chaos in the well deepening process and the use of synergetic principles in analysis and optimal process control based on modern computer-integrated technologies. Introduction of modern tools of catastrophe theory and a phenomenological model for studying dynamics of the well deepening process will enable creation of an effective strategy for automated control of drilling oil and gas wells.

9. Conclusions

1. The main arguments for application of the catastrophe theory for modeling of dynamic stability of the well deepening process were established:

– the well deepening system is nonlinear dynamic, stochastically chaotic;

– the system tends to maintain its steady state for as long as possible;

– the current state of the system depends on the initial conditions;

– the well deepening process is non-reproducible, develops in time, the system trajectories are irreversible;

– the system functions in conditions of priori and current uncertainty of parameters and structure of the object;

– the well deepening process is influenced by various types of obstacles in conditions of absence of shortage of a priori and current information on parameters of the object, geological environment and its structure.

2. Dynamic stability of the well deepening process was studied on the basis of experimental data on change of time *T* spent for drilling 1m of rocks at depth *h* of the well. For a qualitative evaluation of dynamic conditions of well deepening, R. Thom's elemental catastrophe of the «gather» type was used. It was shown that if the system behavior can be described by a canonical cubic equation, then the condition for a stepwise change of these conditions is:

$$dT/dh = d^2T/dh^2 = 0.$$

Solution of this equation is based on the Cardano formulas. Comparison of mathematical models of R. Thom's elementary catastrophes and the complex of models of the Curve Expert software has shown that the phenomenon of emergency in drilling is described more precisely by the Vapor Pressure model.

3. The ability of recognizing dynamic conditions of well deepening on the basis of controlling the change of time spent for drilling 1 m of rock in the well and carry out qualitative assessment of dynamic conditions of well deepening more precisely than with application of the R. Thom's elemental catastrophe was established. The model enables identification of parameters by a single algorithm. According to the results of experimental studies, theoretical conclusions regarding the choice of the model type as optimal for the description of emergencies in drilling were confirmed. It was established that the proposed phenomenological model is adequate to real processes.

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