UDC 65.011.56:51.74:519.685:519.171.2
DOI: 10.15587/1729-4061.2018.140636

DEVELOPMENT AND INVESTIGATION OF METHODS OF GRAPHIC-FUNCTIONAL MODELING OF DISTRIBUTED SYSTEMS

A. Boinik
Doctor of technical sciences,
Professor, Head of Department*
O. Progonnij
PhD, Associate Professor
Department of electric power engineering,
electrical engineering and electromechanics**
O. Kameniev
PhD, Associate Professor*
E-mail: alexstein@kart.edu.ua
A. Lapko
PhD, Associate Professor*
V. Kustov
PhD, Professor, Director***
D. Kuzmenko
PhD, General Director
Scientific and Production Association
«Railwayautomatic», Ltd
Nauky ave., 36, Kharkiv, Ukraine, 61166
O. Shcheblykina
Junior researcher
Department of technological and software***
*Department of automatic and computer
remote control of train traffic**
**Ukrainian State University
of Railway Transport
Feierbakha sq., 7, Kharkiv, Ukraine, 61050
***Scientific and Production
Enterprise «SATEP», Ltd
Tankopiya str., 13/4,
Kharkiv, Ukraina, 61091

У процесі дослідження розроблено метод геометричного моделювання розподілених систем і пов'язаних з ними технологічних об'єктів. В основу методу закладено використання функціональних графів. В контексті дослідження основними відмінностями таких графів є: моделювання технологічних об'єктів розподілених систем виключно вершинами, без застосування ребер для відтворення зазначених об'єктів; використання ребер виключно для відтворення зв'язків між об'єктами. Зважування вершин зазначених графів виконується за допомогою призначених функцій або функціоналів з повною відсутністю виваженості ребер.

На відміну від найближчих аналогів, в основу аналітичної інтерпретації сформованих графічних моделей в пропонованому методі закладені не матриці інцидентності, а параметрично-топологічні матриці суміжності. В таких умовах істотно змінюється принципи присвоєння вагових коефіцієнтів елементів графа: замість позиційного розподілу елементів вагових множин між комірками матриць використовується завдання зазначених елементів в якості аргументів функцій у складі функціональних вершин. При зазначеному підході застосований діагональний спосіб прописування функцій або функціоналів вершин в матриці суміжності. Завдання зв'язків між елементами графа при аналітичній інтерпретації виконується по введеному позиційному принципу із застосуванням додатної або від'ємної логіки. При такому підході досягається можливість аналітичного формування множинних зв'язків між вершинами з довільною кількістю і спрямованістю, що раніше не застосовувалося в складі матриць суміжності. Крім цього, присвоєння елементам графа функціональних залежностей дозволяє відтворювати в складі геометричної моделі не тільки статичних, але й динамічних характеристик об'єктів, що моделюються.

Практична цінність пропонованого методу полягає в підвищенні універсальності і спрощенні процедур автоматизованої конфігурації програмного забезпечення систем керування. Досягнення такого результату можливе за рахунок скорочення обсягу даних, що вводяться, і можливості введення додаткових функцій об'єктів керування без правки вихідного коду. Додатково забезпечується вдосконалення формалізованого складання технічних завдань при розробленні технічної документації та апаратного забезпечення розподілених систем. Крім того, можлива інтеграція методу в існуючі системи САЕ і САПР, що забезпечує нарощування можливостей і створення принципово нових таких систем.

Подальший розвиток запропонованого методу полягає у розв'язанні питань, пов'язаних з оптимізацією розподілу аргументів функцій вершин по комірках парамерично-топологічних матриць

Ключові слова: графічна модель, функціональний граф, параметрично-топологічна матриця, вагові параметри, розподілена система

1. Introduction

An important area of scientific and applied research in the field of technical sciences and the application of relevant results is the modeling of complex control systems and their technological objects. The most common is the variation of the distribution of such systems in space (taking into account both geographical and conditional coordinate features), in particular in transportation, energy, industry, etc. [1–3]. Thus, it is convenient and effective to consider the application of geometric modeling methods. The latter are based, for the most part, on the use of the apparatus of the theory of graphs [4, 5]. The analytical interpretation of the graphic models constructed in this way is carried out with the use of topological or parametric-topological matrices. The basis of these matrices is the systemic anatomical properties of the graphs, such as: incidence, adjacency, cyclicity, etc. [5].

The application of graph-analytic methods for modeling distributed systems and objects lies in two main directions:

– determination and research of system properties of information-control complexes, technological and infrastructure objects with the aim of the further practical usage (during construction, operation, maintenance, repair, etc.);

– direct use, including in terms of integration with the external means of the user interface, in various types of automated design of information management systems, system components of different levels, infrastructural and technological objects.

The most practically useful, demanded and promising is the specific use of graph-analytical modeling. Such modeling is the basis for the formation of new computer-aided design systems (CAD) for various purposes and use as part of well-known computer-aided engineering systems (CAE).

Among them, the key software and hardware tools are designed to automate the configuration of information management systems through distributed objects. The essence of the latter is the formation of the image of the technological object in the system in the form of a geometric model. The mentioned model has to fully reproduce the properties of the object within the functional purpose of the system [6, 7].

In recent years, the theory and methodology of graph-analytical modeling had a significant development. This is explained, above all, by the intensification of the distribution and the volume (array) increase of the functional capabilities of information technologies in all spheres of life and in the management of responsible technological processes in particular. In spite of this, the existing methods and means of graph-analytical modeling have a number of significant drawbacks and limitations on the effectiveness of the application. Among them, the most critical are: the complexity and bulkiness of analytical interpretation of the models of large-scale distributed systems, and the limited ability to reproduce the dynamic characteristics of prototype models.

Thus, the research aimed to simplify the graph-analytic reproduction of distributed systems and reduce its limitations becomes relevant. Hypothetically, this problem should be solved using the properties of functional graphs.

2. Literature review and problem statement

A lot of works have been devoted to the study of methods and means of using the theory of graphs and topological matrices during the modeling of technological systems and objects. The results of the relevant studies are based, first of all, on objects distributed on a plane or in space (railway stations, interconnected technological units, etc.). The applied research topics of recent years come down mainly to the introduction of programmable processing of mathematical models through computing facilities.

One of the latest studies related to the use of graphic modeling of distributed systems is published in the work [8]. It uses an acyclic graph for modeling the directional connections between the components of the distributed system. The reproduction of the dynamic characteristics of distributed structure objects in the work is based on the use of methods of differential topology. On its basis for each element of the graph, an alternating weight is determined, which depends on the current state of the nonlinear system. On the basis of the application of various laws of the distribution of random variables, which include weight coefficients of a graphic model, a set of its static and dynamic properties is reproduced. However, the results of such research can be applied only for «fuzzy» (stochastic) systems, the use of which is limited in the management of responsible technological processes.

A similar approach, but in relation to electronic circuits, is proposed in the work [9]. The graphic models implemented in it based on the fuzzy character reproduce and change the connections between the elements depending on the current state of the electronic keys (transistors). Thus, the dynamic conversion of the link graph determines the operation of the electronic circuit in different modes.

The use of graph-analytic modeling for the formation of cellular automata that reproduce the work of complex distributed systems is considered in the paper [10]. It takes into account the variable nature of the transitions between the elements of the graph on the basis of the dynamic states of cellular automata, implemented on the basis of an example of traffic flow regulation. The analytical interpretation of graphs is based on the classical matrices of adjacency and matrices of routes. Full reproduction of the elements of the graphic model is performed with a pair of such matrices, which complicates the programmable processing of the model.

In the aspect of the use of graph-analytical modeling as the fundamental basis of automated design of technical systems, a corresponding research is published in the paper [11]. It proposes the use of graphic programming languages, which are contained in the corresponding CAD systems of complex systems. But it also suggests the use of typical interfaces that cannot take into account the specificity of each individual system, and there is no analytical representation of the created models.

The use of dynamic graphs for modeling and researching traffic flows is considered in the paper [12]. The process is based on a combination of dynamic graphs and hybrid automata in a single geometric model. On its basis the density of transport protocols is estimated in certain elements of the graphically modeled transport network through the probability characteristics calculated on the basis of Lyapunov's theorem. Thus, the proposed model is a prototype of a functional graph, which can be extended to geographically distributed networks with a differentiated distribution density of any kind of flows (information, transport units, technological fluids, etc.).

A similar problem is solved in the study given in the paper [13]. Unlike the work [12], it uses the distributed status of moving objects traffic instead of its point density as the basis for it. The paper involves a hierarchical structure of a graphic model that has four components. Each of them interprets the characteristics of flows, the places of change of directions, speed and time of their change. The results of the work are integrated into a single geometric model. The result of the model is the simulation of moving objects with different kinematic parameters, which gives the possibility of using it in solving a wide range of tasks.

The paper [14] operates with the use of induced graphs in the problems of designing systems reproduced by the networks of conditional preference (CP-networks). In the paper [14], a method for calculating the triple sequence of the induced graph of the CP-network is proposed. As a result, it is possible to reduce the dimension of graphic models (in terms of the number of vertices and edges) for reproduction of technological systems and objects of arbitrary purpose.

The paper [15] considers the aspects of the use of graphic models for designing and researching information management systems on the railway transport. In particular, the method of graph-analytic designing of software for control systems by means of reproduction of topological development of transport infrastructure objects is proposed. The method is based on the reproduction of the weight parameters of a graphic model in parametric-topological matrices based on the principle of positioning cells. Subsequently, the method of analytical reproduction of graphs of large dimension is proposed by means of determination of indirect sums of matrix blocks. However, the model specializes in reproducing exclusively static characteristics of technological objects. Dynamic properties should be laid down in a constant program code.

A similar approach implemented for the diagnostics of complex dynamic systems based on a weighted decentralized graph is proposed in the paper [16]. The trajectories for transitioning a system to an intermediate state that occurs after failures and malfunctions is simulated with the use of it. The Monte Carlo method determines the probabilistic characteristics of various trajectories of the simulated system behavior.

Moreover, the simulation and optimization of traffic flows on the basis of a directed acyclic graph is proposed in the paper [17]. The model determines the correlation between the behavior of road users, with a built-in model of decision support. This allows increasing the efficiency of managing and controlling the dynamic behavior of distributed systems.

The paper [18] proposes a theoretical multiplicity representation of distributed systems based on the laws of fuzzy logic, which is an alternative to the graphical representation. The routes thus determined between the components of the system are in fact mediated by the interpretation of its graphic model. As a result, it is possible to partially reduce the number of computations with such modeling.

The issues of improving the interpretation of the matrix of geometric models based on random graphs are considered in the paper [19]. Similarly to the paper [15], the block representation of the topological matrix of a continuous graph is proposed. However, the methods of its reverse synthesis are not considered. Moreover, the allocation of multiple weight parameters to graph elements is not taken into account. Instead, the model partially discloses the functional properties of the graph elements due to their digital encryption.

The paper [20] proposes a graphical model for storing file data on geographically distributed carriers in the conditions of deduplication. The model involves the reproduction of the simultaneous use of distributed informational content, and the weight of the graph elements determines the information load of each of the distributed resources. The model can be used to recreate arbitrary characteristics of distributed systems in the conditions of belonging of the respective components to the class of tolerance.

The generalization of methods, models and algorithms of graph-analytical automated designing of infrastructure objects based on the example of railway stations is given in the monograph [21]. Similar to how it is done in the paper [15], the geometric representation of the topological development of an infrastructure object is foreseen in the paper [15]. However, the analytical reproduction is performed not on the basis of topological matrices, but on the basis of separate lists of incidence and weighted parameters of vertices and edges. This reduces the universality of mathematical processing of graphic models in a certain way, which is a significant disadvantage of the proposed approaches.

The monograph [22] explains in some way the updated graph-analytical methods and models proposed in the work [21] in the part of computer realization and scope of application. In particular, it has implemented approaches to the use of these types of simulation in the created simulators and other training complexes for operational personnel related to the management of infrastructure objects (on the example of railway stations).

Either way, it is necessary to generalize the methods, models and means of graph-analytical modeling of distributed technological systems and objects, set forth in the writings [8–22]. In the aspect of the application of these categories in order to research the work and automated design of information management systems, one can distinguish such general limitations and disadvantages:

– lack of the ability to reproduce the necessary completeness of dynamic characteristics, which is necessary for automated designing (configuration) of software tools;

– the complexity of positioning the weight parameters of graphic models on the cells of parametric-topological matrices;

– high probability of errors of model builders for specific objects or systems as a result of bulkiness;

– restrictions on the ratio of the power of sets of weight parameters of graph elements and the dimension of matrix blocks that reproduce graphic models in an analytical way.

Consequently, further improvement of graph-analytical simulation methods is to partially eliminate these restrictions. This is possible due to the use of functional graphs, the vertice and edge functionals of which reproduce both static weight parameters and dynamic properties of the reproduced objects.

3. The aim and objectives of the research

The aim of the research is to develop a method of graph-analytical modeling of geographically distributed complex systems and technological objects, based on the use of functional graphs. The method should provide both a reproduction of the dynamic properties of the modeling system, and to consolidate and simplify the display of static weights.

To achieve the aim, the following objectives are accomplished:

– to determine the basic geometric model and the method of its analytical interpretation in terms of topology, taking into account the properties on the basis of which the graph-functional model of the system will be developed;

– to determine the method of functional reproduction of static and dynamic characteristics, which are modeled by the elements of the graph;

– to develop a method of analytical reproduction of a graph-functional model and to consider variations in its implementation;

– to formulate recommendations and give examples of practical application of the developed method of functional graph-analytical modeling.

4. Materials and methods of graph-functional modeling of distributed technological systems and objects

4. 1. Basic geometric model of a distributed system

Based on these methods and principles, a technological object or system is represented by ordered sets of components of $A=\{a_i\}$, the connections between them $Z=\{z_{i,i+1}\}$ and the functional properties of the components $U=\{u_1\}$.

Thus, each element $a_i \in A$ corresponds to a single element $z_{i,i+1} \in Z$, which connects it to the other element $a_{i,i+1} \in A$. In addition, each element $a_i \in A$ is connected to the set of elements $\{u_{ij1}\} \in U$, which set its properties:

$$\begin{cases} (\forall a_i \in A) \rightarrow (\exists! z_{i,i+1} \in Z): a_i(z_{i,i+1})a_{i+1}, \\ (\forall a_i \in A) \rightarrow (\exists \{u_{ij}\} \in U): a_i(u_{ij})\{f_{ijl}\}, \end{cases} \quad (1)$$

where $\{f_{ij}\}$ is the set of element a_i functions, which is defined by its properties.

According to the formula (1) and the theory of relations [12], the elements $z_{i,i+1}$ define the connection between each pair of elements (a_i, a_{i+1}), while the subsets u_{ij} define the connection between a separated property of the element a_i and the corresponding set of technological functions. Thus, the set Z defines the corresponding relation in the set A, while the set U defines the correlation between the sets A and $F=\{f_{ij}\}$:

$$\begin{cases} (a,a) \in Z, Z \subset A \times A, \\ (a,f) \in U, U \subset A \times F. \end{cases} \quad (2)$$

The regularities and relations (1) and (2) are partially interpreted by means of the weighted (p,q) graph $G=(V,E)$ with a set of vertices $V=\{v_1,v_2,...,v_p\}$ and a set of edges $E=\{e_1,e_2,...,e_p\}$.

The sets A and Z, which are defined by the topology of the distributed technological object, are directly set by the elements of the graph (vertices and edges) and topological connections between them (incidence, adjacency, etc.): $Z \subset V \times E$. When aligning the vertices and edges of the properties of the components of an object or system (set U), the graph G will reproduce the entire set Y:

$$Y \leftrightarrow G = G_Y(V,E): (\{A,Z\} \leftrightarrow \{V,E\}) \wedge (U \leftrightarrow \{g(V,E)\}), \quad (3)$$

where $\{g(V,E)\}$ is the set of weight coefficients of vertices and edges of the graph G:

$$\{g(V,E)\} = \bigcup_{i=1}^{p}\bigcup_{k=1}^{n} g_{ik}(v_i) + \bigcup_{j=1}^{q}\bigcup_{l=1}^{m} g_{jl}(e_j), \quad (4)$$

where n and m is the number of weight parameters of the i-th vertex and j-th edge, respectively.

For the purpose of rational analytical reproduction of the graph G_Y its division into components by two equally powerful sets of cuts $E' \subset E$ and $E'' \subset E$ is used. The bijective mapping of their elements is characterized by the relation of adjacency to the defined reference vertices v_i' according to a certain principle (rule). At the same time, the set $V'=\{v_i'\}$ of the supporting peaks $V'=\{v_i'\}$ is defined by the bijective relation of the incidence with each of the sets of cuts $E'=\{e_i'\}$ and $E''=\{e_i''\}$.

As a result, the graph G_Y is divided into two ordered sets of components $K(E')=\{G_k^{E'}\}$ and $K(E'')=\{G_k^{E''}\}$, formed by the corresponding cuts. The graph G_Y is completely reproduced by the sets $K_H(E') \cap K_P(E'')$ relative to the odd components, formed by the cut E' and the even components, formed by the cut E''.

As a method of analytical interpretation of a geometric model, which is taken as the basis, the method of direct sums, proposed in the work [15] should be used.

Among its variations, it is appropriate to use blocks of parametric-topological matrices (PTM), which combine both topological and parametric (weight) properties of the geometric models.

4. 2. Method of functional reproduction of static and dynamic characteristics of a distributed system

As noted above, all of the graph-analytic models of distributed systems, presented, in particular, in the papers [15–22], reproduce the static properties of the modeled objects. The object dynamic properties are characterized by separate models, which are integrated with the static component at the interface between the application software and the configuration files. In such models, the distributed and the interconnected elements of the system or the technological object are reproduced as vertices and edges of the graph (depending on the functional purpose). In this case, for each element (vertex and edge) of the graph, an ordered set (vector) of weight coefficients that determine the static properties of the modeled element is assigned. The integration of topological and parametric properties of the components of a geometric model within the parametric-topological matrices is performed through the principle of positioning – by dividing the weight coefficients of the corresponding vectors by the cells of the blocks of the corresponding matrices:

$$|M_{PTM\_k}| = \begin{vmatrix} \bar{m}_{11}^k & \bar{m}_{12}^k & ... & \bar{m}_{1b}^k \\ \bar{m}_{21}^k & \bar{m}_{22}^k & ... & \bar{m}_{2b}^k \\ ... & ... & ... & ... \\ \bar{m}_{c1}^k & \bar{m}_{c2}^k & ... & \bar{m}_{cb}^k \end{vmatrix}, \quad \begin{matrix} i=\overline{1,b}, & b \geq 5, \\ j=\overline{1,c}, & c \geq 5, \end{matrix} \quad (5)$$

where $|M_{PTM\_k}|$ is the parametric-topological matrix block, which reproduces the k-th component of the graph analytically:

$$\bar{m}_{ij}^k = \langle n_e, n_v, u^e, u^v, \theta \rangle, \quad (6)$$

where n_e, n_v are the numbers of the edges and vertices of a certain parametric-topological matrix of the graph G_Y, respectively, which correspond to the row and column where the element $\bar{m}_{ij}^k$; is situated; u^e, u^v are the weight parameters of the edges and vertices of the element $\bar{m}_{ij}^k$; θ is the topological property of the edges and vertices of the element $\bar{m}_{ij}^k$.

The values of the elements u^e, u^v are assigned by the rule:

$$u^e = \begin{cases} u_1^e, \text{if } j=1+5h, \\ u_2^e, \text{if } j=2+5h, \\ u_3^e, \text{if } j=3+5h, \\ u_4^e, \text{if } j=4+5h, \\ u_5^e, \text{if } j=5+5h, \end{cases}$$

$$u^v = \begin{cases} u_1^v, \text{if } i=1+5s, \\ u_2^v, \text{if } i=2+5s, \\ u_3^v, \text{if } i=3+5s, \\ u_4^v, \text{if } i=4+5s, \\ u_5^v, \text{if } i=5+5s, \end{cases} \quad h=0,1,...,\left\lfloor \frac{b}{5} \right\rfloor, \quad s=0,1,...,\left\lfloor \frac{c}{5} \right\rfloor, \quad (7)$$

where u_j^e, u_i^v are the weight parameters of the edges and vertices.

The topological property $\theta = 0 \vee \pm 1$ is determined by the type of the topological matrices and corresponds to the relation of the connection of the edge n_e and vertex n_v.

The main disadvantages of this approach are:

– the minimal limitation of the dimension of the parametric-topological matrix blocks (and the corresponding components of the graph) by the capacities of the ordered sets of weight parameters of graph elements;

– the ability to reproduce the topological and parametric properties of the geometric model only on the basis of the parametric-topological matrices, which use the property of the incidence;

– technical bulkiness and complexity of the formation of parametric-topological matrix source blocks; the lack of reproduction of the dynamic properties of the modeled object or system.

To eliminate these disadvantages, the use of a functional graph $G_{YF}\left[V\left(F\right),E\right]$ is proposed. It assumes that the components of a distributed system or technological object are reproduced by functional vertices, the weight of which is defined by the functional $F\left(\underline{U}\right)$, which reproduces both static and dynamic properties of the components of a simulated system or object.

Taking into account the expressions (1)–(7), the formalization of the geometric interpretation of the simulated system (object) will be:

$$Y = A \cap U \cap Z \leftrightarrow G_{YF}\left\{\left[V\left(F\left(\underline{U}\right)\right)\right],E\right\},$$

where

$$F(\underline{U}) = \bigcup_{i=1}^{n} f_i\left(\bar{u}_i\right)$$

is the functional of the distributed components of the system (object), each of which has a technological function $f_i(\bar{u}_i)$, which determines the dynamic functional properties of the corresponding component based on its weight coefficients (parameters) vector $\bar{u}_i$.

The edges $E = \left\{e_j\right\}$ in the graph G_{YF} are unweighted. They only determine the relation between the vertices $V\left(F\right) = \left\{v_i\left[f_i\left(\bar{u}_i\right)\right]\right\}$. However, in general, the graph G_{YF} can be either directional or non-directional or mixed (Fig. 1).

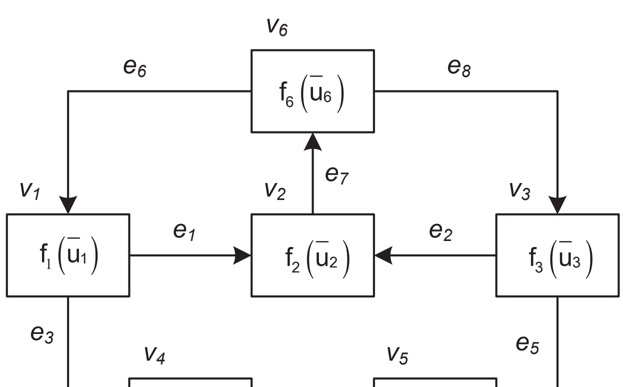

Fig. 1. Example of a graph-functional model G_{YF} of a distributed system with the capacities $[V]=6$ and $[E]=8$

Thus, the method based on the graph-functional model G_{YF} completely reproduces in the geometric sense the whole set $Y = A \cap U \cap Z$. The graph G_{YF}, unlike the graph G_{YF}, simplifies the structure of the graph G_{YF} due to the absence of the weights of the edges and the functional setting of the weight parameters. In addition, the graph G_{YF}, unlike the graph G_Y, reproduces the dynamic properties of a simulated system or technological object.

In terms of the parametric and functional-dynamic properties of each component of the system (object), the corresponding function $f_i(\bar{u}_i)$ should take into account the technological purpose of the component and the logic of its implementation in accordance with the weight parameters $\bar{u}_i$. From the standpoint of the graph anatomy, this function must take into account the degree (valence) of the corresponding vertex v_i, taking into account its sign.

The further processing of the geometric model G_{YF} using a computer should be done with its analytical interpretation.

5. Results of the implementation of the graph-functional modeling of distributed systems

5. 1. Method of analytical reproduction of the graph-functional model

In contrast to the graph G_Y, the geometric model G_{YF} does not provide a direct assignment of weight coefficients to the vertices, and generally does not include such coefficients for the edges (since they are unweighted). This order of things eliminates the mandatory application of the basic matrix of incidence in the analytical reproduction of the model. This is due, firstly, to the lack of the need of the positional distribution of weight coefficients (due to the use of functions $f_i(\bar{u}_i)$), and, secondly, due to the lack of simulation of individual components of the system (object) using edges. Thus, in the general case, any matrix can be taken as the basis for matrix reproduction of the graphical model of G_{YF}, such as matrices of incidence, adjacency, contours, routes, etc.

The most obvious and easiest in terms of addition matrix among the topological matrices is the adjacency matrix. However, its essential disadvantage is the impossibility of direct reproduction of several edges between two adjacent vertices, especially when the assigned edges have different directions. However, in the case of encryption of the degree (valence) of the corresponding vertex, taking into account its sign, in the function $f_i(\bar{u}_i)$ this disadvantage is completely leveled.

In this case, the vector $\bar{u}_i$ contains an ordered subset $\bar{d}_i \subset \bar{u}_i$, $\bar{d}_i = \{p_i^+, p_i^-, p_i^0\}$, where p_i^+, p_i^- and p_i^0 is the number of edges, which are incident to the vertex v_i with positive, negative and neutral valence, respectively. Consequently, in this approach, the function $f_i(\bar{u}_i)$ will indirectly determine the properties of the incidence between vertices and edges in any topological matrix, in particular, the adjacency matrices.

Given that the adjacency matrix is a symmetric square matrix, in it, unlike the incidence matrix, it is expedient to apply not a positional but a diagonal principle of reproduction of the functional properties of the constituent vertices of the graph G_{YF}.

In this case, the parametric-topological matrices, which are based on the adjacency matrix (APTM), which reproduces the parametric and topological properties of the graphic model G_{YF} according to the given rule (and hence the system or technological object modeled by it) will be:

$$\left|M_{APTM\_G_{YF}}\right| = \begin{vmatrix} f_1(\overline{u}_1) & \theta_{21} & \theta_{31} & \ldots & \theta_{n1} \\ \theta_{12} & f_2(\overline{u}_2) & \theta_{32} & \ldots & \theta_{n2} \\ \theta_{13} & \theta_{23} & f_3(\overline{u}_3) & \ldots & \theta_{n3} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \theta_{1n} & \theta_{2n} & \theta_{3n} & \ldots & f_n(\overline{u}_n) \end{vmatrix}, \quad (8)$$

where $\left|M_{APTM\_G_{YF}}\right|$ is the parametric-topological matrix based on the adjacency matrix of the graphic model G_{YF};

$$\theta_{ij} = \theta_{ji} = \begin{cases} 1, \text{if } v_i \updownarrow v_j, \\ 0, \text{if } v_i \parallel v_j, \end{cases}$$

is the topological property of the availability (↕) or the absence (‖) of the adjacency between the i-th and j-th vertices.

For the graph depicted in Fig. 1, the formula 8 is to be like:

$$\left|M_{G_{YF}}\right| = \left|M_{APTM\_G_{YF}}\right| =$$

$$= \begin{vmatrix} f_1(\overline{u}_1) & 1 & 0 & 1 & 0 & 1 \\ 1 & f_2(\overline{u}_2) & 1 & 0 & 0 & 1 \\ 0 & 1 & f_3(\overline{u}_3) & & 1 & 1 \\ 1 & 0 & 0 & f_4(\overline{u}_4) & 1 & 0 \\ 0 & 0 & 1 & 1 & f_5(\overline{u}_5) & 0 \\ 1 & 1 & 1 & 0 & 0 & f_6(\overline{u}_6) \end{vmatrix}.$$

The functions $f_i(\overline{u}_i)$ for this graph reproduce not only the parametric properties of the graph, but also part of its anatomical properties through the vectors $\overline{d}_i = \{p_i^+, p_i^-, p_i^0\}$. Given the singularity of the routes between any pair of adjacent vertices in this graph, these vectors will have the following values for it: $\overline{d}_1 = \{1,1,1\}$, $\overline{d}_2 = \{2,1,0\}$, $\overline{d}_3 = \{1,1,1\}$, $\overline{d}_4 = \{1,0,1\}$, $\overline{d}_5 = \{0,1,1\}$, $\overline{d}_6 = \{1,2,0\}$.

For the directed graph, an alternative method; of setting the number of edges between adjacent vertices, taking into account the direction, without the use of vectors is the encryption of this data in the parameters θ_{ij} and/or θ_{ji}. Then, in general, $\theta_{ij} \neq \theta_{ji}$. Values of θ_{ij} and θ_{ji} should reflect not only the presence or absence of the adjacency, but also the number of edges joining the vertices v_i and v_j, taking into account their direction.

In such conditions, in determining the subordination between the parameters θ_{ij} and θ_{ji} in the expression (8), the basic positive or negative valence principle should be taken as the basis.

In the first case, the parameter θ_{ji} determines the number of edges incident to vertices v_i and v_j, with positive valence for the vertex v_i and negative valence for the vertex v_j. On the contrary, the parameter θ_{ji} determines the number of edges incident to the same vertices, but with the opposite valence. Thus, these parameters will be determined in the formula (8) in the following way:

$$\theta_{ij} = \begin{cases} \left[E_{ij}^{v_i^+, v_j^-}\right], \text{if } v_i \uparrow v_j, \\ 0, \text{if } v_i \parallel v_j \text{ or } v_i \downarrow v_j, \end{cases}$$

$$\theta_{ji} = \begin{cases} \left[E_{ij}^{v_i^-, v_j^+}\right], \text{if } v_i \downarrow v_j, \\ 0, \text{if } v_i \parallel v_j \text{ or } v_i \uparrow v_j, \end{cases} \quad (9)$$

where $E_{ij}^{v_i^+, v_j^-}$ and $E_{ij}^{v_i^-, v_j^+}$ are the sets of edges incident to the vertices v_i and v_j, respectively, with the positive valence for the vertices v_i and negative valence for v_j and vice versa; ↑, ↓ are the conditional noncommutative symbols of adjacency with correspondingly positive and negative incidence of edges with respect to the vertex v_i from v_j.

When using the principle of negative valence in the *ij* cell of the parametric-topological matrices based on the adjacency matrix, the number of edges incident to the vertices v_i and v_j is specified with negative valence for the vertex v_i and positive valence for the vertex v_j. On the contrary, for the cells *ji* the corresponding valence should be reversible. Thus, for this case, the formula (9) is transformed in the following way:

$$\theta_{ij} = \begin{cases} \left[E_{ij}^{v_i^-, v_j^+}\right], \text{if } v_i \downarrow v_j, \\ 0, \text{if } v_i \parallel v_j \text{ or } v_i \uparrow v_j, \end{cases}$$

$$\theta_{ji} = \begin{cases} \left[E_{ij}^{v_i^+, v_j^-}\right], \text{if } v_i \uparrow v_j, \\ 0, \text{if } v_i \parallel v_j \text{ or } v_i \downarrow v_j. \end{cases} \quad (10)$$

In the case of a non-directed graph, there is no need to use both of the vectors $\overline{d}_i \subset \overline{u}_i$, and to establish the subordination between the parameters θ_{ij} and θ_{ji} by the formulas (9) or (10). In this case, it is sufficient to indicate the number of non-directional edges $\left[E_{ij}^{v_i, v_j}\right]$, which are incident to the adjacent vertices v_i and v_j simultaneously:

$$\theta_{ij} = \theta_{ji} = \begin{cases} \left[E_{ij}^{v_i, v_j}\right], \text{if } v_i \updownarrow v_j, \\ 0, \text{if } v_i \parallel v_j. \end{cases} \quad (11)$$

An alternative to using the vectors $\overline{d}_i \subset \overline{u}_i$ for the mixed graph case (Fig. 1) is the structural synthesis of expressions (8), (9), (11) or (8), (10), (11) depending on the adopted base valence of the vertices (positive in the first case and negative in the second case).

In such cases, vector elements $\overline{\theta}_{ij} \neq \overline{\theta}_{ji}$, are used instead of scalar elements $\theta_{ij} \neq \theta_{ji}$. These vector elements are two-element ordered arrays in the presence of adjacency between the vertices I and *j*.

The first element is determined by the expression (9) or (10), depending on the chosen base valence. The second element is determined by the formula (11).

For the case of the basic positive valence, the expressions for determining the parameters will be as follows:

$$\overline{\theta_{ij}^+} = \begin{cases} \overline{\left[E_{ij}^{v_i^+, v_j^-}\right], \left[E_{ij}^{v_i, v_j}\right]}, \text{if } v_i \uparrow v_j \text{ and/or } v_i \updownarrow v_j, \\ 0, \text{if } v_i \parallel v_j \text{ or } v_i \downarrow v_j, \end{cases}$$

$$\overline{\theta_{ji}^+} = \begin{cases} \overline{\left[E_{ij}^{v_i^-, v_j^+}\right], \left[E_{ij}^{v_i, v_j}\right]}, \text{if } v_i \downarrow v_j \text{ and/or } v_i \updownarrow v_j, \\ 0, \text{if } v_i \parallel v_j \text{ or } v_i \uparrow v_j, \end{cases} \quad (12)$$

and for the case of the basic negative valence:

$$\overline{\theta_{ij}^-} = \begin{cases} \overline{\left[E_{ij}^{v_i^-, v_j^+}\right], \left[E_{ij}^{v_i, v_j}\right]}, \text{if } v_i \downarrow v_j \text{ and/or } v_i \updownarrow v_j, \\ 0, \text{if } v_i \parallel v_j \text{ or } v_i \uparrow v_j, \end{cases}$$

$$\overline{\theta_{ji}^-} = \begin{cases} \overline{\left[E_{ij}^{v_i^+, v_j^-}\right], \left[E_{ij}^{v_i, v_j}\right]}, \text{if } v_i \uparrow v_j \text{ and/or } v_i \updownarrow v_j, \\ 0, \text{if } v_i \parallel v_j \text{ or } v_i \downarrow v_j. \end{cases} \quad (13)$$

Thus, the analytical interpretation of the graph G_{YF} depicted in Fig. 1, using the formulas (8), (12) and the parametric-topological matrices based on the adjacency matrix has the form:

$$\left|M^{+}_{G_{YF}}\right| = \left|M^{+}_{APTM\_G_{YF}}\right| =$$

$$= \begin{vmatrix} f_1(\bar{u}_1) & \overline{1,0} & 0 & \overline{0,1} & 0 & 0 \\ 0 & f_2(\bar{u}_2) & 0 & 0 & 0 & \overline{1,0} \\ 0 & \overline{1,0} & f_3(\bar{u}_3) & 0 & \overline{0,1} & 0 \\ \overline{0,1} & 0 & 0 & f_4(\bar{u}_4) & 0 & 0 \\ 0 & 0 & \overline{0,1} & \overline{1,0} & f_5(\bar{u}_5) & 0 \\ \overline{1,0} & 0 & \overline{1,0} & 0 & 0 & f_6(\bar{u}_6) \end{vmatrix},$$

using the formulas (8) and (13) it has the following form:

$$\left|M^{-}_{G_{YF}}\right| = \left|M^{-}_{APTM\_G_{YF}}\right| =$$

$$= \begin{vmatrix} f_1(\bar{u}_1) & 0 & 0 & \overline{0,1} & 0 & \overline{1,0} \\ \overline{1,0} & f_2(\bar{u}_2) & \overline{1,0} & 0 & 0 & 0 \\ 0 & 0 & f_3(\bar{u}_3) & 0 & \overline{0,1} & \overline{1,0} \\ \overline{0,1} & 0 & 0 & f_4(\bar{u}_4) & \overline{1,0} & 0 \\ 0 & 0 & \overline{0,1} & 0 & f_5(\bar{u}_5) & 0 \\ 0 & \overline{1,0} & 0 & 0 & 0 & f_6(\bar{u}_6) \end{vmatrix},$$

As can be seen from the expressions obtained, for the graph, any pair of adjacent vertices of which merely connects an edge, the corresponding vectors $\bar{\theta}_{ij} \neq \bar{\theta}_{ji}$ contain only single or zero elements in each position. For the more general case of a mixed graph (Fig. 2), the value of the corresponding positions can be arbitrary and determined by the capacities of the corresponding sets.

For the above example, the expressions of the parametric-topological matrices based on the adjacency matrix according to the formulas (8), (12) and (8), (13) have the following form:

$$\left|M^{+}_{G_{YF}}\right| = \begin{vmatrix} f_1(\bar{u}_1) & \overline{1,1} & \overline{2,0} & \overline{1,1} \\ \overline{1,1} & f_2(\bar{u}_2) & \overline{0,2} & \overline{0,2} \\ \overline{1,0} & \overline{1,2} & f_3(\bar{u}_3) & \overline{0,2} \\ \overline{1,1} & \overline{0,2} & \overline{1,2} & f_4(\bar{u}_4) \end{vmatrix},$$

$$\left|M^{-}_{G_{YF}}\right| = \begin{vmatrix} f_1(\bar{u}_1) & \overline{1,1} & \overline{1,0} & \overline{1,1} \\ \overline{1,1} & f_2(\bar{u}_2) & \overline{1,2} & \overline{0,2} \\ \overline{2,0} & \overline{0,2} & f_3(\bar{u}_3) & \overline{1,2} \\ \overline{1,1} & \overline{0,2} & \overline{0,2} & f_4(\bar{u}_4) \end{vmatrix}.$$

It should be noted that the matrices formed on the basis of the basic valence (positive or negative) by the formulas (8)–(13) are asymmetric. This follows directly from the properties of the adjacency of the graph elements provided that the edges are oriented between the corresponding vertices. By providing the substantial redundancy to the corresponding matrices, combining the formulas (9) and (10) in various ways, one can bring them to a symmetrical form. In this case, the anatomical properties in the two parts of the matrix will be duplicated.

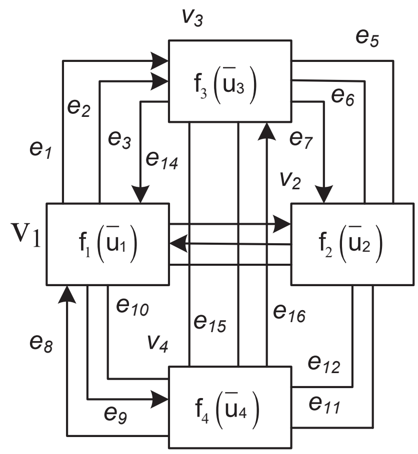

Fig. 2. Generalized example of a graph-functional model G_{YF} of a distributed system with the capacities $[V]=4$ and $[E]=16$

To reproduce distributed systems or large dimension objects by using the developed method, a method of direct sums, which was used for the parametric-topological matrices based on the adjacency matrices in the paper [15], should be used. However, the elucidation of the specifics of its application for the parametric-topological matrices based on the adjacency matrices requires a separate thorough research.

5. 2. Practical application of the method of graph-analytical modeling

The most promising area of application of the developed method of graph-analytical modeling is automated design (configuration) of hardware and software of control systems for distributed technological objects.

This issue is especially relevant to the microprocessor control systems for railway automatics, mainly for the points and signals interlocking. The technological object of such systems is the track development of railway stations, the research of graph-analytical reproduction of which is done in the papers [15–22].

Special attention should be paid to improving the methods of configuring models for testing systems, which should be developed through special procedures in comparison with the main means of information and control systems.

Using the features of assigning the functions $f_i(\bar{u}_i)$ allows not only to adapt the application software to the updated configuration of the technological object (which allows you to make the graphic model G_Y). The above features also provide the addition of new types of management and control objects and allow setting additional dynamic properties to existing objects without correction of the source code.

Thus, the labor costs of software developers are minimized during the preparation of a new implementation project or during the reconstruction of an automated control system.

The application of the proposed method is also possible with the use of separate CAE software complexes (CAD in a broad interpretation). Among these complexes, EPlan attracts special attention [23, 24].

EPlan was developed by EPLAN Software & Service as a unique data management software. It is a modular structured platform with the following main components:

– Electric P8 – creation of electrical circuits;

– Fluid – creation of hydraulic and pneumatic circuits;

– PrePlanning – creation of automation circuits;

– ProPanel – creation of 3D electrical cabinets and switchgear.

In EPlan, it is possible to implement the following basic approaches to formalizing the developer's solutions and the corresponding creation of the project documentation:

1. Creating a project based on conditional graphic designations.

2. Creating a project based on products from the database.

3. Creating a project based on a defined specification.

In particular, when designing microprocessor control systems, EPlan provides extensive capabilities for working with programmable logic controllers (PLC). The PLC and EPlan configuration programs have different procedures for representing the configuration data. When parts of the electrical equipment are presented in EPlan, PLC programming software uses the logical order of representation for programming the PLC. For example, EPlan contains plugs of PLC devices, which are designed for power supply. These device outputs are not required in the PLC configuration program. The PLC setup programs, in contrast, contain information about the interface, such as router information, which is not required in EPlan. The data which is not found by the processing program in the exchange file, since other processing type does not recognize it, will be added when importing. Therefore, the use of the proposed method of preliminary graph-analytical modeling allows using the capabilities of EPlan as a means of CAE optimally at the stage of preliminary planning of a project (Fig. 3).

The device identification both in EPlan and in the PLC setup program is executed either with the PLC type designation, or by specifying the basic data file of the device. Using these properties, the product is also assigned when importing PLC configuration files.

In the tree structure of the PLC navigator dialog box, you can select different types to display the PLC data. In all types, all of the existing PLC data in the project is displayed ($\bar{u}_i$ parameters), namely PLC blocks, PLC devices outputs and $f_i(\bar{u}_i)$. function templates. In this case, both empty PLC blocks and PLC blocks, which contain only function templates, are displayed.

The specific settings of the PLC control units of the bus systems can be exchanged with the various PLC configurations. The PLC data exchange is based on a separated exchange file, which implements one processing program and reads another processing program. This file can be easily exchanged by the design engineer of EPlan and the PLC software engineer. The exchange settings installation is generally implemented for the entire EPlan project using the special dialog interface (Fig. 4).

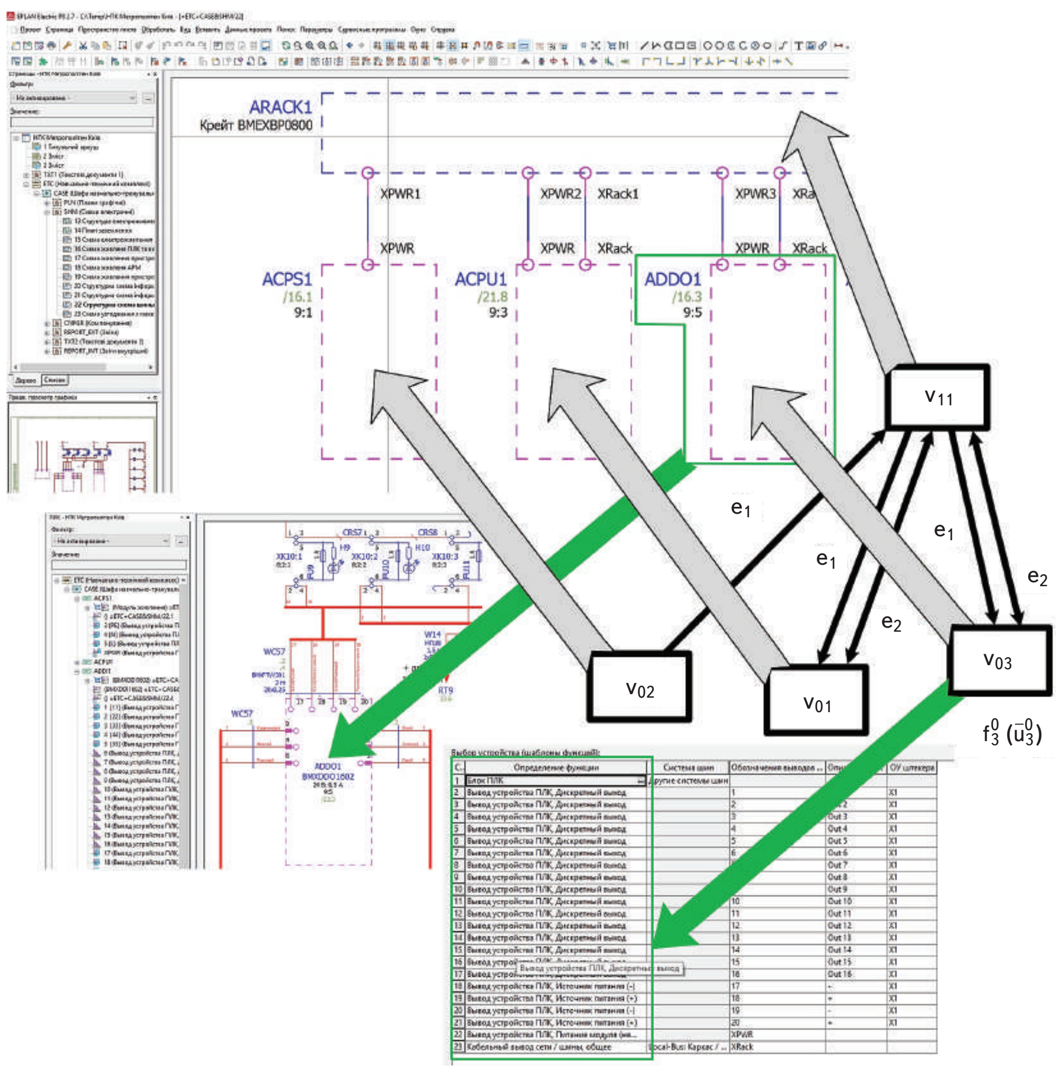

Fig. 3. Example of configuring a PLC by means of EPlan using the method of graph-analytical modeling

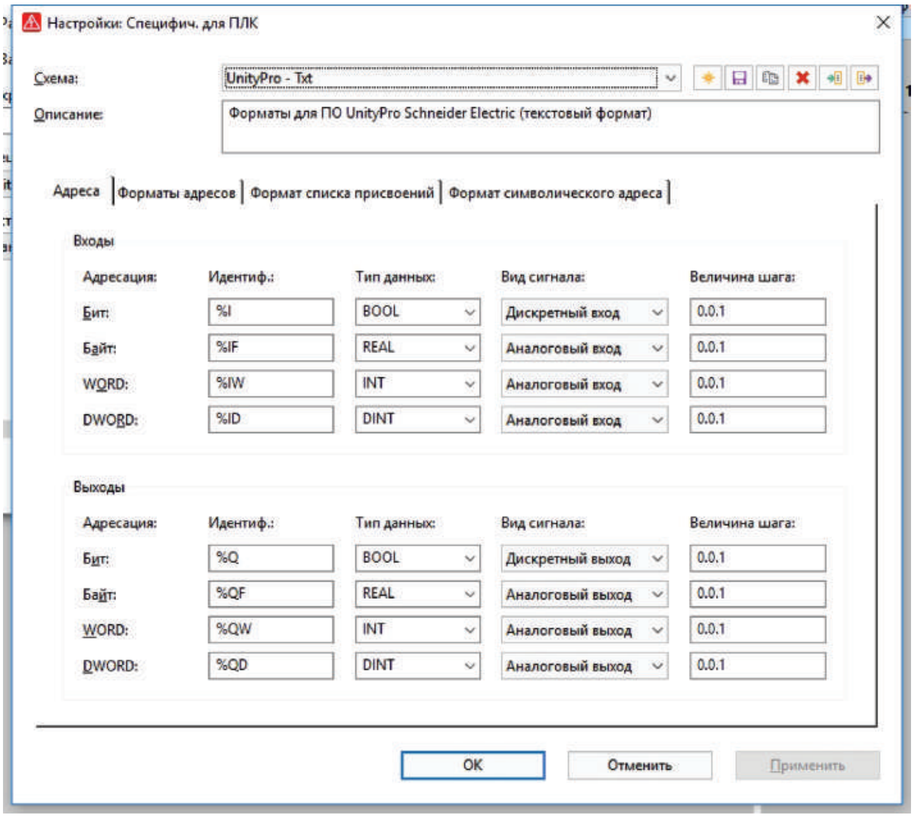

Fig. 4. One of the interface forms for the project software configuration based on the functional vertex

In general, the following configuration data may be subject to change:

– hardware devices data used;

– creation of a frame with product information;

– character table (which may contain a list of assignments, a table of changes, etc.);

– assigning a symbolic program address to a technical support address.

Thus, the integration of the proposed method of graph-functional modeling with the EPlan application package allows you to formalize the process of setting tasks for hardware and software developers. This is done by combining the functional vertices with the components of the projected system and their corresponding interface windows. Similar approaches can be applied to other SAE or CAD systems, taking into account the specifics of such systems.

6. Discussion of the proposed method of graph-functional modeling

Proceeding from the materials of the conducted research, the proposed method of graph-analytical modeling of distributed systems in general is based on the world experience of using graphic models in the modeling of systems, objects and events. The closest models based on the proposed graph-functional model are the geometric models of distributed objects with diverse approaches to analytical reproduction. Such models have been developed in the last decade by specialists in the field of railway transport.

Generally, depending on the functional purpose, such models can be divided into two main groups. The first group is used for researching the technological processes in distributed objects or systems. The second group is used for the structural synthesis of objects, systems or components (software, technical documentation, etc.).

The closest to the proposed method and the models based on it is the graph-analytic method of direct sums based on the use of parametric-topological matrices of incidence. From the scientific point of view, the main fundamental difference of the proposed method is:

the transition from the interpretation of individual elements of systems or technological objects by the edges of the graph to the full reproduction of the corresponding components by the functional vertices;

– the transition from the positional principle of reproduction of the weight parameters of the graph elements to the use of such variables as the arguments of functions laid at the vertex;

– the direct assignment of the dynamic properties of the components of the system or object through the nested functions of the graph vertices;

– the transition from the use of parametric-topological incidence matrices to parametric-topological adjacency matrices in the analytical interpretation of geometric models;

– the indirect assignment of the connections between the components of a system or object is mediated through the purposeful assignment of particular fields or cells of parametric-topological matrices.

In practical terms, the proposed graph-analytic method allows the following:

– forming technical and technological formalized descriptions of distributed systems and objects directly (as technical tasks, technical documentation, etc.);

– developing and configuring application software for automated control systems forming the dynamic properties of the objects of management and control of the systems by assigning the corresponding functions of the graph vertices.

In particular, the method allows optimizing the process of setting tasks to software developers through the formalization of the structure of the projected control system using standard CAD tools. In contrast to the most approximate analogue (based on parametric-topological matrices of incidence), the method has such basic advantages as:

– the possibility of assigning new functions of technological objects directly and introducing new types of objects;

– reducing the amount of data in an array when implementing geometric models;

– simplification of procedures of synthesizing simulated distributed systems (objects) and increasing their level of visibility.

In this case, unlike the classical approaches based on the reproduction of graphs by adjacency matrices, the proposed method implements the possibility of multi-path connections between adjacent vertices, taking into account the direction of the corresponding edges of the graph. In this case, there are many variations of such assignment of connections, both on the basis of positive and negative or mixed logic. A number of examples of such implementation are considered in the research, while the detection of a full range of such variations requires separate research in the subject field. In particular, in the further researches partial-positional distribution of arguments of functions on adjacency matrix cells, inverse mapping of matrices, association of vertices with common functions in separate groups, etc. may be possible.

In this case, the main constraints of the proposed method are the adaptability only to those distributed systems, the elements of which can be reproduced only by the graph vertices.

7. Conclusions

1. As a basic geometric model, which is taken as the basis of the method, a functional graph was formed. In its composition, only the vertices are weighed. The edges of the graph are determined only by the connections between the vertices, which are interpreted as composite technological objects of the distributed system. Instead of the vectors of weight parameters assigned to elements of known graphic models, the proposed vertices are assigned to object functions or functionals, the function arguments become static weight coefficients. The dynamic behavior of reproducible objects is realized by the nature of the given functions.

2. The method of functional reproduction of static and dynamic characteristics of the system objects modeled by a graphic model is determined. Such reproduction is due to the use of functional vertices, the weight filling of which is carried out by the builder of the corresponding model.

3. The method of analytical reproduction of the developed graph-functional model is developed. This method is based on the use of parametric-topological matrices of incidence with the indirect assignment of connections between vertices of the graph. In this case, a number of variations of such a problem are established by the functional division of the individual fields and the cells of the parametric-topological matrix.

4. Recommendations and examples of practical application of the method of graph-functional modeling in the automated design of software of automated control systems are given. In particular, an example of the use of the method for the purpose of formalized drawing up of technical tasks for software developers by combining graph-functional model with the application package of E-Plan CAD.

Thus, a more versatile and user-friendly method of geometric modeling of distributed systems has been developed and proposed. In contrast to the nearest analogues, it uses the functional weight parameters of the graph elements and is based on a slightly different mathematical apparatus of the matrix interpretation. This makes it possible to simplify the procedures of formation of geometric models and to realize the dynamic properties of the simulated objects of the system.

Further research in the direction of graph-functional modeling of distributed systems will lie in determining the number and nature of combinations of methods of indirect assignment of connections between the objects of the distributed system. In addition, in the further improvement of the method the partial positioning of the arguments of the functions of vertices by the cells of the adjacency matrix is hypothetically possible.

References

1. Glinkov G. M., Makovskiy V. A. ASU TP v chernoy metallurgii. 2-e izd., pererab. i dop. Moscow, 1999. 310 p.
2. Rudakova A. V. Problemy upravleniya bol'shimi razvivayushchimisya sistemami // Vestnik Hersonskogo nacional'nogo tekhnicheskogo universiteta. 2010. Issue 2. P. 29–33.
3. Kustov V. F., Kamenev A. Yu. Eksperimental'no-staticheskie modeli raspredelennyh tekhnologicheskih ob'ektov // Metallurgicheskaya i gornorudnaya promyshlennost'. 2013. Issue 2. P. 97–101.
4. Sigorskiy V. P. Matematicheskiy apparat inzhenera. izd. 2-e, stereotip. Kyiv, 1977. 768 p.
5. Giranova A. K. Razrabotka paketa programm dlya provedeniya eksperimentov s rekonfiguriruemymi vychisleniyami // Modeliuvannia ta informatsiyni tekhnolohiyi. 2011. Issue 59. P. 124–129.
6. Improvement of the accuracy of determining movement parameters of cuts on classification humps by methods of video analysis / Panchenko S., Siroklyn I., Lapko A., Kameniev A., Zmii S. // Eastern-European Journal of Enterprise Technologies. 2016. Vol. 4, Issue 3 (82). P. 25–30. doi: https://doi.org/10.15587/1729-4061.2016.76103
7. Kamenev A. Yu. Universal'niy metod konfigurirovaniya programmnogo obespecheniya avtomatizirovannyh sistem upravleniya tekhnologicheskimi procesami // Nauka i proizvodstvo Urala: Nauchno-tekhnicheskiy i proizvodstvenniy zhurnal. 2014. Issue 4. P. 146–150.
8. Cliff O., Prokopenko M., Fitch R. Minimising the Kullback–Leibler Divergence for Model Selection in Distributed Nonlinear Systems // Entropy. 2018. Vol. 20, Issue 2. P. 51. doi: https://doi.org/10.3390/e20020051
9. Zrafi R., Ghedira S., Besbes K. A Bond Graph Approach for the Modeling and Simulation of a Buck Converter // Journal of Low Power Electronics and Applications. 2018. Vol. 8, Issue 1. P. 2. doi: https://doi.org/10.3390/jlpea8010002

10. Małecki K. Graph Cellular Automata with Relation-Based Neighbourhoods of Cells for Complex Systems Modelling: A Case of Traffic Simulation // Symmetry. 2017. Vol. 9, Issue 12. P. 322. doi: https://doi.org/10.3390/sym9120322
11. Model-Based Requirements Management in Gear Systems Design Based On Graph-Based Design Languages / Holder K., Zech A., Ramsaier M., Stetter R., Niedermeier H.-P., Rudolph S., Till M. // Applied Sciences. 2017. Vol. 7, Issue 11. P. 1112. doi: https://doi.org/10.3390/app7111112
12. Chen Y., Guo Y., Wang Y. Modeling and Density Estimation of an Urban Freeway Network Based on Dynamic Graph Hybrid Automata // Sensors. 2017. Vol. 17, Issue 4. P. 716. doi: https://doi.org/10.3390/s17040716
13. Zhang H., Lu F. GSMNet: A Hierarchical Graph Model for Moving Objects in Networks // ISPRS International Journal of Geo-Information. 2017. Vol. 6, Issue 3. P. 71. doi: https://doi.org/10.3390/ijgi6030071
14. Liu J., Liu J. The Treewidth of Induced Graphs of Conditional Preference Networks Is Small // Information. 2016. Vol. 7, Issue 1. P. 5. doi: https://doi.org/10.3390/info7010005
15. Listrovoy S., Panchenko S., Listrova E. Mathematical models in computer control systems railways and parallel computing: monograph. Kharkiv, 2017. 300 p.
16. State Tracking and Fault Diagnosis for Dynamic Systems Using Labeled Uncertainty Graph / Zhou G., Feng W., Zhao Q., Zhao H. // Sensors. 2015. Vol. 15, Issue 11. P. 2031–28051. doi: https://doi.org/10.3390/s151128031
17. Traffic Behavior Recognition Using the Pachinko Allocation Model / Huynh-The T., Banos O., Le B.-V., Bui D.-M., Yoon Y., Lee S. // Sensors. 2015. Vol. 15, Issue 7. P. 16040–16059. doi: https://doi.org/10.3390/s150716040
18. Santone A., Vaglini G. Model Checking Properties on Reduced Trace Systems // Algorithms. 2014. Vol. 7, Issue 3. P. 339–362. doi: https://doi.org/10.3390/a7030339
19. Testing Goodness of Fit of Random Graph Models / Csiszár V., Hussami P., Komlós J., Móri T., Rejtõ L., Tusnády G. // Algorithms. 2012. Vol. 5, Issue 4. P. 629–635. doi: https://doi.org/10.3390/a5040629
20. Lu M., Constantinescu C., Sarkar P. Content Sharing Graphs for Deduplication-Enabled Storage Systems // Algorithms. 2012. Vol. 5, Issue 2. P. 236–260. doi: https://doi.org/10.3390/a5020236
21. Kozachenko D. N., Vernigora R. V., Berezoviy N. I. Kompleksniy analiz zheleznodorozhnoy infrastruktury metallurgicheskogo kombinata na osnove grafoanaliticheskogo modelirovaniya // Zbirnyk naukovykh prats Dnipropetrovskoho natsionalnoho universytetu zaliznychnoho transportu im. akademika V. Lazariana «Transpotni systemy i tekhnolohiyi perevezen». 2012. Issue 4. P. 55–60.
22. Bobrovskiy V. I., Kozachenko D. N., Vernigora R. V. Functional simulation of railway stations on the basis of finite-state automata // Transport Problems. 2014. Vol. 9, Issue 3. P. 57–66.
23. Gischel B. EPLAN Electric P8 Reference Handbook. 4th ed. Carl Hanser Verlag, 2015. 672 p. doi: https://doi.org/10.3139/9781569904992
24. Computer aided design with Eplan electric P8 educational projektovanje primenom računara kroz prikaz programskog paketa Eplan electic P8 educational / Matić D., Lukač D., Bugarski V., Kulić F., Nikolić P. // Journal on Processing and Energy in Agriculture. 2016. Vol. 20, Issue 2. P. 102–105.