THEORETICAL RATIONALE AND IDENTIFICATION OF HEAT AND MASS TRANSFER PROCESSES IN VIBRATION DRYERS WITH IR-ENERGY SUPPLY

Теоретично обґрунтовано процеси радіацій-но-конвективного тепломасообміну між усіма визначальними об'єктами всередині вібрацій-ної сушарки з ІЧ-енергопідведенням. На основі теплового і матеріального балансів визначено рівняння, які описують основні динамічні харак-теристики режиму сушіння олієвмісного зерново-го матеріалу в безперервно діючій ІЧ-сущарці. У зв'язку з тим, що точного аналітичного розв'яз-ку представленої математичної моделі у вигляді системи диференціальних рівнянь у частинних похідних не існує. Запропоноване наближене рішення дозволяє ідентифікувати залежності розподілу температури і вологовмісту зернового та олієвмісного матеріалу за довжиною сушарки для будь-якого моменту часу.

Чисельний розв'язок наведеної математичної моделі можливий лише при наявності визначе-них взаємопов'язаних кінетичних коефіцієнтів. Кінетичні коефіцієнти експериментально прями-ми вимірами знайти не можливо, тому у стат-ті запропоновано метод подолання цих труд-нощів. На основі представленого наближеного аналітичного розв'язку синтезованої математич-ної моделі, з використанням методу обернених задач визначаються комплекси коефіцієнтів за результатами експериментальної ідентифікації процесу зневоднення. В подальшому експеримен-тально ідентифіковані параметричні комплек-си моделі можливо використовувати при аналізі процесу сушіння на наближених розв'язах або для подальшого точного чисельного розв'язку.

На основі експериментальних досліджень зневоднення зерноматеріалу встановлено, що при збільшенні потужності ІЧ-джерела від 400 до 500 Вт час сушіння з початкової вологості матеріалу 11 % до 8,75 % зменшується з 9 до 7 хвилин. Визначено, що критерій Ребіндера, який характеризує вологотеплові характеристики матеріалу, зменшується при зменшенні його во-логості з величини 0,04 при 11 % до 0,01 при 9 %. Це є цікавим з практичної точки зору і дає мож-ливість використовувати отримані результати і математичну модель при енергетичному удо-сконаленні реалізації процесів термічного сушін-ня в типових об'єктах підготовки до переробки олієвмісних зерноматеріалів

Ключові слова: тепломасообмін, інфрачервоне енергопідведення, вібраційна сушарка, олієвмісне зерно, параметрична ідентифікація

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1. Introduction

About 15 % of the total energy consumption in the agro-industrial complex of developed countries accounts for the processes of drying and heat treatment of agricultural materials [1]. Potentially, the most energy-efficient dryers are those with infrared energy, because they provide direct impact of the radiant energy on the moisture in the dried material and reduce losses tied to the spent drying agent. However, in Ukraine the majority of available drying systems and

facilities for highly intensive heat treatment of grain materials with infrared (IR) energy consume an excess specific energy per unit of the obtained product [2, 3].

It is extremely important to introduce innovative solutions aimed at reducing the specific energy consumption of IR equipment for preparation of grain materials for storage or processing, consumption or feeding. The solution is complicated by the lack of unified approaches to the synthesis of the mathematical model of heat and mass transfer processes and the variability of interconnected kinetic coefficients that depend on the physical properties of the interacting substances and energies in analytical models.

Optimal (as to the criteria of energy efficiency and quality) process of material dehydration in dryers with infrared energy supply is impossible without an adequate mathematical model. Therefore, research should be considered relevant if it is aimed at further development and improvement of the mathematical apparatus that characterizes heat and mass transfer processes in IR dryers.

2. Literature review and problem statement

The modes and parameters of equipment for IR heat treatment are substantiated due to the use of mathematical models of the theory of heat and mass transfer, as well as due to physical laws, physical-chemical balance ratios, generalization of experimental data – the theory of similarity, probability, and mathematical statistics [4, 5].

The classical mathematical models of the theory of heat and mass transfer [6] are the most versatile for different types of moisture and heat transfer. However, cumbersome analytical solutions are valid only for constant coefficients, simplified external material form (such as unbounded plate, ball, or cylinder), and uniquely determined boundary conditions. This is possible only with a slow process, or within sufficiently small areas into which the volume of a moist product is divided. The main empirical mathematical models of dehydration of grain material, which are used to describe heat and mass transfer in the grain material, are presented in [7]. But they are considered only for the convective method of heat supply and are applicable only in the range of the investigated parameter values.

Therefore, the engineering calculation of the kinetics of heating and dehydration of moist dispersed materials is based on approximate methods developed with approximate assumptions for specific systems, with taking into account the hydrodynamic state in the apparatus [8, 9]. Thus, studies [10, 11], in particular, use the balance equations for heat and mass transfer to research the kinetics of heating in the drying of grain material under constant or periodic infrared energy supply. Other studies [10, 11], however, simplify analytical calculations when compiling balance ratios and fail to consider all interacting objects engaged in the combined heat exchange.

Studies [12, 13] present the solutions of models shaped as systems of differential equations with partial derivatives, using the grid method or the mathematical computer package. This allows you to determine the length of the process, the moisture and temperature fields of the grain mass and the drying agent in the layer in time for different grain drying conditions. However, it should be noted that numerical simulation always has a partial character (the result is adequate only under certain conditions), which makes it difficult to use the results obtained from the mathematical model.

To determine kinetic coefficients of heat transfer and moisture transfer included in the equations of mathematical models is extremely difficult. This is due to the fact that the coefficients depend on many defining parameters and conditions of the process, and direct measurements can not be obtained.

Scarce references give different values of thermophysical and mass transfer coefficients. Usually, the coefficients are determined approximately with the use of the empirical dependences for the heat and diffusion Nusselt number, which are obtained on the basis of generalizing a large number of experimental studies [14]. Therefore, there are reasons to believe that the lack of certainty of these factors may lead to inaccuracies in simulation.

Another study [15] proves the relevance of the method for solving inverse problems in the theory of heat and mass transfer. The method of inverse problems allows determining the complexes of coefficients included in the solutions of differential equations [16] with the use of experimentally obtained temperatures at several points inside the body or the material layer as well as its average moisture content. Further, the fields of temperature and moisture content of grain materials can be reproduced numerically or analytically. But the use of this method requires at least an approximate solution of a complex mathematical model of processes and indicative values in the complexes of the sought coefficients.

The problem of the theoretical substantiation of the processes of heat and mass transfer in dryers consists in the complex choice between exact but limited in their application empirical models of processes and universal analytical models with insufficiently reliable kinetic coefficients. Therefore, perspective studies must use theoretical and experimental approaches to enhance the versatility of analytical models and the adequacy of empirical dependences. This would allow determining the parameters and operating modes of heat and mass exchangers and synthesizing optimal control systems.

3. The aim and objectives of the study

The aim of this research is to study theoretical and experimental methods of a combined drying process in a vibration dryer with infrared energy supply for its further improvement.

According to the above aim, the following tasks had to be solved:

– to develop an analytical mathematical model of heat and mass transfer processes in a vibration dryer with infrared energy supply, taking into account all interacting surfaces of objects inside the dryer, and obtain its approximate solution;

– to determine experimentally the kinetics of heating and dehydration of the layer of grain material (rapeseed);

– to use the method of inverse problems for identifying the parametric complexes included in the analytical model's solutions, based on the experimental findings on rapeseed drying.

4. Devising a mathematical model of thermal dehydration in the IR energy

Let us consider an experimental plant of our own design. It is a vibrational surface along which there moves grain material from which moisture is removed due to thermal energy transmitted by radiation from infrared (IR) emitters (Fig. 1).
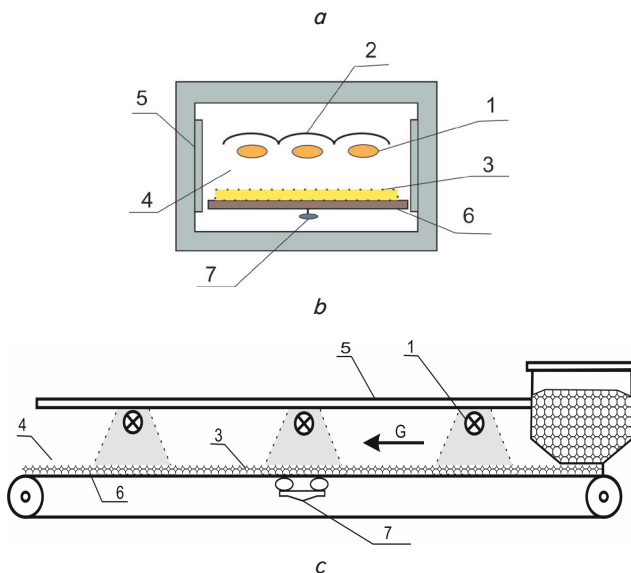
Fig. 1. Experimental infrared dryer: *a* — appearance; *b* — calculation scheme; *c* — technological chart: 1 — infrared heater; 2 — screen; 3 — vibrodiffused layer of grain; 4 — drying agent — air; 5 — side screens; 6 — conveyor; 7 — vibroactivator

Emitters are placed in parallel with the material layer along the entire vibrational transportation surface. A directed flow is ensured due to the screen reflector placed over the emitters and due to the vertical surfaces shielding the side surface of the dryer. The material moves evenly due to the vibrations of the bearing surface of the conveyor belt. In this case, the material perceives the energy of infrared radiation that is partially absorbed and partially reflected, and also by convection transmitted to the air and surrounding surfaces. The thermal energy received by the material is spent on heating the moist material (increasing its enthalpy), evaporation of moisture and is released into the environment: air, side screens, and the conveyor belt. Heat from the material is transmitted both by radiation and convection. The conductive component of the heat transfer to the vibrating surface is incomparably smaller than the convective and radiation components. In the process of drying, the material continuously moves from the place of loading to the place of unloading being heated and dehydrated. The air moves between the belt with the material and the emitters.

In order to facilitate the analysis of this process, we make the following general assumptions in the mathematical description [10, 11]:

– the thermophysical parameters of the material and air do not depend on temperature and in the first approximation are taken as average for the process;

– gradients of temperature and moisture content are viewed regardless of the size of the grain particles (heating and dehydration in individual grains are non-gradient);

– the temperature of the belt, screens and emitters remain the same along the length and in the cross section of the plant (dryer);

– the amount of moisture removed from the material is proportional to the moisture content, and the proportionality factor depends on the temperature linearly.

In accordance with the above description and the assumptions made, we get the following general form of the equation of the heat and material balance for the time $d\tau$ within the segment dy in the direction of motion of the layer of grain material:

$$dQ_0 = dQ_1 + dQ_{12} + dQ_{13} \pm dQ_{14} + dQ_{15}; \qquad (1)$$

$$dQ_{12} = dQ_2 + dQ_{23} + dQ_{24} + dQ_{25}; \qquad (2)$$

$$dQ_{13} + dQ_{14} = dQ_3 + dQ_{34} + dQ_{35} + dQ_w; \qquad (3)$$

$$dQ_{35} = dQ_5 + dQ_{34}; \qquad (4)$$

$$dQ_4 = dQ_{14} + dQ_{24} + dQ_{54} \pm dQ_{34}; \qquad (5)$$

$$dQ_{36} = dQ_6 + dQ_{64}; \qquad (6)$$

$$dM_4 = -dM_3, \qquad (7)$$

where dQ_0 is the energy from the IR emitter, J; dQ_1, dQ_2, dQ_3, dQ_4, dQ_5, and dQ_6 denote energy consumed to heat the elements of the plant, respectively: emitter (lamp, TEH), reflecting screen, material, air, side screens, and transporting means, J; dQ_{12}, dQ_{13}, dQ_{15}, dQ_{25}, dQ_{23}, and dQ_{58} stand for energy transmitted by radiation from the surface of one element to another, according to indices, J; dQ_{14}, dQ_{24}, dQ_{34}, dQ_{36}, dQ_{54}, and dQ_{64} denote energy transmitted by convective (or complex) heat transfer from one element to another, J; dQ_w is energy absorbed by the material and consumed to evaporate moisture from it, J; M is the mass of moisture, kg.

Having determined the values of individual components in the heat and material balance through the known connections [10, 11], we can develop the following system of differential equations (as a mathematical model):

$$P = m_1 c_1 \frac{\partial \theta_1}{\partial \tau} + c_{12} F_{12} \left(T_1^4 - T_2^4\right) + c_{13} F_{13} \left(T_1^4 - T_3^4\right) +$$
$$+ c_{15} F_{15} \left(T_1^4 - T_5^4\right) + \alpha'_{14} F_1 \left(\theta_1 - t\right); \qquad (8)$$

$$c_{12} F_{12} \left(T_1^4 - T_2^4\right) = m_2 c_2 \frac{\partial \theta_2}{\partial \tau} + c_{25} F_{25} \left(T_2^4 - T_5^4\right) +$$
$$+ c_{23} F_{23} \left(T_2^4 - T_3^4\right) + \alpha'_{24} F_2 \left(\theta_2 - t\right); \qquad (9)$$

$$A_\lambda \left[c_{13} F_{13} \left(T_1^4 - T_3^4\right) + c_{23} F_{23} \left(T_2^4 - T_3^4\right) \right] =$$
$$= m_3 c_3 \frac{\partial \theta_3}{\partial \tau} + G_3 c_3 L \frac{\partial \theta_3}{\partial y} + \alpha'_{34} F_{34} \left(\theta_3 - t\right) +$$
$$+ \alpha_6 F_6 \left(\theta_3 - \theta_6\right) \pm c_{35} F_{35} \left(T_3^4 - T_5^4\right) + r \left[m_0 \frac{\partial U}{\partial \tau} + G_0 L \frac{\partial U}{\partial y} \right]; (10)$$

$$m_p c_p \frac{\partial t}{\partial \tau} \pm G_p c_p L \frac{\partial t}{\partial y} = \alpha'_{14} F_1 (\theta_1 - t) +$$
$$+ \alpha'_{24} F_2 (\theta_2 - t) + \alpha'_{34} F_3 (\theta_3 - t) + \alpha'_{54} F_5 (\theta_5 - t); \qquad (11)$$

$$c_{15} F_{15} (T_1^4 - T_5^4) = m_5 c_5 \frac{\partial \theta_5}{\partial \tau} + \alpha'_{54} F_{54} (\theta_5 - t); \qquad (12)$$

$$\alpha'_e F_6 (\theta_7 - \theta_6) = m_6 c_6 \frac{\partial \theta_6}{\partial \tau} + \alpha'_6 F_6 (\theta_6 - t'); \qquad (13)$$

$$\frac{c_7}{Rb} \left(\frac{\partial \theta_7}{\partial \tau} + \frac{G_7 L}{m_7} \frac{\partial \theta_7}{\partial y} \right) = -r \left(\frac{\partial U}{\partial \tau} + \frac{G_4 L}{m_{70}} \frac{\partial U}{\partial y} \right); \qquad (14)$$

$$-\left(m_3 \frac{\partial U}{\partial \tau} + G_3 L \frac{\partial U}{\partial y} \right) = m_4 G_4 \left(\frac{\partial d}{\partial \tau} + G_4 L \frac{\partial d}{\partial y} \right), \qquad (15)$$

where indices in the symbols of the coefficients indicate the objects of heat and mass transfer in accordance with Fig. 1, b; α'_{ij} are coefficients of the complex heat exchange; F_{ij} are the surfaces, m^2; T_i is temperature of the energy source, K; t is air temperature, K; Q is the temperature of the dried material, K; C_{ij} is a reduced radiation coefficient; P is power of the emitter, W; L is length of the conveyor, m; m_i, G_i is the mass of the object and its consumption, kg, kg/s; m_i, c_i is heat capacity of the objects, J/K; A_λ is coefficient of the radiation absorption; r is a reduced heat of vaporization, J/kg; Rb is the Rebinder effect; U is moisture content in the material, kg/kg.

The system can be solved numerically if you set boundary conditions and numerical values of coefficients, which are difficult to calculate. A reduced mathematical description of nonstationary processes allows theoretical analysis of the heat and mass transfer in the existing computer environment and obtaining analytical calculation dependences.

The density of the radiation flux from the radiation surface to the absorbing surface can be represented [10, 11] in the form of the equation similar to the one for convective heat transfer:

$$q_{Bun} = c_{ij} \cdot 10^{-8} (T_1^4 - T_2^4) =$$
$$= c_{ij} \cdot 10^{-8} (T_1^2 - T_2^2)(T_1^2 + T_2^2) = \alpha'_{ij} (\theta_1 - \theta_2), \qquad (16)$$

where

$$\alpha'_{ij} = c_{ij} \cdot 10^{-8} (T_1^2 + T_2^2)(T_1 + T_2)$$

is the linear coefficient of heat transfer under radiation.

Linearizing the radiation components in equations (8)–(10), (12), we rewrite the equation in the following form:

$$m_1 c_1 \frac{\partial \theta_1}{\partial \tau} = P - \alpha_{12} F_{12} (\theta_1 - \theta_2) -$$
$$- \alpha_{13} F_{13} (\theta_1 - \theta_3) - \alpha_{15} F_{15} (\theta_1 - \theta_5) - \alpha'_{14} F_1 (\theta_1 - t); \qquad (17)$$

$$m_2 c_2 \frac{\partial \theta_2}{\partial \tau} = \alpha_{12} F_{12} (\theta_1 - \theta_2) -$$
$$- \alpha_{25} F_{25} (\theta_2 - \theta_5) - \alpha_{23} F_{23} (\theta_2 - \theta_3) - \alpha'_{24} F_2 (\theta_2 - t); \qquad (18)$$

$$m_3 c_3 \frac{\partial \theta_3}{\partial \tau} + G_3 c_3 L \frac{\partial \theta_3}{\partial y} =$$
$$= A_\lambda \cdot \begin{bmatrix} \alpha_{13} F_{13} (\theta_1 - \theta_3) + \alpha_{23} F_{23} (\theta_2 - \theta_3) - \\ -\alpha'_{34} F_{34} (\theta_3 - t) - \\ -\alpha_e F_6 (\theta - \theta_6) \pm \alpha_{35} F_{35} (\theta_3 - \theta_5) \end{bmatrix} +$$
$$+ r \left[m_0 \frac{\partial U}{\partial \tau} + G_0 L \frac{\partial U}{\partial y} \right]; \qquad (19)$$

$$m_p c_p \frac{\partial t}{\partial \tau} \pm G_p c_p L \frac{\partial t}{\partial y} = \alpha'_{14} F_1 (\theta_1 - t) +$$
$$+ \alpha'_{32} F_3 (\theta_3 - t) + \alpha'_{54} F_5 (\theta_5 - t) + \alpha'_{24} F_4 (\theta_2 - t); \qquad (20)$$

$$m_5 c_5 \frac{\partial \theta_5}{\partial \tau} = \alpha_{15} F_{15} (\theta_1 - \theta_5) - \alpha'_{54} F_{54} (\theta_5 - t); \qquad (21)$$

$$m_6 c_6 \frac{\partial \theta_6}{\partial \tau} = \alpha_e F_6 (\theta_7 - \theta_6) - \alpha'_{36} F_{36} (\theta_6 - t'). \qquad (22)$$

In equation (19) we replace the last component, which determines the amount of heat to evaporate the moisture, with its thermal analog from equation (14) and obtain the following:

$$A_7 \left(m_7 - \frac{m_{30}}{Rb} \right) \frac{\partial \theta_7}{\partial \tau} + A_7 L \left(G_7 - \frac{G_{03}}{Rb} \right) \frac{\partial \theta_7}{\partial y} =$$
$$= A_\lambda \begin{bmatrix} \alpha_{13} F_{13} (\theta_1 - \theta_3) + \alpha_{23} F_{23} (\theta_2 - \theta_3) - \\ -\alpha'_{34} F_{34} (\theta_7 - t) - \alpha'_e F_6 (\theta_7 - \theta_6) \pm \\ \pm \alpha_{35} F_{35} (\theta_3 - \theta_5) \end{bmatrix}. \qquad (23)$$

For the established mode, derivatives of time in equations (17)–(22) will equal to zero, and the system of equations for the stationary mode will look as follows:

$$P = \alpha_{12} F_{12} (\theta_1 - \theta_2) + \alpha_{13} F_{13} (\theta_1 - \theta_3) +$$
$$+ \alpha_{15} F_{15} (\theta_1 - \theta_5) + \alpha'_{14} F_1 (\theta_1 - t); \qquad (24)$$

$$\alpha_{12} F_{12} (\theta_1 - \theta_2) = \alpha_{25} F_{25} (\theta_2 - \theta_5) +$$
$$+ \alpha_{23} F_{23} (\theta_2 - \theta_3) + \alpha'_{24} F_2 (\theta_2 - t); \qquad (25)$$

$$\alpha_{15} F_{15} (\theta_1 - \theta_5) = \alpha'_{54} F_{54} (\theta_5 - t); \qquad (26)$$

$$\alpha_6 F_6 (\theta_3 - \theta_6) = \alpha'_{36} F_{36} (\theta_6 - t'); \qquad (27)$$

$$c_7 L \left(G_7 - \frac{G_{03}}{Rb} \right) \frac{\partial \theta_7}{\partial y} =$$
$$= A_\lambda \begin{bmatrix} \alpha_{13} F_{13} (\theta_1 - \theta_3) + \alpha_{23} F_{23} (\theta_2 - \theta_3) - \\ -\alpha'_{34} F_{34} (\theta_7 - t) - \alpha_e F_6 (\theta_7 - \theta_6) \pm \\ \pm \alpha_{35} F_{35} (\theta_3 - \theta_5) \end{bmatrix}; \qquad (28)$$

$$G_p c_p L \frac{\partial t}{\partial y} = \alpha'_{14} F_1 (\theta_1 - t) + \alpha'_{32} F_3 (\theta_3 - t) +$$
$$+ \alpha'_{54} F_5 (\theta_5 - t) + \alpha'_{24} F_4 (\theta_2 - t). \qquad (29)$$

From equations (24)–(27) we determine temperature values for the individual components of the plant (θ_1, θ_2, θ_5, θ_6) by means of the sought values of t and θ_3 and obtain the following:

$$\theta_1 = k_1\theta_3 + k_2 t; \tag{30}$$

$$\theta_2 = k_{31}\theta_3 + k_4 t; \tag{31}$$

$$\theta_5 = k_5\theta_3 + k_6 t; \tag{32}$$

$$\theta_6 = k_7\theta_3 + k_8 t', \tag{33}$$

where k_1–k_8 are dimensionless complexes determined from the constructive F_{ij} and the regime α_{ij} parameters of the plant, which are taken for calculations as constant values:

$$k_1 = \alpha_{13}F_{13}\left(\alpha_{12}F_{12} + \alpha'_{24}F_2 + \alpha_{25}F_{25}\right)\left(\alpha_{15}F_{15} + \alpha_{54}F_{54}\right)/K;$$

$$k_2 = \begin{pmatrix}\alpha_{12}F_{12}\begin{pmatrix}\alpha_{15}\alpha'_{14}F_{15}\left(F_1+F_2\right)+ \\ +\alpha_{54}\begin{pmatrix}\alpha_{15}F_{15}+\alpha'_{14}\left(F_1+F_2\right)+ \\ +\alpha_{25}F_{25}\end{pmatrix}+F_{54}\end{pmatrix}+ \\ +\left(\alpha_{:-;}F_2+\alpha_{25}F_{25}\right)\begin{pmatrix}\alpha_{54}\alpha'_{14}F_1F_{54}+ \\ +\alpha_{15}F_{15}\left(\alpha'_{14}F_1+\alpha_{54}F_{54}\right)\end{pmatrix}\end{pmatrix}/K;$$

$$k_3 = \alpha_{13}F_{13}\left(\alpha_{15}\alpha_{25}F_{15}F_{25} + \alpha_{12}F_{12}\left(\alpha_{15}F_{15}+\alpha_{54}F_{54}\right)\right)/K;$$

$$k_4 = \begin{pmatrix}\alpha_{15}\alpha_{\kappa-;}F_{15}\begin{pmatrix}\alpha'_{14}F_1F_2+\alpha_{13}F_{13}F_2+ \\ +\alpha_{12}F_{12}\left(F_1+F_2\right)+\alpha_{25}F_1F_{25}\end{pmatrix}+ \\ +\alpha_{54}F_{54}\begin{pmatrix}\left(\alpha_{\kappa-;}F_1+\alpha_{13}F_{13}+\alpha_{15}F_{15}\right)\times \\ \times\left(\alpha_{\kappa-;}F_2+\alpha_{25}F_{25}\right)+ \\ +\alpha_{12}F_{12}\begin{pmatrix}\alpha_{15}F_{15}+\alpha'_{14}\left(F_1+F_2\right)+ \\ +\alpha_{25}F_{25}\end{pmatrix}\end{pmatrix}\end{pmatrix}/K;$$

$$k_5 = \alpha_{13}\alpha_{15}F_{13}F_{15}\left(\alpha_{12}F_{12}+\alpha_{:-;}F_2+\alpha_{25}F_{25}\right)/K ;$$

$$k_6 = \begin{pmatrix}\left(\alpha_{:-;}F_2+\alpha_{25}F_{25}\right)\begin{pmatrix}\alpha_{15}\alpha'_{14}F_1F_{15}+\alpha_{54}F_{54}\times \\ \times\left(\alpha_{:-;}F_1+\alpha_{13}F_{13}+\alpha_{15}F_{15}\right)\end{pmatrix}+ \\ +\alpha_{12}F_{12}\begin{pmatrix}\alpha_{15}\alpha'_{14}F_{15}\left(F_1+F_2\right)+\alpha_{54}F_{54}\begin{pmatrix}\alpha_{13}F_{13}+ \\ +\alpha_{15}F_{15}\end{pmatrix}+ \\ +\alpha'_{14}\left(F_1+F_2\right)+\alpha_{25}F_{25}\end{pmatrix}\end{pmatrix}/K;$$

$$k_7 = \frac{\alpha_e F_6}{\alpha_e F_6 + \alpha_{36}F_{36}};$$

$$k_8 = \frac{\alpha_{36}F_{36}}{\alpha_e F_6 + \alpha_{36}F_{36}};$$

where

$$K = \left(\alpha_{\kappa-;}F_2+\alpha_{25}F_{25}\right)\begin{pmatrix}\alpha_{54}F_{54}\left(\alpha'_{14}F_1+\alpha_{13}F_{13}\right)+ \\ +\alpha_{15}F_{15}\left(\alpha'_{14}F_1+\alpha_{13}F_{13}+\alpha_{54}F_{54}\right)\end{pmatrix}+$$

$$+\alpha_{12}F_{12}\begin{pmatrix}\alpha_{54}F_{54}\left(\alpha_{13}F_{13}+\alpha'_{14}\left(F_1+F_2\right)+\alpha_{25}F_{25}\right)+ \\ +\alpha_{15}F_{15}\left(\alpha_{13}F_{13}+\alpha'_{14}\left(F_1+F_2\right)+\alpha_{54}F_{54}\right)\end{pmatrix}.$$

Substituting the obtained values (30)–(33) into expressions (28) and (29), after the corresponding transformations, we get the following system of two interrelated equations:

$$\bar{G}_1 \frac{d\theta_3(y)}{dy} = t - a_1\theta_3 + b_1, \tag{34}$$

$$\bar{G}_2 \frac{dt(y)}{dy} = \theta_3 - a_2 t_2, \tag{35}$$

where

$$\bar{G}_1 = \frac{A_\lambda L G'}{A_\lambda\left(\alpha_{13}F_{13}k_2 + \alpha_{23}F_{23}k_4 + \alpha'_{34}F_{34} - \alpha_{35}F_{35}k_6\right)};$$

$$G' = G_7 - \frac{G_{03}}{Rb};$$

$$a_1 = -\frac{\begin{array}{l}\alpha_{13}F_{13}\left(k_1-1\right)+\alpha_{23}F_{23}\left(k3-1\right)-\alpha'_{34}F_{34}+ \\ +\alpha_{35}F_{35}\left(1-k_5\right)+\alpha_e F_6\left(k_7-1\right)\end{array}}{\alpha_{13}F_{13}k2 + \alpha_{23}F_{23}k_4 + \alpha'_{34}F_{34} - \alpha_{35}F_{35}k_6};$$

$$b_1 = \frac{\alpha_e F_6 k_8 t'}{A_\lambda\left(\alpha_{13}F_{13}k_2 + \alpha_{23}F_{23}k_4 + \alpha'_{34}F_{34} - \alpha_{35}F_{35}k_6\right)};$$

$$\bar{G}_2 = \frac{c_p G_p L}{\alpha'_{14}F_1 k_1 + \alpha'_{24}F_2 k_3 + \alpha'_{34}F_3 + \alpha_{45}F_5 k_5};$$

$$a_1 = -\frac{\alpha'_{14}F_1\left(1-k_2\right)+\alpha'_{24}F_2\left(1-k_4\right)+\alpha'_{54}F_5\left(1-k_6+\right)+\alpha'_{34}F_3}{\alpha'_{14}F_1 k_1 + \alpha'_{24}F_2 k_3 + \alpha'_{34}F_3 + \alpha_{45}F_5 k_5}.$$

Solving equations (34) and (35) with regard to $t(y)$ and $\theta(y)$, after transformations we obtain the following:

$$A\frac{d^2\theta_3}{dy^2} + B\frac{d\theta_3}{dy} + C\theta_3 = D_1, \tag{36}$$

$$A\frac{d^2 t}{dy^2} + B\frac{dt}{dy} + Ct = D_2, \tag{37}$$

where

$$A = \bar{G}_1\bar{G}_2; \quad B = a_2\bar{G}_1 + a_1\bar{G}_2;$$

$$C = a_1 a_2 - 1; \quad D_1 = -b_1 a_2, \quad D_2 = b_1.$$

General solution of equations, under boundary conditions,

$$y=0; \quad \theta_3=\theta_0=\theta_1; \quad t=t_0=t_1; \quad \frac{d\theta}{d\tau}=\frac{dt}{d\tau}=0$$

according to [10] looks as follows:

$$t(y) = c_1'\left(r_1 e^{r_2 y} - r_2 e^{r_1 y}\right) + \frac{D_2}{C}; \tag{38}$$

$$\theta_3(y) = c_2'\left(r_1 e^{r_2 y} - r_2 e^{r_1 y}\right) + \frac{D_1}{C}; \tag{39}$$

where

$$r'_{1,2} = \frac{-B' \pm \sqrt{B'^2 - 4AC}}{2A'},$$

$$c_1' = \frac{t_0 C - D_2}{C(r_1 - r_2)}; \quad c_2' = \frac{\theta_{03} C - D_1}{C(r_1 - r_2)}. \quad (40)$$

To determine the distribution of moisture in the material during drying along the length of the drying chamber in the direction of motion, we use equation (14) for zero derivatives of time:

$$\frac{c_7}{Rb} \frac{G_7 L}{m_7} \frac{\partial \theta_7}{\partial y} = -r \frac{G_4 L}{m_{70}} \frac{\partial U}{\partial y}. \quad (41)$$

The value of the ratio $\frac{G_3}{m_3} L$ characterizes the rate of movement of the grain material because

$$G_3 = \frac{m_3}{\tau}; \quad \frac{L}{\tau} = \upsilon_3,$$

thus, $\frac{G}{m} L = \upsilon,$ and eventually we get the following:

$$\frac{1}{Rb(U)} c_3(U) \frac{d\theta_3}{dy} = -r_0 \frac{dU}{dy}. \quad (42)$$

Having differentiated equation (39), we obtain the following:

$$\frac{d\theta_3(y)}{dy} = K_T \left(r_1 r_2 e^{r_2 y} - r_2 r_1 e^{r_1 y} \right) = F_1(y), \quad (43)$$

where

$$K_T = \frac{\theta_{03} C - D_1}{C(r_1 - r_2)}.$$

Substituting the obtained dependence into equation (41), we get the following:

$$-\frac{dU}{dy} = \frac{c_3(U) \cdot K_T}{Rb(U) \cdot r_0} \left(r_1 r_2 e^{r_2 y} - r_2 r_1 e^{r_1 y} \right), \quad (44)$$

where $c_3(U)$ is heat capacity of the grain material:

$$c_3(U) = U \cdot c_v + (1 - U) \cdot c_{c3}, \quad (45)$$

where c_v is specific heat capacity of water, and c_{c3} is specific heat capacity of the dry grain material.

Provided

$$\frac{Rb(U)}{c_3(U)} = A_R e^{nU},$$

where A_R and n are empirical coefficients, having integrated equation (44)

$$-A_R r_0 \int_{U_1}^{U} e^{nU} dU = K_T r_1 r_2 \left[\int_0^y e^{r_2 y} dy - \int_0^y e^{r_1 y} dy \right],$$

we get the following:

$$U(y) = \frac{1}{n} \ln \left(e^{nU_1} - K_c \left[\frac{e^{r_2 y} - 1}{r_2} - \frac{e^{r_1 y} - 1}{r_1} \right] \right); \quad (46)$$

where

$$K_c = \frac{n \cdot K_T}{A_R \cdot r_0} r_1 r_2.$$

To obtain an analytical expression of the change in the grain moisture content over time, it is necessary to have a dependence of the temperature change in the material in some time. Having substituted the expression of the derivative of grain temperature from the coordinate $\frac{d\theta_3}{dy}$ from equation (43) into equation (19), we obtain the following:

$$A_7 \left(m_7 - \frac{m_{30}}{Rb} \right) \frac{\partial \theta_7}{\partial \tau} + A_7 L \left(G_7 - \frac{G_{03}}{Rb} \right) F_1(y) =$$
$$= A_\lambda \left[\begin{array}{c} \alpha_{13} F_{13} (\theta_1 - \theta_3) + \alpha_{23} F_{23} (\theta_2 - \theta_3) - \\ -\alpha_{34}' F_{34} (\theta_7 - t) - \alpha_e' F_6 (\theta_7 - \theta_6) \pm c_{35} F_{35} (\theta_3 - \theta_5) \end{array} \right]. \quad (47)$$

Let us differentiate equation (47):

$$\frac{dt(y)}{dy} = K_t \left(r_1 r_2 e^{r_2 y} - r_2 r_1 e^{r_1 y} \right) = F_2(y), \quad (48)$$

where

$$K_5 = \frac{t_{04} C - D_2}{C(r_1 - r_2)}$$

and substitute the resulting dependence into equation (19), and we will have the following:

$$m_p c_p \frac{\partial t}{\partial \tau} \pm G_p c_p F_2(y) = \alpha_{14}' F_1(\theta_1 - t) +$$
$$+ \alpha_{32}' F_3(\theta_3 - t) + \alpha_{54}' F_5(\theta_5 - t) + \alpha_{24}' F_4(\theta_2 - t); \quad (49)$$

Equations (47) and (49) as well as (28) and (29) have the same values in the right-hand side, so using substitution (30)–(33) we obtain the equation of the dynamics of change in the parameters $\theta_3(\tau, y)$ and $t(\tau, y)$ in the following form:

$$T_1 \frac{d\theta_3}{d\tau} = t - a_1 \theta_3 + b_1(y); \quad (50)$$

$$T_2 \frac{dt}{d\tau} = \theta_3 - a_2 t + b_2(y); \quad (51)$$

where

$$T_1 = \frac{c_7 m_3}{A_\lambda \left(\alpha_{13} F_{13} k_2 + \alpha_{23} F_{23} k_4 + \alpha_{34}' F_{34} - \alpha_{35} F_{35} k_6 \right)};$$

$$T_2 = \frac{c_p m_p}{\alpha_{14}' F_1 k_1 + \alpha_{24}' F_2 k_3 + \alpha_{34}' F_3 + \alpha_{45} F_5 k_5};$$

$$b_1 = \frac{b_1 - G_3 c_3 L \cdot F_1(y)}{A_\lambda \left(\alpha_{13} F_{13} k_2 + \alpha_{23} F_{23} k_4 + \alpha_{34}' F_{34} - \alpha_{35} F_{35} k_6 \right)};$$

$$b_2 = \frac{\pm G_7 c_7 L \cdot F_2(y)}{A_\lambda \left(\alpha_{13} F_{13} k_2 + \alpha_{23} F_{23} k_4 + \alpha_{34}' F_{34} - \alpha_{35} F_{35} k_6 \right)}.$$

Solution of the system (50), (51) is analogous to the solution of equations (34) and (35) under the following initial conditions:

$$\tau = 0; \quad \theta_3 = \theta_0 = \theta_1; \quad t = t_0 = t_1; \quad \frac{d\theta}{d\tau} = \frac{dt}{d\tau} = 0,$$

where θ_0 and t_0 are the initial values of temperature of the grain and air, which are equal to their values at the input.

$$\theta_3(\tau, y) = \frac{\theta_{03}C - D_1(y)}{C(r_1' - r_2')}\left(r_1'e^{r_2'\tau} - r_2'e^{r_1'\tau}\right) + \frac{D_1(y)}{C}; \qquad (52)$$

$$t(\tau, y) = \frac{\theta_{03}C - D_1(y)}{C(r_1' - r_2')}\left(r_1'e^{r_2'\tau} - r_2'e^{r_1'\tau}\right) + \frac{D_1(y)}{C}; \qquad (53)$$

where

$$r_{1,2}' = \frac{-B' \pm \sqrt{B'^2 - 4AC}}{2A'}; \quad A' = T_1 T_2;$$

$$B' = a_1 T_2 + a_2 T_1; \quad C = (a_1 a_2 - 1);$$

$$D_1(y) = b_2(y) - a_2 b_1(y); \quad D_2(y) = b_1(y) - a_1 b_2(y).$$

Equations (50) and (51) describe the change in temperature of the grain and air with time, along the length of the dryer's chamber. To obtain the dependence of the change in moisture content in the material on the time and coordinates, we differentiate equation (52) by time:

$$\frac{d\theta_3(\tau, y)}{d\tau} = K_\tau(y) r_1' r_2'\left(e^{r_2'\tau} - e^{r_1'\tau}\right), \qquad (54)$$

where

$$K_\tau(y) = \frac{\theta_{03}C - D_1(y)}{C(r_1' - r_2')}$$

and substitute the obtained value in the equation for determining the Rebinder effect:

$$c_7(U)\frac{d\theta}{d\tau} = -Rb(U) r_0 \frac{dU}{d\tau},$$

provided

$$\frac{Rb(U)}{c_3(U)} = A_R e^{nU},$$

having integrated equation (52):

$$-A_R r_0 \int_{U_1}^{U} e^{nU} dU = K_\tau(y) r_1' r_2'\left[\int_0^\tau e^{r_2'y} d\tau - \int_0^\tau e^{r_1'\tau} d\tau\right],$$

we get the following:

$$U(\tau, y) = \frac{1}{n}\ln\left(e^{nU_1} - K_{c'}(y)\cdot\left[\frac{e^{r_2'\tau} - 1}{r_2} - \frac{e^{r_1'\tau} - 1}{r_1}\right]\right), \qquad (55)$$

where

$$K_{c'}(y) = \frac{n \cdot K_\tau(y)}{A_R \cdot r_0} r_1' r_2'.$$

Thus, the basic dynamic characteristics of the drying conditions of grain and oil-bearing materials in a continuously operating plant with IR energy supply can be de-

scribed by equations (52) and (55). Dependences (52) and (55) determine the distribution of temperature and moisture content of grain and oil-bearing materials along the length at any moment of time. To determine the parameters of grain and oil-bearing materials at the outlet it is necessary to accept that $y=L$. Then equations (52) and (55) will determine the change in the final parameters of the material, i. e. moisture content $U_2(\tau)$ and temperature $\theta_2(\tau)$.

Parametric identification of the mathematical model of heat and mass transfer processes in the IR drying of grain materials (with vibrational motion) requires the values of the coefficients substituted in the equation. Theoretically, coefficients can be determined only by iterative calculations by the known reference data.

5. Experimental findings on heat and mass transfer in the IR dryer

The obtained mathematical models can be identified according to experimental data, by comparing the analytical and experimentally obtained dependences [14, 15]. But it is very difficult to determine parameters (such as temperature and moisture content) of the material in the moving layer in the presence of IR radiation and vibration. Therefore, we will determine the final parameters of temperature and moisture content in the grain at the output of the dryer changing the heat treatment exposure. Experimental data have been approximated by polynomials of the second order; the graphic interpretation of the experimental data and calculated curves of the dependence of temperature and moisture content are presented in Fig. 2, 3.
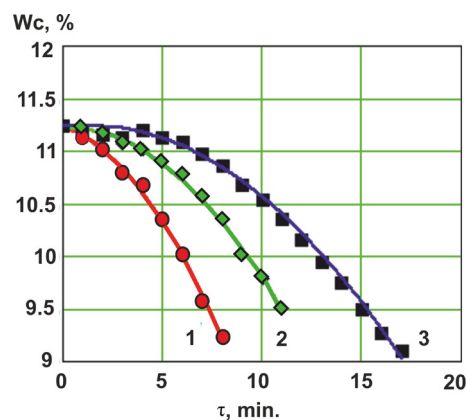

Fig. 2. Changes in the dampness of IR-treated rapeseed in different-weight batches (experimental data and curves of polynomial regressions), P=320 W: 1 — 100 g; 2 — 200 g; 3 — 300 g

In order to determine the Rebinder effect, experiments were carried out in a periodic drying mode, and the changes in temperature and mass were recorded in real time. The dependence of the change in the Rebinder effect on the material dampness is shown in Fig. 4.

As you can see from Fig. 4, the value of the Rebinder effect decreases alongside reducing the moisture content in the grain, which indicates that higher moisture content in the grain material involves higher specific heat consumption for its evaporation.

The change in the moisture content of the material with time (see the drying curve) during the dryer operation in a

continuous mode was determined by the method of successive overlay of experiments, i. e. there were recorded changes in the moisture content during the dryer operation within a certain interval of time.
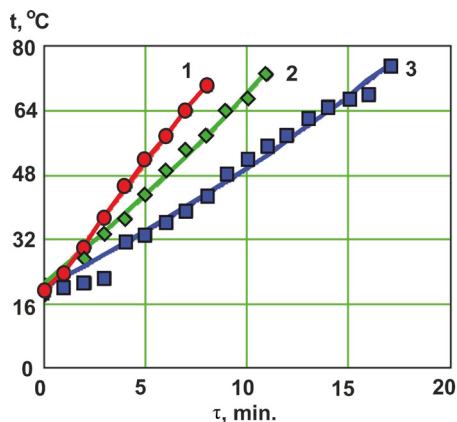

Fig. 3. Thermograms of IR drying of rapeseed in different-weight batches (experimental data and curves of polynomial regressions), P=320 W: 1 — 100 g; 2 — 200 g; 3 — 300 g
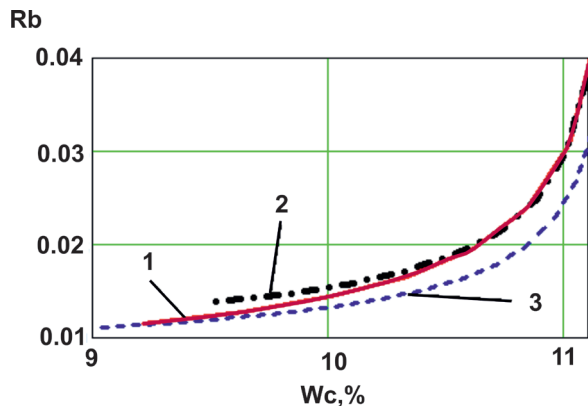

Fig. 4. Dependence of the Rebinder effect on the moisture content in the material, P=320 W: 1 — 100 g; 2 — 200 g; 3 — 300 g

Fig. 5 shows the results of experiments with the graphs of changing moisture content in rapeseed (in a moving layer).
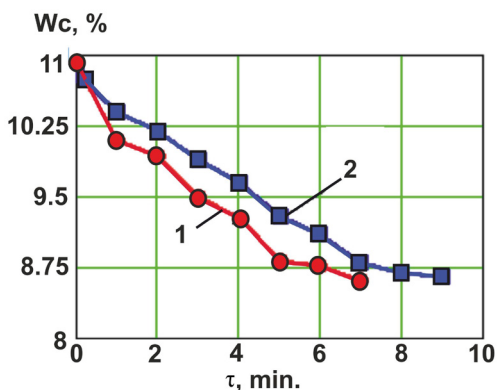

Fig. 5. Changes in the dampness of grain in the moving layer: 1 — P=500 W, 2 — P=400 W

In this case, the analytical dependence can be used to identify mathematical models by comparison with the experimental data.
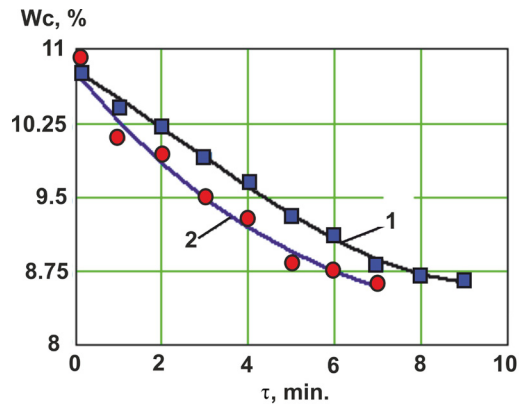

Fig. 6. Experimental data on rapeseed drying and the calculated curves: 1 — P=400 W, 2 — P=500 W

The use of the Leuenberg-Marquard algorithm and the method of parametric identification of heating plants [16] allowed determining the main parametric complexes of models (50) and (53). Thus, in particular, Fig. 6 shows the graphic interpretation of the calculated curves according to the analytical dependence (53) and experimental points.

Fig. 6 shows that the theoretical curves with identified parametric complexes deviate from the experimental points by less than 5 %.

6. Discussion of the results of mathematical modeling and experimental identification of the model

The mathematical model of heat and mass transfer processes is devised by means of analytical modeling based on energy and material balance equations as well as generalizations of experimental data based on the theory of similarity, probability and mathematical statistics. The peculiarity and value of the proposed model consist in the found regularities of heat and material interaction between all objects of heat and mass transfer inside the dryer. The approximate analytical solution presented by this model allows the following:

– to determine the fields of temperature and moisture content of the grain material within time, by the coordinate in the movement direction;

– to determine and clarify the parametric complexes of the model based on the empirical dependences of the kinetics of heating and dehydration of the grain material;

– in the future, to use this model for synthesizing a system of the optimal drying control.

The results of experimental studies are presented in the form of curves obtained by approximation of the experimental data. The found dependences allow summarizing the experimental data in the form of criteria dependences. In particular, Fig. 4 shows the dependence of the Rebinder effect; the use of this dependence allows model (8)–(15) to be solved analytically.

The obtained empirical mathematical models of the kinetics of heating and drying were used for the parametric identification of approximate solutions of the analytical mathematical model. The specified kinetic coefficients of dependences allow further significantly improvement of the accuracy of the numerical solution of the devised analytical model.

The above research findings will be useful to improve plants and technologies of combined IR drying. But their practical use requires rememdering the following: the pre-

sented mathematical models are kinetic, i.e. they do not take into account the distribution of temperature and moisture content inside the seeds, which limits their application when IR energy is highly intensive or the geometric sizes of seeds increase.

In this study, analytical calculations were facilitated due to the linearized coefficients of the radiation heat transfer, regardless the peculiarities of absorption of IR radiation by the material. But the nonlinear or piecewise linear approximation of the coefficients of radiation transfer and taking into account variables of the optical coefficients of the IR-treated material significantly complicate the mathematical calculations and can make obtaining an approximate solution of the system of balance equations impossible.

7. Conclusions

1. The devised mathematical model of heat and mass transfer in the vibration dryer with IR energy supply takes into account the heat transfer by convective and radiation methods between all interacting objects inside the dryer. The proposed approximate solution of the system allows predicting the development of temperature and moisture content fields depending on the power of the IR emitter for further calculation of the energy efficiency in the dryer and synthesizing of the system of optimal process control.

2. The results of experimental studies of the kinetics of heating and dehydration of the grain material have laid the basis for determining how parameters of the IR dryer impact the exposure of the drying process. In particular, if the power of the IR emitter increases from 400 to 500 W, the drying time from the initial dampness of the material of 11 % to the moisture content of 8.75 % decreases from 9 to 7 minutes. It is determined that the Rebinder effect, which characterizes the dampness and heat properties of the material, decreases with a decrease in its moisture content from the value of 0.04 at 11 % to 0.01 at 9 %. This proves that higher moisture content of the material involves higher specific heat consumption for the moisture evaporation.

3. The method of inverse problems was used to identify parametric complexes of finite analytical dependences. The study presents the found parametric complexes and shows that experimental data coincide with the theoretical calculations within the range of 95–98 %.

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