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Розглянуті питання геометричного синтезу просторових шар-нірно-важільних шестиланкових механізмів з лінійним переміщенням кінцевої ланки, що виконують функиію напрямної. Виявлено варіанти компонування механізмів та конструктивні особливості, що забезпечують максимальний хід кінцевої ланки при мінімальних довжинах важелів. Встановлено геометричні параметри механізму в його узагальненому вигляді, які визначають кінематику та компонувальні схеми. Досліджено вплив геометричних параметрів $і$ варіантів компонування на кінематичні параметри механізмів. Отримано залежності, які дозволяють визначити геометричні параметри базового механізму за заданим ходом кінцевої ланки та допустимими кутами передачі у шарнірах повідиів. Подано параметричні залежності, які дозволяють провести точний розрахунок оптимальної геометрії механізму за критерієм мінімізацї довжин повідців при допустимих кутах передачі і необхідному діапазоні переміщень. Запропонована схема розрахунку просторових розмірних ланиюгів для визначення форми деталей. 3D моделюванням виявлена варіативність геометричних параметрів, що дозволило сфорлулювати компонувальні варіанти механізму. Розроблено методику геометричного синтезу та змодельовано у відповідності до цієї методики варіанти просторових шарнірно-важільних шестиланкових механізмів в динаміиі, що дозволило показати особливості руху ланок.

Проведені дослідження виявили можливі шляхи розробок нових варіантів просторових шарнірно-важільних шестиланкових механізмів та розкрили нові можливості при їхньому застосуванні у якості напрямного механізму. Результати досліджень можуть бути використані при розробленні платформ підйомників, маніпуляторах роботів, верстатобудуванні та мехатроніці

Ключові слова: механізм Саррюса, шестиланковий просторовий механізм, напрямний механізм, лінійне переміщення, геометричний синтез

# GEOMETRICAL SYNTHESIS OF SPATIAL SIXLINK GUIDING MECHANISMS 

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## 1. Introduction

Lever mechanisms with a linear motion of moving parts are used in devices and equipment of various areas of mechanical engineering, including robotics, lifting machinery, machine tools, and mechatronics. Despite the complexity of the structure, multifaceted lever mechanisms of linear relocation in some devices have replaced guides with sliding carriages, telescopic mechanisms, and carriages on linear bearings. This is due to various factors: the requirements of layout and minimization of the dimensions, the need to protect open surfaces from friction, the tendency to jam carriages with translational pairs, etc.

The choice of the scheme and structure of these non-standard mechanisms can significantly affect the performance of the devices in general, their layout, dimensions, cost, etc.

Hinged lever mechanisms with translational or near-translational finite-element coupling provide for the implementation of compact structures in a folded state in the absence or minimum number of translational kinematic pairs [1, 2]. This avoids the drawbacks of translational pairs and uses standard camshafts with protected local friction surfaces in the design of rotary kinematics pairs. An increase in the course of the final link is achieved in these mechanisms by using levers.

Flat guide lever mechanisms with rotational pairs are found in devices for a steady movement of the working body of machine tools or platforms of lifts and manipulators of robots. In the designs of lifts and manipulators, flat lever pantograph mechanisms are widely used [3-5], which, however, are not devoid of translational pairs. In woodworking format-cutting machines, combined circuits are employed,
where the movable table rests on the carriage and is supplemented with leverage. The spatial hinged six-link mechanism of translational movement is used in cutting machine tools for linear movement of a disk saw. Lever mechanisms are used in the designs of independent suspensions of wheels of vehicles, where they implement an approaching to the linear center of the wheel trajectory.

However, the scope of applying guiding lever mechanisms unjustifiably limits the need for an individual approach to synthesis and design, as well as a relative complexity of the design. This situation is due to insufficient development of theoretical aspects for analysing design schemes, systematization, and description. The consequence of this is the lack of a standard design methodology, standard solutions, and, finally, the identification of how these mechnisms can be modified and imporved.

Therefore, due to the increasing demand for specialized robotics, compact lifting vehicles, precision manipulators (military subjects, orthopedics, remote surgery, etc.), the study of spatial guiding mechanisms seems to be an important issue.

## 2. Literature review and problem statement

The spatial hinged six-link lever mechanism (SHSLLM), also known as the Sarrus linkage [5], provides an ideal linear trajectory for the end-link movement and can be used to convert the movement, as a folding structure, as a cyclic mechanism, etc. (transport and delivery and load-lifting mechanisms, manipulators, suspension wheels of vehicles, machine tools, etc.). A series of papers is devoted to the synthesis of the SHSLLM, in particular in papers [6, 7]; the possibility of creating an independent suspension of the wheels of the car with the structure of the SHSLLM has been proposed and substantiated, including the aspects of geometric synthesis. However, they do not take into account all geometric parameters and do not perform parametric studies. Also, the options for composing and optimizing geometry with regard to the course of the final link have not been studied. The need to avoid the dead is not taken into account. Kinematical analysis and dynamics equations of this mechanism, derived using the Newton-Euler method, are presented in [8]. Further development of the suspension mechanism of the car on the basis of the SHSLLM is presented in [9], where the possibility of structural modification of the mechanism is grounded in order to improve the conditions of the load of the links.

However, in these works the issues of geometric parametrization of the mechanism remain unclear. Paper [10] analyzes the mobility of a self-installing seven-link mechanism, formed on the basis of the SHSLLM. The properties of compensating for the inaccuracy of the elements of the kinematic chain are shown. However, questions of geometric parametrization are not considered. Paper [11] covers the problems of the geometric synthesis of compact structures with two and three separated degrees of freedom in a folded state of folding structures. The basis of these structures is the use of the Sarrus linkage. An analysis of the kinematics of the multistable Sarrus mechanism with pliable links and elastic hinges is given in [12], which shows the possibility of forming several stable positions. However, these works do not sufficiently elaborate aspects of creating alternative layout schemes and the possibility of modification due to changes in geometric parameters.

In general, to date, the aspects of geometric synthesis of the SHSLLM are not sufficiently developed, and the question of parameterization of the mechanism remains unexplored. Layout variants and design constraints as well as influence on the kinematic and dynamic characteristics of the mechanism remain unidentified. The questions of the geometric synthesis of the SHSLLM as a guiding mechanism and the problem of optimizing its geometry have not been studied.

That is why research in the direction of developing a method of synthesis, parametric studies as well as identification of variants of geometric layouts of spatial mechanisms of linear displacement as a guiding motion need further development.

## 3. The aim and objectives of the study

The aim of the research is to solve the problem of geometric synthesis of spatial-hinged-lever six-link mechanisms with linear displacement of the final link to perform the function of a guide. This will make it possible to calculate and design spatial six-link mechanisms that perform the function of a linear guide, with optimal geometric parameters, taking into account the course and permissible angles of transmission in the joints, and also facilitate the choice of a rational layout.

To achieve the aim, the following objectives were set:

- to determine geometrical parameters of the mechanism in its generalized form, which determine the kinematics and influence the choice of the layout scheme;
- to establish parametric dependencies that help calculate the optimal geometry of the mechanism by the criterion of minimizing the length of the levers at the permissible angles of transmission and the required range of displacements and to investigate the influence of geometrical parameters and features of the layout on the kinematic parameters of the mechanism;
- to find out variants of layout and design features of the SHSLLM, to develop the method of geometric synthesis of SHSLLMs, including calculation of the main structural dimensions of the mechanism elements.


## 4. Parametric studies of the geometry of spatial-hinged-lever six-link mechanisms

## 4. 1. Geometric parameters of the generalized SHSLLM

The Sarrus linkage (Fig. 1) is a variant of the SHSLLM. The final link is connected to the riser by two kinematic chains, each of which is formed by a two-letter group with three rotational pairs. The final link acquires the mobility in the event that the axis of the joints in each separate kinematic chain is parallel. This mobility is the exact translational movement in the direction perpendicular to the axes of all joints.

Since the final link is attached to the riser by two kinematic chains, the SHSLLM relates to mechanisms with parallel kinematics.

Calculation of the number of passive links by the Malyshev formula shows that the mechanism has one passive link. This means that it does not belong to self-installing [13] and is statically uncertain, and to ensure the mobility of a real mechanism, high precision is required for the manufacture
of its parts. It is known that passive bonds contribute to increasing the rigidity of mechanisms with malleable links. In exact mechanics, when constructing kinematic mechanisms it is necessary to adhere to the principle of self-restraint; the use of the SHSLLM is not desirable, because only self-regulating mechanisms help provide an easy move with a reduced accuracy of manufacturing.


Fig. 1. The Sarrus linkage
In the general case, the SHSLLM (Fig. 2) is characterized by the following parameters.

The main parameters on which the laws of motion of all levers (links 2, 3, 4, and 5) and the range of movement (course) of the final link 1 depend are the lengths of the levers $A_{1} B_{1}, B_{1} C_{1}, A_{2} B_{2}$, and $B_{2} C_{2}$, the axial distances $O_{1} O_{1}{ }^{\prime}$ and $\mathrm{O}_{2} \mathrm{O}_{2}{ }^{\prime}$, as well as displacements $\mathrm{O}_{2} \mathrm{~F}$ and $O_{2}{ }^{\prime} G$. The structural parameters on which the stability and stiffness of the structure depend are the angle between the axes of the joints $\beta$ as well as the sizes of $O_{1} C_{1}, O_{2} A_{1}, O_{1}{ }^{\prime} C_{2}$, and $O_{2}{ }^{\prime} A_{2}$.

The mechanism provides the movement of link 1 along the axis $O X$.


Fig. 2. A scheme of the SHSLLM with arbitrary dimensions, interaxial distances and displacements

In [6], the method of the screw calculation proved that the SHSLLM has the highest stability of the structure at an angle between the axes of the hinges of the two chains $\pi / 2$ and loses stability at 0 . A slight deviation from the angle $\pi / 2$ is admissible.

To increase the course of the end-point, which is an indicator of the effectiveness of the guiding mechanism, it is necessary to ensure the possibility of the unhindered passage of the end-link beyond the fixed one (two-way mobility).

Fig. 2 shows the kinematic scheme of the SHSLLM with arbitrary geometric parameters. The lengths of levers $2,3,4$, and 5 are all different, and the interaxial distances $O_{1} O_{1}^{\prime} \neq O_{2} O_{2}^{\prime}$ are different, too. The movement of the final link 1 here is possible along the $O X$ axis without meeting with the fixed link 6 (riser). As shown by the 3D modeling, for the implementation of two-way mobility, the presence of transverse displacements of the axes of the joints of both pairs of levers $O_{2} F=e_{1}$ and $O_{2}{ }^{\prime} G=e_{2}$ is compulsory. With a zero displacement in the lever pair, the mechanism passes the position of uncertainty in which the levers in a given pair are parallel.

The distance $\Omega$ between the parallel segments $O_{1} O_{1}{ }^{\prime}$ and $\mathrm{O}_{2} \mathrm{O}_{2}{ }^{\prime}$ represents the complete transverse displacement of the mechanism and is a complex geometric parameter of the SHSLLM. It can be calculated by the formula

$$
\begin{equation*}
\Omega=\sqrt{e_{1}^{2}+\left(\frac{e_{1}}{\tan \beta}+\frac{e_{2}}{\sin \beta}\right)^{2}} . \tag{1}
\end{equation*}
$$

In the case of $e_{1}=e_{2}=e$,

$$
\begin{equation*}
\Omega=\frac{e}{\sin (0,5 \beta)} . \tag{2}
\end{equation*}
$$

The complete displacement $\Omega$ as well as the individual transverse displacements $e_{1}$ and $e_{2}$ are the basic geometric parameters and necessary initial data for the design of the mechanism.

The Sarrus linkage (Fig. 1) is a variant of the SHSLLM for which $\beta=\pi / 2$, the lengths of all levers are equal, whereas the displacement, interaxial distances and the parameter are equal to zero.
4. 2. Calculation of the optimal geometry of the mechanism by the criterion of minimizing the length of the levers at the permissible angles of transmission and the required range of displacements

In geometric synthesis and layout, it is desirable to optimize the geometry of the mechanism by the criterion of minimizing the sizes of all the links, including the end and the riser. When the end-link load is transverse to the axis of displacement by force and directionaly arbitrary moments, all the links of the SHSLLM work on bending and torsion [9], so minimizing the sizes of all the links increases the stiffness of the mechanism.

Geometric synthesis should be made taking into account the limits of the transmission angles, since when the angle of transmission approaches zero, the mechanism takes a dead position with the uncertainty of the movement of the links [14]. Let us introduce the angles $\alpha_{1}$ and $\alpha_{2}$ respectively in the middle (II) and extreme (I and IV) positions of the levers. The lengths of the levers will be taken differently, which is useful for design or technological reasons. Let us denote the lengths of the levers $a=A B$ and $b=B C$; let $b=k a$, where $k$ is a constructive coefficient. In the case of $k=1$, it is possible to unify the construction of both levers.


Fig. 3. Characteristic provisions of the levers of the two-lever group
the structural parameters $k_{1}$ and $k_{2}$ in the pairs can be different if needed. In this case, the sizes of the levers of both pairs and the transverse displacements $e_{1}$ and $e_{2}$ will be different.
2) The interaxial distances for the two lever pairs should be the same: $O_{1} O_{1}{ }^{\prime}=O_{2} O_{2}{ }^{\prime}=\Delta$. We call such an SHSLLM a normal guiding mechanism. In the opposite case (in the diagram in Fig. 2, $O_{1} O_{1}{ }^{\prime} \neq O_{2} O_{2}{ }^{\prime}$ ), the estimated run decreases by the difference between these distances. In the future, we will only consider the normal SHSLLM.

We set parametric dependences for the geometric synthesis of the SHSLLM. The initial data are the course of the final link $S$ and the parameters for each lever pair: the constructive coefficient $k$ and the permissible angles $\alpha_{1}$ and $\alpha_{2}$.

The dimensions of the links and the transverse displacement $e$ for one lever pair are determined by the following formulas:

$$
\begin{align*}
& a=\frac{S}{\sqrt{8 k\left(\cos \alpha_{1}-\cos \alpha_{2}\right)}}  \tag{3}\\
& e=a \sqrt{k^{2}-2 k \cos \alpha_{1}+1} . \tag{4}
\end{align*}
$$

The calculated transverse displacement $e$ must be provided with structural dimensions the calculation of which is given below.

## 4. 3. Parametric tests of the SHSLLM

Analysis of dependences (3) and (4) helps determine the effect of the coefficient $k$ on the total length of the links $a+b$ and the transverse displacement $e$.

The nature of these influences is shown on the nomograms (Fig. 4,5). The nomograms are calculated for conditional "unit" mechanisms in which geometric parameters are expressed through the course of the final link $S$ when $S=1$ is taken. Curve 1 corresponds to the angles $\alpha_{1}=20^{\circ}$ and $\alpha_{2}=155^{\circ}$; curve 2: $\alpha_{1}=30^{\circ}$ and $\alpha_{2}=145^{\circ}$; curve 3: $\alpha_{1}=40^{\circ}$ and $\alpha_{2}=135^{\circ}$; curve 4: $\alpha_{1}=50^{\circ}$ and $\alpha_{2}=125$.

Changing the coefficient $k$ for a given course of $S$ slightly affects the change in the sum of the lengths of $a+b$, but it causes a significant change in the transverse displacement $e$. For example, when $k$ changes from 1 to 4, the total length of the pair of levers increases by only $25 \%$, and the displacement $e$ is 2.4 times.

In order to ensure the maximum speed of the real SHSLLM with a minimum total length of the levers, it is necessary to conduct a geometric synthesis in accordance with the two rules:

1) Both levers of the pairs of the real mechanism should be calculated with the same course of the final link $S$, but


Fig. 4. The nomogram of the influence of the constructive coefficient $k$ on the transverse displacement parameter $e$ is a pair of levers


Fig. 5. The nomogram of the influence of the constructive coefficient $k$ on the parameter / of a pair of levers
4.4. Calculation of the size of the main structural elements of the SHSLLM

Let us reveal the geometric parameters of the SHSLLM that practically implement the calculated displacements $e_{1}$ and $e_{2}$ of both pairs of levers. Let us consider the scheme
(Fig. 6) of the mutual arrangement of the axes of the fixed hinges and hinges belonging to the finite link of the normal SHSLLM at the midpoint of the final link (corresponding to position II in Fig. 3); $\beta$ is the angle between the axes of the joints of the pairs of levers. Here, the pairs of levers are not shown clearly, and the extreme hinges are schematically depicted in the form of circles inscribed in parallelepipeds 1 and 2 where the points $A$ and $C$ lie on the geometric axes of the extremum hinges in each pair of levers. Also, here and in the future, it is essential to consider the most rational construction of joints with one standard bearing from the point of view of the layout of the power SHSLLM. SHSLLMs are characterized by transverse displacements $e_{1}$ and $e_{2}$ as well as constructive sizes $L_{1}$ and $L_{2}$ - the distances between the conditional planes of the attachment of the extreme hinges of the pairs of levers to the links of the relative translational motion. In order to calculate the dimensional chains, it is recommended to accept the outer face surfaces of the bearings of joints $A$ and $C$ as the indicated planes.


Fig. 6. For calculating the dimensional chains of the SHSLLM

The distances $O_{2} A_{1}$ and $O_{2}{ }^{\prime} A_{2}$ determine the coordinates of the bearing seats in the body, and the distances $O_{1} C_{1}$ and $O_{1}{ }^{\prime} C_{2}$ specify the coordinates of the bearing seats in the final link. They refer to the relationships

$$
\begin{align*}
& O_{1} C_{1}=O_{2} A_{1}-L_{1}+\frac{e_{1}}{\tan \beta}+\frac{e_{2}}{\sin \beta},  \tag{5}\\
& O_{1}^{\prime} C_{2}=O_{2}^{\prime} A_{2}-L_{2}+\frac{e_{2}}{\tan \beta}+\frac{e_{1}}{\sin \beta} . \tag{6}
\end{align*}
$$

Dependencies (5) and (6) make it possible to calculate the dimensional chains of the seats of the bearings of the riser and the terminal.

The above schematics, parametric dependencies and rules allow for the geometric synthesis of the SHSLLM.

## 5. Compilation variants of the SHSLLM and the influence on the kinematics of the links

3D models of conditional mechanisms are performed for illustrating the layering oscillations and the nature of the movement of the SHSLLM units. Fig. 7 shows the sequential positions of the mechanism when the end-link moves from
one extreme position to another. The mechanism is modeled with the following geometric parameters: the first lever pair (Fig. 7, a) $\alpha_{1}=30^{\circ} ; \alpha_{2}=145^{\circ} ; k_{1}=2.5 ; a_{1}=0.172 \cdot S ; e_{1}=0.294 \cdot \mathrm{~S}$ (where S is the course of the final link); the second lever pair (Fig. 7, b) $\alpha_{1}=30^{\circ} ; \alpha_{2}=145^{\circ} ; k_{2}=1.5 ; a_{2}=0.222 \cdot S ; e_{2}=0.179 \cdot \mathrm{~S}$; the angle between the axes of the joints of the groups is $\beta=80^{\circ}$; the axial distances are $\Delta_{1}=\Delta_{2}=0.215 \cdot \mathrm{~S}$.


Fig. 7. The positions of the links of an arbitrary SHSLLM: $a-f$ are the successive positions of the links through the same intervals of movement of the final link within the working range

The character of the movement of the links of this mechanism is illustrated by the graphs of the laws of motion of the levers (Fig. 8, 9).

The graphs of the angular displacements and kinematic parameters are presented as analogues of angular velocities and motion accelerations of long (Fig. 8) and short (Fig. 9) levers - in the function of generalized coordinates (relative movement of the final link). The rotational component of the displacement of the levers in the translational movement of the finite link is smooth, without significant peaks of speed and acceleration, with the reverse of the direction of rotation. The kinematic parameters are maximal at the beginning and at the end of the range of movement of the final link, which causes the need to limit the angles of transmission $\alpha_{2}$ not only due to the danger of approaching the dead position of the mechanism but also in order to limit the maximum of the consolidated mass of the final link. The latter is relevant in the case of using a SHSLLM as part of a fast-acting mechanism or in a variant of a wheel suspension. It is obvious that for $k>1$, the maximum speeds and accelerations are observed for the shorter lever at the beginning and the end of the course of the final link as well as in the middle part. For the longer lever, there is a sharp increase in the beginning and the end of the course of the final link. The diagrams were generated using the SolidWorks Motion program.

Essential due to the necessity to minimize the dimensions of the SHSLLM are questions about the arrangement of the mechanism, which is primarily determined by the design of the hinged units. As the simulation has showed, the minimum axial dimensions of the hinged knots can be achieved with the application of a single bearing (cross-roller or ball-point with a four-point contact) in a hinged knot of the console type [15].

With this arrangement of joints, a pair of levers can have two layers - one-way and two-way joints of the pairs of levers to the links of relative translational movement (Fig. 10). In Fig. 10, the arrow shows the side of connecting the links to the relative translational movement.


Fig. 8. Kinematic diagrams of a long lever of a reciprocal coupled SHSLLM of unilateral joining ( A - angular displacement, B - analogue of velocit0ies, $\mathrm{C}-$ analogue of accelerations by the module)


Fig. 9. Kinematic diagrams of a short lever of a reciprocal coupled SHSLLM of unilateral joining ( $\mathrm{A}-$ angular displacement, $\mathrm{B}-$ analogue of velocities, $\mathrm{C}-$ analogue of accelerations by the module)


Fig. 10. Compounding types of pairs of levers: $a$ - one-way connection; $b$ - two-way connection

The SHSLLM can be arranged on the basis of both pairs of one-sided joints, both duplex or with a combined joining. However, only a couple of two-way connections makes it possible to swap places of long and short levers without affecting the design parameters of the mechanism. Fig. 11 shows the mechanism with the initial (corresponds to position 2 in Fig. 7) and a modified arrangement of the two-way coupling pair levers. Both types of lever pairs allow installing the mechanism in two possible characteristic positions that do not affect its design parameters. This should be taken into account in the layout of the mechanism to avoid interconnection of the links.

$a$

b

Fig. 11. Variants of the SHSLLM with different arrangements of levers in a pair of two-way joining:
$a-$ initial; $b$ - changed; 1 - a short lever; 2 - a long lever
For example, Fig. 12 shows a mechanism with initial and modified characteristic positions of both pairs of levers. Such changes in the position of the levers do not cause any displacement of the final link or a change of the transmission angles and the traction.

The revealed properties favorably expand the variability of structures and allow selecting such in order to avoid interconversion of the links when minimizing the dimensions of the mechanism. To do this in practice, it is advisable to implement the layout variants of the 3D models of the SHSLLM prototypes. The simulation of the conditional prototypes has shown that minimizing dimensions in the layout can only lead to one of the four possible variants of the
characteristic positions of the lever pairs, in which case there will be no interconnection of the links. This should be taken into account and checked for the absence of interconnection across the range of the final link movement in the design of real mechanisms.

$a$

b

Fig. 12. Variants of the SHSLLM with different characteristic positions of both pairs of levers: $a$ - initial; $b$ - changed

In addition, the various characteristic positions of the lever pairs carry the maxima of the velocities of the levers along the movement of the final link, which smoothes the function of the mechanism mass reduced to the final link. Fig. 13 shows graphs of analogues of velocities of short beams of two lever pairs of the SHSLLM with the arrangement and position of groups as shown in Fig. 9, $b$. The maximum speed of one lever approximates the stop of the other and vice versa. The functions of the analogues of the velocities of the SHSLLM levers coincide in the case of location and position of groups as shown in Fig. 9, $a$.

In the case of using identical pairs of levers with the same characteristic positions for $e_{1}=e_{2}=e$ and $\Delta=0$, a geometrically symmetric mechanism with a symmetric characteristic of rigidity relative to this plane can be obtained. Fig. 14 shows a symmetrical SHSLLM with the parameters $\alpha_{1}=35^{\circ} ; \alpha_{2}=140^{\circ}$; $k_{1}=k_{2}=2.25 ; e_{1}=e_{2}=1.53 \cdot a ; \beta=90^{\circ} ; \Delta=0$ with identical pairs of one-way levers.


Fig. 13. Distribution of maximum velocities and points of stops of short levers (curves A and B) of opposite lever pairs in the coordinates of the movement of the final link


Fig. 14. A symmetrical SHSLLM: $a-f$ are the successive positions of the links of the mechanism through identical intervals of movement of the final link within the working range

In the presence of an axial displacement $\Delta \neq 0$, it becomes possible to have a layout in which the projections of two pairs of levers perpendicular to the direction of the plane of the plane will partially overlap. This helps obtain a compact mechanism with minimal transverse dimensions.

## 7. Discussion of the results of research of spatial hinge-lever six-link mechanisms

For efficient synthesis and design of spatial mechanisms, it is expedient to operate the geometric parameters that must be found for a mechanism in its generalized form. These parameters are established by working out the created 3D array of conditional mechanisms and analyzing the spatial kinematic schemes obtained. The study has specified their variational interactions with the constant basic functional indicator of the mechanism of the guide in the final link. The expression of geometric parameters through the course of the final link and the constructive coefficients as well as the introduction of the analysis of permissible angles of transmission in special positions have made it possible to obtain a practically convenient dependence to calculate the minimum dimensions of the links of the mechanism while providing the required course.

Positive is the introduction of a number of concepts, coefficients and constraints that provide for a more systematic approach to the description of this mechanism. The advantage of this approach is to be able to obtain the results that are necessary for the synthesis of the guiding mechanism. The presented dependencies together with the recommendations and described laws, presented in the work, supplemented by calculations of the dimensions of the structural elements of the mechanism, represent the method of synthesizing the guiding mechanism of the SHSLLM. The performed parametrical tests have helped formulate an important conclusion - for the practical implementation of the bilateral mobility of the SHSLLM, it is necessary to provide a significant transverse displacement; the main means for this is an increase in the structural coefficient $k$, which in turn slightly increases the total length of the levers.

The study presents a consideration of possible variants of the layout and illustrative representation of the mechanism in different phases of the movement to extend the idea of the design features. The calculation of the dimensions of the structural elements is useful from a practical point of view; it complements and connects the geometric parameters with the constructive ones. The consideration of the layout variants and revealing their influence on the dynamic characteristics of the mechanism are also essential features of this study.

The implementation of complex geometric tests of the SHSLLM with the use of a guiding mechanism distinguishes this work from the alternatives and provides prospects for developing this topic.

The disadvantages include insufficiently deep parametric and kinematic research, which may be the subject of a separate
study. The development of this direction may also consist in the synthesis of the guiding mechanisms of SHSLLMs that differ in structural and component features, revealing their layout variants.

## 7. Conclusion

1. For a generalized SHSLLM, geometric parameters have been established; besides, concepts of transverse and longitudinal displacements of axes of hinges of lever pairs, interaxial distances, characteristic positions of levers, and a normal directional mechanism have been introduced. Calculations have been made on the size of the levers and transverse displacements by the given course of the final link as well as the permissible transmission angles in the characteristic positions of the levers and the dimensional chains of the SHSLLM, which has helped determine fully its geometry in accordance with the considered layout options. The rules of synthesis are formulated and the notion of normal SHSLLMs is introduced to make it impossible to create inefficient circuits and coordinate the parameters of two lever pairs.
2. Parametrical studying was undertaken on the mechanism presented by the nomograms of the influence of the constructive coefficient of the ratio of the lengths of the levers on the parameter of displacement of the axes and on the total length of the levers. According to the research results, it has been revealed that the most effective way to increase the transverse displacement of the mechanism is to increase the ratio of lengths in both pairs of levers; at the same time, it does not cause a significant increase in the total length of the links. Thus, when the coefficient of the ratio of lengths $k$ varies from 1 to 4, the total length of the pair of levers increases by only $25 \%$ whereas the transverse displacement $e$ increases 2.4 times.
3.3D models of conditional mechanisms simulated in accordance with the given methodology have been studied. The research has confirmed the correctness of theoretical calculations; it has helped reveal the layout variants of the SHSLLM and present an illustrative material that clearly
demonstrates the position of the links throughout the range of displacements. As a result of the kinematic analysis of the 3D models of conditional mechanisms, recommendations have been made on the need to limit the transmission angles in view of the influence on the kinematics of the levers. It is advisable that the angles of transmission in the extreme and middle positions should be no less than $30 \ldots 35^{\circ}$; otherwise, the angular accelerations of the levers are sharply increasing along with the reduced mass of the mechanism in the specified positions. According to the results of the modeling, possible layout variants of pairs of levers, variants of characteristic positions of levers and variants of change of arrangement of levers in a separate pair have been revealed. Recommendations are given for the flow of changes in the characteristic positions on the reduced mass of the mechanism and the effect of the axial displacement on the layout features. In order to smooth the function of the reduced mass of the mechanism, the characteristic positions of the rectangular pairs should be designated as those for which the average positions are shifted in time. The axial offset helps create layout schemes for them to be as compact as possible. Mechanisms that are symmetrical with respect to the longitudinal plane can be obtained at a zero axial displacement.

The method of geometric synthesis of the SHSLLM, which serves as a guiding mechanism, has been developed. The method involves determining the course of the final link optimal for minimizing the lengths of the links of the directional mechanism, taking into account the permissible angles of transmission in the special position of the mechanism. The article presents results of parametric and kinematic research and variants of layout schemes; the formulated rules, recommendations and calculation dependences help carry out geometric synthesis of a guiding SHSLLM with rational layouts and optimal geometric parameters.

The conducted research has revealed possible ways of developing new variants of spatial hinge-lever six-link mechanisms and new possibilities for their application as a guiding mechanism; therefore, it represents both practical interest for developers and scientific interest for researchers in this field.

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#### Abstract

Досліджено динаміку вібращійної машини з дебалансним віброзбудником з урахуванням пружності його з'єднання з електродвигуном асинхронного типу.

Отримано вираз, який описує крутильні коливання пружної муфти в стаціонарних (навколостаціонарних) режимах руху вібраційної машини та формулу для вібраційного моменту для випадку машини з плоским характером коливань робочого органу. Побудовано амплітудно-частотні характеристики коливань привода вібромаиини при використанні «м'яких» та «жорстких» пружних муфт. Показано, що в приводі вібромашини можуть виникати резонансні коливання. Встановлено характер зміни величини вібраційнного моменту (додаткового динамічного навантаження на електродвигун, викликаного коливаннями як несівного тіла вібромашини, так і пружної муфти) в залежності від частоти обертання віброзбудника. Показано, що наявність пружної муфти у приводі вібромашини у певних режимах руху може призводити до підвищення навантаження на двигун, що сприяє виникненню ефекту Зоммерфельда під час пуску. Уточнено критичні частоти приводу вібромашини з дебалансним віброзбудником. Сформульовано рекомендації стосовно вибору власних частот привода для уникнення його резонансних коливань.

За допомогою комп'ютерного моделювання продемонстровано виникнення резонансних крутильних коливань привода вібромашини при проходженні зони її власних частот та пов'язане з иим виникнення ефекту Зоммерфельда; підтверджено ефективність запропонованих рекомендацій для змениення коливань привода.

Отримані результати дозволять уникнути резонансних коливань в приводі вібромашини й тим самим змениити динамічні навантаження у елементах його конструкиії та підвищити надійність $і$ довговічність деталей привода

Ключові слова: вібромашина, дебалансний віброзбудник, коливання привода, пружна муфта, ефект Зоммерфельда


## 1. Introduction

Vibration machines are widely used in various industries. Vibration machines with unbalance drive are the most common primarily due to the simple design and compact size under great disturbance. However, in transient operating modes of such machines, there may be resonant oscillations, accompanied by a significant increase in dynamic loads in structural elements. For example, the
practice of operation of SMZh, VB type vibrating tables for volume compaction of concrete mixes demonstrates frequent failures of drive shafts that interconnect unbalances of individual vibration units [1]. The design of these shafts includes elastic couplings, which may be one of the main causes of destructive oscillations of the drive. Thus, the studies of dynamic processes in vibration machines, taking into account the drive elasticity, are a relevant scientific and applied problem.

