

UDC 338.45:658.588+330(075.8):519.24

DOI: 10.15587/1729-4061.2018.152793

# ALGORITHM FOR SELECTING THE WINNING STRATEGIES IN THE PROCESSES OF MANAGING THE STATE OF THE SYSTEM "SUPPLIER – CONSUMER" IN THE PRESENCE OF AGGRESSIVE COMPETITOR

**O. Domina**

Member of the Board

Scientific Route OÜ

Narva mnt., 7-634, Tallinn, Estonia, 10117

E-mail: info@eu-jr.eu

**D. Lunin**

Director

DKLex Academy Business School

Tartu mnt., 56-2, Tallinn, Estonia, 10115

**L. Mikhailova**

PhD, Associate Professor

Department of Energy and Electrical Systems of the Agro-industrial Complex\*

E-mail: mihajlovaimesg@gmail.com

**O. Niskhodovska**

PhD, Assistant

Department of Economics, Entrepreneurship, Trade and Stock Exchanges\*

E-mail: olenas1308@gmail.com

**O. Barabash**

Doctor of Law, Associate Professor

Department of Administrative and Informational Law

Lviv Polytechnic National University

S. Bandery str., 12, Lviv, Ukraine, 79000

E-mail: olja8877@yahoo.com

**O. Balynska**

Doctor of Law, Professor, Vice-rector

Lviv State University of Internal Affairs

Horodotska str., 26, Lviv, Ukraine, 79007

E-mail: olga\_bal@ukr.net

**Y. Païda**

PhD, Associate Professor

Department of General Legal Disciplines

Kharkiv National University of Internal Affairs

L. Landau ave., 27, Kharkiv, Ukraine, 61080

\*State Agrarian and Engineering University in Podilia Shevchenko str., 13, Kamianets-Podilskyi, Ukraine, 32300

*Розглянуто проблему, пов'язану з пошуком оптимальної стратегії ціноутворення компанією-постачальником у разі появи у неї конкурента, що пропонує нижчу ціну продукції. Виникнення такої проблеми породжує необхідність пошуку оптимального шляху зниження своєї відпускної ціни, з метою не програти в агресивному конкурентному середовищі, що формується новими гравцями, що заходять на ринок зі свідомо кращими пропозиціями. Для вирішення цієї проблеми розроблено алгоритм вибору виграних стратегій на основі оцінювання стратегічних можливостей конкурента в умовах невизначеності.*

*Запропоновано для оцінки вартості товару в системі «постачальник – споживач» використовувати поняття масштабу 1-го рівня. Показано, що при такому поданні стає можливою безрозмірна оцінка ціноутворення товару, незалежно від його виду або натурального грошового вираження. Для формалізованого опису відносин компанії-постачальника і компанії-конкурента запропоновано використання теорії стратегічних ігор, матриця гри в якій формується на основі універсальних рівнянь регресії. Особливістю запропонованих рішень є те, що значення виграшу в матриці гри визначається рішенням оптимізаційної задачі на основі рівняння регресії, що описує вплив транспортних витрат, прибутку та податку на додану вартість (VAT) на ціну гри. Встановлено, що при такому описі має місце гра з сідловою точкою і чистою ціною гри  $z = -0,5$ . На підставі математичного моделювання встановлено, що вибір компанії-постачальника обмежується стратегіями, в якій власний прибуток має знаходитися поблизу середнього або мінімально можливого значення.*

*Розроблено прогнозна модель стратегічних можливостей конкурента в системі «постачальник – споживач», що представляє собою універсальне рівняння регресії. На підставі нього може бути зроблено коригування чисельних показників компонентів ціноутворення товару. Показано, що таке коригування допускає наявність декількох альтернатив, які зводять до нуля переваги конкурента. Обґрунтовано обмеження на одержувані рішення, пов'язані з двома обставинами: припущенні про точність визначення компонентів ціноутворення у конкурента і наявності особливостей оподаткування в міжнародних вантажоперевезеннях*

*Ключові слова: система «постачальник – споживач», масштаб 1-рівня, стратегічні можливості, оптимальна стратегія, ціна гри, рівняння регресії*

## 1. Introduction

The emergence of new competitors for a company with the established channels to sell products to permanent

consumers brings along potential risks. They relate to two essential factors: a lower price for products offered by such new competitors, and the natural desire of a consumer to replace the supplier with the one that proposed lower prices.

Even in cases when consumer has a conservative position in the system of relations with suppliers, the company is forced to search for reserves to reduce the price for its products, in particular to bring down its cost by conducting organizational and technical activities. Such activities include, among others, the processes of technical re-equipment of production, considered as one of the options for strategic planning. The effectiveness of these activities could be assessed by the magnitude of production cost reduction, decreasing material costs, as well as additionally received profit [1]. Its average magnitude over a selected time interval can be defined based on the duration of a period prior to the implementation of activities, duration of enterprise's activities under conditions for implementing measures on its technological development, average efficiency of the enterprise prior to carrying out measures aimed at technological development, technical development effectiveness [2].

If an activity aimed at technical re-equipment is chosen as the best among the available alternatives, it is advisable to use, as its effectiveness, the indicators for a minimum of reduced costs or an investment payback period [3]. Such a way to reduce the price of its product is associated with time and financial costs, which should be minimized to achieve, within a permissible period, advantages, in terms of price against a new competitor.

Another alternative could be the optimization of processes related to supply chain management (SCM) [4]. This way is promising enough, because it is not directly related to changes in the structure of production, which requires raising funds on technical re-equipment, capital for repair and reconstruction, that is, activities aimed at reduction of production costs. However, there are problems of another nature, requiring solutions, specifically, the need to find an optimal interaction between participants of logistics processes. Such interactions are taken into consideration in the framework of generating optimal production volumes and determining optimal prices in the markets under conditions of monopoly or oligopoly. If we consider a special case of oligopoly, duopoly, the principles for making rational decisions are based on finding the equilibrium solutions in the sense of Stakelberg, Cournot, Nash [5]. Diversity in approaches to resolving relevant issues makes it possible to argue about the relevance of search for a rational way to reduce the selling price. It defines the possibility of a company not to lose in an aggressive competitive environment, formed by new players entering the market with offers that are obviously better.

---

## 2. Literature review and problem statement

---

The concept of SCM is presented as a new business ideology, in whose framework an entire complex of tasks is solved, related to planning and managing all types of supply chain operations [6]. Under such an approach, it appears appropriate to take into consideration within this complex the innovative policy of an enterprise. This aspect is typically considered from several perspectives: the impact of knowledge exchange at SCM on the effectiveness of new product introduction and its transfer to logistic business [7, 8], modelling the interaction of processes related to enterprise sustainability, its innovation activity, and market demand within SCM [9].

Despite the obvious fact that theoretically the concept of SCM is very attractive, its readiness for widespread

practical implementation is much debated. Indeed, an analysis of papers [6–9] reveals that within the framework of SCM development, either empirical or purely theoretical aspects prevail. In particular, it concerns the issues related to the study of the impact of innovation activity of an enterprise on improvement in its competitive positions. Confirmation could be found in papers [10, 11] where supply chain management is represented in the form of a functional, integrating the internal and external, in relation to a company, business functions and business processes into a unified business model.

It is obvious that attempts to formalize these processes, as well as the construction of a mathematical apparatus for it, can present significant challenges. Apparently, a partial solution to the problem posed by the complexity to describe the functioning of such systems should be sought in the use of information technologies. This view is reflected, for example, in study [12], in which information systems are considered to be the mechanism of process synchronization inside a company and throughout the supply chain. In other words, there is a symbiosis of design, production, and logistics tasks. Such a view is well-founded, however, the issue on mathematical support for such information systems remains unresolved. In addition, when considering the competitiveness of a enterprise in terms of its capability to minimize the price of the product, one should somehow formalize the notion of an innovation activity, introduced into consideration as one of the main factors for enterprise success in the market [13]. One such optimization model of production planning and delivery of diversified products was proposed in paper [14], and this idea was further advanced in [15]. Special features of solutions, proposed in study [15], is that in order to determine the optimal production plans, as well as its delivery to consumers, and investment in production, it is necessary to obtain equilibrium solutions in the duopoly model. And they define the equilibrium decisions by Cournot if enterprises decide to release products simultaneously and independently of each other, and by Stakelberg if one manufacturer believes that the competitor would behave as a Cournot duopolist. Thus, they take into consideration competition not at the level of individual enterprises but between supply chains. The advantage of such a method is that it allows practical implementation of actual production programs. This to some extent eliminates the issue of excessive theorization on this problem as a dominant approach in the study of the concept of SCM, which acts as a significant factor under conditions of competition.

Logical development of these results is that the model would account for additional significant factors related to the techniques and specifics of product delivery to consumers. Confirmation of this can be found in paper [16], in which the need for dynamic development of distributed logistics systems and the importance of taking into consideration the features of the transportation component is dictated by the conditions of economic globalization. Specifically, it makes sense to argue about the development of approaches to a multifactorial analysis of transportation-logistics systems [17]. The proposed solutions, based on the application of vector optimization of functionals, represent the pattern of representation of a logistic system at the macro level, when each of the modules of this system is represented as a technological object within which the process of a material flow transformation takes place. The advantage, in this case, is linked to the possibility of introducing to the model of a

transportation-logistics system the characteristics of rate of change in the processes that occur within it. This, according to the authors of work [17], makes it possible to take into consideration the dynamic characteristics of the system. In this case, however, significant problems remain that are related to the need for a system analysis into each object involved in the processing of freight flows. In addition, open question here is routing, which also relates to the choice of a transport mode, transportation feature, and a technique to adequately mathematically describe the process.

Solutions, known in this part, relating to multimodal transportation, are based on using ant algorithms [18] and their development, in particular by expanding parametric representation and introduction of weights [19, 20]. Application of such algorithms is quite justified, because owing to convergence there is a guarantee to obtain the optimal solution. However, due to the problem's multifactor nature, the rate of such a convergence cannot be uniquely estimated. Ways to improve the efficiency of ant algorithms could include approaches based on combinatorial optimization [21] and the synthesis of multiple local algorithms that search for optimal solutions [22]. In relation to the problem on automobile transportation, one of such modified algorithms was reported in paper [23]. Specifically, the authors solved a problem on constructing a rational route between points of dispatch and destination based on the modified ant algorithm [18], which was supplemented with a parameter for the function of quality of roads at each section of the route between points. Such a parameter is a product of the membership function, describing the condition of a roadway along the corresponding road section, and an expert estimation of the throughput capacity of this section. In this case, it is concluded that the introduction of such a parameter improves efficiency of the algorithm by accounting for additional important factors, among which: the relief of traffic lines, the level of service in infrastructure, the actual climatic conditions, the probability of an emergency.

Based on an actual example, solutions seem convincing in terms of the practicality of the proposed modification, but as regards unique effectiveness under conditions of the specified multifactor nature they are characterized by an overestimated level of optimism. It is natural that when using a different transport mode conditions for the application of the algorithm would be different. The studies that address the development of principles for intermodal transport selection include [24, 25]. In particular, they considered approaches to the substantiated choice of transportation and principles that form transportation costs. Their priorities include establishing dependences of costs for the freight containers delivery on distances for transportation by road and rail transport. However, the organization and planning of movements of container equipment were not given due attention. Given the importance of this issue, particularly in the context of export-import trade, paper [26] used an example of Odessa region (Ukraine) to propose a scheme of reverse loading of containers, freed from imported goods, as a variant to improve activities of a transport and freight forwarding company. It is an interesting attempt to further develop the ideas, proposed in a given work, in other regions, apparently possessing a number of other conditions. In this case, it would be practically useful to answer the question about how these conditions could be taken into consideration and how they could affect effectiveness of the solutions, proposed in [26]. The lack of effective feedback among all

participants in a transportation process is a drawback, noted in [27]. A conceptual scheme, suggested in this work, must, according to the authors, improve the efficiency of feedback under conditions of maritime transportation. However, the solutions relate to only one type of transportation and only at the level of a conceptual model.

A particular disadvantage of existing areas of research is the consideration of problems related to the functioning of transportation and logistics systems in a single aspect only. For example, paper [28] estimates quality of third-party logistics providers, while their other levels are not considered. Problems of legal regulation in the activities of transport and logistic systems, including as its structural elements the transport-forwarding companies, and an analysis of specificity of consumer protection in the framework of functioning of these systems [29] are not linked to the organization of cargo delivery schemes. All this suggests the presence of unresolved issues relating to analytical solutions regarding the selection of optimal strategies for suppliers. Specifically, such solutions should take into consideration the limitations imposed by the peculiarities of functioning of the systems "supplier – consumer", taking into consideration the legislative regulation of international deliveries.

---

### 3. The aim and objectives of the study

---

The aim of this study is to develop an algorithm for the selection of winning strategies based on the estimation of strategic opportunities of a competitor under conditions of uncertainty. That would make it possible to choose the optimal management strategy in the system "supplier – consumer" by minimizing price advantages of the competitor.

To accomplish the aim, the following tasks have been set:

- to develop an algorithm to estimate a product price in the system "supplier – consumer";
- to build a predictive model of strategic opportunities of a competitor in the system "supplier – consumer" and the selection procedure depending on their alternatives.

---

### 4. Introductory concepts

---

We introduce the following concepts.

Supplier (player 1) is a product manufacturer, selling it to consumer.

Consumer is a company purchasing a product from a supplier.

Aggressive competitor (player 2) is a supplier involved in the fight for a customer, trying to enter the market with advantages that are more favorable in terms of price, and to displace the supplier from the market.

Game is a formalized model of a situation under which two competitors fight for a consumer; it represents a set of rules that describe behavior of players.

Features of the game – the first competitor is a constant supplier of goods to the consumer over a long period of time, the second competitor is a new company that attempts to enter the market, pushing out the first competitor from the system "supplier – consumer" by offering a price advantage.

Consumer loyalty is the attitude of the consumer towards the supplier, tested over a long collaboration; it implies that it does not want to break relations with a regular supplier, and, optionally, provides it with information about player 2.

A player's strategy is the player's unique choice of a solution from a certain valid set.

Vector of input variables  $X$  is the vector whose components are the factors that form a product price:

$$X = \left[ x_{(i)}^{(s)} \right]_1^n, \quad (1)$$

where  $i$  is the number of the column vector's component,  $n$  is the number of the column vector's components,  $s$  is the player's number in the system "supplier – consumer".

Game matrix is a matrix whose elements describe possible winnings in each strategy [30, 31]:

$$Z = \left[ z_{jk} \right] = \begin{bmatrix} z_{11} \dots z_{1N} \\ \dots \dots \dots \\ z_{M1} \dots z_{MN} \end{bmatrix}, \quad (2)$$

where  $j$  is the index that corresponds to the number of an alternative strategy for the first player,  $k$  is the index, which corresponds to the number of an alternative strategy for the second player,  $M$  is the number of available strategies for the first player,  $N$  is the number of available strategies for the second player.

The output variable  $y^{(s)}$  is a scalar magnitude, corresponding to the product price:  $y^{(1)}$  – for player 1,  $y^{(2)}$  – for player 2.

---

### 5. Principle of forming an estimation algorithm of the output variable

---

We introduce the following representation of product pricing.

The price of a product includes 4 main components:

- cost;
- transportation costs (total cost of product delivery);
- value added tax (VAT).

We shall introduce additional concept of the  $l$ -level scale, where  $l$  can accept different values.

At  $l=0$ , scaling represents a transformation of valid values for each pricing component of the four mentioned above, in the range [0; 1]. It is obvious that in this case the range of values for the output variable corresponds to [0; 4].

If the basic unit to which all other components of pricing are reduced is selected to be the cost of the product of player 1, then it is appropriate to represent the vector of input variables in the following form:

$$X^{(s)} = \left( x_0^{(s)} \ x_1^{(s)} \ x_2^{(s)} \ x_3^{(s)} \right), \quad (3)$$

where  $x_0$  is the product cost,  $x_1$  is the transportation cost (total cost of delivery),  $x_2$  is the profit,  $x_3$  is the value added tax (VAT), the index  $s$  is here omitted for simplification, considering the sameness of description for player 1 and player 2.

Output variable in this case is determined as follows:

$$y^{(s)} = \sum_{i=1}^4 x_{(i)}^{(s)}. \quad (3)$$

At  $l=1$ , scaling represents the normalization of values for each pricing component preliminary treated with the  $l$ -level scaling, thus to transfer these magnitudes to the range of

[-1; +1]. In this case, the range of values for the output variable will depend on the ranges of values for an input variable for each of the players. Normalization is performed as follows:

$$x_{ri}^{(s)} = \frac{2x_{ri}^{(s)} - (x_{ri \max}^{(s)} + x_{ri \min}^{(s)})}{x_{ri \max}^{(s)} - x_{ri \min}^{(s)}}, \quad i = 1, \dots, 4, \quad r = 1, \dots, R, \quad (4)$$

where  $x_{ri}^{(s)}$  is the normalized value for the input variables (the scale of level  $l=1$ )

$$x_{ri \max}^{(s)} = \max_r x_{ri}^{(s)}, x_{ri \min}^{(s)} = \min_r x_{ri}^{(s)},$$

$r$  is the index characterizing the number of a conditional experiment, at which the magnitude  $x_{(i)}^{(s)}$ , is determined,  $R$  is the total number of conditional experiments, at which magnitude  $x_{(i)}^{(s)}$ , is determined.

If we assume that one can select a value for  $x_{(i)}^{(s)}$  in the assigned range at own discretion, that is, there are no restrictions for choice, and the basic unit is the product cost of player 1, it is advisable, for example, the following choice for each input variable:

– at  $s=1$ :

$$x_{(0)}^{(1)} = 1,$$

$$x_{(1) \max}^{(1)} = 1, x_{(1) \min}^{(1)} = 0, 2,$$

$$x_{(2) \max}^{(1)} = 1, x_{(2) \min}^{(1)} = 0, 2,$$

$$x_{(3) \max}^{(1)} = 1, x_{(3) \min}^{(1)} = 0, 2;$$

– at  $s=2$ :

$$x_{(0) \max}^{(2)} = 1, x_{(0) \min}^{(2)} = 0, 5,$$

$$x_{(1) \max}^{(2)} = 1, x_{(1) \min}^{(2)} = 0, 2,$$

$$x_{(2) \max}^{(2)} = 1, x_{(2) \min}^{(2)} = 0, 2,$$

$$x_{(3) \max}^{(2)} = 1, x_{(3) \min}^{(2)} = 0, 2.$$

The physical essence of the selected numerical values is as follows:

– for  $s=1$ :

– transportation cost of player 1 is in the range of (20–100) % of the cost of the product of player 1,

– profit of player 1 is in the range of (20–100) % of the cost of the product of player 1,

– value added tax (VAT) of player 1 is in the range of (20–100) % of the cost of the product of player 1.

– for  $s=2$ :

– product cost of player 2 can be less than the cost of the product of player 1 by two times and is in the range of (50–100) % of the cost of the product of player 1,

– transportation cost of player 2 is in the range of (20–100) % of the product cost of player 1,

– profit range of player 2 is (20–100) % of the cost of the product of player 1,

– value added tax (VAT) of player 2 is in the range of (20–100) % of the cost of the product of player 1.

It is possible to assign other numerical values (in % to the cost of the product of player 1).

When assigning such ranges of input variables, the range of values for the output variable is:

- for player 1: [1.6; 4];
- for player 2: [1.1; 4].

Therefore, we obtain a plan for the full factorial experiment:  $R=2^3$  (for  $s=1$ ) and  $R=2^4$  (for  $s=2$ ). These plans are given in Tables 1, 2, respectively.

Table 1

Plan of experiment  $R=2^3$  (for  $s=1$ )

No. of experiment	Input variables, $x_i$							Output variable, $y^{(1)}$
	$x_0^{(1)}$	$x_1^{(1)}$	$x_1^{(1)}$	$x_2^{(1)}$	$x_2^{(1)}$	$x_3^{(1)}$	$x_3^{(1)}$	
1	1	1	+1	1	+1	1	+1	4
2	1	0.2	-1	1	+1	1	+1	3.2
3	1	1	+1	0.2	-1	1	+1	3.2
4	1	0.2	-1	0.2	-1	1	+1	2.4
5	1	1	+1	1	+1	0.2	-1	3.2
6	1	0.2	-1	1	+1	0.2	-1	2.4
7	1	1	+1	0.2	-1	0.2	-1	2.4
8	1	0.2	-1	0.2	-1	0.2	-1	1.6

Table 2

Plan of experiment  $R=2^4$  (for  $s=2$ )

No. of experiment	Input variables, $x_i$								Output variable, $y^{(2)}$
	$x_0^{(2)}$	$x_0^{(2)}$	$x_1^{(2)}$	$x_1^{(2)}$	$x_2^{(2)}$	$x_3^{(2)}$	$x_3^{(2)}$	$x_3^{(2)}$	
1	1	+1	1	+1	1	+1	1	+1	4
2	0.5	-1	1	+1	1	+1	1	+1	3.5
3	1	+1	0.2	-1	1	+1	1	+1	3.2
4	0.5	-1	0.2	-1	1	+1	1	+1	2.7
5	1	+1	1	+1	0.2	-1	1	+1	3.2
6	0.5	-1	1	+1	0.2	-1	1	+1	2.7
7	1	+1	0.2	-1	0.2	-1	1	+1	2.4
8	0.5	-1	0.2	-1	0.2	-1	1	+1	1.9
9	1	+1	1	+1	1	+1	0.2	-1	3.2
10	0.5	-1	1	+1	1	+1	0.2	-1	2.7
11	1	+1	0.2	-1	1	+1	0.2	-1	2.4
12	0.5	-1	0.2	-1	1	+1	0.2	-1	1.9
13	1	+1	1	+1	0.2	-1	0.2	-1	2.4
14	0.5	-1	1	+1	0.2	-1	0.2	-1	1.9
15	1	+1	0.2	-1	0.2	-1	0.2	-1	1.6
16	0.5	-1	0.2	-1	0.2	-1	0.2	-1	1.1

Based on data from Tables 1, 2, one can build the linear regression equations in the form (5), describing the dependence of product price on its constituent components for both players.

$$y^{(s)} = a_0 + a_1 x_1^{(s)} + \dots + a_n x_n^{(s)}. \tag{5}$$

It is obvious that equation (5) is a special case of the Kolmogorov-Gabor regression polynomial, which does not take into consideration factors of the pairwise and the higher-order interactions. Such a simplified structure of the regression polynomial is appropriate due to the specificity in determining an output variable from formula (3). The regression equation, obtained in this case, accurately

describes the influence of all price-forming components on price of the product, and it is easy to verify by substituting in equation (5) the values for input variables for any row of the plan (Tables 1, 2).

Coefficients' weights in the regression equations in this case are derived quite simply:

$$a_i^{(s)} = R^{-1} \sum_{r=1}^R x_{ir}^{(s)} y_r^{(s)}, \quad i=0, \dots, n. \tag{6}$$

However, if we consider the problem of competition in the system "supplier – consumer" as a strategic game, of special interest are not the values for output variables  $y^{(1)}$  and  $y^{(2)}$ , but the difference between them:

$$z_r = y_r^{(2)} - y_r^{(1)}, \tag{7}$$

where  $z_r$  is the price of the game, defined as follows: a win of player 1 equals a loss of player 2.

If, in this case, one solves the problem regarding player 1, which corresponds in essence to its desire to stay in the market, then one can consider that player 2 has sixteen possible strategies. Each of them corresponds to the  $r$ -th row in Table 2. In other words, it is assumed that player 2 operates, when choosing strategies, the extreme values in the range of the input variables. This assumption is justified at this stage, because there is an uncertainty related to the estimation of numerical values for the product pricing components of player 2. Under conditions of such uncertainty, the task of player 1 is to estimate the price of the game and, consequently, to evaluate its potential winnings for each of these sixteen strategies of player 2.

Under such a consideration, the plan of experiment  $R=2^3$  (for  $s=1$ ) for building a regression equation will have a general form given in Table 1, but the column for the output variable will contain the price of the game. Thus, we form 16 tables for the plan of experiment  $R=2^3$ , based on which one can calculate estimates for the coefficients in the regression equation.

Table 3

Plan of experiment  $R=2^3$  (for  $s=1$ ) at estimation

No. of experiment	Input variables, $x_i$							Output variable, $z_r$
	$x_0^{(1)}$	$x_1^{(1)}$	$x_1^{(1)}$	$x_2^{(1)}$	$x_2^{(1)}$	$x_3^{(1)}$	$x_3^{(1)}$	
1	1	1	+1	1	+1	1	+1	4
2	1	0.2	-1	1	+1	1	+1	3.2
3	1	1	+1	0.2	-1	1	+1	3.2
4	1	0.2	-1	0.2	-1	1	+1	2.4
5	1	1	+1	1	+1	0.2	-1	3.2
6	1	0.2	-1	1	+1	0.2	-1	2.4
7	1	1	+1	0.2	-1	0.2	-1	2.4
8	1	0.2	-1	0.2	-1	0.2	-1	1.6

The matrix of estimates for the coefficients in the regression equations, calculated from formula (3), takes the form

$$a^{(1)} = \begin{pmatrix} 1,2 \\ -0,4 \\ -0,4 \\ -0,4 \end{pmatrix}.$$

Therefore, under the existing variant of the choice of range of numerical values for input variables in the natural form all three components of product pricing for player 1 (transportation cost, profit, VAT) exert an equal influence on the price of the game. Particularly, this effect is negative – with an increase in the absolute value for each of these components the price of the game decreases. And that means that the win of player 1 reduces. The physical essence of this result is obvious – player 1 should strive to reduce the magnitude of product pricing components to minimize the advantages of player 2, offering the consumer the lower price for the product. The question arises – which component is to be reduced? Choosing a decrease in the cost by player 1 should not be considered because it entails the implementation of a set of measures related to the financial and time costs. Such activities might include the renewal of equipment, modernization, and technical re-equipment of production, investment in new technology development, etc. If player 2 tries to enter the market aggressively, then such costs in the short term could lead to that player 1 loses the supplier and loses the competition. An option of variation could be a profit margin – under conditions of

the aggressive strategy of player 2, this option is the least expensive, although it should be considered as a temporary measure. One can choose 3 strategies, corresponding to the three levels of values  $x_2^{(1)}$ . In the normalized form, these are:

- strategy 1:  $x_2^{(1)} = -1$ ;
- strategy 2:  $x_2^{(1)} = 0$ ;
- strategy 3:  $x_2^{(1)} = +1$ .

Fixing the magnitude  $x_2^{(1)}$  at these three levels and using the resulting matrix of coefficients estimates, it is possible, based on equation (5), to calculate the win of player 1 when player 2 chooses any of its 16 strategies (Table 4).

**Game matrix**

		Strategies of player 2															
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Strategies of player 1	1	2.4	1.9	1.6	1.1	1.6	1.1	0.4	0.3	1.6	1.1	0.8	0.3	0.8	0.3	0	-0.5
	2	2	1.5	1.2	0.7	1.2	0.7	0.4	-0.1	1.2	0.7	0.4	-0.1	0.4	-0.1	-0.4	-0.9
	3	1.6	1.1	0.8	0.3	0.8	0.3	0	-0.5	0.8	0.3	0	-0.5	0	-0.5	-0.8	-1.3

Table 4 shows that by choosing strategy  $j$ , player 1 ensures a guaranteed winning  $z(x_j^{(1)})$ , equal to the smallest element from set  $L(x_j^{(1)}, x^{(2)})$ ,  $L \subset Z$ :

$$z(x_j^{(1)}) = \min_{x^{(2)}} L(x_j^{(1)}, x^{(2)}). \tag{8}$$

The lowest net price of the game, which maximizes the guaranteed winning of player 1, takes the form

$$\alpha = \max_{x^{(1)}} z(x^{(1)}) = \max_{x^{(1)}} \min_{x^{(2)}} L(x^{(1)}, x^{(2)}). \tag{9}$$

The task of player 2 is to minimize its maximum loss, calculated as follows:

$$z(x_k^{(2)}) = \max_{x^{(1)}} L(x^{(1)}, x_k^{(2)}). \tag{10}$$

In this case, the highest net price of the game, which minimizes the maximum loss of player 2, takes the form

$$\beta = \min_{x^{(2)}} z(x^{(2)}) = \min_{x^{(2)}} \max_{x^{(1)}} L(x^{(1)}, x^{(2)}). \tag{11}$$

The values for  $\alpha$  and  $\beta$  are given in Table 5; boundaries of the respective cells are highlighted with double lines.

**Table 5**  
Calculation of the upper and lowest net price of the game for the matrix of the game, given in Table 3

		$z(x_j^{(1)})$															
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$z(x_k^{(2)})$	1	2.4	1.9	1.6	1.1	1.6	1.1	0.4	0.3	1.6	1.1	0.8	0.3	0.8	0.3	0	-0.5
	2	2	1.5	1.2	0.7	1.2	0.7	0.4	-0.1	1.2	0.7	0.4	-0.1	0.4	-0.1	-0.4	-0.9
	3	1.6	1.1	0.8	0.3	0.8	0.3	0	-0.5	0.8	0.3	0	-0.5	0	-0.5	-0.8	-1.3

Table 5 shows that one can argue about the game with a saddle point and a net price of the game  $z = -0.5$ . However, this result does not suit player 1, which turns out to be a loser. Such a situation is possible if player 2 chooses strategy No. 16, that is, it offers the consumer the lowest possible price, all components of which are minimal (experiment No. 16 in Table 2). Thus, player 1 must find a solution that as a minimum would provide for a price reduction for its product by the magnitude  $z = 0.5$ . In this case, if the consumer is loyal, player 1 can remain in the system “supplier – consumer”, at least for a period of time during which it can take additional measures.

**Table 4**

**6. Building a predictive model of strategic opportunities for player 2 under conditions of uncertainty**

Estimation of actual opportunities for player 1 to minimize

the price of the product can be conducted based on data from Table 5. It shows, for example, that it can ensure a high win  $z = 2.4$  (if player 2 chooses strategy No. 1),  $z = 1.9$  (if player 2 chooses strategy No. 2),  $z = 1.6$  (if player 2 chooses strategies Nos. 3, 5, 9). There are several variants of the worse win when player 2 chooses strategies Nos. 4, 6, 11, 13, 7, 8, 12, 14. If player 2 chooses strategy No. 15, the win of player 1 is equal to zero. However, in this case, the loss of player 2 also becomes zero. Thus, strategy No. 15 does not yield benefits to anyone, that is, the players' odds against the consumer are equalized.

Ranges of opportunities for player 1 for winning are shown in Fig. 1.

It follows from Fig. 1 that the opportunities for player 1 in ensuring a particular magnitude of win fully depends on the strategy which chooses player 2. However, this information is not available for player 1. Therefore, it must either obtain it from any sources, for example, making use of

consumer loyalty, or from its own hidden sources of information, or to somehow eliminate the uncertainty regarding the assessment of possible strategies of player 2. That is, it is necessary to narrow the range of possible strategies for player 2.

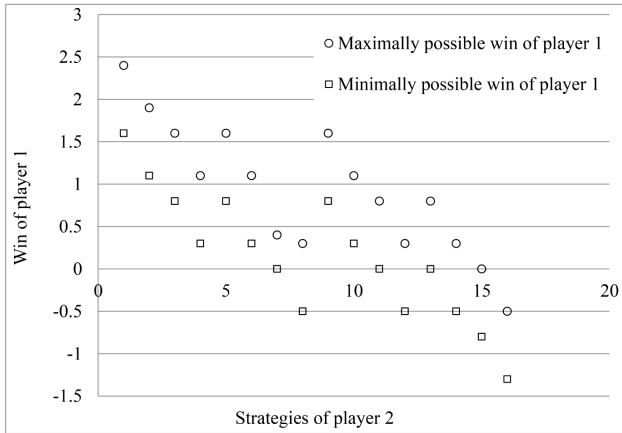


Fig. 1. Maximally and minimally possible wins of player 1

This could employ the regression equation in form (5), in which an output variable is the product price of player 2 ( $y^{(2)}$ ), and the estimates' coefficients are calculated from formula (6). It is obvious that all the conclusions about the structure of this equation for player 1 hold also regarding player 2. The matrix of coefficients' estimates therefore takes the form

$$a^{(2)} = \begin{pmatrix} 2,55 \\ 0,25 \\ 0,40 \\ 0,40 \\ 0,40 \end{pmatrix}.$$

It follows that input variables  $x_1, x_2, x_3$  have the same power in terms of impact on the magnitude of price as is the case for player 1. However, in terms of input variable  $x_0$ , player 2 has a clear advantage. This circumstance is very important, because player 2 has an opportunity to ensure the cost for its product by 2 times lower than player 1.

The opportunities for player 1 to minimize the price of a product are limited because it cannot vary the cost. Variation of profit cannot guarantee an advantage or, at least, minimize the advantages of player 2 (Table 4). Therefore, there remains a possibility to vary the input variables  $x_1$  and  $x_3$  – a value added tax (VAT). On the other hand, player 2 can vary any of the input variables.

In such a situation, the following is important for player 1. If information on the product cost and profit of player 2 is unavailable to player 1 in full, the uncertainty in the estimation of the transportation cost and value added tax can be reduced. Assuming that player 1 does not have its own sources of information regarding pricing of player 2, there remains the one way out. It implies the existence of an analytical descriptions, which makes it possible, based on available information on pricing, to determine, from variables  $x_1^{(2)}$  and  $x_3^{(2)}$ ,  $x_0^{(2)}$  and  $x_2^{(2)}$  (hereafter the designations of input variables have superscripts to identify a player).

On this basis, it is possible to calculate the required values for  $x_1^{(1)}$  and  $x_3^{(1)}$ , and thereby assess the own opportu-

nities to compensate for the price advantage of player 1 (due to low values for  $x_1^{(2)}$  and  $x_3^{(2)}$ ) by reducing transportation cost and the magnitude of VAT related to it. However, in this case, we must consider an important circumstance – VAT reimbursement depends on logistics features in delivery of goods, associated with the factor of internationality.

Analytical solution to this problem could be derived based on regression equation (5), in which the estimates of coefficients correspond to matrix  $a^{(2)}$ . For further transformations, this equation is conveniently rerecorded in its entirety:

$$y^{(2)} = 2,55 + 0,25x_0^{(2)} + 0,4x_1^{(2)} + 0,4x_2^{(2)} + 0,4x_3^{(2)}. \quad (12)$$

Components in a given equation can be rearranged as follows:

$$y^{(2)} = 2,55 + (0,25x_0^{(2)} + 0,4x_2^{(2)}) + (0,4x_1^{(2)} + 0,4x_3^{(2)}). \quad (13)$$

The aim of such a rearrangement is the “clustering” of components in pricing based on the principle of estimation by player 1 “available – non-available”.

We introduce the replacement of variables:

$$\xi_1^{(2)} = 0,25x_0^{(2)} + 0,4x_2^{(2)}, \quad \xi_2^{(2)} = 0,4x_1^{(2)} + 0,4x_3^{(2)}. \quad (14)$$

Considering (14), equation (13) takes the form:

$$y^{(2)} = 2,55 + \xi_1^{(2)} + \xi_2^{(2)}. \quad (15)$$

The transformation of the coordinate system, performed in a factor space of input variables « $x_0^{(2)} - x_1^{(2)} - x_2^{(2)} - x_3^{(2)}$ », into a coordinate system in factor space « $\xi_1^{(2)} - \xi_2^{(2)}$ » is denoted as the  $l$ -level scaling, where  $l=2$ . In this case, the valid values for the ranges of change in the new input variables  $\xi_1^{(2)}$  and  $\xi_2^{(2)}$  can be calculated from the following equations:

$$\xi_1^{(2)} = 0,25 \frac{x_0^{(2)} - 0,75}{0,25} + 0,4 \frac{x_2^{(2)} - 0,6}{0,4}, \quad (16)$$

$$\xi_2^{(2)} = 0,4 \frac{x_1^{(2)} - 0,6}{0,4} + 0,4 \frac{x_3^{(2)} - 0,6}{0,4}. \quad (17)$$

Substituting the extreme values for the ranges of input variables  $x_0^{(2)}, x_1^{(2)}, x_2^{(2)}, x_3^{(2)}$  in the natural form, we obtain valid values for the ranges of change in the new input variables  $\xi_1^{(2)}$  and  $\xi_2^{(2)}$ :  $\xi_1^{(2)} = [-0,65; +0,65]$ ,  $\xi_2^{(2)} = [-0,8; +0,8]$ . The same result will be obtained if we use in formula (14) the extreme normalized values for input variables  $x_0^{(2)}, x_1^{(2)}, x_2^{(2)}, x_3^{(2)}$ .

Such a representation is convenient for practical use when quantifying the opportunities for player 2. For this assessment, it is needed to include in equation (15) the maximally possible ( $y^{(2)}=4$ ) and the minimally possible ( $y^{(2)}=1,1$ ) values for an output variable, and to solve the resulting equation relative to  $\xi_1^{(2)}$ . The straight sections  $\xi_2^{(2)} = \varphi(\xi_1^{(2)})$ , derived in this case, describe the limits of opportunities for player 2 (Fig. 2).

When assigning any value for output variable  $y^{(2)}$  from range  $[1,1;4]$ , we obtain a single particular direct line  $\xi_2^{(2)} = \varphi(\xi_1^{(2)})$ , whose any section will be included within the specified region of constraints.

It should be noted that for player 2 the best variants correspond to the minimum values of the output variable,

that is, the minimum price for its product. Therefore, such strategies would be preferable for it at which direct lines  $\xi_2^{(2)} = \varphi(\xi_1^{(2)})$  are as close as possible to the “lower limit of opportunities”.

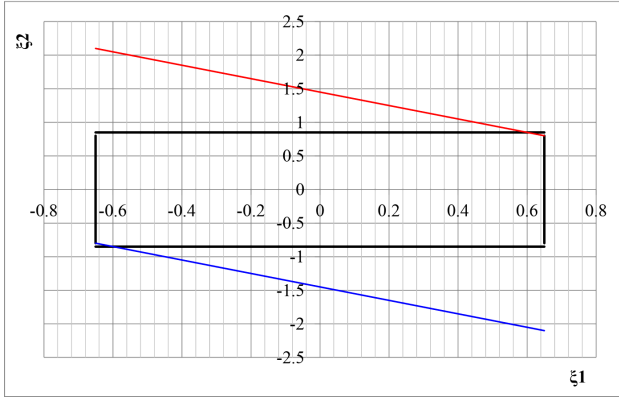


Fig. 2. Estimation chart of opportunities for player 2 in coordinates «  $\xi_1^{(2)} - \xi_2^{(2)}$  »: black rectangle showing the limitations imposed by the range of variation in the input variables  $\xi_1^{(2)}$  and  $\xi_2^{(2)}$ : — — — Linear (Lower limit of opportunities for player 2:  $y=1.1$ ); — — — Linear (Upper limit of opportunities for player 2:  $y=4$ )

The product price of player 1 can be determined from a general regression equation, whose coefficients' estimates were derived based on Table 3:

$$y^{(1)} = 2,8 + (0,4x_1^{(1)} + 0,4x_3^{(1)}) + 0,4x_2^{(1)} \quad (18)$$

In equation (18), components are grouped by the principle similar to (13). Important is the fact that, given the feature of determining an output variable and the use of the orthogonal plan to derive a regression equation, the estimates of coefficients in the input variables will not change at a change in the value for  $y^{(2)}$ . This means that the magnitude  $z_r$  would change only depending on the primary coefficient in equation (6) at  $i=0$ , that is, on coefficient  $a_0$ . Therefore, for any  $y^{(2)}$ , the equation describing the win of player 1 can be constructed if one knows the dependence of form  $a_0=f(y^{(2)})$ . This dependence is linear (Fig. 3) and makes it possible to calculate a value for the initial coefficient in equation

$$z_r = a_0 + \sum_{i=1}^3 a_i^{(1)} x_i^{(1)}, \quad (19)$$

where  $a_i^{(1)}$  are the coefficients' estimates before the input variables derived based on Table 3.

The equation that describes function  $a_0=f(y^{(2)})$  with an accuracy of approximation equal to 1 takes the form:

$$a_0 = -2,8 + y^{(2)}. \quad (20)$$

The result is the obtained system of two equations (19) and (20) in order to assess the win of player 1. If player 1 operates three strategies, corresponding to the three levels of values  $x_2^{(1)}$ , as, for example, is given in Table 4, the use of this system will determine the win at any arbitrary strategy of player 2, other than the 16 strategies given in Table 4. It is obvious that any strategy  $y^{(2)}$  will be located inside the

region of permissible strategies for player 2, assigned by plan  $R=2^4$  (Table 2).

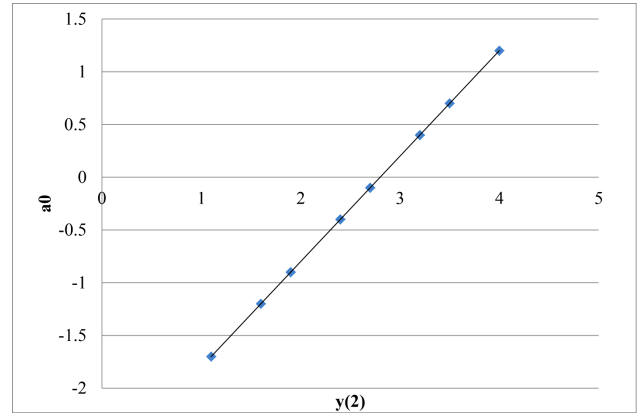


Fig. 3. Function graph  $a_0=f(y^{(2)})$

In the case under consideration, at  $y^{(2)} < y^{(1)}$ , player 1 should determine the conditions under which it might have an equal chance with player 2. To this end, it will suffice to equate the right sides of equations (18) and (15) and to solve the resulting equation relative to  $x_1^{(1)}$ :

$$2,8 + (0,4x_1^{(1)} + 0,4x_3^{(1)}) + 0,4x_2^{(1)} = 2,55 + \xi_1^{(2)} + \xi_2^{(2)}, \quad (21)$$

$$x_1^{(1)} = \frac{b_0}{0,4} - x_3^{(1)}, \quad (22)$$

where

$$b_0 = -0,25 + \xi_1^{(2)} + \xi_2^{(2)} - 0,4x_2^{(1)}. \quad (23)$$

The derived equations (22) and (23) make it possible for player to operate not three strategies depending on the level of values for  $x_2^{(1)}$ , but the larger number of them – at known  $x_3^{(1)}$ , one can choose such  $x_1^{(1)}$ , which equalizes the chances of players. This means that player 1, forced to reduce the magnitude  $y^{(1)}$ , gets an opportunity to operate two variables – the size of its profits and the transportation cost. The ratios between these variables take the form (22) and the choice of a particular solution in the form of an appropriate direct line depends on magnitudes  $\xi_1^{(2)}$  and  $\xi_2^{(2)}$ .

Under actual conditions, it is almost impossible to vary the magnitude  $x_3^{(1)}$ , which is why the problem can be solved relative to variables  $x_1^{(1)}$  and  $x_2^{(1)}$ , assigned by the known magnitude  $x_3^{(1)}$ . The solutions, to be derived, will not differ from solutions relative to  $x_1^{(1)}$  and  $x_3^{(1)}$ , since coefficients in the regression equation before variables  $x_2^{(1)}$  and  $x_3^{(1)}$  are the same.

Upon completion of the described estimation procedures, the resulting values for input variables in the dimensionless form are subject to inverse transformation to the natural form:

– from the scale of level  $l=1$  to the scale of level  $l=0$ :

$$x_n^{(s)} = 0,5 \left[ x_n^{(s)} (x_{n \max}^{(s)} - x_{n \min}^{(s)}) + (x_{n \max}^{(s)} + x_{n \min}^{(s)}) \right]; \quad (24)$$

– from the scale of level  $l=0$  to natural values, assuming that the actual values for magnitudes  $x_i^{(1)}$  are known, at scale of level  $l=0$   $x_0^{(1)} = 1$ , the product price is associated with the pricing components via equation in form (3) and the range of values for the output variable at scale  $l=0$  matches  $[0; 4]$ .



In a general form, the algorithm for choosing a winning strategy is formed by steps given in Table 6.

Table 6

Calculation algorithm for choosing a winning strategy		
Number of algorithm step	Calculation formula	Explanation to operation
1	$X^{(s)} = (x_0^{(s)} \ x_1^{(s)} \ x_2^{(s)} \ x_3^{(s)})$	Scaling at $l$ -level, $l=0$
2	$y^{(s)} = \sum_{i=1}^4 x_i^{(s)}$	
3	$x_n^{(s)} = \frac{2x_n^{(s)} - (x_n^{(s) \max} + x_n^{(s) \min})}{x_n^{(s) \max} - x_n^{(s) \min}}$ , $i = 1, \dots, 4, \ r = 1, \dots, R$	Scaling at $l$ -level, $l=1$
4	$y^{(2)} = 2,55 + \xi_1^{(2)} + \xi_2^{(2)}$	Scaling at $l$ -level, $l=2$ , determined based on consumer loyalty $\xi_1^{(2)} = \varphi(\xi_2^{(2)})$
5	$a_0 = -2,8 + y^{(2)}$	Calculation of win for player 1
6	$z_r = a_0 + \sum_{i=1}^3 a_i^{(1)} x_i^{(1)}$	
7	$y^{(1)} = 2,8 + (0,4x_1^{(1)} + 0,4x_3^{(1)}) + 0,4x_2^{(1)}$	Construction of general equation for product price of player 1
8	$b_0 = -0,25 + \xi_1^{(2)} + \xi_2^{(2)} - 0,4x_2^{(1)}$ $x_1^{(1)} = \frac{b_0}{0,4} - x_3^{(1)}$	Choice of permissible strategies, providing for equal opportunities or the win of player 1
9	$x_n^{(s)} = 0,5 \left[ \frac{x_n^{(s)} (x_n^{(s) \max} - x_n^{(s) \min})}{x_n^{(s) \max} + x_n^{(s) \min}} + \dots \right]$	Transformation from the scale of level $l=1$ to the scale of level $l=0$
10	$x_i^{(1)} = x_0^{(1)} x_{i(t=0)}^{(1)}$	Transformation from scale at level $l=0$ to natural values ( $x_i^{(1)}$ is the value for the $i$ -th input variable in the natural form, $x_{i(t=0)}^{(1)}$ is the value of the $i$ -th input variable in scale at level $l=0$ , $x_0^{(1)}$ is the magnitude of product cost of player 1 in the natural form)

Constraints on the choice of strategies as a preliminary stage of calculations in accordance with the algorithm, given in Table 6, can be defined based on the calculation of the price of the game (Table 5).

**7. Example of application of the winning strategy selection algorithm**

Initial data and the results of scaling at  $l$ -level, required for numerical realization of the algorithm, are given in Table 7.

Table 7

Initial data and the results of scaling at  $l$ -level

Components of product price	Player 1 (s=1)			Player 2 (s=2)			
	Designations for input variables	$y^{(1)}$ y. e.	Scale at level $l=0$	Scale at level $l=1$	$y^{(2)}$ a.u.	Scale at level $l=0$	Scale at level $l=1$
Cost	$x_0^{(s)}$	250	1		?	?	?
Transportation cost	$x_1^{(s)}$	250	1	1	?	?	?
Profit	$x_2^{(s)}$	245	0.98	0.95	?	?	?
VAT	$x_3^{(s)}$	130	0.52	-0.2	?	?	?
<b>Total</b>		<b>875</b>	<b>3.5</b>		<b>750*</b>	<b>3</b>	

Note: \* – the magnitude  $y^{(2)}$  can be determined from the condition of consumer loyalty; it informs player 1 of the product price by player 2

Based on data from Table 6 on the magnitude  $y^{(2)}$ , we determine from equation (15) the values for  $\xi_1^{(2)}$  and  $\xi_2^{(2)}$  (Fig. 4).

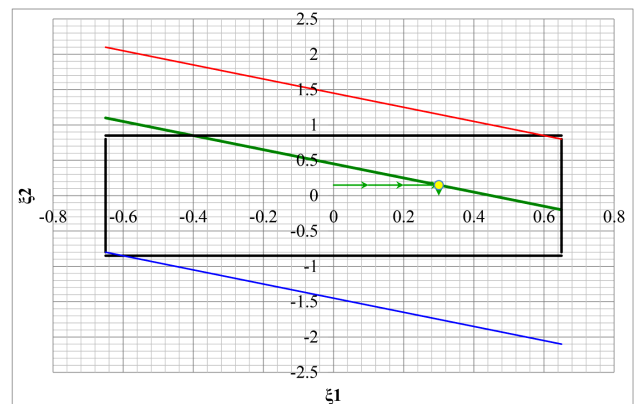


Fig. 4. Calculation of  $\xi_1^{(2)}$  and  $\xi_2^{(2)}$

Based on the results  $\xi_1^{(2)} = 0,3$  and  $\xi_2^{(2)} = 0,15$ , we derive from formula (23) the magnitude  $b_0$ ; and by substituting it in (22), we determine the lower and upper limit of opportunities for player 1 (Fig. 5).

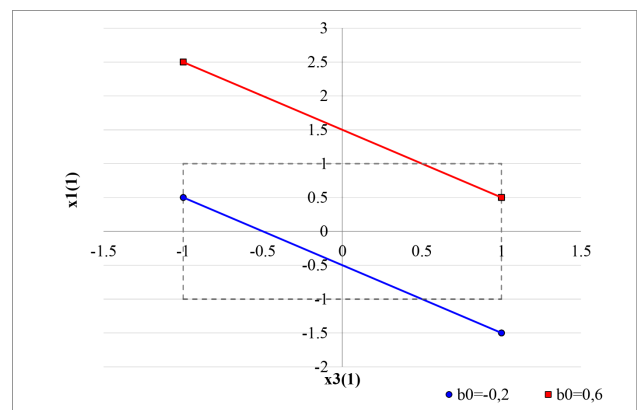


Fig. 5. Ratios between  $x_1^{(1)}$  and  $x_3^{(1)}$  for player 1, which ensures its equal opportunities with player 2. Dashed lines show the constraints imposed by the ranges of input variables (Table 1)

Whatever the value for  $y^{(2)}$ , the range of permissible solutions for player 1 will be located inside the selected range shown in Fig. 5. When substituting a value for  $x_2^{(1)}$  from Table 6 in equations (22) and (23), the result is the linear dependence in form (23) at selected initial data. The corresponding graph is shown in Fig. 6 in green.

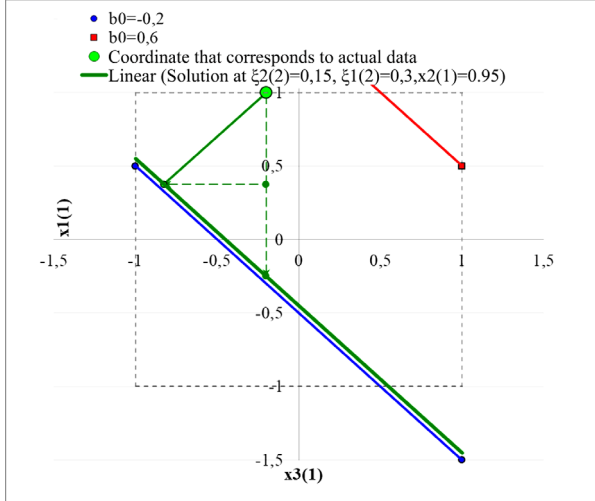


Fig. 6. Relations between  $x_1^{(1)}$  and  $x_3^{(1)}$ , providing for player 1 equal opportunities with player 2, at the assigned initial data (line of equal opportunities)

The resulting graph makes it possible to determine the magnitude by which player 1 should decrease the values for input variables in order to equalize its chances with the player having advantages for price by  $(-0.5)$  units. Fig. 6 shows that, in theory, there are two possible ways (shown in Fig. 6 in green dashed lines):

- by reducing at the same time and by equal magnitude the values for  $x_1^{(1)}$  and  $x_3^{(1)}$ , in this case, this magnitude is 0.884; geometrically, this means the movement along a normal from the initial point at coordinates  $(x_1^{(1)}, x_3^{(1)}) = (-0.2, 1)$  to the line of equal opportunities;
- by not altering the magnitude  $x_3^{(1)}$  and reducing only the magnitude  $x_1^{(1)}$ .

The second variant seems more realizable in practice, because it is almost impossible to change the magnitude  $x_3^{(1)}$ .

It should be noted, however, that if the problem is solved relative to variables  $x_1^{(1)}$  and  $x_2^{(1)}$  at constant  $x_3^{(1)}$ , the first path is preferred. This is due to the fact that player 1 gets more opportunities in terms of variation – both the transportation costs and the size of profit.

## 8. Discussion of results of applying the algorithm for choosing winning strategies

### 8.1. Special features of the obtained solutions, their advantages, and statement of constraints

The obtained results make it possible to approach the problem on choosing the optimal strategies in two ways. The first implies, based on using the derived regression equations of player 1 if player 2 creates benefits in the form of a lower price for the product. For example, it follows from Table 5 that player 1, theoretically, has a chance if not to win, then at least nullify its loss, under the most adverse strategies by player 2.

To do this, it can operate its three strategies if player 2 applies strategies with 1 to 15. A rough quantitative assessment of this possibility can be obtained if one builds a mathematical model in the space of factors ( $s_1-s_2$ ), describing the impact of players' strategies on the win by the first player ( $z$ ). Based on this model, one could resolve the optimization problem on choosing such a strategy for player 1, which would maximize its win. The solutions, to be derived, could also indicate the region of permissible strategies if one varies the input variables  $s_1$  и  $s_2$  as follows:  $s_1=[1; 3]$ ,  $s_2=[1; 15]$ . Normalization of these values can be done by analogy to (4). A procedure to build the model and a technology to solve the optimization problem can be implemented similar to the selection method [32]:

- the model is represented in the form of a second-degree regression polynomial  $z=f(s_1, s_2)$ , whose coefficients' estimates are calculated based on known formulae for central orthogonal composite plans,
  - optimal solutions are searched for at the intersection of a response surface and the surface of constraints generated by the experiment plan, and graphically represent the ridge lines of minima and maxima assigned in the parametric form.
- The solution obtained are shown in Fig. 7–10.

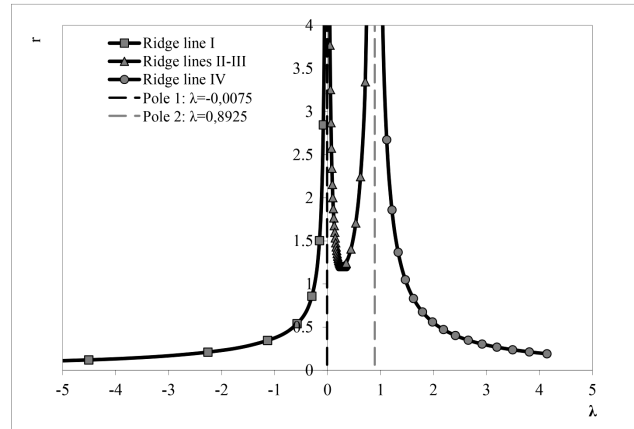


Fig. 7. Description of the system of constraints imposed by the experiment plan in the space of factors ( $s_1-s_2$ ):

$$r(\lambda) = \sqrt{r(\lambda)^2}, \quad r(\lambda)^2 = \sum_{i=1}^n \left( \frac{\eta_i}{2\lambda} \right)^2, \quad \lambda - \text{poles}, \quad \eta_i - \text{linear coefficients of model } z=f(s_1, s_2)$$

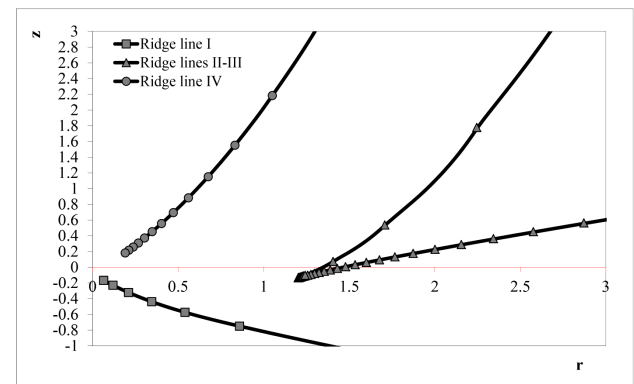


Fig. 8. Dependence of the win of player 1 on the selection of constraints radius:  $z^*(\lambda) = \eta_0 + 2\eta^T s^* + s^{*T} A \eta^*$ ,  $\eta_0$  – initial coefficient of model  $z=f(s_1, s_2)$ ,  $s^*$  – matrix of optimal values for input variables in the normalized form,  $A$  – matrix of nonlinear coefficients of model  $z=f(s_1, s_2)$

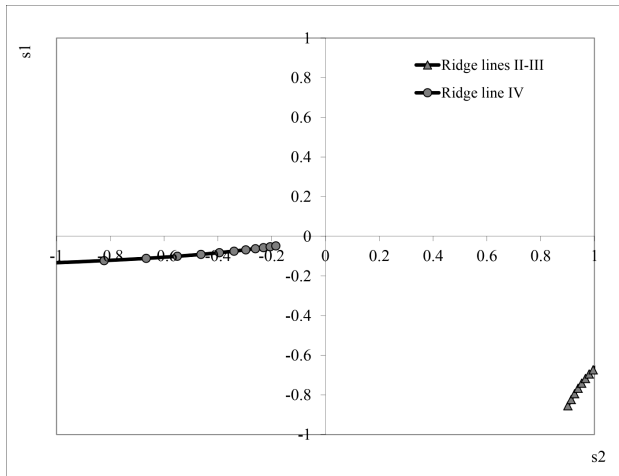


Fig. 9. Set of optimal values for strategies in the normalized form

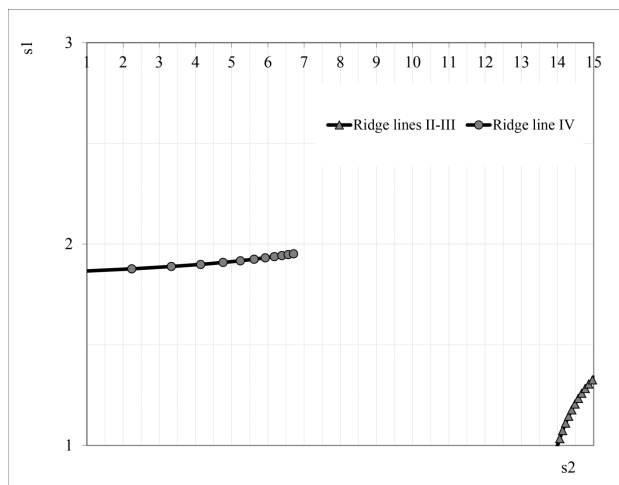


Fig. 10. Set of optimal values for strategies in the natural form

Fig. 8 shows that the solutions that are on ridge line I are unacceptable for player 1, because respective strategies will lead to its guaranteed loss. A zero solution corresponding to equal opportunities for players is shown in Fig. 8 with a red horizontal line. Also unacceptable is part of the solutions along ridge lines II–III within region  $z < 0$ . The remainder of ridge lines II–III and a ridge line IV combed are valid; they are shown as sets of strategies  $s_1$ – $s_2$  in Fig. 9 in the normalized form and in Fig. 10 in the natural form. Based on Fig. 10, one can select a region of acceptable strategies for player 1. For example, it follows that strategy 3 for player 1 is not acceptable. Its choice is limited to strategies that are close to strategy 2 (a profit is chosen near its average value) or strategy 1 (a profit is chosen near its minimum).

When carrying out the numerical estimation of opportunities for player 1 based on the proposed algorithm, it can consider various alternatives for selecting strategies. The results, obtained in this case analytically, resolve the issue related to the quantitative adjustments of pricing components. In this case, the universality of the obtained solutions is very important, ensured by the introduction of the  $l$ -level scale. In particular, such a representation makes it possible to perform a dimensionless estimation of product pricing, regardless of its type or natural cash value. The regression equations, constructed in this case, are universal as well, as they are

based on the orthogonal plans in the planning region, which is guaranteed to cover the actual values for the components of pricing. The latter is ensured by the wide ranges of varied factors [0.2; 1] for transportation cost, profit, and VAT, as well as a wide range of product cost for player 2 – [0.5; 1] in the natural form, reduced to the product cost of player 1.

All this predetermines advantages of the proposed solutions, though we should note considerable constraints imposed on them. The constraints relate to two components:

- accuracy of quantifying the pricing components for player 2;
- taxation regularities in international cargo transportation.

The first limitation can be considered to be a shortcoming of this study because it is not possible to accurately determine magnitudes  $x_i^{(2)}$  and, therefore,  $\xi_i^{(2)}$ . These magnitudes can only be determined applying some interval estimates. Therefore, there is reason to consider them to be fuzzy magnitudes, introducing the notion of uncertainty in the assessment of input variables  $x_i^{(2)}$ . In this case, they must be described by the membership functions while optimal solutions must be found by using methods of fuzzy mathematics [33]. Such solutions should be regarded as a prospect for the further development of this study, they are possible in principle, as they are in the plane of the formalized representation of the problem. The situation with the second component of constraints is much more difficult. To account for them, a detailed analysis is needed in the aspect of legislative regulation of international cargo transportation. This is due to the fact that the magnitude of transportation costs and a possibility for VAT reimbursement depend on the directionality of transportation. That makes it important to consider the fundamentals of taxation when importing goods to the EU and when goods are exchanged within the EU.

### 8.2. Constraints imposed by features of importing the goods

Movement of goods China – EU, United States – the EU.

If goods are imported into the EU from outside the EU (for example, from China, Russia, the United States, or another country that is not in the EU), there is an import of goods to the EU.

When importing goods, VAT is paid at the rate of the country, which will perform customs clearance of the goods. In addition to VAT, a corresponding customs duty is paid.

The size of customs duties is determined based on a uniform, adopted in the European Union, nomenclature, and depends on several factors, primarily on the type of product, where it was made, or where from the goods are supplied (country of origin). Thus, there is always the possibility to determine in advance the possible costs associated with customs payments.

It is worth noting that the rate of customs duties in the EU is uniform while the VAT rate is different in each country. It is also a significant factor when planning commercial profitability of business activities. It should also be noted that companies that have the EU registration as VAT payers can claim the return of VAT paid, that is, the fact of customs clearing could be a base for VAT reimbursement. Some EU countries have regulations, according to which customs procedures can be performed without paying VAT, which means it cannot be reimbursed. At the same time, customs duties are payable on a mandatory basis, except in cases where the duty rate is 0.

Especially important is the following aspect. VAT reimbursement is possible only in cases when a company is registered as a VAT payer. The European Union created a single registry of companies that are VAT payers [34] (Table 8).

Table 8

## VAT rates in the EU

Country	Code	Reduced rate	Standard rate
Belgium	BE	6/12	21
Bulgaria	BG	9	20
Czech Republic	CZ	10/15	21
Denmark	DK	–	25
Germany	DE	7	19
Estonia	EE	9	20
Ireland	IE	9/13.5	23
Greece	EL	6/13	24
Spain	ES	10	21
France	FR	5.5/10	20
Croatia	HR	5/13	25
Italy	IT	5/10	22
Cyprus	CY	5/9	19
Latvia	LV	12	21
Lithuania	LT	5/9	21
Luxembourg	LU	8	17
Hungary	HU	5/18	27
Malta	MT	5/7	18
Netherlands	NL	6	21
Austria	AT	10/13	20
Poland	PL	5/8	23
Portugal	PT	6/13	23
Romania	RO	5/9	19
Slovenia	SI	9.5	22
Slovakia	SK	10	20
Finland	FI	10/14	24
Sweden	SE	6/12	25
United Kingdom	UK	5	20

*Turnover within the EU (movement of goods EU – EU).*

At turnover of goods within the EU, there is a possibility to apply a 0 % VAT rate in accordance with EC Directive 2006/112/EC. However, it must be remembered that the use of a 0 % tax rate is possible only subject to certain conditions:

1. Goods must go from one EU country to another EU country.
2. The seller must have a VAT registration in the country wherefrom the goods are dispatched.
3. The buyer must have a VAT registration in the country where the goods are supplied to.

Compliance with the specified conditions makes it possible to deliver goods within the EU without unnecessary tax burden.

*Income tax (profit tax).*

Another important aspect of taxation in the context of commercial activities is the issue related to corporate income tax.

In different EU countries, the rate of this tax, and the tax system itself, can differ greatly. The inclusion of this tax to the product cost may affect its selling price, as an entrepreneur must add all the extra fees and associated costs to the

product cost. These include, for example, the cost of logistics, storage, etc.

In many ways, a product selling price when selling within the EU countries, from a standpoint of its price competitiveness, could be affected by the optimization of the income tax as a result of the use of a corporate structure, registered in a particular jurisdiction (Table 9).

Table 9

## Income tax rates (profit tax) for the EU companies

Country	Tax rate
Austria	25 %
Belgium	29 % (25 % as of 2020)
Bulgaria	10 %
Croatia	18 %
Cyprus	12.5 %
Czech Republic	19 %
Denmark	22 %
Estonia	Company income is not taxed until profits are distributed. 20 % – only when profits are distributed as dividends.
Finland	20 %
France	33.3 % (36.6 % exceeding €3.5M, 15 % below €38)
Germany	From 22.825 % to 32.925 % depending on municipality
Greece	29 %
Hungary	9 %
Italy	27.9 %
Latvia	Company income is not taxed until profits are distributed. 20 % – only when profits are distributed as dividends.
Liechtenstein	12.5 %
Lithuania	15 %
Netherlands	From 20 % to 25 %
Norway	23 %
Poland	19 %
Portugal	21 %
Slovakia	22 %
Slovenia	22 %
Spain	25 %
Sweden	22 %
United Kingdom	19 %

*Other types of taxes.*

It should be noted that the issues related to business process optimization should also take into consideration the time of distribution of company's profits in favor of its legal owner or a beneficial owner (beneficiary). The rates of such taxes vary depending on the country of residence of the beneficiary and must take into consideration the tax liabilities in connection with the application of international agreements on the avoidance of double taxation.

Because there are no unified tax rates in the EU, an entrepreneur who wishes to conduct business in the territory of the EU must very carefully address the issue of tax optimization of business processes in the context of the planned activity. Proper tax planning and knowledge of important aspects of taxation will make it possible not to only reduce a product selling price, but to also gain a competitive advantage for the business in general. All this, along with taking into consideration the features of logistics in international

transportation (technical-organizational, technological, financial, and economic, legal), can be considered a prospect for the further advancement of this study.

## 9. Conclusions

1. To assess the price of a product in the system “supplier – consumer”, the notion of the  $l$ -level scale could be introduced. Under such representation, a dimensionless estimation of product pricing becomes possible, regardless of its type or natural cash value. The price of a product includes 4 basic components: cost, transportation cost (total cost of delivery), profit, value added tax (VAT). At level  $l=0$ , scaling represents a transformation of actual values for each component of pricing in the range  $[0; 1]$ . In this case, the range of values for the output variable corresponds to  $[0; 4]$ . At level  $l=1$ , scaling represents the normalization of values for each pricing component that were pretreated with scaling at level  $l=0$ , thus to convert these values into the range of  $[-1; +1]$ .

If relations between supplier 1 and supplier 2 are to be represented by the concepts from the theory of strategy games, then, to calculate the win of player 1, one can apply universal regression equations. In this case, a matrix of the game is formed in the following way: player 2 has 16 strategies, in accordance with the plan of a full factorial experiment  $N=2^4$ , player 1 has three strategies, each of which is formed as the maximally possible magnitude of win at three levels of variation in the magnitude of its profit. A maximally possible magnitude of the win is calculated as the difference between a product price from player 2 and that from player 1, and player 1 has 8 strategies, in accordance with the plan of a full factorial experiment  $N=2^3$ . A special feature is that the value for the win in the game's matrix is defined by the solution to the optimization problem based on the regression equation that describes the impact of transportation cost, profit, and a value-added tax (VAT) on the price of the game. In this case, there are only locally optimal solutions derived

at the boundary of the planning region, because regression equations are linear.

The obtained regression equations are universal, as they are based on orthogonal plans in the planning region, within which the actual values for the components of pricing are guaranteed. The latter is ensured by the wide ranges of varied factors  $[0.2; 1]$  for transportation costs, profits, and VAT, as well as by a wide range of product cost for player 2 –  $[0.5; 1]$  in the natural form, reduced to the product cost of player 1.

The proposed variant of the game description showed that we can argue about the game with a saddle point and the net price of the game  $z=-0.5$ . Given that such a variant cannot suit player 1, it must seek, if not to win, then at least to nullify its loss, under the most adverse strategies of player 2. To assess this possibility, a mathematical model was built in the space of factors – strategies of player 1 and player 2 – an output variable in which is the win of player 1. Based on this model, the optimization problem was solved, which made it possible to establish that the strategy, implying the maximum magnitude of profit under conditions of a lower price offered by player 2, is not acceptable for player 1. Its choice is limited to the strategies that are close to alternative ones, when profit is chosen near its average value or a profit is chosen near its minimum.

2. A predictive model of strategic opportunities for a competitor in the system “supplier – consumer” is a universal regression equation, on whose basis it is possible to quantify the components of product pricing for player 2. This makes it possible, by using the obtained system of universal equations for player 1, to adjust the price of its products based on the magnitude of transportation costs and profits, or on the magnitude of transportation costs and VAT. Quantitatively, it is performed in such a way as to nullify the advantages of a competitor. It is shown that the constraints for the obtained solutions are linked to two factors: an assumption about the accuracy in determining pricing components for player 2 and the existence of taxation patterns in international cargo transportation.

## References

1. Makarov F. V. Puti sovershenstvovaniya planirovaniya novoy tekhniki // Voprosy sovershenstvovaniya upravleniya obshchestvennym proizvodstvom. Saratov, 1981. 180 p.
2. Murav'ev A. I. Planirovanie tekhnicheskogo razvitiya ob'edineniya. Moscow: Ekonomika, 1986. 64 p.
3. Demina E. Formirovanie kriteriya celesoobraznosti tekhnicheskogo perevooruzheniya promyshlennogo proizvodstva // Vestnik Kharkivskoho gosudarstvennogo politekhnicheskogo universiteta. Tekhnicheskii progress i effektivnost' proizvodstva. 1999. Issue 95.
4. Oliver K., Webber M. Supply chain management: Logistics Catches up with Strategy / M. Christopher (Ed.) // Logistics: The Strategy Issues. London: Chapman and Hall, 1982. P. 63–75.
5. Intrilligator M. Matematicheskie metody optimizatsii i ekonomicheskaya teoriya. Moscow: Ayris-Press, 2002. 553 p.
6. Supply Chain and Logistics Terms and Glossary. Council of Supply Chain Management Professionals, 2005. 97 p.
7. Sjoerdsma M., van Weele A. J. Managing supplier relationships in a new product development context // Journal of Purchasing and Supply Management. 2015. Vol. 21, Issue 3. P. 192–203. doi: <https://doi.org/10.1016/j.pursup.2015.05.002>
8. Logistics and supply chain innovation / H. Zijm, M. Klumpp, U. Clausen, M. Hompel (Eds.). Springer, 2016. doi: <https://doi.org/10.1007/978-3-319-22288-2>
9. Gualandris J., Kalchschmidt M. Customer pressure and innovativeness: Their role in sustainable supply chain management // Journal of Purchasing and Supply Management. 2014. Vol. 20, Issue 2. P. 92–103. doi: <https://doi.org/10.1016/j.pursup.2014.03.001>
10. Defining Supply Chain Management / Mentzer J. T., DeWitt W., Keebler J. S., Min S., Nix N. W., Smith C. D., Zacharia Z. G. // Journal of Business Logistics. 2001. Vol. 22, Issue 2. P. 1–25. doi: <https://doi.org/10.1002/j.2158-1592.2001.tb00001.x>
11. Lambert D. M. Supply Chain Management: Processes, Partnerships, Performance. 4 th ed. Supply Chain Management Institute, 2014. 463 p.

12. Simchi-Levi D., Kaminsky P. *Designing and managing the supply chain: concepts, strategies, and case studies*. N.-Y.: McGraw-Hill Companies, 2008. 496 p.
13. Fedulova L. Innovacionnoe razvitie: evolyuciya vzglyadov i problemy covremennogo ponimaniya // *Ekonomicheskaya teoriya*. 2013. Issue 2. P. 28–45.
14. Postan M. Ya., Malinovskiy D. A. Model' optimal'nogo planirovaniya proizvodstva i dostavki produktsii predpriyatiya po raspredelitel'nym kanalams // *Metodi ta zasobi upravlinnya rozvitkom transportnih sistem*. 2009. Issue 15. P. 19–28.
15. Kurudzhi Y., Moskvichenko I., Postan M. Method of finding equilibrium solutions for duopoly of supply chains taking into account the innovation activity of enterprises // *Eastern-European Journal of Enterprise Technologies*. 2017. Vol. 3, Issue 4 (87). P. 25–30. doi: <https://doi.org/10.15587/1729-4061.2017.103989>
16. Szyszka G. Sieci logistyczne – nowy wymiar logistyki // *Logistyki*. 2004. Issue 3. P. 5–7.
17. Bosov A., Khalipova N. Formation of separate optimization models for the analysis of transportation-logistics systems // *Eastern-European Journal of Enterprise Technologies*. 2017. Vol. 3, Issue 3 (87). P. 11–20. doi: <https://doi.org/10.15587/1729-4061.2017.103220>
18. Dorigo M., Stutzle T. *Ant Colony Optimization*. MIT Press, 2004. doi: <https://doi.org/10.7551/mitpress/1290.001.0001>
19. Decision of Multimodal Transportation Scheme Based on Swarm Intelligence / Lei K., Zhu X., Hou J., Huang W. // *Mathematical Problems in Engineering*. 2014. Vol. 2014. P. 1–10. doi: <https://doi.org/10.1155/2014/932832>
20. Ramadhani T., Hertono G. F., Handari B. D. An Ant Colony Optimization algorithm for solving the fixed destination multi-depot multiple traveling salesman problem with non-random parameters // *AIP Conference Proceedings*. 2017. doi: <https://doi.org/10.1063/1.4991227>
21. Hassan M. R., Islam M. M., Murase K. A New Local Search Based Ant Colony Optimization Algorithm for Solving Combinatorial Optimization Problems // *IEICE Transactions on Information and Systems*. 2010. Vol. E93-D, Issue 5. P. 1127–1136. doi: <https://doi.org/10.1587/transinf.e93.d.1127>
22. Ashouri M., Yousefikhoshbakht M. A Combination of Meta-heuristic and Heuristic Algorithms for the VRP, OVRP and VRP with Simultaneous Pickup and Delivery // *Brain: Broad Research in Artificial Intelligence and Neuroscience*. 2017. Vol. 8, Issue 2. P. 81–95.
23. Developing the method of rational trucking routing based on the modified ant algorithm / Khalipova N., Pasichnyk A., Lesnikova I., Kuzmenko A., Kokina M., Kutirev V., Kushchenko Y. // *Eastern-European Journal of Enterprise Technologies*. 2018. Vol. 1, Issue 3 (91). P. 68–76. doi: <https://doi.org/10.15587/1729-4061.2018.123862>
24. Konings R., Priemus H., Nijkamp P. (2008) *The Future of Intermodal Freight Transport*. Operations, Design and Policy. Transport economics, management and policy. USA, 2008. doi: <https://doi.org/10.4337/9781848441392>
25. Hanssen T.-E. S., Mathisen T. A., Jørgensen F. Generalized Transport Costs in Intermodal Freight Transport // *Procedia – Social and Behavioral Sciences*. 2012. Vol. 54. P. 189–200. doi: <https://doi.org/10.1016/j.sbspro.2012.09.738>
26. Akimova O. V. An export and import scheme for container delivery by freight forwarding companies // *Eastern-European Journal of Enterprise Technologies*. 2014. Vol. 6, Issue 3 (72). P. 4–10. doi: <https://doi.org/10.15587/1729-4061.2014.28862>
27. Lysyy A., Kotenko V., Yakovtsev S. Conceptual scheme for ensuring the energy efficiency principle in modern container fleet // *EUREKA: Physics and Engineering*. 2018. Issue 6. P. 41–47. doi: <https://doi.org/10.21303/2461-4262.2018.00749>
28. Sweeney E., Evangelista P. 3PL definition and taxonomy. *Technical Focus in Logistics Solutions* // *Journal of the National Institute for Transport and Logistics*. 2005. Vol. 7, Issue 2. P. 9–10.
29. Anan'ev E. Est' u ekspeditorov zakon // *Zhurnal Porty Ukrainy*. 2004. Issue 4. P. 15–17.
30. Blekuell D., Girshik M. A. *Teoriya igr i statisticheskikh resheniy*. Moscow, 1958. 432 p.
31. Korshunov Yu. M. *Matematicheskie osnovy kibernetiki*. Moscow: Energiya, 1980. 424 p.
32. Demin D. Synthesis of optimal control of technological processes based on a multialternative parametric description of the final state // *Eastern-European Journal of Enterprise Technologies*. 2017. Vol. 3, Issue 4 (87). P. 51–63. doi: <https://doi.org/10.15587/1729-4061.2017.105294>
33. Raskin L. G., Seraya V. *Nechetkaya matematika: monogr.* Kharkiv: Parus, 2008. 352 p.
34. VIES VAT number validation. URL: [http://ec.europa.eu/taxation\\_customs/vies](http://ec.europa.eu/taxation_customs/vies)