Розроблено стохастичну модель роботи системи управління запасами матеріалів на судноремонтному заводі (СРЗ). З метою врахування чинників невизначеності та ризику (випадкові моменти прибуття на СРЗ суден, випадкові розміри обсягів ремонтів) для моделювання запропоновано використати апарат марковських процесів зі знесенням. Ці процеси дозволяють врахувати дискретний характер змін чисельності суден, які знаходяться на СРЗ, та безперервний характер коливання рівня запасів матеріалів на складі. При цьому причали СРЗ інтерпретуються як система масового обслуговування. Вважається також, що поповнення запасів матеріалів на складі та використання під час ремонту суден здійснюється безперервно з постійними інтенсивностями, але в залежності від наявності матеріалів на складі. В результаті дослідження сформульована задача стохастичної оптимізації інтенсивностей поповнення запасів матеріалів за критерієм мінімум сумарних середніх поточних витрат СРЗ, які враховують також витрати, що стосуються додаткового простою суден внаслідок відсутності запасів матеріалів на складі під час проведення ремонту.

Модель засновано на сполученні методів теорії запасів і теорії масового обслуговування. Поповнення запасів матеріалів на складі та їх використання здійснюється безперервно з постійними інтенсивностями. Сформульовано завдання стохастичної оптимізації інтенсивностей поповнення запасів матеріалів за критерієм мінімум сумарних середніх витрат заводу в одиницю часу. Доведено, що отримані результати важливі для практики роботи служби постачання СРЗ, оскільки дозволяють формувати стратегію управління запасами матеріалів нас складах СРЗ в умовах нерівномірності у часу виникнення потреби у ремонті суден. З теоретичної точки зору одержані результати демонструють можливість використання апарату марковських процесів зі знесенням для вирішення різних завдань оптимального управління запасами в умовах випадкового попиту на запаси

Ключові слова: судноремонтний завод, система масового обслуговування, запаси матеріалів, ризик простою суден, оптимальне управління запасами

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CONSTRUCTION AND ANALYSIS OF THE MODEL FOR STOCHASTIC OPTIMIZATION OF INVENTORY MANAGEMENT AT A SHIP REPAIR YARD

I. Petrov
PhD, Professor
Department of Sea Transportation
National University
"Odessa Maritime Academy"
Didrikhsona str., 8, Odessa, Ukraine, 65029

M. Postan
Doctor of Economic Sciences,
Professor, Head of Department
Department of Management & Marketing
Odessa National Maritime University
Mechnikov str., 34, Odessa, Ukraine, 65029
E-mail: postan@ukr.net

1. Introduction

Ship repair is a complex and low mechanized industrial sector. Ship repair yards (plants) perform a dock repair, as well as repair of bottom-overboard fittings, pipelines, propeller-rudder system, they replace hull's steel structures, they manufacture fuel equipment for ICE, spare parts for ship equipment and devices, etc. If the repair involves a significant amount of work, an industrial method is applied, based on specialization and cooperation of the repair base, zero stage, aggregate method of repair, automation and integrated mechanization.

Management team at SRY must be able to effectively manage under critical conditions, to adapt to changing market conditions [1], and strive to minimize possible risks. This requires the application of scientific methods for making decisions on inventory management under conditions of uncertainty and risk, accounting for competition, as well as advances information technologies. That gives rise to many new, non-standard scientific tasks because classic models, constructed to study operations, do not take into consideration at all, or to a less degree, the specificity of SRY functioning as a specialized enterprise, as well as its behavior under market conditions. The above necessitates further research in this field.

2. Literature review and problem statement

In paper [2], authors performed an analysis of ship repair market and concluded that small ship-repairing companies increasingly win bids for the repair of ships in Ukraine. The reason for this relates to the fact that large SRY have higher overhead costs, while the lowest price on repair, without compromising its quality, is typically a key requirement from the commissioner. Study [3] suggested an approach to improve effectiveness of managing a SRY, based on the project management methods. Work [4] proposed an entropic model of risk management during implementation of ships repair projects, while paper [5] devised principles for building a risk-oriented strategy to maintain and repair ships. Study [6] suggested a procedure to construct probability trees and to calculate the ratios of probability derivation in order to analyze various organizational and technical tasks in ship repair under conditions of uncertainty and risk. However, the approaches, applied in [3–6], do not take into consideration the dynamics of change in the production situation at SRY, the stochastic character of arrival of ships at SRY, and fluctuations in the volumes of their repair.

Papers [7–11] propose a series of simulation schemes for production processes at SRY, whose implementation could improve the organization of repair operations. Thus, work

[7] suggests a simulation model in terms of discrete events in order to plan and manage the utilization of technological equipment at SRY and supplies of materials to it. In this case, the model takes into consideration the possibility to perform a sensitivity analysis of plans if the initial data changes in the process of implementing plans, which makes it possible to improve equipment utilization. At the same time, the issue related to the supply of materials for repairs was not considered in [7]. In [8], authors propose a decision-making system for operational planning in order to maximize the throughput capacity of SRY and to minimize complete production costs, which helps to avoid internal competition between cost centers at a plant and to improve hardware utilization. Underlying this system is the construction of a common information base to be accessed by all departments at a plant. Paper [9] designed a multiagent information systems (Multi-Agent System) in order to model technological processes in ship repair, which makes it possible to integrate data flows, business processes, and financial flows. However, the issues related to predicting the repairs at SRY based on a given information base have not been addressed in [8, 9], which limits the scope of application of the specified information system.

Production activity of SRY, similarly to that at any industrial enterprise, requires, in order to perform ship repairs, different types of materials and components. Demand for these materials occurs when repairs are performed on the ships in docks at SRY, that is, generally speaking, at random time. Therefore, in order to effectively handle inventories of these materials, it is expedient to employ methods from the theory of inventory control under conditions of random demand. This area of inventory control theory has intensively developed over recent decades. For example, paper [10] examines the model of inventory control in which the time required to execute an order for replenishment is set, while the demand for products is subject to the log-normal distribution. It is unclear, however, how to manage inventory by applying this model if demand is described by any other probability distribution. Work [11] analyses the problem on defining an optimal replenishment policy for perishable products with a backlogged demand and adjusted for inflation. In this case, a change in the discount factor is described using a Markovian process. However, this approach is not applicable in a situation when the demand itself fluctuates randomly. Study [12] provides a series of classic models for the optimal inventory control at random demand; they, however, do not take into consideration the dynamics of change in the fluctuations of stocks over time, thereby reducing their practical significance.

Formally, any SRY could be represented in the form of a multi-channel queueing system (QS), in which the requests for service are the ships themselves with a certain set of repair operations of several types, while the servers are the wharves and docks together with the required equipment.

Even though the queueing theory – QT – includes at present a large number of models for different QS [13], the specificity of individual types of industries still necessitates the construction and investigation of specialized new models. Such systems could certainly include SRY, at which the above specificity manifests itself in a simultaneous description of the production process (arrival of ships and their repair) and the process of materials supplies for conducting repair operations.

At the same time, there is an obvious lack of studies that would address the interaction between a process of a ship arrival and the repair process, on the one hand, and the process of replenishment and consumption of materials, required to execute repair operations, on the other hand. Such a research is necessary to improve operational effectiveness of SRY and to reduce the risk of vessel idling during repairs due to a shortage of materials needed for repairs.

It is known [13] that the construction of mathematical models for QS has commonly employed the apparatus of Markov random processes with a discrete set of states. However, in certain cases, an equally convenient type of the Markovian random processes are the so-called Markovian drift processes. Markovian drift processes are now widely applied in order to model and analyze various logistics systems [14–16], as well as transportation systems [17]. The phase space of such processes represents a direct product of discrete and continuous sets. From an applied point of view, a discrete set describes the dynamics of QS states, defined by discrete variables (the number of ships at docks and in queues to them), while the continuum set can describe, for example, a fluctuation of inventory levels at a warehouse over time. This circumstance makes it possible to state and solve a variety of tasks on optimal inventory management under conditions of uncertainty and risk. This approach could be used in order to solve the problem related to examining an influence of the level of materials stocks at SRY on the dynamics of change in the number of vessels at the yard, and to the construction of an optimization method for the replenishment policy regarding the specified stocks.

3. The aim and objectives of the study

The aim of this study is to state mathematically, and solve, the problem on the optimal inventory control over materials required to repair ships, under conditions of uncertainty about the time of ships arrival at SRY and the volumes of repair operations.

To accomplish the aim, the following tasks have been set:

– to provide a formalized description of SRY in terms of QT and a stock theory, taking into consideration the non-uniformity in ships arrival for repairs and different volumes of repair operations, as well as the consumption of materials, based on the application of an appropriate Markovian drift process (or with “speeds”);

– to derive a system of integral-differential equations for the described probabilistic model of SRY in order to find the joint distribution of the number of ships at docks of SRY under repair and in queue side by side the quantity of materials, components, which are at a SRY warehouse;

– to find, based on the obtained solution:

a) analytical expressions for calculating key performance indicators of SRY as a QS and to estimate average operating costs of SRY for the repair of ships, as well as the costs incurred because of the exhaustion of supplies in warehouse;

b) to construct a model for the stochastic optimization of intensities in the replenishment of materials stocks.

4. Formalized description of SRY in terms of the queueing theory and the theory of stocks

We shall consider SRY that is simplistically described by the following constituent elements:

a) a warehouse for storing M types of materials required for the execution of repair operations;

b) a stock of material of the m-th type, replenished continuously, at intensity U_m;

c) the market of materials suppliers is unlimited;

d) the cost of materials delivery per unit time is proportional to the intensity of the replenishment of a material;

e) each ship, arriving at SRY, requires for her repair a random quantity γ_m of a material of the m-th type, that is, repair of each ship, regardless of other ships, requires a random vector of materials ($\gamma_1, \gamma_2,..., \gamma_M$). For simplicity, we shall assume that all these random variables are mutually independent, and

$$\mathbf{P}\{\gamma_m \le x\} \equiv G_m(x); \quad (1)$$

f) in the process of carrying out repair operations in line with the assigned technology, the intensity of utilization of a material of the m-th type is equal to $W_m > U_m$;

g) each arriving ship takes one of n unoccupied identical and interchangeable docks at SRY, if any; otherwise, it follows the queue to docks; in this case, vessel queue length is limited by the value R.

As regards the above assumptions, the following note should be made.

The modern theory of inventory management considers models with different replenishment policies [12]: continuous replenishment, replenishment by fixed-size batches based on orders, on the current level of existing stock at a warehouse, on the level of demand, etc. Here, only one policy is considered, namely continuous replenishment. This assumption means that restocking is very common, but in relatively small batches; therefore, it could be assumed that the replenishment is performed approximately continuously over time.

Inventory control theory often employs a feedback between a replenishment strategy and the current inventory level. The described modeling scheme allows a situation when, during ship repair, an inventory level of any material (for example, the m-th) is exhausted. In this situation, we shall assume that the repair of the ship continues, but the intensity of stock replenishment (and applying this type of a material to perform repair operations) becomes equal to $U_{0m} \ge U_m$. Specifically, it could be equal to the intensity of utilization of a material of the m-th type W_m, or remain equal to U_m. In the second case, it is obvious that the repair time increases, which could lead to penalties claimed by the shipowner against SRY.

Note that since a repair ends when all M types of operation have been executed, then, provided that all work is performed in parallel, repair time τ of an arbitrary ship is equal to

$$\tau = \max\left(\frac{\gamma_1}{W_1},...,\frac{\gamma_M}{W_M}\right).$$

Hence, from (1), it follows that a repair time of arbitrary ship is a random variable with the distribution function

$$\mathbf{P}\{\tau \le t\} = G_1(W_1 t)...G_M(W_M t) \equiv B(t).$$

These dependences will hold if, during repair of a ship, not such a situation occurs when a warehouse runs out of stock of at least one type of a material used to repair the ship. We shall address a situation below when the exhaustion of materials stocks is possible; we shall examine in detail the two kinds of stock replenishment strategies when a warehouse is empty, namely:

a) $U_{0m}=U_m$,

b) $U_{0m}=W_m$. (2)

For the case a), there is a risk for an additional downtime of the ship under repair due to a decrease in the intensity of stock replenishment; for case b), there will not be ship additional idling.

The ultimate purpose of constructing the described SRY model is to state and solve the problem on finding the optimal values $U_1, U_2,..., U_M$, characterizing the process of materials stocks replenishment at a warehouse, in line with some economic criterion for optimization.

5. Derivation and analysis of the system of differential equations and boundary conditions for finding a stationary joint distribution of the number of ships at SRY and the quantity of materials in stock

We assume that ships arrive at SRY at random points in time, with their flow described by a model of the homogeneous Poisson process with parameter λ. We accept that the random variables γ_m, m=1, 2,..., M are distributed according to exponential laws with average values g_m, m=1, 2,.., M. We introduce the following conditional designations: $\nu(t)$ − the number of ships at SRY at time t; $Z_m(t)$ – inventory level of a material of the m-th type in warehouse at time t.

For simplicity, we shall assume that the storing capacity of a warehouse is large enough, that is, we disregard a possibility to fill the warehouse to capacity.

Given the above assumptions, the random process ($\nu(t)$; $Z_1(t)$,..., $Z_M(t)$) is a Markovian vector drift process. To find the limit probability distribution of this process, it is possible, in principle, to derive an appropriate system of differential equations in partial derivatives, as well as the boundary conditions.

For arbitrary M, however, the mentioned system of equations is too cumbersome and difficult to solve. Below, we give it for particular cases M=1 and M=2, and n=1, that is, for a single dock.

Case M=1. Denote

$$q_k(x) = \lim_{t\to\infty} \mathbf{P}\{x < Z(t) < x + dx\} / dx,$$

$$k \in F = \{0,1,...,R+1\},$$

$$p_k^- = \lim_{t\to\infty} \mathbf{P}\{Z(t) = 0, \nu(t) = k\}, \quad k \in F \setminus \{0\}.$$

In order to determine these functions and constants using the method given in paper [18], it is possible to derive the following system of ordinary differential equations and boundary conditions:

$$U\hat{q}_0'(x) = -\lambda\hat{q}_0(x) + \mu\hat{q}_1(x),$$

$$-V\hat{q}_i'(x) = -(\lambda+\mu)\hat{q}_i(x) + \lambda\hat{q}_{i-1}(x) + \mu\hat{q}_{i+1}(x),$$

$$i = 1,2,\ldots,R,$$

$$-V\hat{q}'_{R+1}(x) = -\mu\hat{q}_{R+1}(x) + \lambda\hat{q}_R(x), \quad x>0, \tag{3}$$

$$-\mu_1\hat{p}_1^- + U\hat{q}_0(0) = 0,$$

$$-(\lambda+\mu_1)\hat{p}_1^- + \mu_1\hat{p}_2^- + V\hat{q}_1(0) = 0,$$

$$-(\lambda+\mu_1)\hat{p}_i^- + \mu_1\hat{p}_{i+1}^- + \lambda\hat{p}_{i-1}^- + V\hat{q}_i(0) = 0, \quad i=2,3,\dots,R,$$

$$-\mu_1\hat{p}_{R+1}^- + \lambda\hat{p}_R^- + V\hat{q}_{R+1}(0) = 0, \tag{4}$$

where $\mu = W/g, \ \ \mu_1 = U_0/g, \ \ V = W-U;$ U_0 equals U or W (see (2)).

The normalization condition for the system of equations (3) and (4) takes the following form:

$$\sum_{i=1}^{R+1} p_i^- + \sum_{i=0}^{R+1}\int_0^\infty q_i(x)dx = 1. \tag{5}$$

Solving a boundary problem (3)–(5) implies certain computational challenges. The standard method to solve it is based on the application of the Laplace transform to the system of equations (3)–(5) and subsequent determining the constants

$$p_i^-, \quad i=1,2,\dots,R+1.$$

As a result of solving it, one could find the basic performance indicators for the described inventory control system, namely:

a) the average quantity of a material in stock at any moment of time:

$$\mathbf{M}Z = \int_0^\infty x\sum_{i=0}^{R+1} q_i(x)\mathrm{d}x; \tag{6}$$

b) the probability of additional stay of a ship under repair due to the lack of a material in stock (for case $U_0=U$):

$$d(U) = \sum_{i=1}^{R+1} p_i^-. \tag{7}$$

6. Statement of a problem on the stochastic optimization of intensities in material stock replenishment

By using indicators (6), (7), it is possible to state a problem on the parameter U optimization, characterizing a material replenishment policy at SRY warehouse. An optimization criterion could be the minimum of average summary costs of SRY per unit time. For case $U_0=U$, these costs are related to a material stock replenishment, its storage at a warehouse, as well as fines for additional stay of repaired ships due to the lack of a material in stock. The analytical expression for these costs takes the following form:

$$\overline{S}(U) = aU + c_1\mathbf{M}Z + c_2 d(U), \tag{8}$$

where a is the cost per unit of a material; c_1 is the daily cost of storing a unit of a material at a warehouse; c_2 is the penalty per unit time for an idling ship due to the lack of a material at a warehouse.

For case $U_0=W$, the specified costs could be represented as follows:

$$\overline{S}(U) = a\left[W\sum_{i=1}^{R+1} p_i^- + U\left(1-\sum_{i=1}^{R+1} p_{i+1}^-\right)\right] + c_1\mathbf{M}Z. \tag{9}$$

Note that the multiplier at parameter a in the right-hand side of (9) defines the average intensity in stock replenishment.

Consider in detail a special case when $R=0$. In this case, system (3) takes the following form:

$$Uq'_0(x) = -\lambda q_0(x) + \mu q_1(x),$$

$$-Vq'_1(x) = -(\lambda+\mu)q_1(x) + \lambda q_0(x), \ x>0,$$

$$-\mu_1 p_1^- + Uq_0(0) = 0,$$

$$-\mu_1 p_1^- + Vq_1(0) = 0,$$

$$p_1^- + \int_0^\infty (q_0(x)+q_1(x))\mathrm{d}x = 1. \tag{10}$$

Summing the first two equations from system (10), following the integration, we obtain equality

$$Uq_0(x) = Vq_1(x), x \ge 0, \tag{11}$$

thus, for example, the first equation from system (10) could be disregarded.

A solution to the system of equations (10), (11) is easily derived via direct integration and takes the form:

$$q_0(x) = q_0(0)e^{-\delta x},$$

$$\delta = \frac{\lambda}{U} - \frac{\mu}{V} > 0,$$

$$q_0(0) = \frac{\mu_1(\lambda V - \mu U)}{\left[g(\lambda V - \mu U) + WU\right]U}, \tag{12}$$

$$p_1^- = \frac{U}{\mu_1}q_0(0) = \frac{\lambda W - (\lambda+\mu)U}{(\lambda+\mu_1)W - (\lambda+\mu)U}.$$

Formulas (12) are valid only under condition

$$\frac{\lambda}{U} > \frac{\mu}{W-U} \ \text{ or } \ U < \frac{\lambda W}{\lambda+\mu},$$

which is required for the existence of the steady-state operation regime of the analyzed inventory control system. Meeting it prevents the accumulation of too many materials at warehouse over time.

By using ratios (11), (12), we obtain

$$\mathbf{M}Z = \int_0^\infty x(q_0(x)+q_1(x))\mathrm{d}x = \frac{WU^2}{\lambda g[\lambda W - (\lambda+\mu)U]}. \tag{13}$$

for case $U_0=U$ and

$$\mathbf{M}Z = \int_0^\infty x(q_0(x)+q_1(x))\mathrm{d}x =$$

$$= \frac{W^2UV}{[\lambda W - (\lambda+\mu)U][\lambda gW - (\lambda+\mu)gU + W^2]} \tag{14}$$

for case $U_0=W$.

Thus, taking into consideration (12), (13), for the replenishment strategy $U_0=U$, an explicit expression for objective function (8) takes the following form:

$$\bar{S}(U)=aU+c_1\frac{WU^2}{\lambda g[\lambda W-(\lambda+\mu)U]}+c_2\frac{\lambda W-(\lambda+\mu)U}{\lambda(W-U)}. \quad (15)$$

It is easy to see that the second term in the right-hand side of expression (15) increases, while the third one decreases with an increase in U. Thus, function (15) indeed reaches a minimum at some positive value for parameter U.

For case $U_0=W$, an explicit expression for objective function (9) takes the form:

$$\bar{S}(U)=\frac{W(W-U)}{(\lambda g+W)W-(\lambda+\mu)gU}\times$$
$$\times\left[a\lambda g+c_1\frac{UW}{\lambda W-(\lambda+\mu)U}\right], \quad (16)$$

where $\mu=W/g$.

Note that, instead of criterion (16), one could consider other optimization criteria, for example, an average current profit of SRY from repair operations, which could be represented as follows:

$$\bar{\Pi}(U)=\left(b\mathbf{M}\sum_{n=1}^{\omega(t)}\gamma_n\right)/t-aU-c_1\mathbf{M}Z-c_2d(U), \quad (17)$$

where $\omega(t)$ – the number of ships whose repair was finished in a time interval (0, t); (b) is the income received by SRY per unit of a repair operation. One could demonstrate by applying the methods of queueing theory that under a steady (statistically equilibrium) regime

$$\mathbf{M}\sum_{n=1}^{\omega(t)}\gamma_n=gt\left(\mu_1\sum_{i=1}^{R+1}ip_i^-+\mu\sum_{i=1}^{R+1}\int_0^\infty iq_i(x)dx\right),$$

therefore, expression (17) takes the form:

$$\bar{\Pi}(U)=b\left(U\sum_{i=1}^{R+1}ip_i^-+W\sum_{i=1}^{R+1}i\int_0^\infty q_i(x)\mathrm{d}x\right)-$$
$$-aU-c_1\mathbf{M}Z-c_2d(U). \quad (18)$$

7. A case of several types of materials

Let $M>1$ and we assume that different types of ship repair operations are performed separately, that is, there are consistently executed operations of the first type of repair, the second, and so on. In other words, each type of repair operations is not performed in parallel. We denote $U_{0m}=U_m$.

Here, we must introduce new designations:

$Z_m(t)$ – stock level of materials of the m-th type, which are at SRY warehouse at time t;

$\nu(t)$ – the number of ships at TSE at time t;

$\alpha(t)$ – the number of the type of repair performed at time t.

Hereafter, we shall confine ourselves to case $M=2$. Denote

$$q_0(x_1,x_2;t)dx_1dx_2=\mathbf{P}\{\nu(t)=0,$$
$$x_1<Z_1(t)<x_1+dx_1,x_2<Z_2(t)<x_2+dx_2\},$$
$$q_{km}(x_1,x_2;t)dx_1dx_2=\mathbf{P}\{\nu(t)=$$
$$=k,\alpha(t)=m,x_1<Z_1(t)<x_1+dx_1,$$
$$x_2<Z_2(t)<x_2+dx_2\},$$
$$k=1,2,...,R+1;m=1,2;x_1,x_2\geq 0;$$
$$q_{k1}^-(x_2;t)dx_2=\mathbf{P}\{\nu(t)=k,\ \ \alpha(t)=1,\ \ Z_1(t)=0,$$
$$x_2<Z_2(t)<x_2+dx_2\},\ \ x_2>0,$$
$$q_{k2}^-(x_1;t)dx_1=\mathbf{P}\{\nu(t)=k,\alpha(t)=$$
$$=2,x_1<Z_1(t)<x_1,Z_2(t)=0\},\qquad x_1>0. \quad (19)$$

Of interest is the limit probability distribution (19) as $t\to\infty$, which is denoted:

$$q_0(x_1,x_2),\ \ q_{km}(x_1,x_2),\ \ q_{k1}^-(x_2),\ \ q_{k2}^-(x_1).$$

To find the specified distribution using a standard method [9–11, 18], based on considering the probability of Markovian process transitions from one state to another one over an infinitesimal time interval, one could derive an appropriate system of differential equations in partial derivatives and boundary conditions.

For example, for case $R=0$, this system of differential equations takes the following form:

$$\left(U_1\frac{\partial}{\partial x_1}+U_2\frac{\partial}{\partial x_2}\right)q_0(x_1,x_2)=-\lambda q_0(x_1,x_2)+\mu_2q_{12}(x_1,x_2),$$

$$\left(-V_1\frac{\partial}{\partial x_1}+U_2\frac{\partial}{\partial x_2}\right)q_{11}(x_1,x_2)=-\mu_1q_{11}(x_1,x_2)+\lambda q_0(x_1,x_2),$$

$$\left(U_1\frac{\partial}{\partial x_1}-V_2\frac{\partial}{\partial x_2}\right)q_{12}(x_1,x_2)=$$
$$=-\mu_2q_{12}(x_1,x_2)+\mu_1q_{11}(x_1,x_2),\ \ x_1,x_2>0. \quad (20)$$

The corresponding boundary conditions take the following form:

$$U_2\frac{dq_{11}^-(x_2)}{dx_2}-V_1q_{11}(0,x_2)=-\mu_1'q_{11}^-(x_2),\ \ x_2>0, \quad (21)$$

$$U_1\frac{dq_{12}^-(x_1)}{dx_1}-V_2q_{12}(x_1,0)=-\mu_2'q_{12}^-(x_1),\ \ x_1>0, \quad (22)$$

$$U_1\frac{dq_{12}^-(x_1)}{dx_1}+U_2q_0(x_1,0)=\mu_2'q_{12}^-(x_1),\ \ x_1>0, \quad (23)$$

$$-V_2\frac{dq_{11}^-(x_2)}{dx_2}+U_1\ q_{12}(0,x_2)=\mu_2'\ q_{11}^-\ (x_2),\ \ x_2>0, \quad (24)$$

$$q_0(0,x_2)=0,$$
$$q_{11}(x_1,0)=0, \quad (25)$$
$$q_{11}^-(0)=0,$$

where

$$V_m=W_m-U_m>0,\ \ \mu_m=W_m/g_m;$$
$$\mu_m'=U_m/g_m,\ \ m=1,2.$$

The system of equations (20) to (25) shall be also supplemented with a normalization condition:

$$\int_0^\infty q_{11}^-(x_2)dx_2+\int_0^\infty q_{12}^-(x_1)dx_1+$$
$$+\int_0^\infty\int_0^\infty(q_0(x_1,x_2)+q_{11}(x_1,x_2)+q_{12}(x_1,x_2))\mathrm{d}x_1\mathrm{d}x_2=1. \quad (26)$$

We shall explain the physical meaning of boundary conditions (21) to (25).

Constraint (21) describes a transition of the process to the state when a repair of the first type is performed, and the stock of a material of the first type is missing at a warehouse, with the intensity of its use during the repair became equal to the intensity of its replenishment, that is U_1. The stock of a material of the 2-nd type is replenished at intensity U_2.

Constraint (22) describes a transition of the process to the state when a repair of the 2-nd type is performed, the stock of a material of the 2-nd type at a warehouse has been exhausted, and the intensity of its use during the repair became equal to the intensity of its replenishment, that is U_2. The stock of a material of the 1-st type is replenished at intensity U_1.

Constraint (23) represents a transition of the process to the state when:

a) a repair of the 2-nd type is finished at a zero level of stock of a material of the 2-nd type at a warehouse;

b) ship repair is completed and the ship leaves SRY; replenishment of the stock of a material of the 2-nd type starts at a warehouse at intensity U_2;

c) the stock of a material of the 1-st type at a warehouse is replenished at intensity U_1.

Finally, constraint (24) reflects a transition of the process to the state when a repair of the 1-st type is completed at a zero level of stock of a material of the 1-st type at a warehouse; its stock is replenished at a warehouse at intensity U_1; repair of the 2-nd type begins.

Conditions (23) represent the impossibility for the process to enter the following states:

a) the lack of stock of a material of the 1-st type at a warehouse at the time immediately after the repaired ship leaves SRY (that is, upon completion of a repair of the 2-nd type);

b) the lack of stock of a material of the 2-nd type at a warehouse at the time immediately after another ship is due for repair;

c) the lack of stock of a material of the 2-nd type at the time immediately after completion of the 1-st kind of repair in the absence of stock of a material of the 1-st type at a warehouse.

The boundary value problem (20)–(26) could be solved by the method of the Laplace transform. Denote

$$q_0^{**}(s_1,s_2)=\int_0^\infty\int_0^\infty\exp(-s_1x_1-s_2x_2)q_0(x_1,x_2)\mathrm{d}x_1\mathrm{d}x_2,$$

$$q_{1m}^{**}(s_1,s_2)=\int_0^\infty\int_0^\infty\exp(-s_1x_1-s_2x_2)q_{1m}(x_1,x_2)\mathrm{d}x_1\mathrm{d}x_2,$$

$$m=1,2,$$

$$q_{11}^{-*}(s_2)=\int_0^\infty\exp(-s_2x)q_{11}^-(x)\mathrm{d}x, \quad (27)$$

$$q_{12}^{-*}(s_1)=\int_0^\infty\exp(-s_1x)q_{12}^-(x)\mathrm{d}x,\quad Res_1,Res_2>0.$$

We first apply the Laplace transform to equations (15) considering conditions (25). Following the standard transformations, we arrive at the following system of equations relative to representations (27):

$$(\lambda+s_1U_1+s_2U_2)q_0^{**}(s_1,s_2)-\mu_2q_{12}^{**}(s_1,s_2)=U_2q_0^*(s_1,0),$$

$$-\lambda q_0^{**}(s_1,s_2)+(\mu_1-s_1V_1+s_2U_2)q_{11}^{**}(s_1,s_2)=-V_1q_{11}^*(0,s_2),$$

$$-\mu_1q_{11}^{**}(s_1,s_2)+(\mu_2+s_1U_1-s_2V_2)q_{12}^{**}(s_1,s_2)=$$
$$=U_1q_{12}^*(0,s_2)-V_2q_{12}^*(s_1,0),$$

$$Res_m>0,\quad m=1,2, \quad (28)$$

where the following designations are used:

$$q_0^*(s,0)=\int_0^\infty e^{-sx}q_0(x,0)\mathrm{d}x,$$

$$q_{11}^*(0,s)=\int_0^\infty e^{-sx}q_{11}(0,x)\mathrm{d}x,$$

$$q_{12}^*(s,0)=\int_0^\infty e^{-sx}q_{12}(x,0)\mathrm{d}x,\quad Res>0.$$

Let us transform by Laplace the boundary conditions (21) to (24) taking into account conditions (25):

$$V_1q_{11}^*(0,s_2)=(\mu_1'+s_2U_2)q_1^{-*}(s_2),$$

$$V_2q_{12}^*(s_1,0)=(\mu_2'+s_1U_1)q_2^{-*}(s_1)-U_1q_2^-(0),$$

$$U_2q_0^*(s_1,0)=(\mu_2'-s_1U_1)q_2^{-*}(s_1)+U_2q_2^-(0),$$

$$U_1q_{12}^*(0,s_2)=(\mu_1'+s_2V_2)q_1^{-*}(s_2). \quad (29)$$

The determinant of a system of three equations (29), as it is easy to see, equals

$$\Delta(s_1,s_2)=(\lambda+s_1U_1+s_2U_2)\times$$
$$\times(\mu_1-s_1V_1+s_2V_2)(\mu_2+s_1U_1-s_2V_2)-\lambda\mu_1\mu_2. \quad (30)$$

The corresponding determinants for finding unknown functions $q_0^{**}(s_1,s_2),q_{12}^{**}(s_1,s_2),q_{11}^{**}(s_1,s_2)$ are:

$$\Delta_0(s_1,s_2)=U_2q_0^*(s_1,0)(\mu_1-s_1V_1+s_2V_2)\times$$
$$\times(\mu_2+s_1U_1-s_2V_2)-\mu_2[\mu_1V_1q_{11}^*(0,s_2)-$$
$$-(\mu_1-s_1V_1+s_2U_2)(U_1q_{12}^*(0,s_2)-V_2q_{12}^*(s_1,0))],$$

$$\Delta_{11}(s_1,s_2)=-V_1q_{11}^*(0,s_2)(\lambda+s_1U_1+s_2U_2)\times$$
$$\times(\mu_2+s_1U_1-s_2V_2)+\lambda\{U_2q_0^*(s_1,0)(\mu_2+s_1U_1-s_2V_2)+$$
$$+\mu_2[U_1q_{12}^*(0,s_2)-V_2q_{12}^*(s_1,0)]\}, \quad (31)$$

$$\Delta_{12}(s_1,s_2)=(\lambda+s_1U_1+s_2U_2)\{[U_1q_{12}^*(0,s_2)-$$
$$-V_2q_{12}^*(s_1,0)](\mu_1-s_1V_1+s_2U_2)-$$
$$-\mu_1V_1q_0^*(s_1,0)\}+\lambda\mu_1U_2q_0^*(s_1,0).$$

Thus, by using ratios (29) to (31), we obtain

$$q_0^{**}(s_1,s_2)=\Delta_0(s_1,s_2)/\Delta(s_1,s_2),$$

$$q_{12}^{**}(s_1,s_2)=\Delta_{12}(s_1,s_2)/\Delta(s_1,s_2),$$

$$q_{11}^{**}(s_1,s_2) = \Delta_{11}(s_1,s_2)/\Delta(s_1,s_2). \qquad (32)$$

The derived solution contains four unknown functions

$$q_0^*(s_1,0),\ \ q_{11}^*(s_1,0),\ \ q_{12}^*(s_1,0),\ \ q_{12}^*(0,s_2).$$

These functions are expressed, by using boundary conditions (29), through two unknown functions $q_1^{-*}(s_2),\ q_2^{-*}(s_1)$, which are determined by using a condition of analyticity of functions (27) in region $\mathrm{Re}\,s_1,\ \ \mathrm{Re}\,s_2 \geq 0$, that is, the condition for matching zeros at denominator and numerators in fractions (32). The result is a certain boundary value problem for functions of two complex variables. This computational procedure is described in details in monograph [19]. The remaining unknown constant $q_2^-(0)$ is determined from the normalization condition (26).

Similar to the case of a single type of materials, one could also state the problem on $U_1,\ U_2$ parameters optimization in order to minimize the average intensity of costs related to the supply of materials and losses due to idle ships caused by the interruption of repair, that is, a function of the form

$$\bar{S}(U_1,U_2) = a_1U_1 + a_2U_2 + +c_{11}\mathbf{M}Z_1 + c_{12}\mathbf{M}Z_2 + c_2d(U_1,U_2), \qquad (32)$$

where $d(U_1,U_2)$ is the probability of additional stay of ships under repair because of the lack of materials in stock, and

$$d(U_1,U_2) = \int_0^\infty q_{11}^-(x)dx + \int_0^\infty q_{12}^-(x)dx + q_2^-(0) = = q_{11}^{-*}(0) + q_{12}^{-*}(0) + q_2^-(0);$$

$\mathbf{M}Z_i$ is the mean quantity of a material of the i-th type at a warehouse, and

$$\mathbf{M}Z_1 = \int_0^\infty\int_0^\infty x_1(q_0(x_1,x_2) + q_{11}(x_1,x_2) + +q_{12}(x_1,x_2))\mathrm{d}x_1\mathrm{d}x_2 + \int_0^\infty xq_{12}^-(x)\mathrm{d}x,$$

$$\mathbf{M}Z_2 = \int_0^\infty\int_0^\infty x_2(q_0(x_1,x_2) + q_{11}(x_1,x_2) + +q_{12}(x_1,x_2))dx_1dx_2 + \int_0^\infty xq_{11}^-(x)\mathrm{d}x;$$

a_i is the unit cost of a material of the i-th type; c_{1i} is the daily cost of storing a unit of a material of the i-th type at a warehouse.

8. Example of solving a problem on the stochastic optimization of restocking intensities

We give a numerical example that illustrates the described problem on stochastic optimization, confining ourselves to case M=1, R=0, that is, we consider the problem of minimization of function (13) for different values of parameter W and at the following initial data: a= 5 thousand monetary units per ton, c_1=0.08 thousand monetary units per ton per day, c_2=10 thousand monetary units per day, g=0.5 tons.

Calculations are performed applying the software package Microsoft Excel. The results of calculations are given in Table 1.

Table 1

Calculation results on a stochastic optimization model

No. of entry	Value for parameter W, tons per day	Optimal value for parameter U, tons per day	Minimal value for function (13), thousand monetary units per day
1	0.15	0. 005	0. 22438
2	0.20	0. 050	0. 32500
3	0.25	0. 070	0. 35843
4	0.30	0. 074	0. 37904
5	0.35	0. 077	0. 03962
6	0.40	0. 079	0. 41105
7	0.45	0. 081	0. 42440
8	0.50	0. 082	0. 43673
9	0.55	0. 084	0. 44839
10	0.60	0. 085	0. 45964

Data from Table 1 show that with an increase in values for the intensity of material utilization during repair operations (parameter W), the optimal values for the intensity of material replenishment at a warehouse (parameter U) grow slower, and this growth has a limit.

9. Discussion of results of examining the model of stochastic optimization of inventory control at SRY

The present study shows that the proposed approach to optimizing inventory control at SRY makes it possible to minimize the expected operating costs of SRY under conditions of the random arrivals of ships at SRY and the randomness in the volumes of repair operations at each ship. This is accomplished by finding the analytical dependences of the respective cost components on the desired control parameters (that is, intensities of restocking). In this case, there appears a possibility to account for the losses by SRY caused by penalties on the part of shipowners due to the extra downtime of ships because of the lack of materials at a warehouse. At the same time, for the case of several kinds of materials (and related types of repairs), the implementation of this approach involves certain analytical difficulties, which, however, could be overcome by specialized methods for solving boundary value problems for functions of several complex variables.

The described scheme of SRY operation modelling could form the basis for the development of an appropriate simulation model. This would be justified in cases where it is necessary to take into consideration the non-Markovian character of the random processes of ships arrivals at the yard (in particular, their arrival on a predefined schedule) and the volumes of required repair of ships. It should be noted, however, that solving the problem on stochastic optimization in this case requires a significant volume of computation. In such a situation, it appears most effective to apply a combination of analytical and simulation approaches within the framework of the so-called directed simulation calculations [20].

From the point of view of an inventory control theory, applying the Markovian drift process makes it possible to not only take into consideration the random fluctuations in demand, but also to take into account the formation of demand related to the transportation process, that is, to the operation of ships, which in some (generally speaking, random) time would be in need of repair.

Note that the described methodical approach could also be used to solve the task on choosing the SRY by a shipowner to repair a ship based on the criteria obtained above, for example, the minimum probability (5) or the average total current costs (6), (28).

It is of practical and theoretical interest to further generalize the results obtained, for example, for a case of different materials replenishment strategies (delivery in individual fixed batches, periodic replenishment, deliveries dependent on the current level of stock at a warehouse, etc.).

10. Conclusions

1. It has been proven that the formalized description of SRY operation in the form of QS makes it possible to simultaneously take into consideration the non-uniformity in ships arrivals for repairs, different volume of repair operations, and to plan the respective cost of materials.

2. It has been shown that when interpreting SRY operation as a queueing system in terms of the Markovian drift process there appears a possibility to derive an appropriate system of differential equations in partial derivatives with boundary conditions for finding a stationary joint probabilistic distribution of the number of ships at SRY and the quantity of materials at an SRY warehouse. Solving this boundary value problem makes it possible to obtain analytic expressions for different objective functions that evaluate the efficiency of inventory control over materials at an SRY warehouse, for example, the average total cost per unit of time for replenishment and keeping a stock, or the average profit by SRY per unit of time.

3. The solution to the specified system of differential equations was derived using the Laplace transform and the theory of boundary value problems from the theory of functions of complex variables. An analytical solution in the terms of the Laplace transform makes it possible, easy enough, to calculate the desired performance indicators for the examined inventory control system as a function of the desired control parameters.

4. Based on the solution derived, we have obtained analytical expressions for calculating key performance indicators of SRY operation as a queueing system (the average level of inventories at a warehouse, the probability of additional idling of ships under repair due to the lack of materials in stock, etc.). We have stated a problem on determining the optimal values for the intensities of restocking at a warehouse based on one of the two economic criteria: a minimum of average current costs and a maximum of the average current profits by SRY. Solving these optimization problems makes it possible to choose such a strategy to manage materials stocks that would minimize average current costs or maximize the average current profit by SRY.

5. It has been demonstrated that in contrast to existing stochastic models of inventory control, the proposed stochastic model makes it possible to simultaneously describe a production process (that is, repair of ships) and the process to manage the inventory of materials required for repair operations, which makes it possible to consider, when building a strategy for restocking, the uncertainty related to the yard's load in terms of repair operations.

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STUDY INTO CONDITIONS FOR THE INTERACTION BETWEEN DIFFERENT TYPES OF TRANSPORT AT INTERMODAL TERMINALS

V. Petrushov
PhD, Associate Professor*
E-mail: vvpetrushov@gmail.com
O. Shander
PhD, Associate Professor*
E-mail: o.e.shander@gmail.com
*Department Management of Operational Work
Ukrainian State Academy of Railway Transport
Feierbakha sq., 7, Kharkiv, Ukraine, 61050

Проведеними дослідженнями в організації взаємодії різних видів транспорту на інтермодальних терміналах встановлено, що для досягнення ефективного функціонування перевантажувальних терміналів необхідним є удосконалення технологічного процесу роботи терміналу. Зокрема, за умови задоволення основних вимог–безперервність, ритмічність, паралельність та поточність усіх операцій, максимальне суміщення при високій якості безумовного використання. Доведено, що досягнення відповідних умов можливе при використанні дескриптивної моделі двохпортального терміналу, функціонування якого забезпечується процесами самосинхронізації руху автоматизованих платформ, здійснюючих перевезення контейнерів між автомобільним та залізничним порталами.

Встановлено, що створення досконалих комп'ютерних моделей для потреб організації взаємодії різних видів транспорту на інтермодальних терміналах як проектно-конструкторську задачу треба вирішувати у поєднанні дескриптивних та аналітичних моделей. В даних моделях виділяються програмні та апаратні компоненти, забезпечуючі умови здійснення концепції самосинхронізації руху навантажувачів. Зокрема встановлено, що самосинхронний підхід управління забезпечує велику ступінь узгодження при функціонуванні контейнерного терміналу та дозволяє збільшити паралельність процесів, тобто одночасне здійснення подій у системі.

Показана можливість формалізації процесів самосинхронізації засобами мереж Петрі. Цей математичний апарат дуже зручний для моделювання динамічних дискретних систем та дозволяє дослідити послідовне виконання всіх процесів, що відбуваються на інтермодальному терміналі. На основі моделювання доведено, що середній простій контейнера на терміналі зменшується, що дозволяє збільшити переробну спроможність та зменшити питомі витрати на переробку контейнера на терміналі.

Таким чином, є підстави стверджувати, що цілком можливою є розробка технологічно завершених термінальних структур "морський порт–залізничний портал–автомобільний портал" у різних конфігураціях. Тип конфігурації залежить від обраних логістичних маршрутів доставки вантажів, застосувавши для цього наведену методику організації роботи двохпортального терміналу

Ключові слова: самосинхронізація, мережа Петрі, інтермодальні перевезення, контейнерний термінал

1. Introduction

The most important direction of the transport policy of countries in the context of the globalization of international relations is the search for an optimal combination of conditions for functioning of the main international transport corridors. The development of a national network of international transport corridors, which are parts of the Crete international transport corridors and which correspond to the norms and standards of the European Union, provide conditions for attraction of additional volumes of transportation. The XXI century challenges the development of relations in the field of continental transport in the new Europe-Asia format. The main modern trend in the world transport system is the development of mixed freight transportation. International practice suggests that two thirds of international