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ANALYSIS OF THE EFFICIENCY OF SPACE-TIME ACCESS IN THE MOBILE COMMUNICATION SYSTEMS BASED ON AN ANTENNA ARRAY

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Проводиться аналіз ефективності різних методів і критеріїв просторово-часової обробки сигналів адаптивної антенної решітки з метою виявлення алгоритму придатного для використання при організації просторово-часового доступу у системах мобільного зв'язку. Показано, що в основу всіх методів покладена оцінка комплексного вектора вагових коефіцієнтів, що включаються в трактах прийому кожного з антенних елементів і керовані за тими чи іншими алгоритмами.

Показано, що більш конструктивним для використання в задачах просторово-часового доступу є рекурсивні процедури, що дозволяють здійснювати корекцію вектора вагових коефіцієнтів в динамічній, в тому числі нестаціонарній завадово-сигнальній обстановці. Це особливо важливо для зв'язку з мобільними абонентськими станціями і дозволяє скоротити час на обробку викличних сигналів за рахунок швидкої збіжності рекурсивних процедур.

Проведено порівняльний аналіз алгоритмів Уідроу-Хоффа та Калмана-Б'юсі. Показано, що процедура Калмана-Б'юсі поряд з оптимальністю щодо складної сигнально-завадової обстановки характеризується максимально коротким часом збіжності до сталого стану. Збіжність процедури забезпечується на інтервалі часу, що відведений для дії сигналів виклику абонентських станцій у мобільній мережі.

Запропонована модель по дослідженню впливу початкових умов на ефективність просторово-часового доступу за параметром швидкості збіжності алгоритму адаптивної просторово-часової обробки сигналів у антенній решітці.

За рахунок налаштування комплексного вектору вагових коефіцієнтів на основі використання інофрмації о напрямках приходу сигналів виклику абонентських станцій у мобільній мережі вдається наблизити значення вектору до оптимального.

Отримано результати розрахунків показника відношення сигнал/(завада+шум) від кроку збіжності для різних алгоритмів адаптивної антенної решітки.

Показано, що для лінійної чотирьохелементної адаптивної антенної решітки завдяки вдалому початковому вибору значення комплексного вектору вагових коефіцієнтів дозволило істотно поліпшити перехідні характеристики алгоритмів. Це також надало змогу збільщити значення відношення сигнал/(завада+шум) на виході антени до 4 дБ

Ключові слова: просторово-часовий доступ, адаптивна просторово-часова обробка, антенна решітка, комплексний ваговий коефіцієнт

1. Introduction

In recent years, there has been a substantial development of wireless technologies of subscriber access, designed to improve the quality of services of wireless Internet and mobile communication systems. In the near future, with the implementation of mobile connection technologies of the 5th generation, the load on the mobile connection lines is supposed to increase substantially by 2020, which is caused both by an increase in the number of users and by the emergence of new services related to high-speed data transmission.

An increase in the volume of transmitted information requires from both equipment manufacturers and operators to provide services, ensure fairly high throughput of networks. Since the boundary of throughput is determined by a number of limiting factors of the environment of radio waves propagation, the solution of this problem is becoming crucial for operators of mobile services.

The main reasons for a decrease in performance of a mobile communication system and its throughput is the existence of inter-channel interference, caused by the increasing number of users, interferential freezing of signals, signals delay during the radio waves propagation, which is caused by multi-beams, as well as by users' mobility.

Studies aimed at improving the performance of wireless systems are currently carried out all over the world. Mobile operators are looking for new ways to maximize the effectiveness of the allocated frequency resources for their networks and to increase their profitability. Introduction of modern technologies of adaptive antenna arrays in mobile communication systems promise a great increase in the system performance in terms of throughput, expansion in coverage area and an increase in qualitative indicators, which in general will ultimately lead to increasing effectiveness of using the allocated frequency resource.

Such measures for the implementation of adaptive antenna arrays are concentrated on the space-time processing of a digital signal, which is regarded as the evolution of already implemented traditional signal processing methods using the technologies of multi-antenna arrays (MIMO – Multiple Inlet Multiple Outlet).

It is worth noting that the implementation of antenna arrays with adaptive spatial processing signals in a mobile communication systems is carried out in conjunction with the implementation of space-time access (SDMA – Space-Division Multiple Access) and is a non-trivial complex scientific and applied problem, which implies increasing performance of access in mobile communication systems based on the use of the methods of adaptive space-time signal processing while providing a desired quality of services and constancy of the basic algorithms of mobile network functioning.

The access of many users to the common resource of telecommunication systems ensures the maximum quantitative coverage of consumers of information services. The well-known methods include the methods of access with fixed resources and the methods of providing resources at request. The problems of the access to the common resource of the base station of a wireless telecommunication system are solved based on differentiation of the signals of subscriber stations (SS). Certain physical parameters are used to differentiate between these signals.

The methods of TDMA (Time Division Multiple Access), FDMA (Frequency Division Multiple Access) and CDMA (Code Division Multiple Access) have already been implemented in today's mobile systems. They are used separately or in various combinations. However, spatial parameters themselves have long been used in wireless communication technologies in the problems of diversified reception, Massive MIMO, 3D-Beamforming, in the construction of micro- and femtocells, etc. But it is worth noting that in most cases they are used in a passive form, ensuring the necessary energy in mobile communication lines. Their active use that extends the space of access parameters is most effective.

One of the most important challenges of mobile communication systems is limited capabilities for multiple access. The problem of access to the common resource of WC is normally solved based on the separation of frequency-time parameters of signals of subscriber stations.

The methods of space-time access of SS to the WS resources are based on the use of the algorithms for space-time processing of received signals that are implemented based on N-element adaptive antenna arrays.

The essence of the problem of space-time access of each SS to the WC resources in the ascending channel lies in group using the receiving antenna array of WC, at which for each SS, an individual distribution of the structure of the received area of a signal is formed with the help of selection of the VWC $\vec{W}_i$. Thus, as many variants of distribution are created at the same time, as many SS signals are received at a given moment. Each of these distributions provides the maximum optimal signal/interference+noise ratio for reception the SS signal and establishing zero levels of reception for other SS.

An important merit of space-time access (STA) is that this method is perfectly combined with the other methods, such as frequency-temporal, code, different organizational methods, supplementing them successfully, and expands the potential quality of signal reception due to the expansion of decision-making space. In addition, STA has a number of advantages in comparison with other methods of processing. Thus, STA is not related to the additional loss of time and frequency resources. Moreover, using STA, it is possible to implement the reuse of operating frequencies when reception of different signals at the same frequency.

In addition, when using STA, non-linearity and limitations of a dynamic range have a significantly smaller influence. The application of these methods makes it possible to save the parameters of useful signals, requirements for the communication channels, and communication modes without increasing the allocated frequency band.

Space-time methods can allow both group and individual processing the signals of multiple sources. In the latter case, the own diagram of AA orientation is formed in the direction of each source. Thus, the opened possibilities when using STA and STMP directly affect the very concept of the use of a mobile telecommunication system.

That is why scientific studies that deal with the development of the systemic scientific and technical solutions in terms of improving the performance of access in mobile communication systems are relevant. This can be achieved through the use of antenna arrays and the methods for adaptive space-time signal processing while providing a desired quality of services and constancy of the basic algorithms of mobile network functioning.

2. Literature review and problem statement

At present, the interest in the issues of implementation of the space-time access and space-time signal processing in a wireless communication system is becoming increasingly important. Thus, papers [14–20] focus on the new analytical approaches to the aspects of modernization of the methods for space-time signal processing in adaptive antenna arrays.

Study [14] introduced the adaptive algorithm of chaotic formation of beam, which is the new adaptive method for the synthesis of diagrams of antenna array orientation. The developed adaptive method is based on optimization of the algorithm of minimum root mean square deviation using the chaos theory and ensures quick adaptation of the antenna array by the orientation diagram, a decrease in the impact of noises of reference channel and improvement of surveillance opportunities. In this paper the authors do not consider the methods for the formation of a direct reference channel, but rather perform the analysis of the noise level in it. This assumption decreases the accuracy of calculation of weight coefficients of the antenna array.

Paper [15] introduced the method for beam formation with the algorithm of minimum distortionless variance (MVDR – Minimum-Variance Distortionless Response) in combination with linear antenna arrays. The research was conducted for the two scanning processes – by azimuth and of angle height – and is used to produce the desired signal and suppression of interference and noise. It was shown that the proposed method achieves the signal/(interference+noise) ratio better than the existing methods. In this paper the authors modernize the Capon method for a linear antenna array with the known geometry of the location of antenna elements. The resolution of Capon method depends on the width of the main beam of antenna array and is associated with the restriction of the Rayleigh separation of signals. To increase resolution, it is necessary to use antenna arrays with a large number of antenna elements, which can be considered a rather problematic solution.

In paper [16], the Chebyshev distribution in combination with different variants of the algorithm of least root mean square deviation (LMS – Least Mean Square) is used for the formation of adaptive radiation for an intelligent antenna. The results show that using these hybrid algorithms, it is possible to obtain an insignificant level of side petals. However, the authors do not consider the scenario of adaptive generation of antenna array beam with a different inter-element distance. It was established in the paper that the characteristics of the SLMS (Sign Least Mean Square) and TDSLMS (Sign Least Mean Square with Chebyshev Distribution) improve specifically for large antenna arrays with a large number of antenna elements. A pitch of location of antenna elements is important, as at the pitch size of 0.02, adaptive beam formation is not achieved. Therefore, the pitch size is an essential parameter for the performance of adaptive beam formation, which is completely dependent on the configuration of the antenna array, which is a significant restriction.

Paper [17] deals with the multiple aspects of signal processing in an antenna array and the formation of adaptive radiation. The study includes a detailed description of different schemes of beam formation, adaptive algorithms for the correction of necessary values of weight coefficients in antenna elements, the methods for estimation of directions of signal arrival, including their comparison by the performance indicators.

In paper [17], authors emphasize that the hardware implementation of adaptive algorithms is a complicated task, since the adaptive formation of the orientation diagram is performed in real time mode, and inexpensive, but high-speed algorithms are required with preference to LMS-based algorithms. However, the study did not explore the issues of computational complexity of the algorithms and memory requirements that are of interest in terms of using these algorithms in mobile communication systems.

Paper [18] contains the review of the existing studies, related to multiple space-time access (SDMA). The basic features and shortcomings of the various existing and new methods for multiplexing the multiple access are explored. A new model for the clustered orthogonal signature multiple access for the next generation of cell networks was proposed. The proposed concept involves the use of enhancement of orthogonality of SDMA bundles in order to improve spectral efficiency for future cellular networks. However, comparing various methods of access, the authors do not highlight the features of using the SDMA method along with adaptive beam formation in the ascending channel considering dynamic relocation of mobile users. They emphasize the potential expansion of possibilities of access during the 3D-beam formation in the vertical and azimuth planes, providing an additional degree of freedom.

In paper [19], a reliable algorithm with restricted LMS, based on maximizing SINR for the worst case, is developed. The updated vector of weight coefficients was obtained by the method of descent gradient and the Lagrange method of multipliers. It was shown that the proposed recursive algorithm provides excellent resistance to the inconsistencies of the vector of weight coefficients at a small size of training dataset, has a high convergence rate, consistent approximation of the average output signal power to interference plus noise ratio (SINR) to optimal.

The article explores the descending wireless connection line, but does not take into consideration the peculiarities of the space-time access of a mobile subscriber to a mobile network.

Article [20] explores an adaptive antenna array with arbitrary configuration of the antenna elements with a maximum signal to noise ratio (SNR) at the outlet. Analytical solution for the optimal weight vector of AAA is obtained if the inlet process is determined by the matrix of correlations of noise and vector of useful signal. Based on this solution, the assessment of the weight vector was obtained with the use of a limited number of samples of input noise, which can be either more or less than the number of elements in an array. It was shown that the computational complexity of the proposed assessment is proportional to the number of noise samples, the number of external noise sources and square of the number of elements in an array. However, the model of calculation of weight coefficients at the initial stage was not explored in this study.

In scientific papers [14–20], addressing the space-time signal processing in antenna arrays, attempts were made to use actively the spatial-polarization parameters of signals and antennas for the solution of various problems, including space-time methods of access. However, many of them have a partial, autonomous character, not associated with the systemic problems of mobile communication networks. Thus, in papers [14–20], the studies of AAA functioning are based on the use of the asymptotic algorithms that are optimal for stationary signal-interference situation. They explore the results of subsequent development of the methods for analysis of AAA functioning with taking into consideration the implementation restrictions, existence of mutual influence between antenna elements, during the synthesis of an access procedure at the uncertainty of initial data about signal-interference situation. The criteria of synthesis of adaptive antenna arrays in problems of space-time access, explored in papers [14–20], do not take into consideration dynamic non-stationary interference-signal situation by the quality and performance parameters, which is quite relevant for mobile communications systems. In the analysis of the procedure of space-time access in the ascending channel to the resources of the base station, many authors explore a multi-beam antenna array as an antenna of the base station.

The enumerated sources [14–20] did not perform the integration of the problem of adaptive space-time processing into the general systemic problem of the space-time access for mobile communication systems.

That is why the method based on the organization of the individual STA for each reception of a specific SS proves to be relevant, in this case the signals of other SS should be considered interfering. Thus, to ensure STA at azimuth transitions of SS, one can propose an adaptive antenna array, the orientation diagram of which is adjusted according to the dynamics of spatial changes of the received signals of SS. In this case, the group STSP with a large volume of calculations with restriction for reception quality is implemented in order to solve an electrodynamic problem.

The result of this attempt is the possibility to obtain not only a new positive effect from the proposed method of space-time access, but also the possibility of the integrated decisions, which will ensure the growth of general systemic efficiency and productivity of a mobile network. It will also create prerequisites for saving the radio-frequency spectrum, providing high noise immunity, electromagnetic compatibility and, accordingly, the possibility of providing high-quality services to information consumers.

3. The aim and objectives of the study

The aim of this research is to select an effective method of adaptive space-time signal processing in the antenna array regarding its implementation in the problems of space-time access in the mobile communication systems.

To accomplish the aim, the following tasks have been set:

– based on different criteria for the evaluation of space-time processing, to make the assessment of the effectiveness of the methods for synthesis of the antenna array in terms of the suitability of their using in mobile communication systems;

– to discover the regularities of construction of algorithms of space-time processing in terms of the convergence rate;

– to identify the influence of initial conditions during establishing the vector of weight coefficients on the characteristic properties of an adaptive antenna array by the indicator of signal-to-noise+interference ratio for a linear 4-element AAA during the action of one and two interference for the algorithm of LMSD and MOSN.

4. Evaluation of effectiveness of the methods for synthesis of antenna array suitable for mobile communication systems in organization of space-time access

The criterion of the least root mean square deviation (LMSD) that was proposed by Widrow [11] is most popular during solution of various problems of radio communication. VWC is derived from recurrent formula [3–11]:

$$\vec{W}(k+1)=\vec{W}(k)+2\mu[y_e(\mathrm{k})-\vec{W}(k)^T\vec{X}(k)]\vec{X}(k), \qquad (1)$$

where k is the discreet time; μ is the pitch coefficient; $y_e(k)$ is the reference signal at moment k; $\vec{X}(k)$ is the vector of inlet impacts at moment k.

Popularity of LMSD criterion is explained by the fact that the algorithms synthesized based on LMSD have a fairly simple structure. The solutions obtained in this way fit well with common criteria, by which mobile telecommunication systems operate.

The features of operation in multi-beam communication lines impose certain limitations on the application of a certain algorithm, synthesized by LMSD criterion. For the operation of this algorithm, it is necessary to have information about the signal structure, however, this information may not always be known. Thus, the algorithm, synthesized by LMSD criterion, can be effectively used in the cases where the structure of the radiated and received signal is the same and exactly known.

However, this criterion can be helpful in development of procedures that have the recurrent form. These are, in particular, the procedures of stochastic approximation, linear or nonlinear filtering, as well as the Kalman-Bucy procedures, the methods of recurrent convolution of the correlation matrix.

The algorithms of adaptive antenna arrays using a priori information about the direction of arrival of a useful signal, are synthesized by the criterion of maximum output ratio of useful signal power to the sum of interference and noise powers (MOSN) [1, 4]:

$$\vec{W}(k+1)=\vec{W}(k)-2\mu[\vec{X}_n^T(k)\vec{W}(k)\vec{X}_n(k)-\vec{V}_c], \qquad (2)$$

where

$$\vec{V}_C^{\,T}=\left[A_{1C}e^{(-j\phi_1)},A_{2C}e^{(-j\phi_2)},...,A_{NC}e^{(-j\phi_N)}\right]$$

is the vector of the signal wave front; $A_{iC}=\vec{E}_i(\Theta_C,\Phi_C)\vec{\varepsilon}_C$ is the vector characteristic of orientation of the i-th AA by electromagnetic field intensity, $\vec{E}_i(\Theta_C,\Phi_C)$ is the electromagnetic field intensity; $\vec{\varepsilon}_i$ is the single vector of coming wave polarization; μ is the pitch coefficient.

In the algorithms, synthesized by the criterion of MOSN, the direction of signal arrival is used as an information parameter. However, configuring VWC is carried out by interferences, in the absence of radiation of a useful signal, otherwise it could be suppressed. This restriction is not always possible to implement or requires additional costs, which makes it undesirable.

For the AAA algorithms, synthesized by the criterion of minimum output power (MOP) [10], the value of the output signal power of antenna array is used as the objective function.

$$E\left\{y^2(t)\right\}=\vec{W}^T\cdot R_{xx}^{-1}\vec{W}, \qquad (3)$$

where R_{xx} is the matrix of space correlation of the inlet mixture of signals and interferences.

Direct minimization (3) can be performed by different methods of unconditional optimization [1].

In this case, VWC will be derived from expression:

$$\vec{W}_{opt}=sR_{xx}^{-1}\vec{V}_y, \qquad (4)$$

where s is the normalizing coefficient; $\vec{V}_y$ is the control vector, which assigned the predicted angle of signal arrival.

The expression for determining VWC can be written down in the recurrent form:

$$\vec{W}(k+1)=\vec{W}(k)-$$
$$-2\mu[\vec{W}^T(k)\vec{X}(k)][\vec{X}(k)-\vec{W}(k)(\vec{W}^T(k)\vec{X}(k))]. \qquad (5)$$

The methods based on reversion of the sample co-variation matrix of received signals are asymptotic and related to large losses of time for picking and processing the statistics for assessment and circulation of the co-variation matrix. Recurrent methods based on gradient procedures under stationary conditions and under other equal conditions have the same efficiency. However, the advantage of the recursive methods is the dynamic character that makes it possible to obtain the VWC assessment in real time, which is consistent with the dynamics of the communication process in mobile systems.

In a non-stationary signal-interference situation (SIS), there are space-time changes of signal and interference parameters. These changes can occur due to the impacts of the wave propagation environment, transition of the SS, a receiver, or a transmitter of interference and for other reasons. For such dynamic signal-interference environment, the equation of the state of VWC will take the general form [1, 4]:

$$d\vec{W}(t)/dt = F(t)\vec{W}(t)+G(t)\vec{\nu}(t), \qquad (6)$$

where $F(t)$ and $G(t)$ are, respectively: matrices of state and intensities of changes of signal and interference parameter (elements f_{ij}, $i,j=\dim W(t)$, are the magnitudes, inverse coefficient of mutual correlation between *i* and *j* components); $\vec{\nu}(t)$ is the vector that generates white Gaussian noise of the model (6), with spectral power density N_u.

The recursive adaptive algorithms do not require large expenditures for computing the matrices and are limited to a small amount of permanent memory and RAM. The sequence of solutions in recurrent procedures of evaluation calculation $\hat{W}_{\text{opt}}(t)$ is starting from the earlier stage of obtaining a solution (4). This sequence starts from the earlier stage of finding the extremum of gradient of the function that contains magnitudes W(t) as an argument.

All this makes it possible to perform analysis considering the transitional AAA, as well as of non-stationary statistics X(t). Recursive methods are based on the sequential iteration procedure of finding solutions for VWC in the direction that is opposite to the gradient of function of quality indicator $\Delta(k)$. In the general case, a discrete gradient algorithm which a pitch takes the form

$$W(k)=W(k-1)-\mu\nabla(k), \qquad (7)$$

where μ is the coefficient (pitch constant), which takes into consideration the speed of extremum search.

Algorithm (7) shows that the value of VWC at the k-th step equals to the values of VWC at the previous $(k-1)$-th step with the adaptive addition $\mu\nabla(k)$, which depends on discrepancy of difference $y_e(k)-y(k)$. Procedure (7) at value $\Delta t=(t_{k+1}-t_k)\to 0$ transits in the continuous one, which can be represented in the form of a differential equation

$$dW(t)/dt=-\mu\nabla(t). \qquad (8)$$

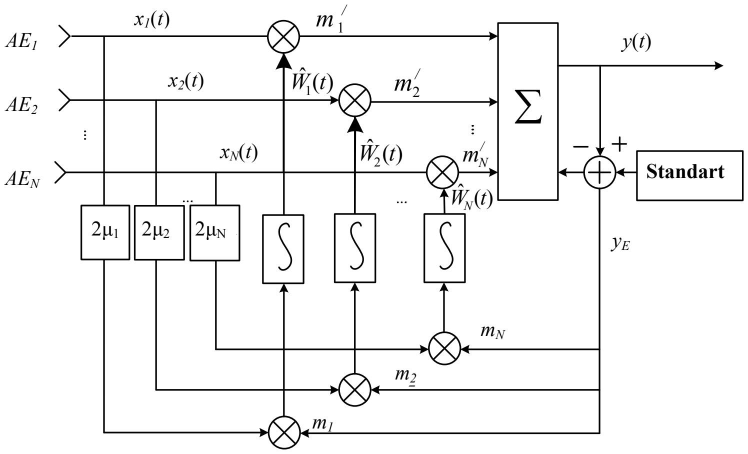


Fig. 1. Main control circuit of VWC of AAP, synthesized by the Widrow algorithm

This algorithm at a continuous procedure takes the form of the differential equation of Widrow-Hoff [1,4]:

$$d\hat{W}(t)/dt=2\mu\left[y_E(t)-y(t)\right]X(t)=2\mu\nu(t)X(t). \qquad (9)$$

Fig. 1 shows the main circuit of discreet algorithm *N* – dimensional AAA, constructed in accordance with (9).

We will note one important fact that differs the AAA using the LMSD criteria from the AAA, in which MOSN criterion is adopted.

In the AAA, synthesized by the MOSN criterion, a reference signal that determines the position of the main maximum of DS is introduced for each reception channel; it must be strictly consistent with the position of the antenna elements of the array.

In the studied Widrow-Hoff algorithm, a reference signal $y_e(t)$ is introduced to the general reception channel after the total adder (Fig. 1).

That is why there is no explicit dependence of the AAA algorithm on the location of antenna elements. This makes it possible to use this algorithm at the random or unknown location of receiving elements of an array or unknown direction of arrival of signal and interferences, for example when using the AAA on moving objects.

The procedures of type (1), (7)–(9) are nothing else than the procedures of stochastic approximation [12], the condition of convergence and stability of which is provided by the corresponding selection of coefficient μ. At the same time, the selection of a pitch constant μ is not considered reasonable, because this leads to an increase in errors. That is why it is necessary to consider the statement of the problem and its solution from the standpoint of the theory of recurrent linear and nonlinear filtering in the space of states. The methods and algorithms of stochastic approximation, including (1), (7)–(9), are a special case of the more common procedures, such as filtering procedures of Stratonovich, Kalman-Bucy, etc. [8]. It is known that the algorithms of the types (1), (7)–(9) during satisfaction of Dvoretsky conditions coincide with probability 1 to the estimated value and are a recursive method of finding conditional mean. In other words, the procedures (1), (7)–(9) are assigned to evaluate the constant random magnitudes, for which the state of estimation of magnitude VWC can be determined through simple differential equation

$$dW(t)/dt=0. \qquad (10)$$

If we assume a random change in time, the model of the state of VWC and parameters of direction of signal and interference arrival, it is necessary to express the VWC state in the form of multidimensional (N-dimensional) stochastic differential equation

$$dw_i(t)/dt=-\alpha_i(t)w_i(t)+\sum_{j=1}^{N} b_{ij}(t)\xi_m^{(i)}(t)$$

$$\text{at } i,\ j=\overline{1,N} \qquad (11)$$

where $a_i(t)$ are non-random magnitudes that characterize the rate of change of VWC $\mathbf{a}_i$ (t)= =$\mathbf{t}^{-1}_{\text{corr}}$; $\mathbf{t}_{\text{corr}}$ is the correlation interval of these changes; $\xi_m^{(i)}(t)$ are the components of white Gaussian noise that generates the noise of the model with uniform spectral power density V_M^i; $b_{ij}(t)$ are the elements $N\times N$ of matrix $B(t)$ which generates noise and determines intensity of changes in each of VWC components.

Random processes that are described by stochastic differential equations of the type (11) refer to the Markovian class and coefficients $a_i(t)$ and $\beta_{ij}(t)=\sum_{k=1}^{\infty} b_{ik}(t)b_{jk}(t)$ are called accordingly coefficients of wear and diffusion of these processes. Obviously, expression (1) is a particular case of (11) at $a_i(t)=\beta_{ij}(t)=0$.

To obtain the evaluation of VWC by using standard procedures for linear and nonlinear filtering, it is needed, in addition to the equation of state, to assign the equation of observation. The equation of observation will be represented in the form

$$y(t)=W^T(t)x(t)=W^T_{opt}(t)X(t)+\nu_c(\mathrm{t}), \tag{12}$$

where $X(t)$ is the vector, obtained based on (2) from vector $x(t)$ without considering isotropic noise $\nu_i(t)$; $\nu_c(t)$ is the noise of observation, created as result of the weighed sum of noises $\nu_i(t)$, as well as the result of different approximation errors, accepted in the model.

Noise of observation will be also approximated by Gaussian "white" noise with spectral power density V_C. The algorithm for obtaining evaluation of VWC for this case is standard and is determined from the stochastic differential equation

$$d\hat{w}_i/dt=-a_i(t)\hat{w}_i(t)+\sum_{j=1}^{N}K_{ij}(t)F_j^{\backprime}(\hat{w}_t,t), \tag{13}$$

where $F_j^{\backprime}(\hat{w}_t,t)=dF_j(\hat{w}_t,t)/d\hat{w}_t$ is the N-dimensional vector-column, obtained through derivatives by observation time of logarithm of function of likelihood

$$F(w_t,t)=\frac{1}{V_H}\left[2\mathrm{y}_E(\mathrm{t})y(w_t,t)-y^2(w_t,t)\right], \tag{14}$$

$K_{ij}(t)$ is the variance of estimation error that is described by the Riccati equation:

$$\begin{gathered}dK(t)/dt=B(t)V_SB^T(t)+K(t)A(t)+\\+A^T(t)K(t)-K(t)F^{//}(\hat{w}_t,t)K(t),\end{gathered} \tag{15}$$

here

$$F''(\hat{w}_t,t)=d^2F(\hat{w}_t,t)/d\hat{w}_t^{(i)}d\hat{w}_t^{(i)}$$

is the square $N\times N$ matrix, $A(t)$ is the diagonal matrix with elements $a_i(t)$.

With regard to (4), values of the derivatives in expression (14) are determined in the following way:

$$F'(\hat{w}_t,t)=\begin{bmatrix}F_1\\F_2\\\dots\\F_N\end{bmatrix};\quad F''(\hat{w}_t,t)=\begin{vmatrix}F_{11}&F_{12}&\dots&F_{1N}\\F_{21}&F_{22}&\dots&F_{2N}\\\dots&\dots&\dots&\dots\\F_{N1}&F_{N2}&\dots&F_{NN}\end{vmatrix},$$

where

$$F(w_t,t)=\frac{1}{V_S}\left[2y_E(t)y(w_t,t)-y^2(w_t,t)\right];$$

$$F_j=2V_S^{-1}\left[y_E(t)-y(w_t,t)\right]x_j(t)=2V_S^{-1}\nu(t)x_j(t);$$

$$F(w_t,t)=2V_S^{-1}\nu(t)x_j(t);$$

$F_{ij}(t)$ is the element of matrix $F^{\backprime\backprime}(\hat{w}_t,t)$, which stands on the ij-th place, $F_{ij}=-x(t)x_j(t)$.

Given this, equation of VWC estimation takes the form

$$\begin{gathered}d\hat{w}_i(t)/dt=-\alpha_i(t)\hat{w}_i(t)+\\+\sum_{j=1}^{N}2K_{ij}(t)V_S^{-1}\left[y_E(t)-y(t)\right]x_i(t)=\\=-\alpha_i(t)\hat{w}_i(t)+2V_S^{-1}\nu(t)\sum_{j=1}^{N}K_{ij}(t)x_i(t)\end{gathered} \tag{16}$$

or in the vector form

$$d\hat{W}(t)/dt=A(t)\hat{W}(t)+2V_S^{-1}\nu(t)K_{ij}(t)X(t). \tag{17}$$

Equation (17) is matched with a block diagram of the algorithm of estimation of VWC of AAA shown in Fig. 2 or the Kalman-Bucy filter. The considered problem of the VWC assessment is linear both by statement (17), (13), and by solution (15)-(17) and corresponds to the linear variant of the Kalman-Bucy filter. A distinctive feature of the resulting solution is that a posterior variance $K_{ij}(t)$ appeared to be dependent on observation results due to existence of matrix $F^{\backprime\backprime}(\hat{w}_t,t)$. In contrast to classical solutions, here one needs to calculate the values of matrix $K_{ij}(t)$ in real time scope.

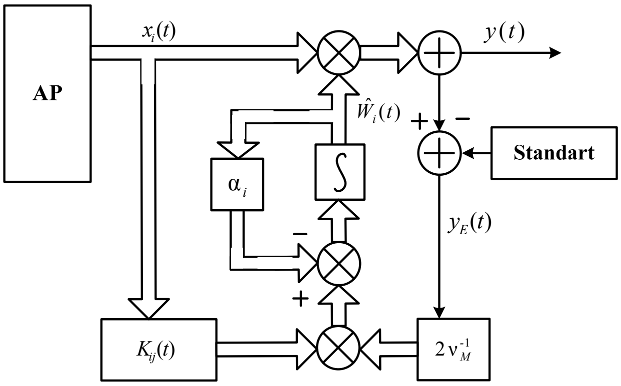


Fig. 2. Block diagram of the algorithm of VWC of AA assessment

Thus, for mobile communication systems, it is possible to recommend algorithms AAA, synthesized based on the methods of Kalman filtration of VWC formation.

5. Regularities in the construction of algorithms of space-time processing based on a convergence rate indicator

Let us identify the regularities of construction of the Widrow-Hoff algorithm and Kalman-Bucy algorithm. The diagrams of dependences of SINR on time are presented in Fig. 3. Curve 1 corresponds to the case of using Widrow-Hoff algorithm. Curve 2 – Kalman-Bucy algorithm.

At the same time, as shown by the analysis of the Kalman-Bucy procedure, along with optimality for a complex signal-interference situation, it is characterized by a maximum short time of convergence to the consistent state on interval $\tau\leq\tau_{\mathrm{corr}}$. That is, during choosing discretization pitch $\Delta t=0{,}1\tau_{\mathrm{corr}}$ the transition process finishes after 3–10 discretization steps.

Thus, convergence of the Kalman-Bucy procedure is ensured on the time interval that is allocated for the action of calling signals.

Fig. 4 represents the diagrams of dependences of SINR on the number of antenna elements N. The dependence diagrams were constructed at the value of signal/interference P_s/P_{int} dB for different values of signal/noise ratio P_s/P_{int}.

Curve 1 (Fig. 4) corresponds to value P_s/P_N=10 dB, curve 2 corresponds to P_s/P_N=20 dB and curve 3 corresponds to P_s/P_N=30 dB.

An analysis of AAA of small and large dimensionality shows that with an increase in the number of the antenna elements, efficiency of interference suppression at first sharply increases, then this increase stops and becomes proportional to the number of elements N. In this case, SINR at the outlet of AAA essentially depends on the signal/noise ratio at its inlet.

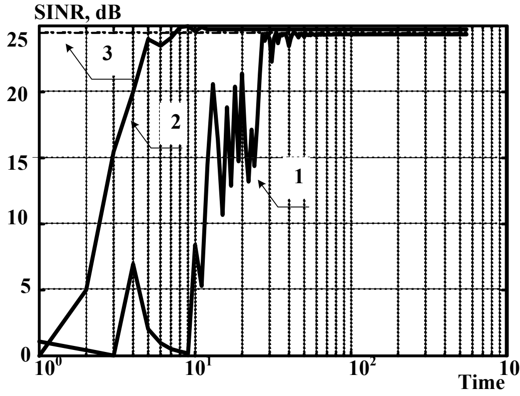


Fig. 3. Dependence of SIHR at the outlet of the 8-element of AAA on time for the algorithm Widrow-Hoff (curve 1), Kalman-Bucy (curve 2), the potential value of SINR, dB (curve 3)

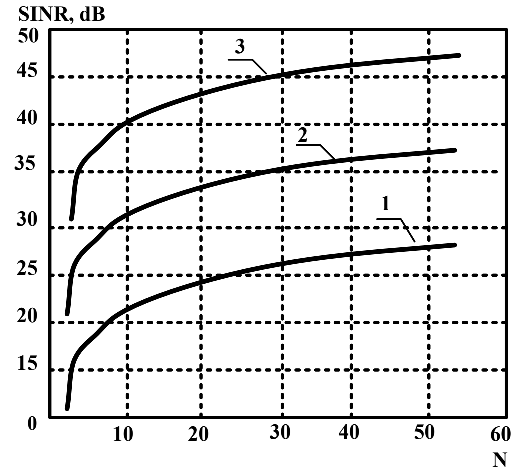


Fig. 4. Dependence of SINR at the outlet of AAA on the number of antenna elements N

The feasibility of effectiveness analysis of recursive procedures according to the analysis of procedures based on random estimates is justified by that both of these problems have a common statement of the problem's solution and general criteria of effectiveness.

In the present research, to estimate the vector of weight coefficients of the antenna array, it was proposed to use the recursive processing methods in the space of variables of state, which, unlike the well-known asymptotic algorithms, is optimal in a dynamic, in particular, non-stationary interference-signal situation by the quality and performance speed indicators. That has made it possible to achieve a sustainable mode of this processing in the interval of action of calling signals without changing the network operation modes.

6. Establishment of the influence of the initial conditions during finding the vector of weight coefficients on characteristic of the quality of adaptive antenna array

It is possible to determine the influence of the initial conditions by introducing the assumption that transition characteristics improve at the appropriate choice of the initial value of VWC ($\vec{W}_0$) [1, 4]. The relevance of this assumption is proved by the fact that the estimation of VWC in the algorithms with feedback occurs by successive approximations in time, that is, iteratively.

In this sense, a good approximation would be $\vec{W}_0$, which provides maximum strengthening of a useful signal at full suppression of interference. However, under actual conditions, it is next to impossible to obtain such initial approximation, since the number and directions of arrival of interference are a priori unknown and often change over time. More realistic is the assumption about the existence of some information about a useful signal: a signal form, the direction of arrival, frequency-time characteristic, or how interference exceeds a useful signal by power. In this case, the estimation of the direction of arrival of a useful signal can be an informative parameter, by which the most accurate tuning of VWC is possible. The block diagram of the algorithm of STA of SS signals for solving this problem is shown in Fig. 5.

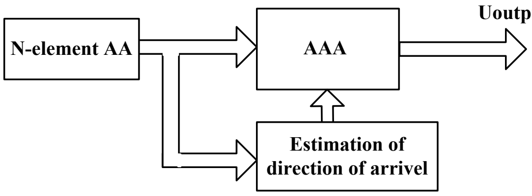


Fig. 5. Structure of the algorithm of space-time access of SS signals with preliminary determining the direction of arrival

For analysis of the influence of the initial conditions, we will use the values of VWC in the consistent state (at $k \to \infty$) [1, 4]:

$$W(k+1) = \left[I - \mu R_{xx}\right]^{K+1} W(0) + \mu \sum_{i=0}^{K} \left[I - \mu R_{xx}\right]^{i} R_{xc}, \tag{18}$$

where μ is the generalized pitch constant; R_{xx} is the correlation matrix of accepted signals of interferences and noise

If there is no useful signal, this expression takes the form:

$$W(k+1) = \left[I - \mu R_{ii}\right]^{k+1} W(0) + \mu \sum_{i=0}^{K} \left[I - \mu R_{ii}\right]^{i} H. \tag{19}$$

With the right choice of the pitch constant, other signals in exponential equations approach their respective optimal VWC, and the norms of zero summands are equal to zero.

That is why the rate of procedures convergence is usually determined as the rate of descending of these norms, which, in turn, is associated with the minimized own numbers of matrices R_{xx} and R_{ii}. Indeed, moving to coordinate systems in which R_{xx} and R_{ii} are diagonal, we obtain

$$E\{\vec{W}(k+1)\} =$$

$$= Q^{-1}[I-\mu\Lambda]^{k+1}Q\vec{W}(0)+\mu Q^{-1}\sum_{i=0}^{k}[I-\mu\Lambda]^{i}Q\vec{R}_{xc}, \tag{20}$$

$$E\{\vec{W}(k+1)\} =$$

$$= Q_1^{-1}[I-\mu\Lambda_1]^{k+1}Q_1\vec{W}(0)+\mu Q_1\sum_{i=0}^{k}[I-\mu\Lambda_1]^{i}Q_1\vec{H}, \tag{21}$$

where Q,Q_1 are the matrices of transformation of coordinates, the columns of which are eigenvectors R_{xx} and R_{ii}, and Λ,Λ_1 are the diagonal matrices, the elements of which are own numbers R_{xx} and R_{ii}. However, it is known [1, 4], that the above convergence ("strong convergence"), which is determined from the conditions of closeness of the adjusted vector to the optimal by the norm.

$$\left\|\vec{W}(k)-\vec{W}_{opt}\right\|\to 0 \tag{22}$$

is not necessary for the initial adjustment at correctly selected $\vec{W}_0$. That is why it is possible to separate two main characteristics of convergence – suppression of interferences (I) in term of closeness to the adjusted VWC to optimal (II). It is obvious that the convergence in the sense (I) depends both on the selection of $\vec{W}_0$, and on the current interference situation and is determined by the smallest own number of the correlation matrix of input signals that satisfies the condition

$$\lambda_i>\lambda_{\min}=\sigma_{Ni}^2. \tag{23}$$

Convergence in sense (II) is determined by the implementation of condition (22) and depends on $\lambda_{\min}$ (that is, it depends on $\vec{W}_0$ only). Therefore, based on conditions (22) and in order to perform the requirement to maximize SINR, we will obtain the following. To achieve the greatest convergence rate in the sense of closeness to the adjusted VWC to optimal, the initial VWC must be chosen by the agreement with a useful signal. That is $\vec{W}_0$ should ensure the orientation of maximum CO in the direction of arrival of a coming signal.

Considering that a useful signal is determined by expression $x(k)=x_s(k)+x_{int}(k)+x_N(k)$ we will obtain that the original VWC takes the form

$$\vec{W}_0=k_n\vec{V}_c^{*}, \tag{24}$$

where k_n is the constant coefficient;

$$V_c^T=[1,e^{i\varphi_{c1}},e^{i\varphi_{c2}},\ldots,e^{i\varphi_{c(N-1)}}]$$

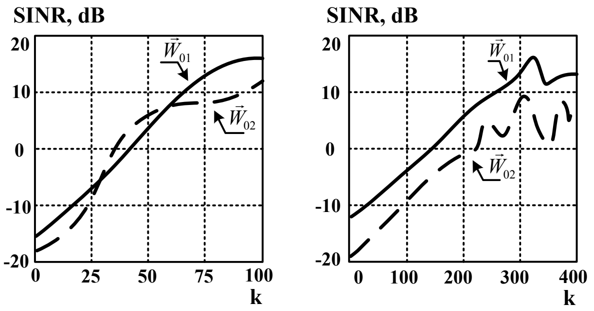
is the vector of direction.

It is obvious that the direction vector V_c^T can be known in advance or determined by the methods of direction of a useful signal arrival according to the diagram in Fig. 5.

We performed the simulation in mathematic package of applications for numerical analysis MATLAB 16.0.

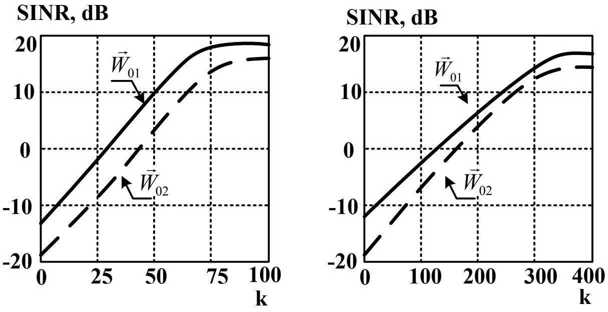
Step-by-step values of SINR in the presence at the inlet of linear 4-element AAA ($d_i=\lambda_c/2$) of one and two interferences are presented in Fig. 6, 7 for the algorithm LMSD and MOSN, respectively.

Step-by-step dependences of SINR at the outlet of AAA are presented at the existence at inlet of the array of two interferences with arrival angles $\theta_{\text{int}1}=15^{\circ}$ $\theta_{\text{int}2}=45^{\circ}$ for which P_{int1}/s_N^2=40 dB, P_{int2}/s_N^2=34 dB.



a *b*

Fig. 6. Step-by-step dependences of SINR at the outlet of the 4-element AAA for the LMSD algorithm: *a* – at the action of one interference ($\theta_1=15^{\circ}$) with the ratios – P_s/σ_N^2=13 dB, P_{int}/σ_N^2=40 dB; *b* – at the action of two interferences ($\theta_1=15^{\circ}$; $\theta_2=45^{\circ}$) with the ratios – P_{int1}/σ_N^2=40 dB, P_{int2}/σ_N^2=34 dB



a *b*

Fig. 7. Step-by-step dependences of SINR at the outlet of the 4-element AAR for algorithm MOSN: *a* – at the action of one interference ($\theta_1=15^{\circ}$) with ratios – P_s/σ_N^2=13 dB, P_{int}/σ_N^2=40 dB; *b* – at the action of two interferences ($\theta_1=15^{\circ}$; $\theta_2=45^{\circ}$) with ratios – P_{int1}/σ_N^2=40 dB, P_{int2}/σ_N^2=34 dB

In this case, we consider $\vec{W}_0$, which satisfies (24), as well as another boundary case when $\vec{W}_0$ determined the original characteristic of the orientation (CO), constant for all angles of signals arrival (isotropic CO). The true meaning of VWC is designated as $\vec{W}_{01}$, $\vec{W}_{02}$ – isotropic DS. The powers of a useful signal, interference and noise at the inlet of AAA met the ratios: P_s/s_N^2=13 dB, P_{int}/s_N^2=40 dB.

It follows from the analysis of calculation dependences (Fig. 7, 8) that the selection of $\vec{W}_0$ in the form (29) makes it possible not only to improve significantly the transient characteristics of the algorithms, which is in practice also quite significant, since it increases the possible SINR up to 4 dB for a four-element AA.

7. Discussion of results of assessing the effectiveness of space-time access using antenna arrays

The novelty of the methods lies in considering *a priori* data about the directions of arrival of signals at the initial stage, which ensures a reduction of the interval of convergence of the process of adaptation of control algorithms of adaptive antenna array and the necessary quality of the algorithm of space-time access.

It is known from the process of solving many calculation problems that the effectiveness of the solution itself (convergence rate, residual errors, etc.) is largely determined by a good selection of initial conditions. The problems of STA, convergence rate of the algorithm of adaptation of AAA plays a fundamental role in the sense that all processes concerning STA (detection of the calling signal of SS, determining the direction of the signal arrival, estimation of VWC of AAA, communication quality assessment, etc.) must be completed within minimum time, which is prior to the service provided by the SS. The most important restriction of the STA problem is the selection of an initial value of VWC of AAA.

Thus, based on the performed assessment, there are a number of comments on the choice of initial values for VWC:

1. An increase in SINR at the outlet of AAA is explained by the fact that at arbitrary choice of $\vec{W}_0$, it is not possible to meet the conditions of convergence of the adjusted VWC to optimal, even at the number of iterations that is higher by an order, which exceeds the value necessary to achieve this convergence in the sense of interference suppression.

2. As it can be seen from the diagrams (Fig. 6, 7), convergence in the sense of interference suppression practically does not depend on $\vec{W}_0$, but is rather determined by the parameters of algorithms and an interference situation, that is, conditionality of the correlation matrices of input signals.

3. For the LMSD algorithm, as well as for the MOSN algorithm used in the absence of a useful signal, maximum value of SINR is achieved when choosing the initial VWC, coordinated with the useful signal wave front (even at the four-element AA, SINR is achieved at such $\vec{W}_0$ that is by 4 dB larger than the case of $\vec{W}_0$, which determines the isotropic CO). In this case, convergence rate in the sense of closeness of the adjusted VWC to optimal increases significantly. The procedures with such VWC can be used in the mobile communication systems with STA at the known direction of signal arrival.

The conducted evaluation of research into the methods of space-time processing, suitable for the problems of organization of the space-time access of SS in mobile communication systems, is continuation of the studies conducted earlier.

A particular advantage of this study, carried out using the well-known approaches and criteria for the assessment of the quality of adaptive antenna arrays, is the proposed model taking into consideration a-priori information about the directions of signal arrival at an early stage. This ensures a reduction of the interval of convergence of the process of control algorithms adaptation of the adaptive antenna array and the required quality of the space-time algorithm.

Further improvement of the study reported here will be aimed at assessing the accuracy of information on the direction of signals arrival, because the accuracy of the vector of the wave front of a signal will depend on the resolution degree of the methods for estimating the directions of useful signal arrival.

The approach, presented in the research, will enable further determining and selection of the most effective methods of STSP for future implementation of the STA in mobile communication systems.

4. The rate of VWC changes $W(t)$ should be coordinated with the rate of a change of a signal-interference situation, and the range should be consistent with the dynamic range of changes of the level of signals and interferences, as well as phase ratios in various elements of the AAA. Obviously, the ideal situation is when the rate of VWC changes is infinitely high, while the dynamic range of changes of amplitude and phase characteristics is unlimited. However, in practice, based on the technical implementation capabilities and other reasons, it is necessary to limit these characteristics, which, generally speaking, leads to a corresponding decrease in the effectiveness of the AAA. In this sense, we talk about the AAA with limitations [4].

An important factor in solving the problems of the present research is the restrictions on the design, configuration and dimensions of an antenna array, the spectral composition of signals and interference and other parameters that are involved in the synthesis of the orientation diagram. In terms of the problem of the space-time access, the ultimate purpose of using the AAA is to provide the necessary qualitative characteristics (to maximize) of useful signals at the outlet of an antenna.

The disadvantages of this research include the lack of evaluation of effectiveness of different methods for determining the direction of arrival of calling signals of mobile users during determining the wave front of vector V_c^T, depending on the current signal-interference situation. In this regard, we did not perform the estimation of computational complexity of the algorithms for the calculation of the vector of direction of calling signals, which also determines the time of the space-time access of a mobile station to the resources of a base station in a mobile network.

The subsequent research into evaluation of effectiveness of space-time access in mobile communication systems using antenna arrays will involve using different methods for estimation of determining the direction of signal arrival: Rayleigh and super-Rayleigh spatial distribution of the SS signals, taking into consideration the complex signal-interference environment, as well as the models of radio waves propagation in the analysis of energy of signals and interference.

9. Conclusions

1. Each of the methods in the synthesis of AAA (LMSD, MOP, MOSN) involves finding the estimations of the vector of weight coefficients that are at the outlet of antenna elements of the antenna array based on the correlation matrix (or its circulation) of signals and interferences that characterize the current signal-interference situation. Using the criterion of LMSD, the Kalman-Bucy procedure was developed, particularly, based on the recurrent convolution of the correlation matrix.

In contrast to traditional methods of STA, based on the methods for synthesis of the antenna orientation diagram (AOD) and accompanied by the beam of a received signal, the proposed solution does not require consideration of the AOD, but rather implies finding the best at the current time signal-interference ratio that satisfies the optimality criterion. It allowed processing calling signals within the minimum period of time that does not exceed 6–8 s, which is required to achieve a sustainable mode of the adaptive procedure of STA.

It is important that the effectiveness of the Kalman-Bucy algorithm in the STA problem is determined by the implementation possibility of the existing technical and technological basis without changing communication modes in the mobile network. Implementation of the Kalman-Bucy algorithm at the organization of STA does not depend on the other used methods, which is the result of involving additional resource set of spatial and temporal parameters.

2. Recursive procedures of STSP implementation are more reasonable for using in the STA problems. This makes it possible to adjust VWC in real time scope, which is especially important for communication with mobile SS and variable parameters of the radio channel. The methods of Widrow-Hoff are relatively simple, however 50–150 steps of discretization are required for their convergence, which may be unacceptable for the STA problems, since the STA problem itself needs to be completed before the time of rendering the SS service, that is, for $t = 2 \div 5$ sec.

Thus, to estimate the vector of weight coefficients of an antenna array, it was proposed to use the recursive procedure of Kalman-Bucy in the space of state variables. In contrast to the well-known asymptotic signal processing algorithms, the recursive algorithm of Kalman-Bucy is optimal in a dynamic, in particular, non-stationary situation by the indicators of quality and performance speed. It made possible to achieve the sustainable processing on the interval of the action of calling signals, that is, for 3–10 steps of discretization rates without changing the common modes of the mobile network operation.

3. The selection of the initial value of VWC of AAA is the most important restriction in the STA problem. It is known from the process of solving many calculation problems that the effectiveness of the solution itself (convergence rate, residual errors, etc.) is largely determined by an appropriate choice of initial conditions.

A priori information about the directions of arrival of a signal from the SS in the problems of spatial access is required both on the ascending section for the rapid formation of weight coefficients of AAA and accompaniment of angular motion of SS, and on the descending section for the synthesis of the adaptive DS of AA in the direction of SS.

Obviously, the direction vector V_c^T must be known in advance or determined by the methods of estimation of the direction of useful signal arrival. This makes it possible to significantly improve the transient characteristics of the algorithms, and as shown in this research, quite essentially, since it increases the value of the signal/interference+noise ratio up to 4 dB for a four-element AA.

The quality of space-time signal processing increases in proportion to the number of AA. For mobile communication systems, it is possible to recommend the AAA algorithms, synthesized based on the methods of Kalman filtering of the formation of VWC with the number of antenna elements that is equal to 4…16.

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