### ENGINEERING TECHNOLOGICAL SYSTEMS

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Розглянуто можливість здійснення порівняльного оцінювання ефективності процесу подрібнення в барабанному млині при традиційному усталеному та самозбудженому автоколивному режимах руху внутрішньокамерного завантаження.

Побудовано математичну модель параметрів ударної дії молольного завантаження на подрібнюваний матеріал. Застосовано аналітико-експериментальний метод візуального аналізу картин течії у поперечному перерізі обертової камери.

Чисельно за допомогою наближених процедур встановлено динамічний ефект зростання середніх сум вертикальних складових ударних імпульсів та середніх сум потужностей таких складових при самозбудженні автоколивань.

Експериментально встановлено технологічний ефект суттєвого зниження енергоємності та деякого зростання продуктивності виявленого автоколивного процесу подрібнення порівняно із характеристиками традиційного усталеного процесу. Для цього було використано ситовий аналіз продукту подрібнення та вимірювання оборотності руху завантаження і потужності приводу обертання барабана.

Як приклад, було розглянуто процес подрібнення цементного клінкера при ступені заповнення камери завантаженням 0,45, відносному розмірі кульових молольних елементів 0,026 та повному заповненні подрібнюваним матеріалом проміжків між молольними тілами. Встановлено, що при самозбудженні автоколивань енергоємність подрібнення знижується на 27,2 %, а продуктивність зростає на 6,7 %.

Встановлені в роботі ефекти дозволяють прогнозувати раціональні параметри автоколивного процесу подрібнення в барабанному млині

Ключові слова: барабанний млин, внутрішньокамерне завантаження, ударна дія, самозбудження автоколивань, енергоємність подрібнення

### 1. Introduction

Drum-type mills are widely used in many industries for fine grinding of granular materials. To reduce the energy intensity of such equipment, they use a variety of energyexchanging devices in the form of protruding elements that set a part of the milling fill into pulsating flow. However, the accelerated abrasive wear of such systems greatly limits their effectiveness.

At the same time, under certain conditions, there occurs the self-excitation of auto-oscillations of the intrachamber fill. Utilization of this effect makes it possible to improve the intensity of grinding at mills that are executed based on conventional design solutions with a smooth-surfaced chamber.

Given the above, it is a relevant task to predict the dynamic action of the fill, as well as the technological and energy efficiency of the self-oscillatory grinding process in a tumbling mill. UDC 621.926.5:539.215:531.36 DOI: 10.15587/1729-4061.2019.155461

# REVEALING THE EFFECT OF DECREASED ENERGY INTENSITY OF GRINDING IN A TUMBLING MILL DURING SELF-EXCITATION OF AUTO-OSCILLATIONS OF THE INTRACHAMBER FILL

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### 2. Literature review and problem statement

Flow modes of granular fill in a rotational cylindrical chamber substantially affect the implementation of manufacturing processes and energy consumption by the drives of machines of the drum type [1]. Modeling the hydrodynamics of such regimes is of interest when studying a variety of rotary systems [2]. The applied relevance of the issue on forecasting the workflows of such equipment has constantly attracted an enhanced research attention to describing a possible unstable behavior of the processed granular medium. Significant complexity of this task necessitates the perfection of traditional, as well as the application of new, theoretical and experimental research methods.

The problem of modeling the flow of a granular fill inside a rotating drum was solved by analytical methods. Paper [3] determined the boundary between a transition of the passive zone of filling into the active zone. Parameters for the flow of a sliding layer of the fill were established in [4]. Study [5] described, based on an analytical and experimental method of research, the patterns in the fill flow in the cross section of the drum. However, the results obtained refer only to the stable steady modes of medium flow.

Contemporary tumbling mills commonly employ protruding elements at the surfaces of operating chambers in order to enhance the circulation and reduce the share of passive part of the fill. Numerical modeling of flow, under the action of lifters, of individual particles of the fill was performed, based on the principle of similarity, in paper [6] by applying the discrete elements method (DEM). Study [7] reported an experimental and numerical comparative analysis of the impact of three different profiles of lifters on power of the mill's drive.

However, during operation of tumbling mills, their lifters experience considerable abrasive wear that leads to the total loss of functional properties. Paper [8] demonstrated, based on the derived results of numerical simulation and laboratory experimental studies, a significant decrease in the efficiency of the grinding process due to a rapid wear of the protruding parts in the drum's chamber lining. The influence of lifters' shape on the lining's wear was investigated in [9] numerically, using the DEM method applying a model of the filling's shear impact energy.

The efficiency of the grinding process at a tumbling mill is determined mainly by the intensity of the impact action from elements of the intrachamber milling fill on the particles of the ground material. Influence of the impact action was studied using a variety of theoretical and experimental methods.

Analytical modeling of the impact action of the fill was performed in paper [10]. The authors determined a magnitude of the maximum impact force considering the height of lifters and the coefficient of friction. The dynamic processes in a tumbling mill were studied in [11] analytically and by using a laboratory model. The authors demonstrated good convergence between the derived theoretical and experimental results in determining the potential energy and stresses of an impact.

There have been many attempts to numerically solve the task on determining the impact action of fill in the chambers of the drum-type mills. A review of several numerical methods to model the impact action of fill is given in [12]. Much attention is paid to the application of DEM method. The paper considers new variants of DEM and, as an alternative to it, numerical algorithms for simulating the impact manifestations. In [13], authors used the method of DEM and applied a universal model of grinding for the numerical investigation into energy of the impact action of fill at a self-grinding mill. A mathematical model for the numerical determination of surface impacts of the ball mill's fill was proposed in [14]. Paper [15] applied the DEM method to numerically model the energy of impact interaction between particles in a granular fill of the rotating drum's chamber. Study [16] provides an overview of numerical algorithms, based on DEM method, to determine the impact action of fill at ball mills and self-grinding mills and its effect on the drum's chamber lining wear. A three-dimensional simulation, using the DEM method, of the extreme manifestations of energy of the fill impact action at a self-grinding mill was performed in [17]. Paper [18] proposed an improved DEM method to model an impact action of the ball mill fill.

However, the initial conditions for the considered problem are not known in advance, while boundary conditions are of non-physical nature. That predetermines a significant constraint to the accuracy of numerical calculations, the results of which do not meet practical needs.

Impact processes in the behavior of fill in a rotating drum have also been investigated by experimental methods. Paper [19] studied experimentally the influence of the filling impact action on energy intensity of self-grinding mills. The effect of such an impact action on energy of grinding at ball mills was experimentally investigated in [20]. Study [21] addressed experimentally the effects of multiple repeated impacts between particles in the fill on the process of grinding in tumbling mills. The influence of impact action on the energy consumption and performance of self-grinding mills was investigated experimentally in [22]. Authors of [23] conducted an experimental analysis of the dependence of impact action energy on the geometric parameters of ball fill.

However, the technical complexity of instrumental control over the dynamic behavior of a granular fill in the rotational chamber, predetermined by the limitations in the separating capability of measuring equipment, reduces the reliability and accuracy of results obtained from experimental research.

Given the limited possibilities of numerical and experimental methods, there were several attempts at comparing the results of their application when determining the impact action of the intrachamber fill. Paper [24] studied the effect of impact action on the process of grinding in tumbling mills using a numerical simulation by DEM method and an experiment that employed laboratory models. Energy of the filling's impact action was studied with the use of a numerical kinetic model and an experimental laboratory setup in [25]. The authors demonstrated good convergence between calculated and experimental results. Paper [26], based on DEM method using the models of balance between the masses of fill, studied numerically the distribution of energy of the impact action in a tumbling mill's chamber. The results of modeling were compared to experimental data. Research into the energy of surface impacts of fill was performed in [27] experimentally and by using numerical methods.

The applied interest to the characteristics of impact action of fill is also predetermined by its effect on the intensive wear of the drum's chamber protective lining. The attempt to establish such an influence on the lining of a tumbling mill, depending on changes in geometric parameters of lifters, was implemented in [28] by using three-dimensional numerical simulation. Applying a three-dimensional simulation by using the DEM method in order to resolve such a problem was reported in [29]. Paper [30] employed the DEM method to study the influence of impact interaction between particles in a fill on wear. Numerical analysis of the process of the lining's wear due to the shear impact action of grinding bodies was performed in [31]. The results of theoretical calculation were compared with experimental data. In [32], authors investigated experimentally the effect of impact interaction between the fill and the lining on wear. Influence of the geometrical characteristics of fill on the lining's wear under the action of an impact fill was experimentally studied in [33]. Paper [34] suggested a theoretical-experimental model to predict wear as a result of the impact action.

Consequently, the insurmountable computational difficulties and low reliability of instrumental control limit the effectiveness of studying the impact action of a granular fill in a rotating drum. The derived results from analytical modeling, numerical calculations, as well as experiments, approach the actual modes of the dynamic behavior of the examined medium only in terms of qualitative characteristics and external attributes. Moreover, they address only a steady stable flow of filling mostly at drum rotation with a moderate speed; failing to model the self-oscillatory processes. In terms of quantitative indicators, these data substantially diverge.

Paper [35] constructed an analytical model of quality conditions and stability factors for the steady motion of a machine unit whose operating machine is a permanently rotating drum with a granular filling. The proposed method makes it possible to predict the manifestation of factors related to the self-excitation of auto-oscillations in the considered dynamic system. However, such a model does not make it possible to quantitatively evaluate the impact action of an intrachamber fill.

At present, there are no generalized models for determining the impact action of a granular fill in the rotating drum that account for the wide-range variations in system parameters. The lack of such models is particularly evident for the case of self-excitation of auto-oscillations in the intrachamber fill at drum-type mills and the influence of the chamber fill pulsations' impact action on a grinding process.

### 3. The aim and objectives of the study

The aim of this study is to construct a mathematical model for the impact action of the intrachamber fill in a tumbling mill for the conventional steady and the self-oscillatory flow modes, and the impact of such an action on the technological and energy parameters of the process of grinding. That would make it possible to establish characteristics of the dynamic action of milling fill and to predict the effectiveness of implementation of the self-oscillatory process of grinding.

To accomplish the aim, the following tasks have been set: – to perform an analytical-experimental simulation of impact action of the intrachamber fill in a tumbling mill;

 to establish parameters of the impact action under the steady conventional and the self-oscillatory motion modes of the milling fill;

 to perform an experimental modeling of the influence of flow modes of the fill on the grinding process in a tumbling mill;

– to quantify the impact of the filling's self-oscillations on performance and energy consumption of the grinding process.

### 4. Procedure to study the influence of behavior of the intrachamber fill on the grinding process in a tumbling mill

At conventional rotation speed of a tumbling mill, which is 0.7...0.9 of the critical ( $\psi_{\omega}$ =0.7...0.9), the chamber is filled through circulation flow under a three-phase mode, with the formation of a solid-state zone in the cross section, a non-free falling zone, and a sliding layer zone (Fig. 1). Conventional grinding is carried out mainly via the impact and erasing action of the milled fill on the ground material's particles. Technological outcomes of the grinding process as a result of such actions are roughly comparable.

For the conventional steady operation mode of a tumbling mill (Fig. 2), the impact action takes place at surface BC of the transition zone of non-free fall 2 (ABC) to sliding layer zone 3 (BCDE). Erasing action is implemented in sliding layer 3.

Under condition for the increased rotation speed of a tumbling mill, which is about 1...1.2 from critical ( $\psi_{\omega} \approx 1...1.2$ ), there occurs the self-excitation of self-oscillations of granular fill of the operating chamber without lifters [35]. In this case, the fill executes oscillatory flows under a two-phase mode with the formation in the chamber's cross section of a non-free fall zone and a pulsating zone (Fig. 3). In this case, grinding is due to the impact action only.



Fig. 1. Pattern of the filling's steady flow with a relative size of particles d/(2R)=0.01...0.03at the degree of filling  $\kappa=0.45$  of the chamber rotating at relative speed  $\psi_{\omega}=0.75$ 



Fig. 2. Estimation schematic to determine the filling's impact action at steady flow when  $\psi_{\omega}=0.75$ ,  $\kappa=0.45$  and d/(2R)=0.01...0.03 (in line with the flow pattern in Fig. 1): 1 - solid-body flow zone, 2 - non-free fall zone, 3 - sliding layer zone

For the self-oscillatory mode (Fig. 4), the impact action occurs at surface *BCE* that includes section *BC* of the transition between non-free fall zone 2 (*ABC*) and solid-body zone 1 (*ABCD*) and section *CE* of the contact between pulsating zone 3 (*ACE*) and the chamber's surface.

The accepted qualitative model of the filling's behavior makes it possible to perform a quantitative analysis of the dynamic action.



Fig. 3. Sequential patterns of the filling's flow over time *t* for a single period of self-oscillations  $T_{op}$  with a maximum span when  $\kappa = 0.45$ , d/(2R) = 0.01...0.3 and  $\psi_{\omega} = 1.1$ : a - t = 0;  $b - t = T_{op}/12$ ;  $c - t = 2T_{op}/12$ ;  $d - t = 3T_{op}/12$ ;  $e - t = 4T_{op}/12$ ;  $f - t = 5T_{op}/12$ ;  $g - t = 6T_{op}/12$ ;  $h - t = 7T_{op}/12$ ;  $i - t = 8T_{op}/12$ ;  $j - t = 9T_{op}/12$ ;

 $k - t = 10 T_{op} / 12; / - t = 11 T_{op} / 12; m - t = T_{op}$ 



Fig. 4. Estimation schematic to determine the filling's impact action under self-oscillatory flow when  $\psi_{\omega}$ =1.1,  $\kappa$ =0.45 and d/(2R)=0.01...0.03 (in line with the flow patterns in Fig. 3): 1 – solid-body flow zone, 2 – non-free fall zone, 3 – pulsating zone

## 5. Analytical modeling of impact action of the milling fill on a ground material

It is convenient to estimate technological results of the grinding process at conventional and self-oscillatory modes of a mill by performing a comparative analysis of parameters for the filling's impact action only.

When a grinding particle  $M_v$  hits the respective surface of the transition between zones of the filling's flow, there occurs a jump-like ultimate change in its speed over a small time  $\tau$ . In this case, the contact's surface is exposed to impact force  $\overline{F}_v$ . A measure for the impact interaction is the impact pulse:

$$\overline{S}_{v} = \int_{1}^{\tau} \overline{F}_{v} \mathrm{d}t.$$

Duration of the impact is very low  $(\tau \approx 10^{-2}...10^{-4} \text{ s})$ . Since the impact pulse  $S_v$  has a finite value, the impact force module of the milling body could be quite large  $(F_v \rightarrow \infty \text{ at} \tau \rightarrow 0)$ , which ensures the implementation of grinding process via the impact action. In this case, effect of the non-impact forces can be disregarded.

It is expedient to estimate the technological effect of impact action not based on a value for the impact force, but rather based on its pulse, work, and power.

An impact pulse corresponds to a change in the amount of motion  $\Delta \bar{Q}_v$  by a particle of mass  $m_v$  over the time of impact:

$$\bar{S}_{v} = \Delta \bar{Q}_{v} = m_{v} \left( \bar{u}_{v} - \bar{v}_{v} \right), \tag{1}$$

where  $\bar{v}_v$  is the speed of a particle before the impact,  $\bar{u}_v$  is the speed of a particle after the impact.

Work of the impact force corresponds to a change in the kinetic energy  $\Delta T_v$  of a particle during impact:

$$A_{\rm v} = \Delta T_{\rm v} = \frac{1}{2} m_{\rm v} \left( \bar{u}_{\rm v} - \bar{v}_{\rm v} \right)^2.$$
 (2)

The largest, in terms of magnitude, and defining, in terms of effectiveness of the implementation of an operating process, component of the impact action is vertical. The impact under consideration is a complex non-perfect interaction between particles. Then the expressions for the vertical component of the impact pulse and work of the vertical component of the impact force, according to (1) and (2), can be approximately represented in the form:

$$S_{vh} = m_v K_{vhv} v_{hv}, \tag{3}$$

$$A_{\rm vh} = \frac{1}{2} m_{\rm v} K_{\rm vhv}^2 v_{\rm hv}^2, \tag{4}$$

where  $v_{hv}$  is the vertical component of particle's velocity  $\overline{v}_v$ before the impact,  $K_{vhv} = 0...1$  is the coefficient of loss of the vertical component  $v_{hv}$  of velocity  $\overline{v}_v$  during impact.

Assuming:

$$v_{hv} = \sqrt{2gh_v}$$

expressions (3) and (4) take the form:

$$S_{\rm vh} = \sqrt{2g} m_{\rm v} K_{\rm vhv} \sqrt{h_{\rm v}},\tag{5}$$

$$A_{\rm vh} = gm_{\rm v}K_{\rm vhv}^2h_{\rm v},\tag{6}$$

where  $h_v$  is the height of particle's fall before the impact, g is the gravitational acceleration.

We shall further consider the vertical component of the impact action of particles at the surface of transition between zones of the filling's motion for N ( $\nu$ =1.2,...,N) grinding particles over an arbitrary time span  $\Delta t$ . Then, according to (5) and (6), expressions for the sum of vertical components of the impact pulses and the sum of work of the vertical components of impact forces over  $\Delta t$  take the following form:

$$S_{h}^{\Delta t} = \sqrt{2g} \sum_{\nu=1}^{N} m_{\nu} K_{\nu h \nu} \sqrt{h_{\nu}}, \qquad (7)$$

$$A_h^{\Delta t} = g \sum_{\nu=1}^N m_\nu K_{\nu h\nu}^2 h_\nu. \tag{8}$$

The mean, over time  $\Delta t$ , value for the total power of the vertical components of impact forces corresponds to the average value for the sum of work of such forces  $A_h^{ut}$  per unit time (1 s) and is equal to:

$$P_h = A_h^{ut} = \frac{A_h^{\Delta t}}{\Delta t}.$$
(9)

For the case of the conventional steady mode of operation (Fig. 3), the sum of the vertical components of impact pulses over  $\Delta t$  (7) can be calculated from formula:

$$S_{sh}^{\Delta t} = \sqrt{2g} m_s^{\Delta t} K_{vhs} \sqrt{h_{sf}}, \qquad (10)$$

where  $m_s^{\Delta t} = \sum_{v=1}^{N} m_{vs}$  is the sum of masses of *N* particles,  $K_{vhs}$  is the average coefficient of loss of the vertical component of particles' velocity in a non-free fall zone, which can be determined using a method of the filling's flow visualization,  $h_{sf} = \cos \varphi_{sf} \sum_{i=1}^{k} h_{sfi} / k$  is the average height of fall of *N* particles in a non-free fall zone,  $h_{sfi}$  is the elementary linearized trajectory of fall of a separate particle in a non-free fall zone,  $\varphi_{sf}$  is the angle of inclination to vertical of the averaged linearized trajectories of falling particles in a non-free fall zone.

The sum of the vertical components of impact pulses under a steady mode (10) for a single rotation of the drum is:

$$S_{sh}^{tn} = \sqrt{2g} m_s^{tn} K_{chs} \sqrt{h_{sf}}, \qquad (11)$$

where  $m_s^{tn} = mK_{ts}$  is the sum of masses of grinding particles that execute an impact within a single turn of the drum;  $m = \pi R^2 L \kappa \rho$  is the mass of the milling fill; R is the radius of the drum's chamber; L is the length of the drum's chambers;  $\kappa$  is the degree of filling the chamber with a fill;  $\rho$  is the density of the milling fill at rest;  $K_{ts}$  is the loading's turnover under a steady mode.

Turnover describes the number of periods of circulation of the fill over a single drum rotation:

$$K_t = \frac{2\pi}{t_c \omega},$$

where  $t_c$  is the duration of circulation period of a fill in the rotating drum's chamber,  $\omega$  is the angular speed of the drum. Turnover can be determined based on an experimental analysis of the fill flow patterns. Fig. 5 shows the resulting quasi-static dependence of turnover  $K_t$  on relative rotation speed  $\Psi_{\omega} = \omega \sqrt{R/g}$  at a degree of filling the chamber  $\kappa$ =0.45 for a granular fill with a relative size of the particles d/(2R)=0.01...0.03.



Fig. 5. Dependence of fill turnover  $K_t$  on relative rotation speed  $\psi_{\omega}$  at  $\kappa$ =0.45 and d/(2R)=0.01...0.03

The mean, over a single rotation of the drum, value for the sum of the vertical components of impact pulses under a steady mode per unit time is:

$$S_{sh}^{ut} = \frac{S_{sh}^{tn}}{T_s},\tag{12}$$

where  $T_s = 2\pi/\psi_{\omega s}\sqrt{R/g}$  is the period of drum rotation under a steady mode,  $\psi_{\omega s} = \omega_s \sqrt{R/g}$  is the relative rotation speed of the drum under a steady mode,  $\omega_s$  is the angular velocity of the drum under a steady mode. Then, taking into consideration (11) and (12),

$$S_{sh}^{ut} = \frac{1}{\sqrt{2\pi}} gm K_{ts} \psi_{\omega s} K_{vhs} \sqrt{\frac{h_{sf}}{R}}.$$
 (13)

For the case of the self-oscillatory mode of operation (Fig. 4), the sum of the vertical components of impact pulses over  $\Delta t$  (7) can be approximately calculated from expression:

$$S_{oh}^{\Delta t} = S_{ofh}^{\Delta t} + S_{oph}^{\Delta t} = \sqrt{2g} \left( m_{of}^{\Delta t} K_{vhof} \sqrt{h_{of}} + m_{op}^{\Delta t} K_{vhop} \sqrt{h_{op}} \right), (14)$$

where  $S_{ofh}^{\Delta t}$  is the sum of the vertical components of impact pulses of particles in a non-free fall zone over  $\Delta t$ ;  $S_{oph}^{\Delta t}$  is the sum of the vertical components of impact pulses of particles in a pulsating zone over  $\Delta t$ ;  $m_{of}^{\Delta t} = \sum_{v=1}^{N_f} m_{vo}$  is the sum of masses of  $N_f$  particles in a non-free fall zone, which execute an impact over  $\Delta t$ ;  $m_{op}^{\Delta t} = \sum_{v=1}^{N_p} m_{vo}$  is the sum of masses of  $N_p$  particles in a pulsating zone, which execute an impact over  $\Delta t$ ;  $K_{ohof}$  is the average coefficient of loss of the vertical component of velocity by particles in a non-free fall zone;  $K_{vhop}$  is the average coefficient of loss of the vertical component of velocity by particles in a pulsating zone,  $h_{of} = \cos \varphi_{of} \sum_{i=1}^{n} h_{ofi}/n$  is the average height of fall of particles in a non-free fall zone,  $h_{ofi}$  is the elementary linearized trajectory of fall by a separate particle in a non-free fall zone,  $\varphi_{of}$  is the angle of inclination to vertical of the averaged linearized trajectories of falling particles in a non-free fall zone,  $h_{op}$  is the average height of fall by particles in a pulsating zone.

The sum of the vertical components of impact pulses under a self-oscillatory mode (14) for a single drum rotation is:

$$S_{oh}^{tn} = \sqrt{2g} \left( m_{of}^{tn} K_{vhof} \sqrt{h_{of}} + m_{op}^{tn} K_{vhop} \sqrt{h_{op}} \right), \tag{15}$$

where  $m_{of}^{in} = m(1-\kappa_{op})K_{to}$  is the sum of masses of particles in a non-free fall zone, which execute an impact within a single drum rotation;  $m_{op}^{in} = m\kappa_{op}(\psi_{oop}/\psi_{oo})$  is the sum of masses of particles in a pulsating zone, which execute an impact within a single drum rotation;  $K_{to}$  is the turnover of fill under a self-oscillatory mode (Fig. 5);  $\kappa_{op}$  is the mass fraction of the pulsating zone in the mass of the entire fill;  $\psi_{ooo} = \omega_o \sqrt{R/g}$  is the relative speed of rotation of the drum in self-oscillatory mode;  $\omega_o$  is the angular velocity of drum under a self-oscillatory mode;  $\psi_{oop} = 2\pi\sqrt{R/g}f_{op}$  is the relative circular frequency of the filling self-oscillations;  $f_{op}$  is the cyclic frequency of self-oscillations.

Parameters for self-oscillations can be determined experimentally using a method for the visualization of a filling's flow. The mass share of pulsating zone  $\kappa_{op}$  is calculated based on an analysis of flow patterns. The cyclic frequency of selfoscillations  $f_{op}$  is established from an analysis of transient flow modes. The absolute mass and the mass share of a non-free fall zone and a pulsating zone are variable parameters over the period of self-oscillations:  $m_{of}^m = \operatorname{var}, m_{op}^m = \operatorname{var}$  and  $\kappa_{op} = \operatorname{var}$ .

The mean, over a single drum rotation, value for the sum of the vertical components of impact pulses under a selfoscillatory mode per unit time is:

$$S_{oh}^{ut} = \frac{S_{oh}^{tn}}{T_o},\tag{16}$$

where  $T_o = (2\pi/\psi_{\omega o})\sqrt{R/g}$  is the period of drum rotation under a self-oscillatory mode. Then, taking into consideration (15) and (16):

$$S_{oh}^{ut} = \frac{1}{\sqrt{2}\pi} gm \begin{bmatrix} K_{to} \left(1 - \kappa_{op}\right) \psi_{\omega o} K_{vhof} \sqrt{\frac{h_{of}}{R}} + \\ + \kappa_{op} \psi_{\omega op} K_{vhop} \sqrt{\frac{h_{op}}{R}} \end{bmatrix}.$$
 (17)

The average sum of the vertical components of impact pulses (17) over a period of self-oscillations varies from a minimum value when  $\kappa_{op} = 0$ :

$$S_{ohmin}^{ut} = \frac{1}{\sqrt{2\pi}} gm K_{to} \psi_{\omega o} K_{vhof} \sqrt{\frac{h_{of}}{R}},$$
(18)

to a maximal value when  $\kappa_{op} = \kappa_{op \max}$ 

$$S_{ohma}^{ut} = \frac{1}{\sqrt{2\pi}} gm \begin{bmatrix} K_{to} \left(1 - \kappa_{op \max}\right) \psi_{\omega o} K_{vhof} \sqrt{\frac{h_{of}}{R}} + \\ + \kappa_{op \max} \psi_{\omega op} K_{vhop} \sqrt{\frac{h_{op}}{R}} \end{bmatrix}.$$
(19)

Expressions for the mean value of total power of the vertical components of the impact forces of fill can be derived by analogy to the above procedures for the formalization of average sums of the vertical components of impact pulses. An expression for the total work of the vertical components of impact forces for a single drum rotation under a steady mode, taking into consideration (7), (8) and (11), takes the form:

$$A_{sh}^{tn} = gm_s^{tn} K_{vhs}^2 h_{sf}.$$
 (20)

The average total power of the vertical components of impact forces, taking into consideration (9), (12), (13) and (20), is:

$$P_{sh} = \frac{1}{2\pi} gm K_{ts} \Psi_{os} \sqrt{\frac{g}{R}} K_{vhs}^2 h_{sf}.$$
 (21)

An expression for the total work of the vertical components of impact forces for a single drum rotation under a self-oscillatory mode, taking into consideration (7), (8) and (15), takes the form:

$$A_{oh}^{tn} = g \Big( m_{of}^{tn} K_{vhof}^2 h_{of} + m_{op}^{tn} K_{vhop}^2 h_{op} \Big).$$
(22)

The average total power of the vertical components of impact forces under a self-oscillatory mode, considering (9), (16), (17) and (22), is:

$$P_{oh} = \frac{1}{2\pi} gm \sqrt{\frac{g}{R}} \left[ K_{to} \left( 1 - \kappa_{op} \right) \psi_{\omega o} K_{vhof}^2 h_{of} + \right] + \kappa_{op} \psi_{\omega op} K_{vhop}^2 h_{op} \right].$$
(23)

The average sum of power (23) over a period of self-oscillations changes, similar to (18) and (19), from a minimum:

$$P_{ohmin} = \frac{1}{2\pi} gm \sqrt{\frac{g}{R}} K_{to} \Psi_{\omega o} K_{vhof}^2 h_{of}, \qquad (24)$$

to a maximal value:

$$P_{ohmax} = \frac{1}{2\pi} gm \sqrt{\frac{g}{R}} \begin{bmatrix} K_{to} \left(1 - \kappa_{opmax}\right) \Psi_{oo} K_{vhof}^2 h_{of} + \\ + \kappa_{opmax} \Psi_{oop} K_{vhop}^2 h_{op} \end{bmatrix}.$$
 (25)

The expression for the ratio of average values for the sum of the vertical components of impact pulses under a self-oscillatory and a steady mode per time unit, taking into consideration (13) and (17), takes the form:

$$\frac{S_{oh}^{ut}}{S_{sh}^{ut}} = \frac{K_{to} \left(1 - \kappa_{op}\right) \psi_{\omega o} K_{vhof} \sqrt{h_{of}} + \kappa_{op} \psi_{\omega op} K_{vhop} \sqrt{h_{op}}}{K_{ts} \psi_{\omega s} K_{vhs} \sqrt{h_{sf}}}.$$
 (26)

Ratio (26) over a period of self-oscillations changes  $(S_{oh}^{ut}/S_{sh}^{ut} = \text{var})$ , taking into consideration (18) and (19), from a minimum:

$$\frac{S_{ah}^{ut}}{S_{sh}^{ut}\min} = \frac{K_{to}\psi_{\infty}K_{vhof}}{K_{ts}\psi_{\infty}K_{vhs}}\sqrt{\frac{h_{of}}{h_{sf}}},$$
(27)

to a maximal value:

.

$$\sum_{\substack{oh\\ sh max}}^{ut} = \frac{K_{to} \left(1 - \kappa_{op max}\right) \Psi_{\omega o} K_{vhof} \sqrt{\frac{h_{of}}{h_{sf}}} + \kappa_{op max} \Psi_{\omega op} K_{vhop} \sqrt{\frac{h_{op}}{h_{sf}}}}{K_{ts} \Psi_{\omega s} K_{vhs}}.$$
 (28)

The expression for the ratio of average values for the total power of the vertical components of impact pulses under a self-oscillatory and a steady mode, taking into consideration (21) and (23), takes the form:

$$\frac{P_{oh}}{P_{sh}} = \frac{K_{to} \left(1 - \kappa_{op}\right) \psi_{\omega o} K_{vhof}^2 h_{of} + \kappa_{op} \psi_{\omega op} K_{vhop}^2 h_{op}}{K_{ts} \psi_{\omega s} K_{vhs}^2 h_{sf}}.$$
(29)

Ratio (29) over a period of self-oscillations changes  $(P_{oh}/P_{sh}=var)$ , taking into account (24) and (25), from a minimum:

$$\frac{P_{oh}}{P_{sh}}_{\min} = \frac{K_{to}\psi_{oo}K_{vhof}^2/h_{of}}{K_{ts}\psi_{os}K_{vhs}^2/h_{sf}},$$
(30)

to a maximal value:

$$\frac{P_{oh}}{P_{sh}}_{max} = \frac{K_{to} \left(1 - \kappa_{op \max}\right) \psi_{\omega o} K_{vhof}^2 \frac{h_{of}}{h_{sf}} + \kappa_{op \max} \psi_{\omega op} K_{vhop}^2 \frac{h_{op}}{h_{sf}}}{K_{ts} \psi_{\omega s} K_{vhs}^2}.$$
 (31)

Ratios of the impact parameters under a self-oscillatory and a conventional steady mode (26) to (31) are conveniently used for a comparative assessment of the impact action of a milling fill under these modes.

### 6. Results of determining an influence of the fill flow modes on parameters of the impact action

According to the acquired experimental data, the geometric and kinematical parameters for the flow of a fill accept the following values:  $K_{vhs}=0.4$ ,  $K_{vhof}=0.75$ ,  $K_{vhop}=1$ ,  $h_{of}/h_{sf}=1.54$ ,  $h_{op}/h_{sf}=1.46$ ,  $\psi_{\omega s}=0.75$ ,  $\psi_{\omega o}=1.1$ ,  $\psi_{\omega op}=1.31$  (at  $f_{op}\approx 2$  Hz),  $K_{ts}=1.63$  (at  $\psi_{\omega s}=0.75$ ),  $K_{to}=1.11$  (at  $\psi_{\omega o}=1.1$ ),  $\kappa_{opmax}=0.0662$ .

Then the extreme magnitudes for ratios of average sums of the vertical components of impact pulses and the total power of such pulses for the self-oscillatory and steady modes per unit time, according to (27), (28), (30) and (31), acquire the following approximate values:  $(S_{oh}^{ut}/S_{sh}^{ut})_{min} = 2.32,$  $(S_{oh}^{ut}/S_{sh}^{ut})_{max} = 2.39, (P_{oh}/P_{sh})_{min} = 5.41, (P_{oh}/P_{sh})_{max} = 5.7.$ 

 $\begin{array}{l} (S_{oh}^{ut}/S_{sh}^{ut})_{\max} = 2.39, (P_{oh}/P_{sh})_{\min} = 5.41, (P_{oh}/P_{sh})_{\max} = 5.7.\\ \text{Charts for dependences } S^{ut}/S_{sh}^{ut} \text{ for the variable values}\\ \text{for numerator } S_{sh}^{ut} \text{ and } S_{op}^{ut} \text{ over a single period of self-oscillations } T_{op} = 1/f_{op} \text{ are shown in Fig. 6.} \end{array}$ 



Fig. 6. Dependence of ratios of average sums of the vertical components of impact pulses for the self-oscillatory and steady modes per unit time  $S/S_h$  over a period of self-oscillations  $T_{op}$  when  $\kappa$ =0.45 and d/(2R)=0.01...0.03



Charts of dependences  $P/P_{sh}$  for the variable values for

numerator  $P_{sh}$  and  $P_{oh}$  over a period of self-oscillations are

shown in Fig. 7.

of power of the vertical components of impact pulses for the self-oscillatory and steady modes per unit time  $P/P_h$  over a period of self-oscillations  $T_{op}$ when  $\kappa=0.45$  and d/(2R)=0.01...0.03

Numerical values for the dynamic parameters of the fill action indirectly characterize the influence on the operating process of milling.

### 7. Experimental modeling of influence of the fill action on the grinding process

For a comparative estimation of influence of the impact action of the milling fill on the performance of the grinding process in a tumbling mill under the self-oscillatory and conventional steady modes, we applied a laboratory ball mill.

The ground material used was the cement clinker, prepared by preliminary crushing, with a relative size of particles  $d_m/(2R) < 0.0059$ . The milling bodies used were steel balls with a relative size of  $d_b/(2R) < 0.026$ .

The degree of filling the chamber with a milling fill was  $\kappa = 0.25$ . The degree of filling the fill with particles of the ground material was  $\kappa_m = 0.4$ , which corresponded to the compete filling of gaps between ball milling bodies at rest with the particles.

Duration of the grinding process was 30 min.

Performance of the grinding process was estimated based on values for sifting through a sieve with a cell size of 0.08 mm. Performance was defined in relative units in line with expression  $C=1-(m_r/m_m)$ , where  $m_r$  is the mass of residue of the shredded material in a sieve after sifting,  $m_m$  is the total mass of a batch of the shredded material prior to sifting.

Comparison of performances for different modes could be conveniently performed based on ratio:

$$\frac{C_o}{C_s} = \frac{1 - m_{ro}/m_m}{1 - m_{rs}/m_m},$$
(32)

where  $C_o$  and  $C_s$  are the performances under the self-oscillatory and conventional steady modes,  $m_{ro}$  and  $m_{rs}$  are the masses of residue in a sieve for such regimes.

Energy intensity of the grinding process was determined from ratio  $E=P_d/C$ , where  $P_d$  is the power of rotation drive of the filled drum. Comparison of energy intensities for different modes was carried out according to ratio:

$$\frac{E_o}{E_s} = \frac{P_{do}/C_o}{P_{ds}/C_s},\tag{33}$$

where  $E_o$  and  $E_s$  are the energy intensities at self-oscillatory and steady modes,  $P_{do}$  and  $P_{ds}$  are the powers of drive for such regimes.

The power of rotation drive of the filled drum can be determined using an experimental dependence of torque on the resistance to rotation. Such a torque is conveniently represented as a relative drive momentum  $\psi_{M0.5}$ :

$$\Psi_{M0.5} = \frac{M}{M_{\text{max}0.5}},$$

where *M* is the absolute value for torque,  $M_{max0.5}$  is the absolute value for the conditional maximum torque when filling the chamber by half, which corresponds to an imaginary fill distribution in the cross section of the drum's chamber in the form of a perfect solid-body segment, rotated relative to the original position at a right angle. Fig. 8 shows the resulting dependence of relative torque of rotation drive of the filled drum  $\psi_{M0.5}$  on relative rotation speed  $\psi_{\omega}$  at a degree of filling the chamber  $\kappa$ =0.45 for a granular filling with a relative size of particles d/(2R)=0.01...0.03.

Fig. 9 shows the resulting dependence of relative power of rotation drive of the filled drum  $\psi_{P0.5} = \psi_{M0.5} \psi_{\omega}$  on  $\psi_{\omega}$  at  $\kappa = 0.45$  for d/(2R) = 0.01...0.03.



Fig. 8. Dependence of relative torque of rotation drive of the filled drum  $\psi_{M0.5}$  on the relative rotation speed  $\psi_{\omega}$ when  $\kappa$ =0.45 and d/(2R)=0.01...0.03



Fig. 9. Dependence of relative power of rotation drive of the filled drum  $\psi_{M,5}$  on relative rotation speed  $\psi_{\omega}$ at  $\kappa$ =0.45 and d/(2R)=0.01...0.03

Quasi-static dependences of dynamic parameters (Fig. 8, 9) are suitable for the quantitative estimation of power and energy characteristics of the drive.

According to the acquired experimental data, the technological and energy parameters for the grinding process in a tumbling mill accept the following values:  $C_s=0.435$ ,  $C_o=0.464$ ,  $P_{ds}=0.532$  (at  $\psi_{\omega s}=0.75$ ),  $P_{do}=0.413$  (at  $\psi_{\omega o}=1.1$ ).

Then the extreme magnitudes for ratios of the registered performances and energy intensities, according to (32) and (33), acquire the following values:  $C_o/C_s=1.067$ ,  $E_o/E_s=0.728$ .

### 8. Discussion of results of studying the influence of the filling's self-oscillations on parameters of the grinding process

The derived numerical results have made it possible to quantify the magnitude of the impact action of a milling fill at auto-oscillations self-excitation in comparison with a conventional steady mode. It turned out that at self-oscillations the approximate value for average sum of the vertical components of impact pulses by particles in a fill per unit time grows larger than 2 times, and the value for average sum of power of these pulses – larger than 5 times. The above shows a significant growth in the impact action of fill at auto-oscillations self-excitation due to a reduction in the share of the passive solid-body flow zone, an increase in the share of active zone of non-free fall and the formation of an active pulsating zone.

We have experimentally quantified the magnitude of energy consumption and performance of the grinding process in a tumbling mill at auto-oscillations self-excitation in comparison with a conventional steady flow mode of the fill. It turned out that at self-oscillations specific energy intensity decreases by larger than 25 %, and productivity rises by more than 6 %. The above indicates a substantial decrease in energy intensity and a certain increase in the process productivity at auto-oscillations self-excitation due to the increased impact action of the fill and a decrease in the power of rotation drive of the filled drum.

Such results from the dynamic and technological studies were derived at a degree of filling the chamber 0.45, relative size of globular elements in the fill 0.026, and when the gaps between milling bodies were completely filled with the ground material. We estimated two modes of cement clinker grinding in a tumbling mill: conventional steady at relative rotation speed  $\psi_{\omega}=0.75$ , and self-oscillatory at  $\psi_{\omega}=1.1$ .

The disadvantages of the applied approach to evaluating the impact of self-oscillations on the operating process include the underestimation of geometrical criteria for the similarity of system under consideration.

In the future, it is advisable to elucidate the qualitative and quantitative effects of the degree of filling the chamber on the self-oscillatory action. That would make it possible to establish rational conditions for the self-excitation of the fill pulsations in order to implement the auto-oscillatory grinding processes in tumbling mills of different types.

### 9. Conclusions

1. We have formalized the dynamic parameters for impact action of the tumbling mill's fill using an analytical-experimental method to visually analyze the patterns of flow in the cross section of the chamber. We have approximately numerically estimated the magnitude of impact pulse and its power based on values for average sums of the vertical components of impact pulses by particles in the fill and average sum of power of such pulses per unit time.

2. It was established that the ratio of impact pulses of fill for the self-oscillatory and steady motion modes is 2.32...2.39. Such a dynamic effect was registered at a degree of filling the chamber 0.45, relative size of globular elements in the fill 0.01...0.03 and when the gaps between milling bodies were completely filled with a ground material. The ratio of power of such pulses, under such conditions, is 5.41...5.4.

3. We have formalized the influence of the filling's flow modes on the grinding process in a tumbling mill using a sieve technological analysis of the product of grinding and measuring the dynamic parameters for a rotation drive. We have experimentally evaluated performance and energy intensity of grinding under the conventional and self-oscillatory modes based on the characteristics of two cases when the cement clinker was ground.

4. It was established that the specific energy intensity of grinding for the self-oscillatory mode is 27.2 % lower than the energy intensity of a steady conventional mode. Such a technological effect was registered at a degree of filling the chamber 0.45, relative size of globular elements in the fill 0.026, and when the gaps between milling bodies were completely filled with a ground material. Performance efficiency for the self-oscillatory regime under such conditions is 6.7 % higher than that of the conventional mode.

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