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DEVELOPMENT OF THE METHOD FOR GEOMETRIC MODELING OF *S*-SHAPED CAMBER LINE OF THE PROFILE OF AN AXIAL COMPRESSOR BLADE

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Розроблено метод геометричного моделювання S-подібної середньої лінії профілю лопатки осьового компресора, яка подається складеною кривою та формується з трьох ділянок. Кожна з ділянок моделюється у натуральній параметризації та застосуванні певних законів розподілу кривини вздовж дуги модельованої кривої. Кривина вхідної ділянки підпорядковується лінійному закону, двох інших – квадратичному закону розподілу від довжини власної дуги. Стикування ділянок відбувається із забезпеченням другого порядку гладкості, який передбачає рівність значень функцій, похідних та кривин в точці спряження. На відміну від існуючих методів, побудову середньої лінії профілю лопатки компресора пропонується виконувати безпосередньо в решітці, для якої відомі осьова протяжність, кут установки профілю та його хорда. При цьому геометричні кути входу та виходу потоку приймаються за вихідні дані. Надання середній лінії профілю лопатки S-подібної форми сприятиме безградієнтності руху робочої речовини на виході із решітки профілів, а, отже, зниженню втрат енергії в компресорі. На підставі запропонованого методу розроблено програмний код, який, окрім цифрової інформації по модельованій середній лінії профілю лопатки компресора, також видає отримані результати в графічному вигляді на екран монітора комп'ютера. Проведені розрахункові дослідження підтвердили працездатність запропонованого методу моделювання середніх ліній профілів лопаток осьових компресорів. Метод може бути корисним організаціям, які займаються проектуванням осьових компресорів газотурбінних двигунів

Ключові слова: осьовий компресор, профіль лопатки, геометричне моделювання, середня лінія, натуральна параметризація

1. Introduction

Axial compressors are widely used in the constructive schemes of aviation, marine and locomotive turbine engines. This type of compressors is also used in stationary gas turbine plants to drive electric generators at electric power stations and blowers in main gas pipelines at natural gas pumping. Modern compressors are characterized by decreasing the number of stages with a simultaneous increase in pressure and efficiency.

Efficiency of axial compressors essentially depends on the degree of geometric perfection of blade units. The existence of even a minor inconsistency between the geometry of the compressor blades and the flow of working medium leads to the emergence of additional losses of energy.

In order to enhance the efficiency of a compressor, the measures related to the change of the geometry blades are applied: lean and sweep [1], wing sweep [2], windage [3], a change in the configuration of the leading edges [4], the use of profiles with «controllable» diffusivity [5, 6], etc. The research in this direction is carried out by introducing these changes of the geometrical shape of the blade edge to the methods of designing these important components of flow-through parts of axial compressors. Thus, paper [7] proposes the method for improving the profile of axial compressors blades by solving an optimization problem and construction of compressor cascades with optimal profiling of blade crowns.

Designing blade units of compressors is a complex multi-stage process, the key component of which is the construction of the cascades of profiles. Traditionally, these profiles are formed by distribution of optimized aerodynamic profiles along some camber line. The camber line must ensure the specified angles of the flow inlet into the cascade of profile and outlet out of it. This is achieved by the respective inclination angles of tangents at the initial and end points of the line.

At an increase in the velocity of the flow of the working medium, the designers of compressors began to give camber lines *S*-shaped form, which leads to the so-called «controllable» diffusivity. It ensures gradient-free motion of working medium at the outlet of inter-blade channels.

Advances of science and technology contribute to the implementation in production of technologically complex industries of the innovative highly productive technological equipment, provided with computing tools that enable the implementation of spatial displacements of a cutting tool.

Such equipment makes it possible to produce products of a very complex geometric shape, but, in turn, requires the development of sophisticated mathematical support and software. Under modern conditions, the blades of axial compressors are produced at high-rate technological machining centers with numerical program control. These high-tech machining centers have their specific hardware and software means. Controlling programs of these machining centers are numeric models of the corresponding technological processes. In this regard, these machining centers set extremely high requirements for the quality of geometric modeling of the planar and spatial contours of machined parts.

Thus, one of the most effective approaches to enhancing effective indicators of flow-through parts of axial compressors is modeling the profiles of plane cross-sections of blades with the *S*-shaped camber line. This leads to the relevance of research towards the development of the methods for the mathematical description of camber lines of the profiles of axial compressor blades of the *S*-shape type, which will contribute to enhancement of effective indicators of compressor of turbine engines.

2. Literature review and problem statement

The practice of creation of the blade units of compressors uses different analytical methods of modeling camber lines of blades profiles, which differ among themselves mainly by mathematical dependences, laid at the basis for these methods. Some of them use the arcs of circles, combination of two arcs of circles of different radius, the segments of exponential curves, polynomial curves, sections of hyperboles, fractional-rational curves, etc.

From an early stage in the development of projects of axial compressors and up to now, the camber line of blade profiles is often described by the arc of the circle [8]. In this case, the relative coordinate $\bar{y}$ of the median line was determined by expression:

$$\bar{y}=\sqrt{\bar{x}-\bar{x}^2-\frac{\cos^2\chi}{4}}-\frac{\mathrm{ctg}\chi}{2},$$

where χ is the parameter related to the geometric angles of the flow inlet and outlet from the blade unit, which in the case of application of the circle arc should be equal.

However, the assigned angles of the flow inlet and outlet can be achieved by the special location of this arc. But, in this case, there is no possibility to ensure the necessary value of the stagger angle of profile in the cascade. And it is completely clear that it is impossible to give the *S*-shape to the camber line by the arc of the circle.

The connection of two arcs of circles makes it possible to construct a compound S-shaped curve, to ensure the assigned angles of the flow inlet and outlet, as well as the inflection and the angle of inclination of tangent at the point of the compound curve, assigned by a designer [9]. However, it is not possible to influence the stagger angle of the profile in the cascade, besides, there is a jump-like change of the curvature radii at the connection point of the circles' arcs. The similar situation was not critical at low velocities of the flow of working medium.

Parabolic curves not higher than second degree ones are normally used at the analytical representation of the camber line of the profile of an axial compressor blade. An example of the application of the parabola can be found in paper [10]. It is known that the second-degree parabola is a unimodal curve, so, it is clear that it may not ensure the construction of a S-shaped camber line.

In paper [11], it is proposed to describe the camber line of the profile of an axial compressor blade by the fraction-rational function, taken in the form of:

$$y(x)=a\frac{x(1-x)}{1+bx},$$

where

$$a=\frac{1}{x_f^2}\frac{f}{c};\quad b=\frac{1-2x_f}{x_f^2}.$$

In these expressions, f means the maximum deflection of the camber line of the profile, x_f is the distance between the point of maximum deflection from the beginning of the camber line, which is measured along the chord of profile c.

The above fractional-rational function does not make it possible to plot a *S*-shaped line either.

Papers [12, 13] introduced a parametric approach to the description of the camber line of the profile of an axial compressor blade, although a different mathematical apparatus is applied in this case.

Thus, in paper [12], it is proposed to represent the camber line of the blade profile with a polynomial of the fifth degree in the following form:

$$Y=a+bX+cX^2+dX^3+fX^4+eX^5.$$

Coefficients a, b, c, d, f, e of this polynomial, as well as angles γ_1 and γ_2, which are related to the angles of the flow inlet and outlet, respectively, are the solution to the system of eight equations. The first five equations correspond to the geometric representation of the camber line in the coordinate system, in which the initial and the end point of the camber line are on the horizontally located axis of abscissas, and the axis of ordinates is oriented perpendicularly to this axis. The sixth and the seventh equation of the systems imply that the input section of the median line is described by the arc of the circle. And only the eighth equation makes it possible to give the *S*-shape to the output section of the modeled camber line. Regarding the method, described in paper [12], it can be noted that the camber line of the profile of the compressor blade is not fully described by a polynomial curve. The inlet section is presented by the arc of the circle. In the place of connection of the arc of a circle with a polynomial curve, the curvature break will occur, that is, the connection of the curves will be performed with provision of the first order of smoothness. In addition, polynomial curves of high degrees are sometimes characterized by so-called oscillatory effects, which in the case of camber lines of blades profiles is an undesirable phenomenon.

In paper [13], the parametric description of the camber line is performed using Bezier curves. Note that the Bezier curves, the mathematical base of which is Bernstein polynomials, are very popular when describing the objects with a complex geometric shape. An interest in these curves significantly increased with a wide spread of the computer technology in many areas of science and technology.

The Bezier curves are also used by the authors of papers [14, 15], both for direct modeling of camber lines of blades profiles (paper [14]), and for approximation of the contours of the compressor blades.

In addition to Bezier curves, *B*-splines [16] and *NURBS*-curves [17, 18] are applied in the representation of camber lines of the profiles of compressor blades. All the listed curves are characterized by the following feature. The shape of the resulting curves is determined by the coordinates of the initial and end points of the modeled lines, as well as some totality of intermediate points. It is common to call these intermediate points controlling vertices. These controlling vertices outline the shape of the modeled curve in first approximation. It should be noted that the Bezier curves, *B*-splines, and *NURBS*-curves have a certain analytical background, and therefore there is no need to memorize the coordinates of each point of the modeled curve. This makes it possible to create quite complex curved objects with a small number of controlling vertices. However, appropriate location of control vertices is a time-consuming and ambiguous task. The solution to this problem depends on the skill of an expert not only in the geometry of curves, but also in his subject area.

Due to the increased load on the steps of axial compressors, a significant increase in the velocity of the motion of working medium, the requirements for camber lines of the profiles of blades have changed significantly. These lines started to be given the *S*-shape.

Paper [9] proposed the method for geometric modeling of a two-section *S*-shaped curve, which is described by two exponential dependences with respect to camber lines of the profiles of axial compressor blades. During modeling the curves, the following boundary conditions are used:

– at $x=0$: $y=0,\ y'=\mathrm{tg}\alpha_1,\ y''=P\cdot(y'')_{extr}$;

– at $x=1$: $y'=\mathrm{tg}\alpha_2,\ y''=Q\cdot(y'')_{extr}$,

where α_1, α_2 are the angles of inclination of tangents to the modeled curve at its initial and end points; P and Q are some parameters that are smaller than unity, which determine the fraction of an extreme value of the second derivative at the initial and end points of the modeled curve, respectively.

To ensure the influence on the second derivative of the exponential curve, its expression was taken in the form of:

$$y''=b(x-s)e^{a(x-s)},$$

where a and b are the unknown coefficients.

The extreme value of the second derivative y'' appears, when $y'''=0$. This corresponds to coordinate x, which is determined by expression:

$$x_{extr}=s-1/a,$$

and directly the value of the second derivative in this case will be equal to:

$$y''_{\mathrm{ext}}=-b/ae.$$

The expression for the first derivative and directly the equation of the curve is derived by double integration of the equation of the second derivative:

$$y'=\frac{b}{a^2}e^{a(x-s)}\left[a(x-s)-1\right]+c;$$

$$y=\frac{b}{a^3}e^{a(x-s)}\left[a(x-s)-2\right]+cx+d.$$

It should be noted that the authors of paper [9] did not coincide the problem of the degree of smoothness of a compound curve at the junction point of the sections of exponential curves. Under the specified conditions, the equality of the values of the function and its derivative will be ensured at the connection point, which corresponds to the first degree of smoothness. This is a consequence of the fact that at modeling the camber line of the blade profile by the sections of exponential curves, the problem of ensuring the equality of curvature at the connection point is not even addressed.

Paper [9] also proposes a method for representation of the camber line of the profile of the compressor blade by a polynomial curve, the equation of which is taken in the following form:

$$(x-h)^2=4a(y''-k)$$

or in another way:

$$y''=\frac{1}{4a}(x-h)^2+k,$$

where h is the parameter, which on the axis of abscissas determines the location of the point, where the second derivative must reach a maximum (minimum) value; k is the value of the second derivative, which should be reached at this point. Both parameters are determined in the process of modeling the camber line of the profile of the compressor blade.

It is possible to obtain the expression for the first derivative by double integration of the second derivative, and then the expression directly for function:

$$y'=\frac{1}{12}(x-h)^3+kx+b;$$

$$y=\frac{1}{48a}(x-h)^4+\frac{k}{2}x^2+bx+c.$$

Thus, to plot the camber line, it is necessary to find five coefficients: a, b, c, h and k. During modeling the camber line, we desire to obtain a maximum value of the second derivative at the initial point.

Completing the analysis of literature sources, it should be noted that not all the methods of description of camber lines of the profile of a compressor blade make it possible to obtain the *S*-shape. Those methods that allow giving the *S*-shape to the camber line do not ensure the second order of smoothness at the connection point of the curves. In addition, they do not use the stagger angle of the profile in the cascade as a source magnitude. However, this angle is a very important geometric parameter that significantly affects the aerodynamic quality of the cascade of profiles, in particular, in terms of losses of energy of the working medium.

3. The aim and objectives of the study

The aim of present research is to develop the method for geometric modeling *S*-shaped camber lines of the profiles of plane cross-sections of the blades of axial compressors, composed by a curve. The sections of this curve are represented in the natural parameterization with the assigned laws of curvature distribution on the length of the proper arc. In this case, the second order of smoothness in the nodes of connection of the sections should be provided.

To achieve the set aim, the following tasks were set:

– to determine the position of supporting points, angles of inclination of tangents in them for subsequent drawing a compound *S*-shaped curve through them;

– to construct the input section of the camber line of the profile of the compressor blades using natural parameterization and the linear law of curvature distribution;

– to construct the rest of the sections of a compound curve with the use of the quadratic law of curvature distribution on condition of ensuring the second order of smoothness in the places of connection of the sections;

– to implement the proposed method of modeling the camber line of the profile of the axial compressor blade in the form of the computer code with the visualization of the obtained results on the computer monitor screen.

4. Modeling the camber line of the profile of the axial compressor blade using natural parameterization of curves

The curvature of curve k is determined by the following dependence:

$$k(s) = d\phi/ds,$$

where ϕ is the angle of inclination of the tangent.

In differential geometry, there are expressions to determine the angle of the tangent inclination and coordinates of the points of the modeled curve depending on the length of its arc:

$$\phi(s) = \phi(0) + \int_0^s k(s)\mathrm{d}s; \qquad (1)$$

$$x(s) = x(0) + \int_0^s \cos\phi(s)\mathrm{d}s;$$

$$y(s) = y(0) + \int_0^s \sin\phi(s)\mathrm{d}s, \qquad (2)$$

where $\phi(0)$, $x(0)$, $y(0)$ are the angles of inclination of the tangent and the coordinates of the initial point of the modeled curve.

Usually the integrals in expressions (2) are not taken analytically; their computation is possible only by the numerical methods of integration.

4. 1. Location of supporting points of the camber line

Modeling a camber line starts by arranging four points based on certain conditions. These are points 0–3, shown in Fig. 1. We will note that points 0, 1, 3 are applied when plotting an ordinary, rather than a *S*-shaped camber line. The curve in the section between points 0 and 3 can be drawn provided that the axial length of cascade B, the chord of profile b_3 and stagger angle β_{st} – the angle of mounting the profile in the cascade – are known. The specified geometric parameters are found as a result of performing the gas-dynamic calculation of the compressor.

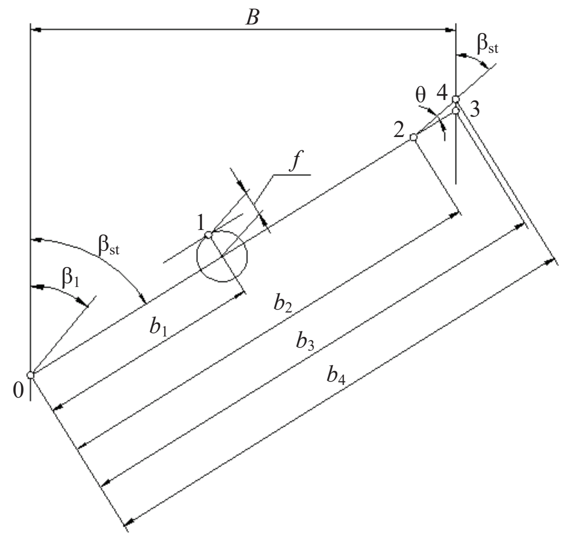

Fig. 1. Source data to modeling the camber line of the profile of the axial compressor blade

The position of point 1 is determined by its distance b_1 from the input point of the camber line (point 0), which is measured along straight line 03 and is taken as some of its fraction. Value f is an arrow of inflection of the camber line. The values of the specified parameters are accepted based of the analysis of the statistical data that are optimized in different organizations of compressor profiles.

During modeling the camber line of the profile of a compressor blade, the geometric angles of the flow inlet in grating β_1 and outlet β_2 from it are applied. The values of these angles are taken by the results of the gas-dynamic calculation of the compressor.

Point 2 is determined as the point of inflection of the camber line. The existence of inflection of this line is caused by the desire to give *S*-shape to a profile. As follows from consideration of Fig. 1, point 2 is on section 03, its location is determined by magnitude b_2, which is the part of segment b_3. Angle θ, which causes the deviation of section 24 from the direction, determined by straight line 03, is assigned at point 2. The length of segment 02 and angle θ are assigned by a designer of the compressor. Appropriate values of these parameters are found by the results of the calculations of spatial viscous flow of the working medium in the cascades of profiles. The purpose of these calculations is to minimize energy losses in the blade units of the compressor.

It should be noted that the introduction to consideration of point 2, and angle θ led to the appearance of point 4 and, consequently, an increase in the chord of the profile, which, as a result, will be equal to b_4. To preserve the magnitude of the axial length of the cascade of profiles B, it is necessary to reduce the original value b_3. These actions are implemented by using the appropriate iterative process.

Thus, for modeling the *S*-shaped camber line, it is necessary to draw a curve through points 0, 1, 2 and 4. At all these points, the angles of inclination of tangents are known. Note that at points 1 and 2 these angles converge with the stagger angle.

Since we will need the coordinates of the specified four points for further calculation, we will determine the dependences for finding them. We will locate the coordinate origin at zero point, then $x_0=0$, $y_0=0$.

From geometric considerations that follow from studying Fig. 1, the expressions for determining the coordinates of points 1–3 will take the form:

$$x_1 = b_1 \cos\beta_{st} - f \sin\beta_{st};$$

$$y_1 = b_1 \sin\beta_{st} + f \cos\beta_{st};$$

$$x_2 = b_2 \cos\beta_{st};$$

$$y_2 = b_2 \sin\beta_{st};$$

$$x_3 = b_3 \cos\beta_{st};$$

$$y_3 = y_2 + (x_3 - x_2)\tan(\beta_{st} + \theta).$$

4. 2. Modeling the input section of the camber line

The desired camber line will be constructed as a curve composed of three sections that are given in the natural parameterization. These are the sections, located between points 0 and 1, 1 and 2 and 2 and 4. Connection of the sections will take place at points 1 and 2 with ensuring the second order of smoothness, which implies the equality of the values of the function and its derivatives, as well as the equality of curvature, at the points of junction.

Geometric modeling of the sections of the compound camber line of the profile of the compressor blade will be performed based on the research results presented in [19].

The first (input) section of the compound curve, located between points 0 and 1, will be modeled with the use of a linear law of distribution of curvature k on the length of contour arc s, taken in the form of:

$$k(s) = as + b, \tag{3}$$

where a and b are the unknown coefficients, which are to be determined in the process of modeling the curve.

After the substitution of the equation of the linear law of curvature distribution in the form of (3) in expression (1), we will obtain the dependence of the distribution of angle the tangent on the arc length:

$$\phi(s) = \phi(0) + \frac{as^2}{2} + bs.$$

Since the angles of inclination of tangents at points 0 and 1 are known, the following expression is true:

$$\phi_1 = \phi_0 + \frac{aS_{01}^2}{2} + bS_{01},$$

where S_{01} is the length of the arc, plotted between points 0 and 1. Coefficient a is determined from expression:

$$a = \frac{2}{S_{01}}\left(\frac{\Delta\phi}{S_{01}} - b\right),$$

where $\Delta\phi = \phi_1 - \phi_0$.

With respect to coefficient a, expression for the distribution of the angle of inclination of the tangent to the curve, depending on the length of the arc will take the following form:

$$\phi(s) = \phi_0 + \frac{s^2}{S_{01}}\left(\frac{\Delta\phi}{S_{01}} - b\right) + bs. \tag{4}$$

We will note that angles ϕ_0 and ϕ_1 are determined by angles β_1 and β_{st}. Specifically, they equal:

$$\phi_0 = \pi/2 - \beta_1,$$

$$\phi_1 = \pi/2 - \beta_{st},$$

where angles β_1 and β_{st} are measured in radians.

After substituting the coordinates of point 1 in expressions (2) and taking into account (4), we will obtain two equations:

$$x_1 = x_0 + \int_0^{S_{01}} \cos\left(\phi_0 + \frac{s^2}{S_{01}}\left(\frac{\Delta\phi}{S_{01}} - b\right) + bs\right) ds;$$

$$y_1 = y_0 + \int_0^{S_{01}} \sin\left(\phi_0 + \frac{s^2}{S_{01}}\left(\frac{\Delta\phi}{S_{01}} - b\right) + bs\right) ds,$$

through the numerical solution of which we will determine two unknown magnitudes: the length of arc S_{01} and coefficient b.

At the arbitrary assigning the values of the arc length S_{01} and coefficient b, we will obtain coordinates $x(s)$ and $y(s)$ of some point, located at distance d from point 1, which is determined by expression:

$$d = \sqrt{(x(s) - x_0)^2 + (y(s) - y_0)^2}. \tag{5}$$

By varying the arc length S_{01} and coefficient b, it is possible to find such their values that will enable determining deviation d with predetermined accuracy. It is possible to make the process of selecting magnitudes S_{01} and b focused in character, if we accept the expression for determining deviation d as an objective function and apply to it one of the methods of minimization of functions of many variables.

To find the minimum of function (5), we applied the algorithm proposed in paper [20], which makes it possible to minimize the function of several variables by the method of direct search, that is, to do without calculating the derivatives of the minimized function. The practice of the calculations showed high efficiency of this algorithm for solving the problems of minimization of the function of many variables.

4. 3. Modeling the second and the third sections of the camber line

Upon completion of modeling the first section of the camber line, one starts plotting its second part, located between points 1 and 2. A distinctive feature of modeling the section of the camber line is the fact that distribution of curvature k along contour length s is presented by the parabolic dependence of the second degree:

$$k(s) = as^2 + bs + c, \tag{6}$$

in which unknown coefficients a, b and c are determined in the process of modeling the second section of the camber line.

The connection of these two sections at point 1 is carried out on the condition of the equality of coordinates, angles of inclination of tangents, that is, derivatives and curvature. This corresponds to the second order of curves connection. It is assumed that at point 1, curvilinear coordinate s is equal to zero. As a result of plotting the first section of the pressure side of the profile, the curvature of the curve at point 1 becomes the known magnitude, which is why coefficient c is assigned the value that is equal to the curvature of the curve at the end point of the first outline section, that is, $c=k_1$.

Since the angles of inclination of tangents at points 1 and 2 are known (moreover, they are equal), it is possible to write it down like it was done for the initial section of the camber line:

$$\phi_2 = \phi_1 + \frac{aS_{12}^3}{3} + \frac{bS_{12}^2}{2} + k_1 S_{12}$$

and to find an expression for the calculation of coefficient a for the second section of the compound curve:

$$a = \frac{3}{S_{12}^2}\left(\frac{\Delta\phi}{S_{12}} - \frac{bS_{12}}{2} - k_1\right).$$

In this expression, we imply by S_{12} the length of the arc curve, located between points 1 and 2.

The dependence that sets the distribution of the angle of inclination of tangent on the second section of the camber line of the profile of a compressor blade, taking into account coefficients a and c, will take the form:

$$\phi(s) = \phi_1 + \frac{3s^3}{S_{12}^2}\left(\frac{\Delta\phi}{S_{12}} - \frac{bS_{12}}{2} - k_1\right) + \frac{bs^2}{2} + k_1 s,$$

where

$$\Delta\phi = \phi_2 - \phi_1.$$

After substituting this expression to equations (2), which are written down in respect of point 2, we will obtain the following dependences to determine the coordinates of point 2 using a numeric method:

$$x_2 = x_1 + \int_0^{S_{12}} \cos\left(\phi_1 + \frac{3s^3}{S_{12}^2}\left(\frac{\Delta\phi}{S_{12}} - \frac{bS_{12}}{2} - k_1\right) + \frac{bs^2}{2} + k_1 s\right) \mathrm{d}s;$$

$$y_2 = y_1 + \int_0^{S_{12}} \sin\left(\phi_1 + \frac{3s^3}{S_{12}^2}\left(\frac{\Delta\phi}{S_{12}} - \frac{bS_{12}}{2} - k_1\right) + \frac{bs^2}{2} + k_1 s\right) \mathrm{d}s.$$

The length of arc S_{12} between points 1 and 2, as well as coefficient b, is determined as a result of solving the problem of minimization of the magnitude of deviation d of the obtained intermediate end point of the outline from point 2.

The third step of modeling the camber line of the profile of an axial compressor blade involves plotting the section of the curve between points 2 and 4. We also applied the quadratic law of curvature distribution to ensure the second level of smoothness of junction of the sections of the curves at point 2.

Without violating the accepted system of points numbering, we will denote the length of the arc between points 2 and 4 as S_{24}, the angle of the tangent inclination to the modeled curve at point 4 as ϕ_4. The magnitude of this angle will be equal to:

$$\phi_4 = \pi/2 - \beta_2.$$

The curvature of curve k_2 at point 2 can be determined by expression (6), since after completion of modeling the second section, the values of coefficients a, b and c of this expression and the length of arc S_{12} become known.

These coefficients, as well as the length of the curve arc S_{24} between points 2 and 4, are found in the similar way as it was considered above for the second section of the modeled line.

5. Results of modeling the camber line of the profile of axial compressor blade

Based on the proposed method for plotting camber lines of the profiles of axial compressor blades, we developed a computer code that makes it possible to calculate and visualize the obtained results on a computer monitor screen.

Fig. 2 shows the results of visualization of the text example of the camber line of a blade profile. The source data for these calculations are accepted by the results of the gas and dynamic calculation of the compressor, presented in paper [21]. Tangents are denoted by section of straight lines at points 0, 1, and 4. In point 2, the tangent coincides with straight line 03 (Fig. 1).

Because the output section of the camber line, which is located between points 2 and 4, is shown in Fig. 2 in a small scale and almost coincides with straight line 03, its curvilinear character is difficult to observe. In this regard, the output section of the camber line was substantially increased and shown in Fig. 3. In this figure, the curvilinear character of section 24 and its connection with the previous section at point 2 are clearly observed. It is clear that it is possible to estimate this connection visually as junction of first smoothness when the values of the function converge and there is a general tangent.

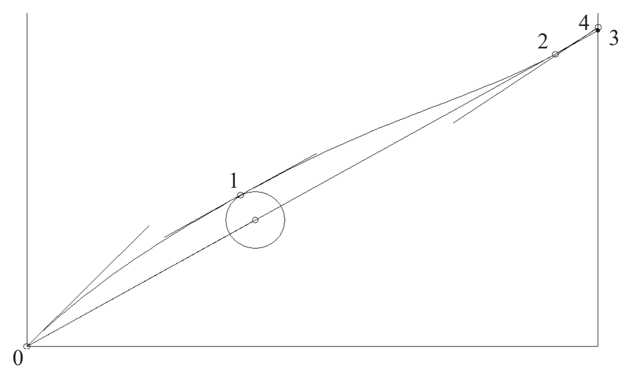

Fig. 2. Test example of camber line

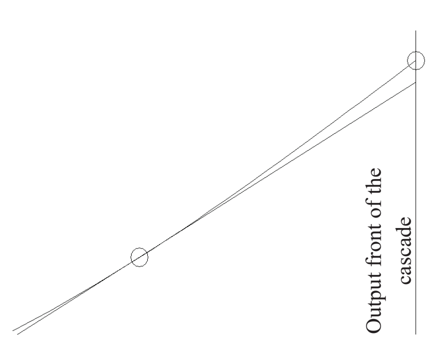

Fig. 3. Output section of the camber line of the blade profile

To prove that junction occurs with the second order of smoothness, we will plot the diagram of distribution of curvature of the compound curve, shown in Fig. 2. To do this, we will supplement the computer code with the unit of curvature calculation and visualization.

Fig. 4 shows the graph of arc curvature distribution depending on the relative length of the arc. The numbers on it mark the points, numbering of which correspond to the designations applied in Fig. 2. Consideration of Fig. 4 reveals that at the points of connection of the sections of the camber line of the compressor blade profile (these are points 1 and 2), the curvature of the adjacent sections is equal, which proves the fact of junction with the second order of smoothness.

Analyzing the graphic dependences of curvature distribution on the separate sections, it is possible to note that the diagram of curvature has a linear character in the first section and a curvilinear character on the second and third sections. Regarding the third section, it is possible to add that the degree of curvilinearity of the diagram of curvature distribution is not very high, which is explained, firstly, by a relatively small length of this section, and secondly, which is more important, the magnitude of the differences between the angles of tangent inclination at points 2 and 3 is small.

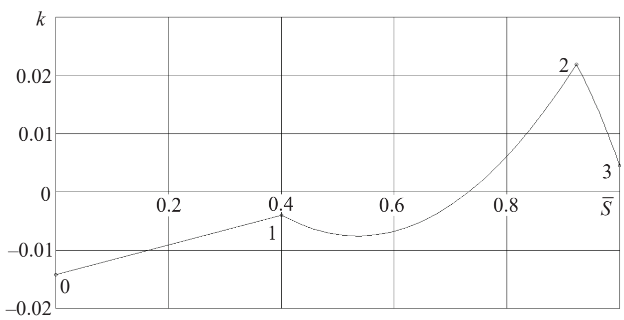

Fig. 4. Diagram of distribution of curvature depending on the relative length of the arc

The graphic dependence of curvature distribution on the second section of the modeled curve has an inflection, which is natural, because the angles of inclination of tangents at points 1 and 2 are equal, and therefore, a smooth transition between two parallel straight lines is realized.

Finishing consideration of graphical information presented in Fig. 2, it should be noted that, at first glance, the graphs of curvature are somewhat unfavorable in nature. This is explained by the fact that the scale factor of the coordinate horizontal and vertical axes is drastically different and it imposes a certain impact on the nature of passage of a compound curve.

Therefore, all of the above geometric parameters fully satisfy the conditions of modeling median lines of the profiles of axial compressors blades. However, it should be noted that the selection of the values of geometric parameters must be approached reasonably. Primarily, this refers to the location of point 1, at which there is the maximum arrow of deflection of the camber line of profile of the axial compressor blade.

6. Discussion of the method for modeling camber lines of the compressor blade profiles

In this paper, we proposed a new method for geometric modeling of camber lines of the profiles of axial compressor blades. The specific feature of the method is that the modeled camber line is presented as a compound curve that passes through all of the assigned supporting points at providing the assigned angles of tangent inclination in them. In this case, the second order of smoothness of junction of the curves of separate sections is achieved.

It is necessary to approach reasonably the selection of source data. An inflection of the curve at the input section of the camber line is possible at a small difference between the stagger angle and the geometric angles of the flow at the inlet. A similar situation may occur at the introduction of the overrated value of deflection arrow of the camber line or very close location of the point of maximum rise of the camber line to the front side of the cascade.

The curves of curvature distribution at the points of connection of separate sections have a jump-like change of the derivative, depending on the curvature along the length of the arc. It should be noted that this is not critical, but it can be eliminated.

Subsequent development of the proposed method for modeling the camber lines of the profiles of axial compressor blades is related to the development of measures that would provide the third order of smoothness of junction of separate sections of a compound curve. The third order of smoothness, except those that are inherent in the second order of connection of the sections, has to be ensured at connection points by the equality of the derivatives of the curvature. The implementation of these measures, of course, will lead to certain mathematical difficulties associated with an increase in the degrees of the laws of curvature distribution of separate sections.

7. Conclusions

1. We developed the method for geometric modeling the *S*-shaped camber lines of the profiles of plane cross-sections of axial compressors blades in the form of a curve composed of three sections. Each section is represented in natural parameterization and applying the laws of curvature distribution depending on the length of the proper arc.

2. It is proposed to plot the camber line directly in the cascade of profiles taking into account the stagger angle, axial length of the blade crown and the chord of profiles. Geometric parameters, necessary for finding the coordinates of supporting points and angles of tangent inclination, are determined according to the data of the gas-dynamic calculation of the compressor.

3. The input section of the camber line of the compressor blade profile is modeled using the linear law of curvature distribution on the length of the arc. The unknown coefficients of the law of curvature distribution and the length of the arc are determined in the process of modeling a curve by means of solving the problem of minimizing intermediately obtained end points of the section from the specified supporting point 1. The second and third sections of the compound curve are plotted using the quadratic law of curvature distribution. The second order of smoothness is ensured at the points of connection of the sections of the compound curve. Like in the case of the input section, the unknown coefficients of quadratic laws of curvature distribution and the length of the arcs are determined by solving the problem of minimization of intermediately obtained end points of the sections from the assigned points 2 and 4.

4. The method for modeling the camber line of the profile of a *S*-shaped axial compressor blade is implemented in the form of a computer code. Performed calculations proved the efficiency of the method. Subsequent improvement of the proposed method is associated with ensuring the third order of smoothness of junction of the sections at conjugate points.

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