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Досліджено розподіл рівнів звукового тиску в резонаторах Гельмгольца в широкому діапазоні частот. Проведено комп'ютерне моделювання звукового поля в резонаторі методом кінцевих елементів та експериментальні дослідження.

Встановлено наявність багатьох резонансних частот в резонаторі та показано розподіл максимумів і мінімумів рівнів звукового тиску в об'ємі резонатора. Виявлено, що розподіл резонансних частот резонатора не відповідає гармонійному закону. Це дає змогу розглядати резонансні властивості резонатора аналогічно до коливань мембрани чи дзвона. Друга резонансна частота резонатора в 6–9 раз вище першої резонансної частоти, що відповідає резонансу Гельмгольца. Моделювання звукового поля в резонаторі показало наявність вузлових ліній в розподілі звукового тиску як в об'ємі резонатора так і горлі. Встановлено, що кількість вузлових ліній для перших частот на одиницю менша за номер резонанса.

Спільним для всіх розподілів є те, що при наближенні точки вимірювання до краю горла резонатора рівень звукового тиску зменшується. Також при дослідженнях встановлено, можливість створення резонансу лише в об'ємі резонатора без яскраво виражених вузлових ліній в горлі.

Порівняльний аналіз між експериментальними даними та даними комп'ютерного моделювання показав високий рівень достовірності отриманих результатів. Похибка в визначенні резонансної частоти становила не більше 0,8 %. Даний факт дозволяє в подальшому при визначенні звукового поля в системах резонаторів користуватися комп'ютерним моделюванням замість ресурсозатратних експериментальних досліджень.

Наявність багатьох резонансів в резонаторі Гельмгольца дозволяє проводити побудову широкосмугових приладів, що можуть базуватися на використанні даного типу резонаторів

Ключові слова: резонатор Гельмгольца, резонансні частоти, звукове поле, метод кінцевих елементів

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EXPERIMENTAL STUDY INTO THE HELMHOLTZ RESONATORS' RESONANCE PROPERTIES OVER A BROAD FREQUENCY BAND

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1. Introduction

Research into analysis of human sound sensations began in the mid-19th century [1]. Even though a given work belongs to the field of psychoacoustics, a need arose during its execution to use and register the phenomenon of resonance. It was shown that an elastic body (a string, a stretched membrane) could resonate not only to the sound, equal in height to its natural tone, but also to the overtones. To prove it, sensitive devices were used, namely, glass or metal balls with a narrow neck or tubes – the Helmholtz resonators. In terms of electric acoustics, they represent an acoustic oscillating system, consisting of flexibility, mass, and active resistance. In this case, the flexibility is the air inside the container, the mass is the air that fills the narrow resonator throat, and the attached mass of air adjacent to the end of the throat. The presence of active resistance is predetermined by the friction between air and the walls of the throat and by losses in the oscillatory energy due to the radiation of sound by an open end of the throat [2].

The Helmholtz resonators were used to analyze the spectra of complex sounds before the advent of computing technology, they were also applied in temple structures for the correction of acoustic properties of premises.

One of the areas where resonators could be employed is the construction of focusing systems for acoustic medical instruments or flaw detection devices with simultaneous amplifying properties. In addition, the existence of multiple resonance frequencies in the resonator makes it possible to use them in broadband acoustic systems.

2. Literature review and problem statement

Although the Helmholtz resonators were initially designed to analyze a complex sound, they are currently used to suppress noise in the ducts of ventilation systems, exhaust pipes at internal combustion engines [3]. Now they are starting to be used as the base for materials of the new generation – acoustic metamaterials that enable the effective focusing of ultrasonic waves for medical diagnostic instruments using the flat systems [4]. They are also applied in the architectural acoustics, in ultrathin structures to absorb sound [5]. It should be noted, however, that papers [3–5] investigate the resonator's properties only near the first resonance frequency.

Study [6] explores a field inside the resonator, analyzes the calculation of the first resonance frequency based on known formula (1) and by applying a finite element method. The existence of multiple resonance frequencies in the resonator was also demonstrated. However, the difference is that this work investigated resonators with a volume in the form of a cylinder, thereby the nodal lines are either parallel or perpendicular to the plane of the neck's cross-section. Whereas the distribution of nodal lines in a spherical resonator is much more complex.

Research into the higher modes of the spherical resonator's oscillations was reported in [7], however, modelling was carried out disregarding the presence of a neck.

Paper [8] did analyze the Helmholtz resonator oscillations under the influence of Lamb waves over a wide frequency band, but it did not specify the multi-resonance character of oscillations in these resonators.

Study [9] analyses the sound-absorbing properties of resonators only near the first resonance frequency as well.

Ultrasonic therapeutic emitters are used in devices to treat a series of diseases: peripheral nervous system, musculoskeletal system, internal organs, dental, obstetrics and gynecology, as well as eye-related. Given this, there is a variety of emitters. They are typically built in the form of discs, made from high-quality piezoceramics of zirconate-lead titanium, for example, PZT-8, and are placed in a waterproof shell made of aluminum or stainless steel [10], the reverse side of the disc is bordered by air. Contact with the human skin is enabled either through a thin layer of contact liquid or via an acoustically transparent bag with water, which can take the shape of an irradiated body part. Radiation mode could be either continuous or pulse. The working frequency range from 0.88 MHz to 3 MHz was initially captured using the single-frequency emitters.

Note that the generation of oscillations at frequencies 0.88 MHz and 2.64 MHz involves piezo transducers with the same thickness, the third harmonic is only for generating 2.64 MHz [11]. However, to emit the same power as at the first harmonic, a three-time larger voltage amplitude of HF generator is needed. In the instrument BTL-07, made by a Czech firm, a converter could emit two separate sound frequencies of 1 MHz and 3 MHz when switching a voltage frequency of HF generator.

A major shortcoming of the therapeutic emitters that emitting a single sound frequency, especially when working at maximum permissible intensities and direct contact with the skin, which is not mentioned in specifications, is the requirement to move them during a procedure. This relates to the need to avoid local damage due to the possible formation of standing waves and "hot spots".

For a more uniform cross-sectional beam of the ultrasound field, there were suggestions to use sources with a wide range of radiation. The converters should have been the piezoelectric transducers with a variable thickness. Not to mention the problem on creating a broadband emitter with a sufficient intensity under continuous mode, such converters are unacceptable because different parts of tissues at the beam's cross-section are exposed to different frequencies. Thus, a promising area is to use an acoustic system based on the Helmholtz resonators that would make it possible to utilize not only the third harmonics, but also higher harmonics. This could significantly expand the possibilities of using ultrasonic therapy in dermatologic and cosmetic medical practice that requires high frequencies.

3. The aim and objectives of the study

The aim of this work is to experimentally study the resonant properties of the Helmholtz resonators. Detection of the existence of resonant frequencies that differ from the known Helmholtz resonance frequency, and establishment of patterns in a change in the levels of sound pressure within the resonator's volume. Resonators are represented in the form of a spherical container, of different size and with a different throat length. Frequency range of emission is from 20 Hz to 10 kHz.

To accomplish the aim, the following tasks have been set:

– to carry out an experimental study to establish the existence of several resonances;

– to build a computer model to analyze a sound field in the resonator and to determine the natural frequencies of air oscillations in the resonator.

4. Measurement of resonance frequencies

We studied three Helmholtz resonators (Fig. 1), placed in free space. Sound wave was propagating from a sound source, located at a distance of 1.2 m from the resonator. The resonator itself was placed both perpendicularly to the direction of a sound propagation and along the line of propagation. The resonators were made of glass, their geometric dimensions are given in Table 1.

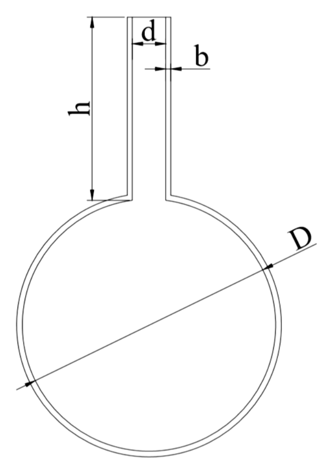


Fig. 1. General view of resonator

When conducting the experimental research, a sound field within and around the resonator was generated by an electrodynamic loudspeaker, which received a linear frequency-modulated (LFM) signal in the frequency range 20–10,000 Hz. To estimate the nonlinearity of the Helmholtz resonator, LFM signal was sent both with an increasing and a decreasing frequency. The difference between resonance frequencies was not significant, suggesting a linear measurement mode.

Measurements were carried out using two microphones. One of them was installed directly in the resonator, and the microphone's size was negligible compared with the linear dimensions of the neck and cavity, the second – at the same distance from the sound source outside of the resonator. That made it possible to control the influence of resonator on the sound field generated. The microphone signals were digitized by a two-channel ADC and were processed using a spectral analysis. In this case, we computed the spectra of signals and estimated the resonance frequencies.

Table 1

Geometrical dimensions of resonators used in the experiment

Resonator No.	V, ml	D, mm	d, mm	h, mm	b, mm
1	250	82	32	63	2.5
2	500	105	30.5	70	2.5
3	1,000	125	30.5	180	3.0

It turned out that the resonators exhibited a large number of resonance surges, whose height and quantity strongly depended on the location of the measuring microphone and the geometry of the resonator. Table 2 gives the resonance frequencies and normalized, relative to the first resonance (resonance of Helmholtz), sound pressure levels that correspond to resonator No. 1 (Table 1).

Table 2

Resonator No. 1. Frequency of resonances and their levels

No. of entry	Resonance frequency, Hz	Measuring point height, mm						
		20	40	60	80	100	120	140
		Power spectral density level, dB/Hz						
1	306	0	–1	–2	–2	–5	–10	–20
2	2,070	–37	–47	–	–	–28	–28	–36
3	2,990	–36	–	–40	–40	–40	–29	–37
4	4,252	–43	–	–	–	–	–39	–41

Note: dash "–" shows the absence of resonance or that its level was below –50 dB

Tables 3, 4 give similar results for resonators No. 2 and No. 3 (Table 1).

Theoretical value for a resonance frequency is described by expression:

$$f_{theor} = \frac{c}{2\pi}\sqrt{\frac{S}{hV}}, \tag{1}$$

where c=330 m/s is the speed of sound in air; $S = \frac{\pi d^2}{4}$ is the throat cross-sectional area, m^2; $V = \frac{\pi D^3}{6}$ is the volume of the resonator's cavity, m^3; h is the height of the throat, m.

Table 3

Resonator No. 2. Frequency of resonances and their levels

No. of entry	Resonance frequency, Hz	Measuring point height, mm								
		20	40	60	80	100	120	140	160	180
		Power spectral density level, dB/Hz								
1	204	0	0	0	–1	–1	–3	–7	–12	–22
2	1,871	–26	–30	–42	–29	–21	–15	–15	–16	–25
3	2,426	–27	–31	–44	–29	–30	–31	–22	–25	–32
4	3,478	–37	–46	–59	–45	–34	–40	–37	–32	–39
5	4,112	–36	–46	–50	–43	–	–34	–	–32	–35
6	6,380	–41	–42	–47	–41	–50	–47	–55	–	–
7	7,861	–	–43	–	–	–	–47	–	–45	–47

Note: dash "–" shows the absence of resonance or that its level was below –60 dB

Table 4

Resonator No. 3. Frequency of resonances and their levels

No. of entry	Resonance frequency, Hz	Measuring point height, mm															
		15	30	50	70	90	110	120	140	160	180	200	220	240	260	280	300
		Power spectral density level, dB/Hz															
1	99	0	0	0	0	0	0	0	0	0	–1	–2	–3	–6	–9	–14	–17
2	893	–39	–42	–42	–44	–45	–45	–39	–28	–24	–23	–22	–22	–23	–25	–29	–32
3	1,681	–36	–41	–46	–53	–44	–33	–33	–28	–26	–29	–43	–32	–24	–23	–28	–31
4	1,947	–27	–32	–40	–40	–34	–38	–35	–33	–26	–25	–30	–48	–35	–37	–35	–38
5	2,625	–38	–44	–59	–56	–	–49	–37	–32	–35	–38	–29	–30	–	–37	–35	–38
6	3,037	–34	–40	–54	–58	–45	–45	–38	–43	–36	–46	–46	–43	–44	–41	–42	–41
7	3,568	–45	–51	–60	–56	–56	–71	–44	–39	–86	–43	–49	–	–42	–	–47	–
8	3,978	–41	–58	–42	–42	–49	–45	–54	–	–	–	–	–	–	–	–	–
9	4,060	–40	–47	–55	–58	–61	–45	–43	–50	–43	–52	–44	–	–45	–75	–43	–43

Note: dash "–" shows the absence of resonance or that its level was below –75 dB

Calculation and measurement results are given in Table 5.

Table 5

First resonance frequencies obtained from calculation and experimentally

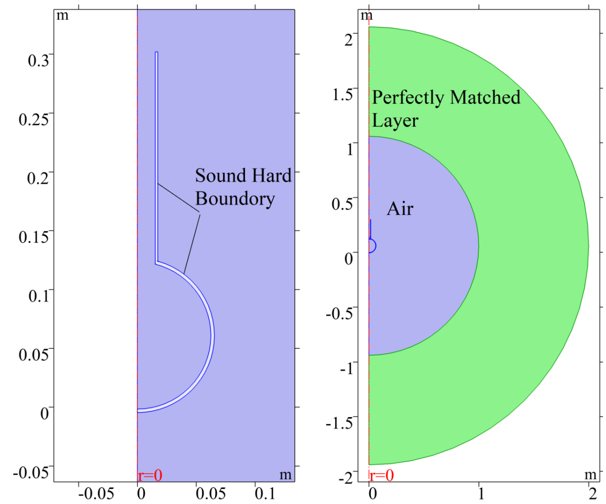
Type of resonator	Calculation frequency, Hz	Measured frequency, Hz
Resonator No. 1	349.2	306
Resonator No. 2	217.9	204
Resonator No. 3	104.6	99

Thus, the first resonance frequency, obtained in the experiment, corresponds to the Helmholtz resonance.

5. Computer simulation of resonators

For the further research into the natural oscillation frequencies of the Helmholtz resonator, we built a computer model of resonator No. 3 in the programming environment COMSOL Multiphysics (Fig. 2).

The geometrical dimensions of the resonator correspond to data from Table 1 for resonator No. 3. Because the resonator model is axisymmetric, the calculation of natural modes of oscillations employed 2-D modeling with an axis of symmetry. Space in and around the resonator was filled with air. The resonator's walls are acoustically hard. At a distance of 1 m from the resonator we assigned the perfectly absorbing layer with a thickness of 1 m. By using a finite element method, we calculated natural frequencies of the resonator and built the oscillation modes of the sound pressure (Fig. 3).



a *b*

Fig. 2. Geometry of the resonator computer model: *a* – general view of the model; *b* – central part of the model (resonator)

The results of comparing the results from measurements of the sound pressure and from calculations are shown in Fig. 4, 5 (a maximum of the sound pressure level is taken as 0 dB).

Based on the results obtained we could argue that computer simulation of the field in the Helmholtz resonator is in good agreement with the experimental results. It is shown that for the Helmholtz resonance the amplitude of sound pressure within the volume, and partly in the throat, does not change. Then, in the throat, the amplitude decreases towards the hole, with the speed of a decrease in the sound pressure level growing when approaching the throat.

It was also found that the second resonance frequency (Fig. 4, *b*) is 7–9 times higher than the Helmholtz resonance frequency. The dependence of sound pressure level on the microphone location height at this frequency is more complex. There is a minimum of sound pressure in the upper third of the volume, and then a sharp increase in the sound pressure in the middle of the throat, which gradually decreases toward the hole.

The sound pressure at the third resonance frequency has two minima, one is in the center of the volume, and the second is in the bottom third of the throat.

At higher resonance frequencies, we observe the existence of maxima and minima of the sound pressure levels. In all cases, however, the sound pressure decreases significantly towards the neck.

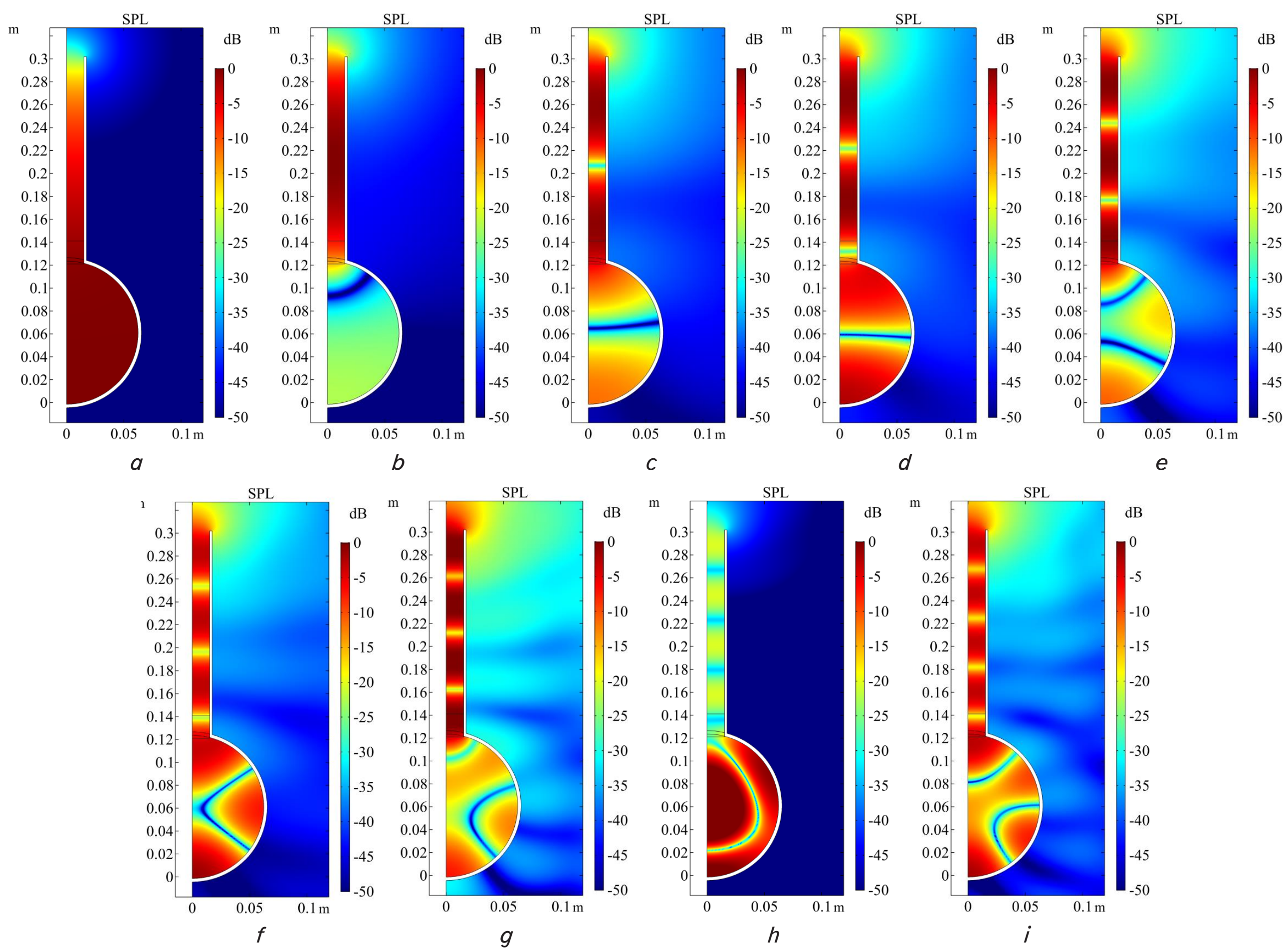


Fig. 3. Distribution of sound pressure levels in resonator No. 3 at the first nine resonance frequencies: *a* – 99 Hz; *b* – 892 Hz; *c* – 1,693 Hz; *d*–1,947 Hz; *e* – 2,644 Hz; *f* – 3,038 Hz; *g* –3,592 Hz; *h* – 3,962 Hz; *i* – 4,075 Hz

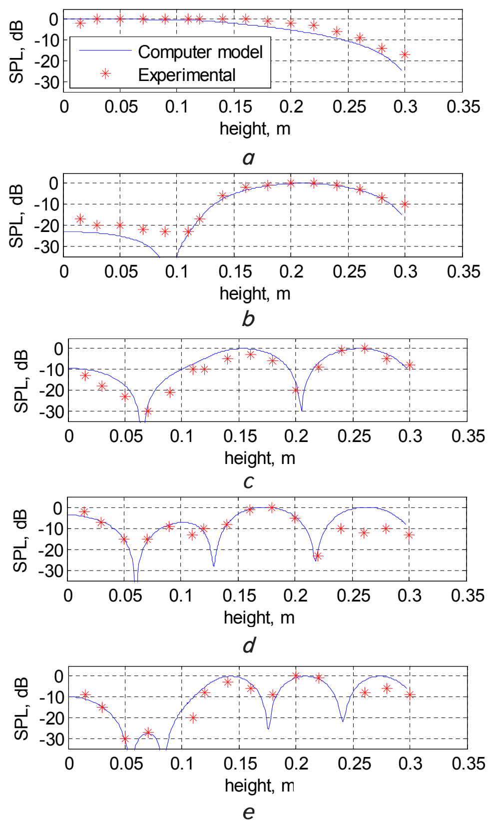


Fig. 4. Comparison of simulation results and experimental data for resonator No. 3, the first five harmonics (the frequency of computer simulation is shown in brackets): *a* – 99 (99) Hz; *b* – 893 (892) Hz; *c* – 1,681 (1,693) Hz; *d* – 1,947 (1,947) Hz; *e* – 2,625 (2,644) Hz

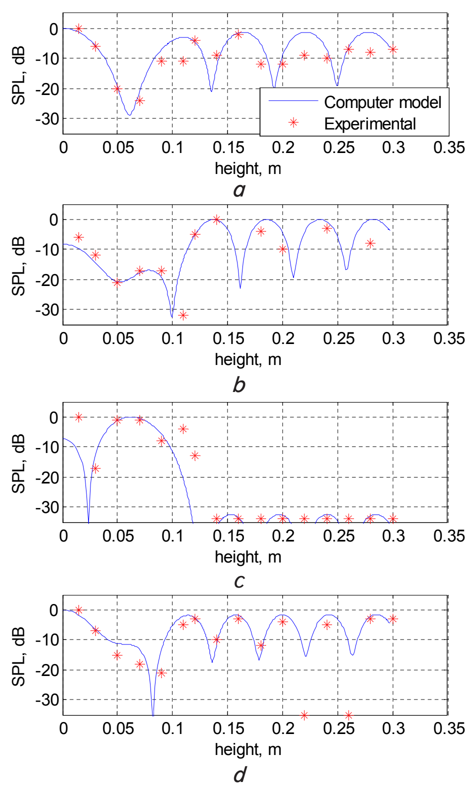


Fig. 5. Comparison of simulation results and experimental data for resonator No. 3, the sixth–ninth harmonic (the frequency of computer simulation is shown in brackets): *a* – 3,037 (3,038) Hz; *b* – 3,568 (3,592) Hz; *c* – 3,978 (3,962) Hz; *d* – 4,060 (4,075) Hz

At the 8-th resonance frequency (Fig. 5, *d*), the resonance is observed only at points in the volume and it does not affect the throat. Thus, at this frequency, the resonance phenomenon is demonstrated only by the volume of air in the cavity.

6. Discussion of results from simulation and obtained experimentally

The multiplicity of resonances could be explained by the fact that the Helmholtz resonator is a system with distributed parameters; similar systems are a string, a membrane, or a bell. In this regard, as is the case for a membrane or a bell, a series of resonance frequencies is not harmonic in contrast to a string or an air column in the pipe.

The study conducted has shown good consistency between simulation results and experimental data. This fact makes it possible in the future to continue computer simulation of the systems built using these resonators, without time-consuming experiments. Discrepancy in the calculation of resonance frequency did not exceed 0.8 %, but determining the sound pressure levels at some points at high frequencies exhibited a significant error. Such surges could be associated with errors in determining the coordinates for the sound receiver and dimensions of the microphone capsule itself.

While conducting computer simulation, of significant importance is the temperature and atmospheric pressure of environment. These parameters affect the speed of sound in the medium and, consequently, the resonance frequency.

Application of a given model is only possible for resonators made from acoustically hard materials. The model does not take into consideration the resonance phenomena associated with the fluctuations of the resonators' walls or the passage of sound through walls.

The results obtained could be interesting when designing instruments and devices for ultrasonic diagnosing and therapy or flaw detection. In order to focus and amplify sound over a wide frequency band.

In the future we plan to study a joint sound field from multiple resonators in order to investigate a possibility to focus sound at different resonance frequencies. In addition, it will be interesting to study the broadband sound amplification when using different resonators.

7. Conclusions

1. Applying a method of computer simulation, we were able to obtain a sound field in the resonator, as well as determine the natural resonance frequencies of a given resonator. It turned out that this type of the resonator has multiple resonance frequencies, rather than one as previously thought. The distribution of these resonance frequencies is not harmonic in character. The second resonance frequency is 7–9 times higher than the first. For the resonances of low order (n=4÷6) the number of nodal lines is a unity less than the resonance number. However, there are resonances where only the air volume resonates within the volume of the resonator without the participation of air in the throat. In this case, the number of nodal lines decreases dramatically.

2. By conducting an experimental study, we have confirmed the existence of many resonance frequencies, which coincided, at error not exceeding 0.8 %, with those derived from simulation using a finite element method. The presence of multiple resonance frequencies makes it possible to exploit this phenomenon to build instruments and devices with a high bandwidth.

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