Розглянуто необхідність кількісної оцінки суспільних процесів, тенденцій та дій соціальних інституцій, що здійснюють керування. Визначено, що інформаційна відкритість та наявність інструментів обробки і інформування формує умови ефективного адміністрування. Проведено аналіз трендів – часових рядів динаміки індикаторів. Для аналізу і узагальнення обмежено перелік пріоритетних показників чотирма та обрано данні звітів Світового Банку, Obozrevatel.ua, Transparency International. Введено уніфіковану норму, яка забезпечить співвимірність показників, що формують висновок про стан, тенденції та процеси. Введено одну із можливих норм, що застосовуються для евклідових просторів. Обґрунтовано систему базисної оцінки інтегрального показника за даними про ефективність урядування, якість регулювання, політичну стабільність та відсутність насильства та індекс CPI. Отримані вирази оцінки прогнозних значень інтегрального показника. Запропоновано у якості оцінки для аналізу ефективності дій публічного адміністрування використовувати нижню межу. Промодельовані процеси оцінювання і отримано п'ятирічний часовий ряд для інтегрального показника. Представлено зв'язок похибки інтегрального показника, розміру вікна ковзання, стрибків першої та другої похідних узагальненого часового ряду та допустимої похибки, як обмеження нерівність. Запропоновано вводити систему представлення і відображення часових рядів у безрозмірних, обмежених просторах поворотом на кут навколо спільної вісі. Введені означення, доведено теорему про збереження локальних значень відносних розмірів та похибок. Показано вплив квадратичної форми на локальну відносну похибку інтегрального показника. Проведено прогнозування розвитку та дана оцінка дій публічного адміністрування за оцінкою інтегрального показника

Ключові слова: інтегральний показник, ефективність квадратичної норми, норма розвинення, обмеження нерівність норми, геометрична нерівність

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SYNTHESIS OF A TREND'S INTEGRAL ESTIMATE BASED ON A TOTALITY OF INDICATORS FOR A TIME SERIES DATA

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1. Introduction

It is common knowledge that the existence of tools to quantify the trends, processes, and activities of social institutions that perform public administration is the key to informational openness and new possibilities for analysis and successful management [1].

Representation of quantitative estimates that demonstrate the implementation of the economic policy [2] makes it possible to control the progress of economic processes based on the totality of materials [3] from statistical reporting or based on the totality of quantitative indicators that are determined by standard or the established list [4]. Typically, volumes of data on the dynamics of economic indicators, which were accumulated and are systematically collected for studying social processes [5], management effectiveness, and corruption signs, are too large [6]. Their perception is especially complicated for ordinary citizens [7]. In this regard, the task on finding the ways of compact representation [8] requires its statement and solution. Not less important are similar problems for other subject areas, such as oil and gas transportation, fluctuations in currency exchange rate, etc.

In addition to the above, even though at present in the studies of the dynamics of socio-economic processes [9] the regulating documents mainly determine the systematic set of quantitative and qualitative indicators, homogeneous non-systematic data are also applied.

In relation to these facts, in order to create the reliable environment of quantitative indicators [12] in the course of the study and to construct the method of the compact integral representation, we will select data [13], obtained and published in official sources by external independent experts [14]. To analyze the trends and forecasts of tendencies and risks, we will choose long [5] and sometimes even twenty-year-long terms, within which the dynamics of indicators of public administration efficiency is observed, because such steps lead to increased reliability. Under these circumstances, like in most cases, we will apply the same titles for of indices and numerical data from the World Bank reports [14]. Furthermore, to assess the influence of the share of «shadow» economy, the data about its dynamics from such sources as Obozrevatel.ua – 2017 [7] and those that are given in comparison with the data on the corruption perception index (CPI) by Transparency International for 2012–2017 are selected. However, the increasing number of information sources of indicators leads to heterogeneity and violates the condition of comparability, which greatly complicates the awareness and transparency of perception even for trained

experts and systems of expert estimates. To avoid this kind of problems, according to ISO 37001:2016 «System of management of corruption combat measures, the list of priority indicators» that is represented by a time series [12] was quantitatively limited, but this neither solves the problem of transparency and simplicity, nor creates the conditions for prognostic estimates and conclusions.

In terms of the world globalization, the problem of the quantitative and qualitative analysis of a large number of numerical flows when working with metadata actualizes the task of constructing the estimation methods. One of them is to form an integral indicator by the totality of indicators – time series. The main obstacle that impedes the solution of the problem of the formation of the integral indicator is the absence of the model of the established law of quantitative relationship between the indicators. Direct application of multi-factor approximation methods to the solution of this problem is not possible, because quantitative data on the magnitude of the integral indicator and the kind of function of integral indicator are not known at the same time. In addition, the problem of transparent simultaneous representation of the selected totality of indicators for expert analysis has not been solved.

Today, the paradigm of «coordination», the theoretical basis of which was laid down in [15], is one of the most effective for the construction of the models and control of the organizational and economic systems. It should be noted that its application becomes successful only with a comprehensive selection of quantitative indicators. However, the methods for systematic analysis and mathematical statistics, which appear as an effective tool for constructing models of analysis and synthesis, not suitable for the integral estimation by the set of time series of indicators, including public administration. Due to the mentioned above, the practical problem of the formation of the method for construction of the integral indicator as the tool for assessing the efficiency of control and corruption in the society is attractive for analysis. The list of indicators in line with standard [12] includes: the management efficiency, regulation quality, political stability, absence of violence and the CPI index [13, 14]. The ability to solve the problem of the integral indicator to the formation of likely results is assessed in terms of the existence of a system of quantitative indicators according to the data of statistical reporting [13] that are recognized as reliable. There exist some examples of the successful construction of nonlinear models for integral estimation by the system of four or five types of indicators. For example, the deflection of functionals from quadratic forms, which are minimized [16] or represented by artificial and recurrent networks [17], or assess the integral indicator of adequacy [21], was constructed based on the known physical relation. Even though they do not apply directly to the solution of the problem of generalization of four time series, they are an example of solving the problem of integral estimate. It is impossible to search for and research into the influence of the factors as causes of a decrease in the indicators of development and the list of activities or inactivity of institutions, executive and legislative authorities using the methods of coordinative management. One of the reasons for this is the absence of the unified method, which makes it possible to unite the totality of time series into a single indicator – a time series.

In connection with the above, the research into the capability and usability of the methods of applied mathematics and the introduction of new cybernetic paradigms to the construction of an integral estimate is relevant. The implementation of the requirements of transparency, unambiguity and simplicity as the necessary requirements will make it possible to give objective information to the society through the media.

Thus, unlike the problem of approximation of time series, for which numerical data are known and the task is to establish the type of function and numerical coefficients, there arises a fundamentally new problem. Its essence and difference is in the need to construct the function, the form of which is unknown by the data of the time series, the procedure of determining and the enumeration of which was set by the standards or other regulatory documents.

2. Literature review and problem statement

Step-by-step moves toward market reforms, ensuring the structural reconstruction of economies are implemented when using modeling tools [8] and the assessment of corruption level [4, 5]. The absence of tools and practice of estimation of corruption dynamics, and it as the phenomenon inherent in transformational economies [4], are among the factors that hinder the deepening international economic and political integration and undoubtedly prevents the development [5].

Today, there are indexes, indicators of corruption control, proposed by the World Bank. Their time series show some effectiveness of the models of anti-corruption institutions [5]. However, despite this, rethinking the state strategies in this area after the formation of the new anticorruption institutions shows that there is a significant amount of controversy [9]. These controversies refer to both normative and legal support and organizational mechanism, and in particular the methods and criteria for evaluation of the corruption level [10, 11].

We will separate for the analysis only the latter – methods and criteria for evaluation of the corruption level [10, 11]. Based on the methodology of evaluation of the World Bank and the standard [12], we will limit the consideration only to three time series, which will involve corruption perception index (CPI) of Transparency International [13, 14]. Thus, the consideration involves four independent time series: governance effectiveness, regulation quality, political stability and absence of violence, and the CPI index.

The main unresolved contradiction and the problem of construction of the method for integral estimation is the absence of the known law of the relations between these four time series, as well as the lack of numerical data on the time series for the integral indicator.

In addition, it should be noted that changing the influence of the vectors of information policy is very important in these periods. In this regard, the need for and focus on the increasing access to the information concerning government services and indicators of corruption generates the need to determine the dynamics of quantitative indicators [6] by means of visual mapping. Its essential component, which also requires the quantitative estimation and transparent mapping, is the course of the budget process and public procurement for the state and municipal institutions, including those with credible declaration of the revenue of officials and the negative attitude to the corruption manifestation in the society. It is not possible to achieve positive changes and results in the implementation of these processes using

the old methods of step-by-step trial and errors, as well as the old tools [15, 16]. It is especially true for the conditions of limited gold-currency and energy resources, time limits and military aggression. Under such conditions, the solution of mutually related organizational, technical and economic problems requires modeling and mapping the results using the modern means [15–21].

Another unresolved contradiction is the inability of modern visual tools for both simple and transparent to understand mapping of the totality of indicators – time series. According to them, the nonlinear objects combined in systems and subsystems were generalized, and analysis of their functioning is reduced to the analysis of the quantitative generalized indicator [16]. That is why the direct application of the ideas proposed as early as in the 1970s and in the further developed papers [22, 23], requires their improvement. Recent papers that focus on the analysis of time series, for example [24, 25], demonstrate new results in comparison with the techniques for the construction of the models of the type of autoregression with running mean with endogenic variables. This procedure of the construction of mathematical models and the algorithm for the estimation of unknown parameters based on time series analysis has the advantages due to the fact that it uses the Bayesian approach [24], but its distribution on nonlinear models is complicated. The attempts to generalize the processes of construction of the models and prediction for nonlinear models of time series and numerical examples are found in article [26]. As shown in [26], the proposed generalized scheme of prediction of non-stationary interconnected time samples of the series is effective to describe almost currently known combined, hybrid or decomposition model. The success of its implementation is caused by the introduction of the identification based on the methods «Caterpillar»-SSA, a quick orthogonal search, the method of group consideration of arguments. However, despite its advantages over the models of autoregression with the running mean, such procedures do not make it possible to unite the models of time series into a unified generalizing indicator and to construct the model of a nonlinear object [27].

At present, there is an experience on the basis of which the common representation of nonlinear objects by the description paradigm was replaced with a super modern paradigm [16]. Their representation by the recurrent network, which extended the boundaries of its capabilities due to the application of the recurrent approximation method is not less effective [17]. The introduction of the comparative theory of the scalar multifactor evaluation including identification [19], together with the introduced three-level comparator [20], forms the basis to evaluate the errors of the model [20] and to control its construction based on the multifactor relevance criterion [21]. Such a formulation of the problem of the synthesis of controlling influence and application of the modern comprehensive approach to the control of the techno-economic systems was successfully implemented to solve specific technical problems [28–33]. However, to state and to ensure their solution, special criteria [28], tools [29], operation [30], which were represented by the unified analytical expression were used: effectiveness [31], adequacy [21, 33], vector-indicator [29–31], analytical learning [32] and calibration of the network [33].

Thus, the experience of implementation of papers [29–32] demonstrated that construction of the method that will ensure the formation of the expression of the integral criterion of the quantitative estimate and the illustration of the course of socio-economic processes for the determined list of indicators is decisive [29–31]. In this regard, to unite the benefits of the prescription control paradigm [16], representation of the integral criterion with the unified expression [17] is a reasonable and logically grounded problem. The development of the specified direction of mathematical substantiation of the cybernetic approach to the construction of the system of estimation of public administration effectiveness to combat corruption in the society, the control and information effectiveness will be ensured only due to the use of a special method for the formation of the integral indicator. Its implementation, together with the use of network technologies and analytical leaning [17], recurrent calibration [33] or graphical representation and presentation through the operation of matrix algebra opens up new opportunities and stimulates the search for new forms of representation of time series [34].

Generalization of the analysis of these sources makes it possible to state that the main unresolved contradiction and the problem of construction of the method for the integral estimation is the absence of the known law of connection between time series, as well as the absence of numerical data about time series for the integral indicator.

3. The aim and objectives of the study

The aim of this study is to construct a method for the synthesis of an integral estimate that represents it as a single expression for the specified list of four indicators – time series – and quantitatively evaluates, visually illustrates the course of social-economic processes, including estimates for combating corruption in the society.

To accomplish the aim, the following tasks have been set:

– to explore the applicability of the unified metrics to the construction of the trend of the general efficiency index based on the idea of evaluation of the lower boundary and to establish the properties of relative error for the constructed trend;

– to explore the applicability of the unified metrics to the synthesis of the trend based on the idea of representation in the space with rotating coordinate planes and to establish the relation between the dimensions of the sliding window, jumps of first and second derivatives of the original time series and generalized and permissible error.

4. Application of the unified metric to the construction of the trend of the general index of public administration efficiency

4. 1. Application of the unified metric to the construction of the trend of the general index of efficiency based on the idea of a lower boundary assessment

To specify and simplify understanding about the examined object and to avoid arguments about the number of indicators that are necessary to select for analysis, we will use the norm of a modern standard [12], the data by the World Bank and Transparency International about time series. This legislatively determined problem statement will make it possible to eliminate extra factors and will allow focusing on the merits of the essence of research into the aspects of applied mathematics and cybernetics.

To implement the approach to constructing adequate models [25, 26, 28] of techno-economic systems, we will normalize the indicators. The norm will be selected based on positiveness of estimates in conventional monetary units

and positiveness of estimates of government's actions. We will take into consideration the heterogeneity of the content of indicators – time series, which is selected as the basis today [21]: effectiveness of governance, regulation quality; political stability; absence of violence; and *CPI* index. In regard to the above, we will select, for each *i*-th time series, its own magnitude of the norm $|X_i|_{\max}$. However, taking the maximum value on the interval of the region of determining $|X_i|_{\max}$ as the norm (Table 1), it will be necessary to re-determine it during every change of the sampling interval and during the transition from one factor to another factor. The latter is definitely a downside to it, but this norm ensures high sensitivity to local changes of factors [21]. Let us assume that the estimates of indicators published in reports [13, 14] of the World Bank and Transparency International are reliable, then according to them, we will build the evaluation, which will be shown in TableWe will limit the consideration to the example published in reputable data sources for one country [13, 14], because the data themselves do not play a substantial role. This approach is substantiated by the fact that this paper explores the possibilities of the construction of the method of integral estimate rather than dynamics of the corruption level.

We will introduce the designation for the integral index *GIE*. Let us note that its role may also be defined as a synonym of the name «general index of efficiency» by analogy to the name that is used by standard ISO 37001:2016 and the data from the World Bank and Transparency International. According to the purpose of the study, it is necessary to construct the method, which makes it possible to find the function that establishes the relations between the integral *GIE* index and normalized dimensionless indicators: governance efficiency, regulation control – X_1; political stability – X_2; absence of violence – X_3; *CPI* index – X_4. We will designate the sought for function as follows: $GIE=f(X_1, X_2, X_3, X_4)$.

We will apply the general approach used in problems of applied mathematics, functional analysis, studies of operations and natural sciences – method of assumptions. The main statements of postulates will be selected:

– at zero result of implementation of management activity – governance efficiency – its integral indicator (general index of efficiency) *GIE* is zero;

– at zero management quality (regulation), its integral indicator (general index of efficiency) GIE is also zero;

– under conditions when each of the factors at the same time is equal to zero, integral indicator (general index of efficiency) *GEI* is equal to zero.

We will implement the subsequent search within these assumptions. Based on these statements, the general index of efficiency *GIE* will be developed into a Maclaurin series, which will be represented in linear approximation:

$$GIE=\sum_i^N \frac{\partial GIE}{\partial X_i}\bigg|_{x_i=0} \Delta X_i=\sum_i^N \frac{\partial GIE}{\partial X_i}\bigg|_{x_i=0} X_i=C\sum_i^N \delta_i X_i;$$

$$C=\left(\sum_i^N \frac{\partial GIE}{\partial X_i}\bigg|_{x_i=0}\right);\ \ \delta_i=C^{-1}\frac{\partial GIE}{\partial X_i}\bigg|_{x_i=0}. \qquad (1)$$

According to the properties of geometric inequality, we will estimate the lower boundary of the general index of efficiency:

$$GIE=C\sum_i^N \delta_i X_i \geq C\prod_{i=1}^N \left(X_i\right)^{\delta_i}.$$

The latter makes it possible to obtain the estimate of dynamics of indicators of development of governance processes with precision to the constant multiplier. Accepting this group of four indicators of analysis in first approximation as equivalent and for metrics $N^n=\sum_{i=1}^n |X|_i^n$, the estimate of the general index of efficiency according to the geometric inequality will be presented as:

$$GIE=\frac{1}{4}\left(|X_1|^4+|X_2|^4+|X_3|^4+|X_4|^4\right)\geq X_1X_2X_3X_4.$$

For the case when the impact of various factors is not equal, but determined by weight coefficients, the least value of effectiveness can be estimated using the modified form of the same geometric inequality:

$$\left(|X_1|^4+|X_2|^4+|X_3|^4+|X_4|^4\right)\geq$$
$$\geq\left(\frac{1}{k_1}\right)^{k_1}\left(\frac{1}{k_2}\right)^{k_2}\left(\frac{1}{k_3}\right)^{k_3}\left(\frac{1}{k_4}\right)^{k_4}X_1^{4k_1}X_2^{4k_2}X_3^{4k_3}X_4^{4k_4}.$$

Thus, due to the properties of the power function, the resulting expression gives the upper and lower boundary of efficiency indicator, which taking into account the designations of the CPI index, we will write down:

$$X_1X_2X_3X_4\leq GIE\leq 4X_1X_2X_3X_4. \qquad (2)$$

Table 1

Indicators of state administration according to data [13, 14] and based on an integral estimate

Indicator	1996	2009	2010	2011	2012	2013	2014	2015	2016
Governance efficiency [14]	28	22	24	21	32	31	40	35	32
$X_1$10	7.	5.5	6	5.25	8	7.75	10	8.75	8
Regulation quality [14]	34	32	34	30	30	30	29	30	36
$X_2$10	9.4	8.9	9.4	8.33	8.3	8.33	8.06	8.33	10
Political stability and absence of violence [14]	43	34	45	44	42	21	6	5	6
$X_3$10	9.56	7.56	10	9.78	9.33	4.67	1.33	1.11	1.33
Index *CPI* [13]	–	–	–	–	26	25	26	27	29
X_4	–	–	–	–	0.897	0.86	0.897	0.93	1
Index *GIE*	–	–	–	–	0.694	0.3499	0.1198	0.087	0.106

Thus, based on the introduced norm, the determined weight coefficients and the geometric inequality of the above, the estimation of the upper and lower boundaries of the general index *GIE* (2) was substantiated. Such a wide interval gives an advantage to the pessimistic assessment – lower boundary, which subsequently should be used as an estimate of the general index of efficiency *GIE*.

To assess the impact of errors in the estimation of output indicators, we will designate their relative magnitudes as ε_i. Under these conditions, the relative error of the integral indicator will be determined as a relative error of indirect measurement:

$$\Delta GIE = GIE\sum_{i=1}^{4}\frac{\Delta X_i}{X_i} = GIE\sum_{i=1}^{4}\varepsilon_i. \quad (3)$$

Due to the selected norms, the highest maximum possible value of the error of integral indicator will be determined as the sum of magnitudes of relative errors. To determine the impact of the magnitude of the interval of averaging and dynamic properties of the integral indicator on the local value of error, using the quadratic form of the expansion in the Taylor ranks in the neighborhood of moment t_0 and write down:

$$\Delta GIE = \left(\begin{array}{l} GIE_0 + \sum\limits_{i=1}^{4}\dfrac{\partial GIE}{\partial X_i}\dfrac{dX_i}{dt}\Big|_{t_0}\Delta t + \\ + \sum\limits_{j=1,i=1}^{4,4}\dfrac{\partial^2 GIE}{2\partial X_j \partial X_i}\dfrac{\partial X_j \partial X_i}{dt^2}\Big|_{t_0}\Delta t^2 \end{array}\right)\times$$
$$\times\sum_{i=1}^{4}\left(\varepsilon_{i0} + \frac{d\varepsilon_i}{dt}\Big|_{t_0}\Delta t + \frac{d^2\varepsilon_i}{2dt^2}\Big|_{t_0}\Delta t^2\right). \quad (4)$$

Thus, the averaging interval, together with the dynamic properties of the integral indicator and the estimation of the error of impact factors stated the requirements for the maximum time of estimation of each of the magnitudes of indicators. Expansion (4) also establishes the general relationship between the size of the sliding window, jumps of first and second derivative of the generalized time series, and admissible error of the integral indicator. To set the expressions for practical calculations, we will assume that time is a continuous magnitude, error ΔGIE is integral together with the square, we will select the sliding window of the width $\delta\tau$ and introduce the norm:

$$\|\Delta GIE\| = \frac{1}{\delta\tau}\left[\int_{t_0-\delta\tau/2}^{t_0+\delta\tau/2}\Delta GIE^2\,dt\right]^{1/2}. \quad (5)$$

Applying norm (5) to equation (4) we will obtain:

$$\left|GIE_0 + \frac{dGIE}{dt}\Big|_{t_0}\Delta t + \frac{d^2GIE}{2dt^2}\Big|_{t_0}\Delta t^2\right|_{\min}\times$$
$$\times\sum_{i=1}^{4}\left\|\varepsilon_{i0} + \frac{d\varepsilon_i}{dt}\Big|_{t_0}\Delta t + \frac{d^2\varepsilon_i}{2dt^2}\Big|_{t_0}\Delta t^2\right\| \le \|\Delta GIE\| \le$$
$$\le\left|GIE_0 + \frac{dGIE}{dt}\Big|_{t_0}\Delta t + \frac{d^2GIE}{2dt^2}\Big|_{t_0}\Delta t^2\right|_{\max}\times$$
$$\times\sum_{i=1}^{4}\left\|\varepsilon_{i0} + \frac{d\varepsilon_i}{dt}\Big|_{t_0}\Delta t + \frac{d^2\varepsilon_i}{2dt^2}\Big|_{t_0}\Delta t^2\right\|. \quad (6)$$

The last inequality is equivalent to the expression that determined the upper and lower boundary, which limit the value of the norm of a relative error and associate it with the properties of the formed time series for the pessimistic estimation of the integral indicator and the length of the sliding window:

$$\frac{GIE_0}{\left|GIE_0| + \left|\frac{dGIE}{dt}\Big|_{t_0}\Delta t\right| + \left|\frac{d^2GIE}{2dt^2}\Big|_{t_0}\Delta t^2\right|\right\|_{\max}}\times$$
$$\times\sum_{i=1}^{4}\left\|\varepsilon_{i0} + \frac{d\varepsilon_i}{dt}\Big|_{t_0}\Delta t + \frac{d^2\varepsilon_i}{2dt^2}\Big|_{t_0}\Delta t^2\right\| \le$$
$$\le\frac{\|\Delta GIE\|}{\left|GIE_0| + \left|\frac{dGIE}{dt}\Big|_{t_0}\Delta t\right| + \left|\frac{d^2GIE}{2dt^2}\Big|_{t_0}\Delta t^2\right|\right\|_{\max}} \le$$
$$\le\sum_{i=1}^{4}\left\|\varepsilon_{i0} + \frac{d\varepsilon_i}{dt}\Big|_{t_0}\Delta t + \frac{d^2\varepsilon_i}{2dt^2}\Big|_{t_0}\Delta t^2\right\|. \quad (7)$$

Thus, under conditions of existing possibilities to regulate the sizes of the sliding window, its magnitude should be chosen taking into consideration the jumps of first and second derivative of the generalized time series and admissible error.

4. 2. Application of the unified metric to the synthesis of a trend based on the idea of representation in space with rotating coordinate planes

To implement the second approach to constructing adequate models that generalize the set totality of four time series, we will also apply normalization of indicators. Based on the fact of positiveness of estimates, which are realized in conventional monetary units and positiveness of estimates of actions of public administration, we will select one of the possible Euclidian norms: the maximum value on the interval.

For the further presentation of the idea, we will assume that we have a common axis of time for the whole system of four factors that are legislatively determined at present [12]. Under these conditions, the general expression of the *n*-th indicator as a time series for mapping into the plane, after multiplying by multiplier $\exp(i\pi n/2)$ will be turned by angle $\pi n/2$:

$$X'_n(t) = \exp\left(\frac{i\pi n}{2}\right)X_n(t),$$

where *i* is the imaginary unit.

The choice of the angle values from inequality $\alpha < \pi/2n_{\max}$ makes it possible to introduce time series in the form of the totality of displays in the first octant. The latter allows making quick express reviews at acute or blunt angles during the movement of the sliding window simultaneously for the whole selected totality of indicators. Let us represent such totality as follows:

$$GIE = \prod_{n=0}^{3}X'_n(t) = \prod_{n=0}^{3}\exp(i\alpha n)X_n(t). \quad (8)$$

It should be noted that the proposed method is simple and clear in representation in 3D modeling. However, the formed time series has the deformed magnitude of absolute error, but the magnitude of local relative error when

displaying the integral series or the components of time series is retained. To ensure consistent terminology, we will introduce the definitions.

Definition 1.

The quarter space that was formed as the intersection of semi-spaces and limited by two positive directions of the time axis and the indicator axis, which was turned counterclockwise by the angle of 90 degrees, will be called the plane of representation of time series.

Definition 2.

The first plane in the selection in the counterclockwise direction among the representation planes with the common time axis will be called the reading plane.

Definition 3.

Dihedral angle between the reading plane and the plane of representation of time series that was counted in the counterclockwise direction will be called rotation angle.

Definition 4.

The pane of representation, turned by the rotation angle of 90 degrees, will be called the plane of mapping of time series.

Theorem. The choice of the value of rotation angle α by the magnitude that was selected from range $[\alpha] < \alpha < \pi/2n_{\max}$, ensures retaining the relative error for all the components of displays of components of numerical series regardless of a change in the order of presentation and the magnitude of the indicator.

The proof. We will represent the increment of general multiplier $\exp(i\alpha n)X_n(t)$ of integral indicator, represented by expression (8), as follows:

$$\Delta\left[\exp(i\alpha n)X_n(t)\right] = \exp(i\alpha n)\frac{dX_n(t)}{dt}\Delta t.$$

Under these conditions, absolute error and the general multiplier have common multiplier $\exp(i\alpha n)$. The rate of changing the general multiplier was multiplied by the increment of time as the derivative of the scalar, and the magnitude of general multiplier always match by direction since both magnitudes are scalars. In the case of displaying these two magnitudes as a scalar sum onto the representation plane, these two summands are projected onto it. The latter is equivalent to multiplication by cosine of the same rotation angle and does not change their ratio:

$$\frac{\Delta\left[\exp(i\alpha n)X_n(t)\right]}{\exp(i\alpha n)X_n(t)} = \frac{\exp(i\alpha n)\frac{dX_n(t)}{dt}\Delta t}{\exp(i\alpha n)X_n(t)} = \frac{\frac{dX_n(t)}{dt}\Delta t}{X_n(t)} = \frac{\Delta X_n(t)}{X_n(t)} = \varepsilon_n(t) \quad (9)$$

since they are the common multiplier.

The consequences of the theorem are:

– first, in the range of values $[\alpha] < \alpha < \pi/2n_{\max}$, there are no selected values of angles of rotation of the plane of representation;

– second, the representation quality is determined by the angle of sensitivity threshold $[\alpha]$ and resolution of the tool of visual analysis of represented mappings.

In addition, it should be noted that the application of the quadratic norm as one of four norms, which is applied in Euclid spaces, proves that representation of the general estimate of time series in the form of (8) does not change the local error of output series. We will prove the last statement by direct calculation, determining the components by (9):

$$\varepsilon_{GIE}(t) = \left[\sum_{n=1}^{4}\varepsilon_{GIEn}^2(t)\right]^{1/2}. \quad (10)$$

If the magnitude of the indicator is normalized, its maximum value is equal to unity. Under these conditions, the value of the maximum possible absolute error is also equal to unity, and thus the normalized with the same norm of the value of relative maximum possible error is equal to unity. Then the estimate of error by expression (10) gives 2. The estimate of the maximum possible error by expression (1) will be 4, and hence, the synthesis of the form of presentation of the indicator by expression (8) is more effective.

The use of the operation of rotation by the angle makes it possible to represent information in a compact way, but such vector representation leads to an increase in resulting relative error. Its magnitude lies within the interval [2, 4].

5. Discussion of modeling results and further development

As a result of modeling, the lower boundary was calculated from formula (2) of the general index of efficiency *GIE*, the results of which are shown in the bottom line of Table 1. Analysis of dynamics of the general index of efficiency demonstrates that for the data regarding Ukraine in 2015, it reached its lower limit of 0.08702, and in 2016 it began to grow and increased by magnitude of 0.019 reaching the value of 0.106667. The latter makes up almost twenty percent of the magnitude of the *GIE* indicator of 2015. Its estimated value will be 0.13784 at the end of 2017 and will reach value 0.144 ± 0.09 at the end of 2018.

Synthesis of the estimate that was performed based on the proposed method for the data of the World Bank and Transparency International, is not the only possible area of application. Such integral estimates can be spread to other time series under conditions when they in their totality describe one object. Completing this paper, it should be noted that the validity of the method is difficult to assess when the reliability of the input information is unknown. However, in this case, an indirect estimate is important and obvious. Thus, the International Monetary Fund, evaluating the government activity, came to a similar conclusion about the increase in indicators. The decision of the Fund «to allocate the loan support to Ukraine» should be assessed as an indirect feature that by the trend is consistent with the growth in the integral indicator of the index of efficiency *GIE* (Table 1).

No doubt, since the aim of this work was to synthesize a simple and transparent method for construction of the estimate of the integral indicator – the general index of efficiency *GIE*, the analysis relied only on the known numerical values. The introduction of any existing transformations of these data, firstly, brings in an additional error component to be estimated, secondly, decreases transparency of calculations. Transition to a description of these parameters by using the theory of fuzzy sets seems problematic. Despite the examples of effective application to financial analysis, which are given in article [35] or its other sophisticated applications to description, prediction and optimization of nonlinear objects of drones-transformers [36], such transition significantly complicates calculations, because it requires the

existence of effective methods of construction of membership functions. The latter is caused by the level of development of modern methods for selection of laws for approximation and ensuring its adequacy [12].

It is obvious that the direct application of the temporal trends of certain indicators and indicators that were defined by international standards [12] is a complex task. Substantiation of the type of function and the search for approximation parameters, including the least squares method [37] or the method of group considering the argument [38], today is the problem, solved for a limited interval. The results of the application of spline approximation [39] or recurrent approximation [40] along with the use of the idea of maximizing conformity [21], or the comparative idea of identification of the product by Kolmogorov-Gabor polynomials [19] does not simplify the problem either. Due to the presented results of the synthesis of the estimate by the unified expression of the time series of integral indicator, providing a simple spatial representation of group information about the components of time series and determining the conditions of the absence of a curve of a local relative error, one should expect the following directions of development. Above all, it is the development of inter-transformation tools and agreement of deductive and inductive thinking with informal images [41, 42]. Secondly, the introduction of the multilevel monitoring [43] with the simultaneous search for and implementation of error-control modeling [44] will open the way to the formation of effective methods for the evaluation of the integral indicator in the problems of public administration. One should expect that the search for new possibilities of using the methods of approximation of functions with constraints [45] and success of separate applications to the solution of technical problems [46] along with the introduction of the Chebyshev approximation [47] play an important role in the formation of methods for integral estimation by time series data.

Thus, the proposed method for the estimation of the general index of efficiency of public administration to combat corruption in the society, *GIE*, has some advantages in comparison with the other specified possible approaches due to their simplicity and clarity. However, it is necessary to expect that during the high detailing of analysis, that is, increasing the number of factors of analysis by more than four, this method starts losing sensitivity to the influence of particular factors as a result of comparability of the maximum possible error with the magnitude of increment in the integral indicator *GIE*. Under these conditions, the search for the methods of direct application of recurrent networks together with the use of the algorithm of analytical learning [17], the idea of adequacy maximizing [16], tools of the vector-indicator [17], will obviously make it possible to construct the algorithm of description of the fragments of the past of time series in the form of a black box. It is expected that under these conditions, trends will have a lower error and greater sensitivity, but will not make it possible to predict their future course.

7. Conclusions

1. The method of integral estimate that represents it with a unified expression for the specified list of four indicators – time series, and can serve as a general index of efficiency of control of the indicator-index was substantiated. The uniform norms that lead all of the impact factors to comparable conditions were applied. Even though the original impact factors are different in dimensions and physical nature, the time series that were formed from them are dimensionless magnitudes of the determined limited range. Non-dimensionality and limitation of the integral indicator provides for the representation and transparent mapping (without distortion of the magnitude of relative error) to form a conclusion on the state of progress of the actions and forecasting results.

2. A pessimistic estimate or an estimate of the minimum value of the integral indicator of efficiency of public administration is represented as the product of the basic system of four normalized indicators that were determined by the standard. Representation of the integral indicator by the idea of assessment of the lower boundary in the form of the product reduces the error by two times. Dynamic properties of the integral indicator, the averaging interval, estimates of the error of impact factors generate the requirements to a maximum time of evaluation of each of the output indicators in the form of constraints of inequalities. Such constraints link the requirements for the accuracy of integral indicator and quadratic norms of properties of the original time series. Representations in space with rotating coordinate plates make it possible to establish a link between the size of the sliding window, jumps of first and second derivatives of generalized time series and the admissible error. This representation on the plane of mapping makes it possible to represent the information in a compact way, but the vector representation leads to an increase in the resulting relative error. Its magnitude lies in the range [2, 4], which is still less than the maximum possible error of the sum of four indicators, which is equal to 4.

3. The research into dynamics of integral indicator using the proposed method based on data from 2014–2017 according to reports of the World Bank and Transparency International has revealed the capability of the formed method to reproduce in practice the qualitative correspondence with their general conclusion. The direct application of the developed method to calculations has made it possible to obtain a prognostic value for the integral indicator of 0.13784 by the end of 2017, and, by the end of 2018, it would equal 0.144 ± 0.09.

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