Проаналізовано теоретичні особливості захоплюючої здатності валків при простому процесі прокатки в сталому режимі. Показано, що в залежності від умов деформації відношення максимального кута захоплення до коефіцієнту тертя може дорівнювати, бути меншим або більшим 2.

Проведено експериментальне дослідження, в якому здійснювали прокатку свинцевих ступінчастих зразків з вимірюванням випередження на кожній сходинці. Результати досліду показують, що в граничному випадку деформації випередження більше нуля, тобто є достатній запас сил тертя для подальшого збільшення обтиснення, але це неможливо.

Проаналізована рівновага всіх горизонтальних сил в осередку деформації. Показано, що в кожному перетині втягуючі контактні горизонтальні сили витрачаються не тільки на подолання виштовхуючих, але і на врівноваження поздовжніх внутрішніх сил, які виникають в результаті пластичної деформації металу.

Розроблено силовий метод оцінки поздовжньої сталості процесу тонколистової прокатки. Показником сталості є критерій, який визначається з епюр розподілу нормального контактного напруження та напруження тертя. Показано, що при позитивному значенні критерію процес прокатки відбувається в сталому режимі, при негативному значенні – сталий процес неможливий та у разі його нульового значення – настає граничний випадок деформації.

Проведено теоретичне дослідження з визначення максимального кута захоплення за різних умов тонколистової прокатки. Показано, що відношення максимального кута захоплення до коефіцієнту тертя практично не залежить від товщини штаби, діаметру валків і коефіцієнту тертя та дорівнює 1,43–1,44. Зменшення захоплюючої здатності валків пояснюється дією в осередку деформації, крім контактних, ще і внутрішніх сил

Ключові слова: рівновага сил, тонколистова прокатка, осередок деформації, захоплююча здатність, критерій сталості

UDC 621.771.011

DOI: 10.15587/1729-4061.2019.160487

CONSTRUCTION OF A FORCE METHOD FOR ESTIMATING THE LONGITUDINAL STABILITY OF THE PROCESS OF THIN SHEET ROLLING

R. Romaniuk
PhD, Associate Professor*
E-mail: r22roma@i.ua

K. Levchuk
PhD, Associate Professor*
E-mail: r22romar@gmail.com

Yu. Hasylo
PhD, Associate Professor
Department of Welding Technology and Equipment**
E-mail: hasylo_yurii@i.ua

D. Chasov
PhD, Associate Professor
Department of Mechanical Engineering**
E-mail: 0969995009@ukr.net

O. Kriukovska
PhD, Associate Professor*
E-mail: olga.krukovska@i.ua

*Department of Labor Protection and Life Safety**
**Dniprovsk State Technical University
Dniprobudivska str., 2, Kamenskoe, Ukraine, 51900

1. Introduction

One of the important technological parameters that defines the stability of the rolling process is the gripping capacity of rolls. There are conditions for gripping a metal with a working tool at the initial moment when the strip edges touch rollers and under a steady mode.

Ensuring reliable and stable grip is a mandatory condition for the rational technological process. Increasing the gripping capacity of roll makes it possible to not only improve the performance of rolling mills, but also improve the workability of metal, which reduces the number of internal defects.

Disruption in the longitudinal stability of the rolling process leads to strip skidding in the rolls, and in some cases it is the cause of major accidents at rolling mills.

In the theory of rolling there is the notion of «longitudinal stability of the process.» It is associated with the gripping capacity of rolls under a steady mode and the boundary gripping angle $\alpha_y^{\max}$. As it is known from the theory of rolling, the condition for gripping a metal with rollers under a steady mode takes the following form:

$$\alpha_y \leq 2 f_y, \quad (1)$$

where α_y is the gripping angle under a steady mode; f_y is the friction coefficient at a deformation site.

This inequality was derived from the condition that the sum of contact stretching forces is sufficient to overcome the sum of contact ejecting forces. In this case, the following assumptions are accepted:

– normal contact stresses at a deformation site are averaged;

– friction stresses are proportional to the normal contact stresses and are defined by a Coulomb model;

– the rolling is flat, that is, there is no strip enlargement at a deformation site.

This condition indicates that the maximum gripping angle cannot exceed the double magnitude of the coefficient (or angle) of friction under a stable rolling process.

In a more general form, inequality (1) can be represented as follows:

$$\frac{\alpha_y^{\max}}{f_y} = n, \qquad (2)$$

where n is the coefficient, which characterizes the inclination angle of the resultant of a normal contact stress.

Therefore, formula (1) was derived under important assumptions, and practical significance of the boundary gripping angles in practice is very high, for example, when developing modes of deformation at rolling mills.

In addition, as it is known, the process of sheet rolling proceeds with tensions (front and rear). Therefore, it is not appropriate to apply ratio $\alpha_y^{\max}/f_y$, as an indicator for the stability of this process.

Thus, it is a relevant task to undertake a study aimed at the further forward creep of theoretical bases for determining the stability of deformation during sheet rolling. Ultimately, the improved method would make it possible to reliably determine the longitudinal stability of the process when developing and advancing deformation modes at industrial sheet mills.

2. Literature review and problem statement

To determine the factor n in formula (2), different authors conducted a significant amount of theoretical and experimental research. We shall address some of them in detail.

The theoretical study, reported in [1], shows that under uniform distribution of the normal contact stress at a deformation site, $n=2$. If the resultant of these stresses deviates towards the release of a metal from rollers, then $n>2$, if it deviates towards the intersection of entering the deformation site, then $n<2$.

When analyzing the equilibrium of contact forces at a deformation site, the authors obtained a boundary condition for a stable rolling process, which takes the following form:

$$\alpha_y^{\max} = 1.41 f_y. \qquad (3)$$

However, it has not been widely used as some researchers criticized it pointing to the incorrect analysis of the scheme of acting forces, though the author stuck to his point of view.

It was concluded in the studies, when using different theoretical models of friction, that under a boundary case of rolling, $n>2$.

Experimental data by several researchers also give reason to assume that the value for coefficient n may significantly differ from two. The results of these studies are also described in paper [1].

Thus, in the experiments involving the rolling of nickel ingots, $n=1.16$, and at deformation of copper – $n=1.22$.

The experiments, which were carried out at the blooming mill 1150 of Novo-Tagil steel plant (Russia), involved the rolling of steel bars on a smooth barrel and in caliber. Based on the results from these experiments, the magnitude of n changed within 1.25–1.35.

According to studies, when rolling blooms in rolls with a rolling diameter of 886 mm, the n ratio was 1.37 at regular gripping, and 1.42 – with a pre-compression on a cone [1].

There are known experiments, which examined the gripping capacity of rolls under laboratory conditions. The experiments were carried out at a two-roll mill at a rolling speed of 0.3 m/s. The maximum gripping angle was determined by rolling lead stepped-wedge-shaped samples based on the magnitude of reduction at which the first spins occur. The ultimate thickness of strips changed in the range from 0.5 mm to 50 mm; the initial thickness of the samples was chosen accordingly. The results of experiments showed that the coefficient n was within 1.5–1.9 [1].

In other experiments, when rolling wedge-shaped and rectangular lead samples, the ratio $n=1.4$–1.45 was obtained [1].

Such a discrepancy between experimental data and the accepted theoretical conditions questions the legitimacy of certain provisions from the theory of rolling and requires their refinement. It primarily refers to equality (1), according to which the boundary gripping angle is determined only by the magnitude of coefficient of external friction. But the experiments, which were reported in [1], show that the gripping capacity of rollers depends on the thickness and width of deformed metal as well.

Thus, the results from theoretical and experimental studies reveal that the theoretical condition (1) does not always correctly reflect the actual pattern for the longitudinal stability of the rolling process. Experiments established that the boundary gripping angle under a stable mode is affected not only by the conditions of friction in a contact between a tool and a metal, but by the geometric parameters and the strip strained state at a deformation site as well.

Given the above, we can conclude that the theory of gripping the strip with rolls requires further forward creep and elucidation of some provisions.

Paper [1] also describes cases of the force balance loss at a deformation site with a significant strip's forward creep, followed by its subsequent skid in rolls. It would seem that a deformation site has a sufficient reserve of friction forces for a stable process, however, a small increase in the gripping angle leads to sample skidding.

There is no any convincing explanation for the results of these experiments in the theory of rolling, although there are some assumptions. For example, the loss of metal equilibrium in rolls in the presence of a forward creep is due to a plastic stretching of the strip owing to the action of longitudinal stretching stresses σ. These stresses reach the limit of metal strength σ_T, the result is its grip with rollers is disrupted and there occurs the skidding of a strip, even in the presence of the forward creep.

However, if $\sigma=\sigma_T$, the average pressure of metal on rolls will be smaller than the forced strength limit, which is energetically impossible at simple rolling process. In this regard, the proposed explanation is not convincing.

We shall analyze known experimental data in more detail. Fig. 1, *a* demonstrates results from an experimental study at rolling lead samples of thickness 11.4–12.7 mm and a width of 50 mm for the resulting thickness of 1.0–1.1 mm [1, 2]. The rolling was performed on steel dry rolls with a diameter of 194.6 mm, $f_y \approx 0.25$. The forward creep was measured by the method of core prints.

As noted in [1], the experiments revealed the following: when rolling a sample with reduction $\Delta h=11.2$ mm ($\alpha_y=0.34$ rad), the forward creep was $S=10.1$ %, and at a slight increase in reduction to $\Delta h=11.6$ mm ($\alpha_y=0.35$ rad) another sample started skidding in the rolls. Thus, there was a loss of equilibrium of metal in rolls at the forward creep close to 10 % and $\alpha_y=0.35$ rad.

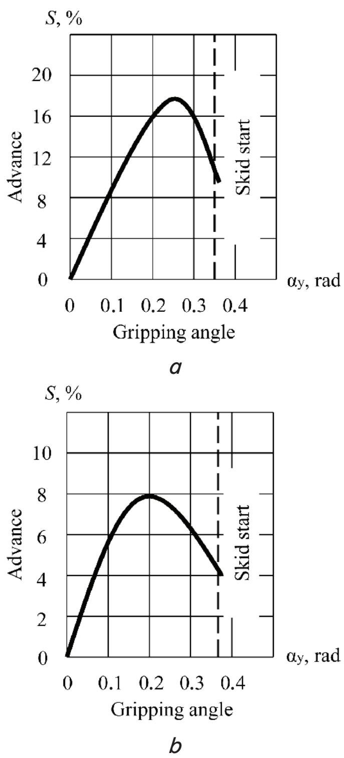


Fig. 1. Dependence of forward creep on gripping angle based on experimental data: *a* – the ultimate sample thickness 1.0–1.1 mm; *b* – the ultimate thickness of samples 3.75 mm

Similar experimental data were acquired when deforming lead stepped samples for the resulting thickness of 3.75 mm. The results of these experiments are shown in Fig. 1, *b*. The rolling was performed in steel rough rolls with a diameter of 210 mm, $f_y \approx 0.28$. The height of the last step corresponded to the boundary reduction at rolling a wedge-shaped sample. As can be seen, the loss of strip equilibrium occurred at an forward creep of about 4 % and gripping angle $\alpha_y = 0.37$ rad (Fig. 1), that is, gripping for the conditions of friction (1) is not achieved. Paper [1] also argues that the cause for the loss of strip equilibrium in the presence of forward creep is the significant longitudinal stresses that occur at rolling with gripping angles that are larger than the coefficient of friction.

Given that the above experiments are unique, and will be subject to further analysis, while the theory does not provide their convincing explanation, we performed our own experimental research involving determining the forward creep by a core method. The results of these experiments are given below.

Thus, we note that during experimental studies, considered above, one observed, at rolling lead thin samples, a loss of strip equilibrium in rolls in the presence of the forward creep. In this case, there are doubts about the fact that in all experiments the longitudinal stretching stress σ would reach the limit strength of the deformed metal. Therefore, the explanation that a strip slows down in the rolls under condition $\sigma = \sigma_T$ is, probably, not valid.

Even though the above experimental data were obtained at the end of the last century, there has not been any reasonable explanation for them up to now.

Next, we shall deal with studies into stability of the sheet rolling process in the longitudinal and transverse directions.

Paper [3] proposed a theoretical condition for the course of a stable process of sheet rolling in the longitudinal direction. However, it takes into consideration only the kinematic parameters for a deformation site and indicates that the range of possible values for parameter γ_c/α_c is limited to the length of a plastic contact between the strip and the roll, more precisely, by the presence at the same time of plastic regions of forward creep and lagging. This condition takes the form:

$$\frac{x_{1n}}{l_c} - \frac{x_1}{l_c} < \frac{\gamma_c}{\alpha_c} < 1 - \left(\frac{x_1}{l_c} - \frac{x_{0n}}{l_c} \right), \qquad (4)$$

where γ_c is the angle that describes the position of a neutral intersection, rad.; α_c is the angle of an elastic-plastic contact between the strip and the roll, rad.; l_c, x_1 are, respectively, the length of the arc of contact between the strip and the roll and its increment along the line that connects the centers of rotation of the rollers, which is a consequence of the elastic compression of rolls and an elastic recovery of the strip, mm; x_{0n}, x_{1n} are, respectively, the increase in the length of arc of the contact between a strip and the roll that is the result of elastic compression and elastic recovery of the strip, mm.

In the course of a theoretical analysis of a given condition, work [4] concluded that at a maximal value for the gripping angle, $\alpha_c^{\max}$, in this case, $q/2k = 0.53 - 0.55$, it probably shows under which parameters for the process there is a fracture in the strip, that is, the stable rolling process is disrupted significantly earlier than it follows from expression (4).

Experimental data that are also given in [4] indicate that the optimal values for relative specific tensions are in the range:

$$0.17 \le \frac{q}{2k} \le 0.49, \qquad (5)$$

where q is the specific tension of the strip, MPa; k is the deformation resistance to pure shear, MPa.

The practical operation of industrial rolling mills shows, in contrast, that $q/2k \approx 0.15$.

Thus, as it follows from the data above, there are significant differences in the conditions for the course of a stable process of sheet rolling.

Issues on ensuring the quality of a rolled thin sheet were addressed in [5, 6], and in others. A study into the instability of strip section at wide-strip mills of hot rolling and the ways for its stabilization was reported in [7, 8]. Some tasks on modeling and automated control over thickness, tension, and flatness of strips were successfully resolved in papers [9–11].

An interesting forward creep is a technique for regulating the cross thickness of a thin strip by using roll profiling was proposed in [12]. As shown by results from the experimental rolling, a deviation in the cross thickness of silicon steel decreased to 8 μm owing to a given method. It is obvious that the profiling of rolls requires additional costs, it can therefore be applied only to critical components.

Issues on the dynamic fluctuations of strip in the rolls, as well as working rolls, were examined in papers [13, 14]. They suggested theoretical models that make it possible to predict these phenomena when setting up equipment for sheet rolling. Study [15] shows that it would not do to ignore the effect of strip's transverse displacement as well. In that work, the authors studied the effect of tension and taper angle of the roll on a lateral flow of metal by drawing a grid on the surface of samples.

The impact of dynamic phenomena on the kinematic, force, and energy parameters of the process of sheet cold rolling was considered in study [16]. It is shown that different variations of these parameters negatively affect the operation of equipment and the quality of finished products.

Thus, there are significant forward creeps in the theory and practice of ensuring the required geometry and flatness of the thin sheet, the transverse displacement of the strip in rolls, the dynamics of the deformation process. However, there are the remaining unresolved aspects that relate to the longitudinal stability of a simple process of rolling and the deformation by tensions.

The reason for this is probably the lack of interest from industrial specialists in improving the modes of deformation at rolling mills. For it is much easier to use the data provided by equipment manufacturers, given in technological manuals or technical literature.

As regards the scientists, insufficient attention to the issues on the longitudinal stability of sheet rolling process can be explained by the inadequate financing of their studies and by weak ties with enterprises at which they could undertake their experimental research.

One of the directions for improving the reduction modes at thin sheet rolling mills is to increase the front and rear tensions during deformation in order to reduce energy costs of the process. This is very important because it could help bring down the cost of products and improve an enterprise competitiveness. However, there may occur the slipping of metal in the rolls at the same time, which would lead to emergencies.

Therefore, it is expedient to continue developing the issue on the longitudinal stability of the thin sheet rolling process; this is the aim of our study.

3. The aim and objectives of the study

The aim of this study is to devise a method for estimating the longitudinal stability of deformation based on the force interaction between a metal and rollers. This would make it possible to improve the accuracy of determining the stability of process when developing and improving the modes of rolling at industrial thin sheet mills.

To accomplish the aim, the following tasks have been set:

– to perform a theoretical analysis of the interaction between a metal and rollers during thin sheet rolling and to devise a procedure for determining the horizontal contact forces and the longitudinal internal forces caused by the plastic deformation of metal;

– to define a criterion under which it is possible to assess the stability of a simple process of sheet rolling that could be used for the modes of deformation with tensions as well;

– to investigate, based on the theoretical treatment of contact stresses diagrams, the reliability of determining the boundary conditions for rolling based on a given criterion;

– to study theoretically the occurrence of the boundary conditions for a simple process of sheet rolling considering the equilibrium at a deformation site of the contact and internal forces.

4. Materials and methods to study stability in a simple process of thin sheet rolling

4. 1. Experimental study to determine the boundary conditions of the process of rolling

The experiments were conducted at a laboratory two-roll mill of Dnipro State Technical University (Ukraine). The rolls are made from steel, diameter is 195 mm, their surface is rough enough (purity class 8), speed of rolling is 0.3 m/s, the coefficient of friction at a deformation site is $f_y \approx 0.25$. By using a dividing head, we applied cores at the surface of rolls at a distance of 10 mm from each other. The purpose was to ensure that at each section (step) during rolling it is possible to detect the forward creep. The samples used were the lead stepped samples (grade C1) of width 60 mm. The stepped surface was obtained by planing. Dimensions of the sample steps, and other parameters for experimental rolling, are given in Table 1.

Table 1

Dimensions of stepped samples, geometrical parameters for rolling, and values of forward creep, under conditions of the performed experiment

Number of sample's step	1	2	3	4	5	6	7	8
Step height, mm	7	10	12	14	15	16	16.5	17
Step length, mm	15	15	15	20	25	25	30	30
Reduction, mm	2.8	5.8	7.8	9.8	10.8	11.8	12.3	12.8
Gripping angle, rad.	0.17	0.244	0.283	0.317	0.333	0.346	0.355	0.362
Forward creep, %	5.42	8.75	8.3	7.5	6.52	5.05	4.0	slipping

The rolling was intended for the resulting thickness of $h_1 = 4.2$ mm. The number of rolled stepped samples is 5. To know in forward creep at which step one might expect the slipping of a metal in rolls, we preliminary rolled the wedge-shaped samples.

It follows from Table 1 that the general shape of the dependence of forward creep on gripping angle qualitatively coincides with the theoretical one (Fig. 1). At gripping angle $\alpha_y = 0.355$ rad., the forward creep is about 4 %, which indicates a sufficient force of friction at the contact between a metal and rollers to proceed with a stable rolling process. A transition to the sample's next step ($\alpha_y = 0.362$ rad.) led to the slipping of the strip in rolls. In this case, the ratio $\alpha_y^{\max} / f_y$ is equal to 1.42.

Thus, the results from our experiment completely confirm that the stable process of rolling is disrupted in the presence of forward creep when the condition for gripping is not met (1).

4. 2. Theoretical analysis of force equilibrium at a deformation site

In order to search for the cause of discrepancy between a theoretical realization of the stability of a simple process of thin sheet rolling and experimental data, we analyzed the equilibrium of horizontal forces at a deformation site.

Let us separate the current volume of metal in the region where the metal contacts the rollers (Fig. 2, *a*) and replace the action of the roller on the metal with horizontal contact forces Q_x, and the action of the removed part of the metal – with internal forces (due to plastic deformation of metal) $Q_{x\,vn}$ (Fig. 2, *b*).

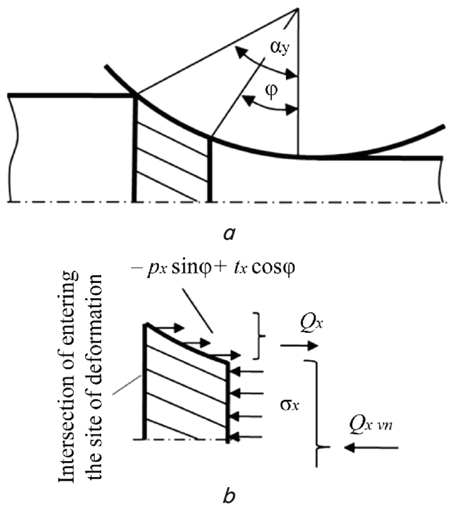


Fig. 2. Analysis of equilibrium in horizontal forces: *a* – deformation site; *b* – separated current volume of metal

As a result, the following equality holds:

$$-2\int_{\varphi}^{\alpha_y} p_x \sin\varphi \mathrm{d}\varphi Rb + 2\int_{\varphi}^{\alpha_y} t_x \cos\varphi \mathrm{d}\varphi Rb - \sigma_x h_x b = 0, \qquad (6)$$

where φ is the current intersection of a deformation site, rad.; p_x are the normal contact stresses, MPa; t_x is the friction stress, MPa; R is the radius of rolls, mm; b is the width of the rolled strip, mm; σ_x are the longitudinal internal stresses, MPa; h_x is the thickness of a strip in the current intersection, mm.

Given that $h_x = h_1 + R\varphi^2$ (h_1 is the ultimate thickness of a strip, mm), then we obtain in the dimensionless form:

$$-2\int_{\varphi}^{\alpha_y} \frac{p_x}{2k} \sin\varphi \, \mathrm{d}\varphi + 2\int_{\varphi}^{\alpha_y} \frac{t_x}{2k} \cos\varphi \, \mathrm{d}\varphi - \frac{\sigma_x}{2k}\left(\frac{h_1}{R} + \varphi^2\right) = 0. \qquad (7)$$

Note that the forces do not act in the intersection of entering a deformation site. Analyze in more detail the components included in equality (7):

$$-2\int_{\varphi}^{\alpha_y} \frac{p_x}{2k} \sin\varphi \mathrm{d}\varphi + 2\int_{\varphi}^{\alpha_y} \frac{t_x}{2k} \cos\varphi \mathrm{d}\varphi = 2Q_x^*$$

is the current resultant horizontal contact force in the dimensionless form, which includes the components that eject and retract a metal from/into rolls;

$$\frac{\sigma_x}{2k}\left(\frac{h_1}{R} + \varphi^2\right) = Q_{xvn}^*$$

is the current longitudinal internal force in the dimensionless form, caused by the plastic deformation of a metal.

It follows from expression (7) that these forces are directed opposite one another, in addition, force Q_x^* is active, because it contributes to gripping the metal by rollers, and force Q_{xvn}^* is the resistance force, which is a response to the action of the active force. These forces are in equilibrium, thereby emphasizing the stationarity of the rolling process.

To determine these forces, the source data are the contact stresses diagrams, which can be derived theoretically when solving a differential equation of equilibrium by T. Karman [17, 18]. According to this theory, when mapping all the acting forces on the selected element at a deformation site onto the x axis, we obtain:

$$-2p_x \sin\varphi d\varphi\, Rb + 2t_x \cos\varphi d\varphi\, Rb + \left(\sigma_x + d\sigma_x\right)\left(h_x + dh_x\right)b - \sigma_x h_x b = 0;$$

$$-2p_x \sin\varphi d\varphi Rb + 2t_x \cos\varphi d\varphi Rb + \sigma_x h_x b + d\sigma_x h_x b + \sigma_x dh_x b + d\sigma_x dh_x b - \sigma_x h_x b = 0.$$

Since $d\sigma_x dh_x b \approx 0,$ we obtain:

$$-2p_x \sin\varphi d\varphi Rb + 2t_x \cos\varphi d\varphi Rb + d\sigma_x h_x b + \sigma_x dh_x b = 0.$$

We obtain in the dimensionless form:

$$2kRb\left\{\begin{array}{l} -2\dfrac{p_x}{2k}\sin\varphi d\varphi + 2\dfrac{t_x}{2k}\cos\varphi d\varphi + \\ +\dfrac{1}{R}\left[\dfrac{\sigma_x}{2k}dh_x + \dfrac{d\sigma_x}{2k}h_x\right] \end{array}\right\} = 0;$$

$$-2\frac{p_x}{2k}\sin\varphi\, d\varphi + 2\frac{t_x}{2k}\cos\varphi\, d\varphi + \frac{1}{R}\left[\frac{\sigma_x}{2k}dh_x + \frac{d\sigma_x}{2k}h_x\right] = 0.$$

Considering $h_x = h_1 + R\varphi^2$ and $dh_x = 2R\varphi d\varphi,$ we obtain:

$$-2\frac{p_x}{2k}\sin\varphi d\varphi + 2\frac{t_x}{2k}\cos\varphi d\varphi + +2\frac{\sigma_x}{2k}\varphi d\varphi + \frac{d\sigma_x}{2k}\left(\frac{h_1}{R} + \varphi^2\right) = 0. \qquad (8)$$

Treating a deformation site as a set of infinitesimal elements, the following integral equality holds:

$$-2\int_{\varphi}^{\alpha_y} \frac{p_x}{2k}\sin\varphi \mathrm{d}\varphi + 2\int_{\varphi}^{\alpha_y} \frac{t_x}{2k}\cos\varphi \mathrm{d}\varphi + +2\int_{\varphi}^{\alpha_y} \frac{\sigma_x}{2k}\varphi \mathrm{d}\varphi + \int_{\varphi}^{\alpha_y} \frac{\mathrm{d}\sigma_x}{2k}\left(\frac{h_1}{R} + \varphi^2\right) = c.$$

Since in the intersection of entering a deformation site $\varphi = \alpha$, then $c = 0$, we finally obtain:

$$-2\int_{\varphi}^{\alpha_y} \frac{p_x}{2k}\sin\varphi d\varphi + 2\int_{\varphi}^{\alpha_y} \frac{t_x}{2k}\cos\varphi d\varphi + +2\int_{\varphi}^{\alpha_y} \frac{\sigma_x}{2k}\varphi d\varphi + \int_{\varphi}^{\alpha_y} \frac{d\sigma_x}{2k}\left(\frac{h_1}{R} + \varphi^2\right) = 0. \qquad (9)$$

When comparing the last equality with (7), one may notice that the first two components are the horizontal contact forces, others – the longitudinal internal forces. Thus, when solving the equation of equilibrium one can determine these forces.

5. Results of theoretical research into stability of the simple process of sheet rolling considering the internal forces at a deformation site

5. 1. Theoretical analysis of the longitudinal internal force under different conditions of rolling

We shall analyze the distribution of longitudinal internal force Q_{xvn}^* across a deformation site for different cases of rolling.

As an example, we shall consider the results from theoretical studies related to the estimation of change in the normal contact stresses and stresses of friction lengthwise the deformation site at cold rolling of thin strips. Note that when solving a Karman equation, the author applied a friction model that takes into consideration both slipping at a deformation site and a direct proportional relationship between contact stresses. Accordingly, for the regions of lagging and a forward creep, formulae for determining the stresses of friction take the form [19]:

$$t_x = -fp_x\left|\frac{h_0}{h_0-h_n}\right|\left(\frac{h_n}{h_x}-1\right);\; t_x = -fp_x\left|\frac{h_1}{h_n-h_1}\right|\left(\frac{h_n}{h_x}-1\right). \quad (10)$$

Thus, when rolling under conditions when R=300 mm, h_0=0.3 mm, α_y=0.017 rad., f_y=0.044 (h_0 is the initial thickness of a strip, mm), the results from calculation of contact stresses take the form shown in Fig. 3, *a, b*.

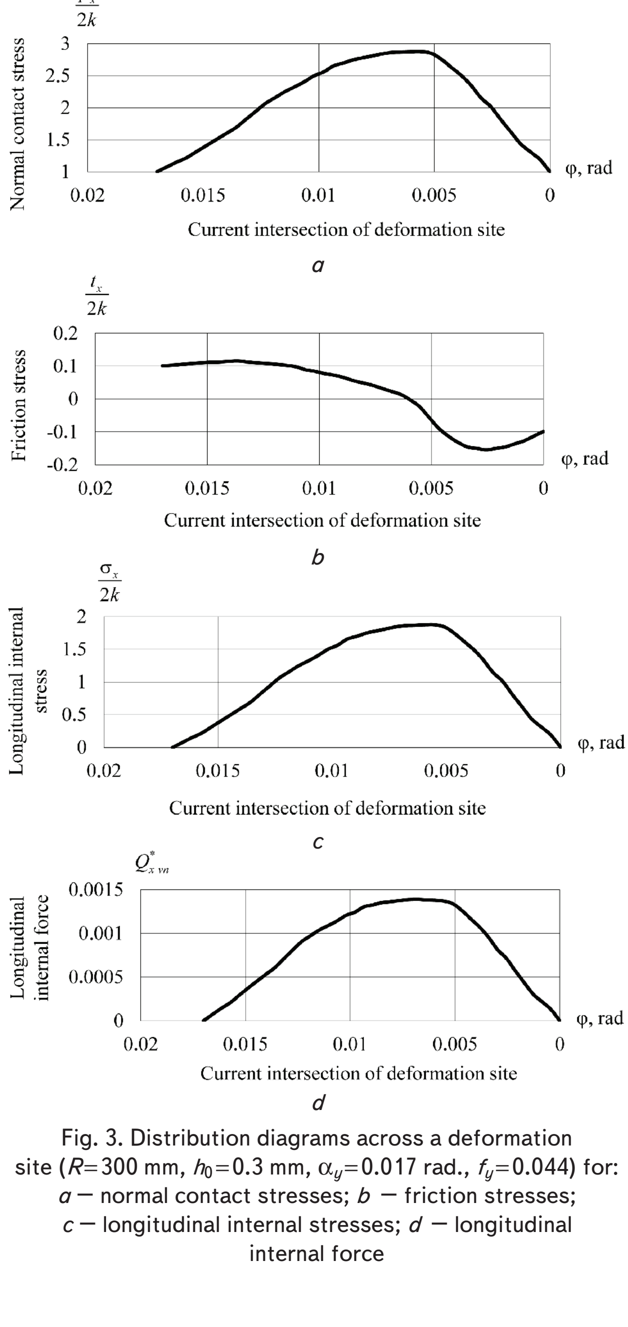


Fig. 3. Distribution diagrams across a deformation site (R=300 mm, h_0=0.3 mm, α_y=0.017 rad., f_y=0.044) for: *a* – normal contact stresses; *b* – friction stresses; *c* – longitudinal internal stresses; *d* – longitudinal internal force

The current longitudinal internal force at each intersection of a deformation site is determined from formula:

$$Q_{xvn} = \sigma_x h_x b. \quad (11)$$

When the distribution diagram of a normal contact stress is known, it is easy to find from the equation of plasticity the distribution σ_x lengthwise the deformation site:

$$\sigma_x = p_x - 2k, \quad (12)$$

or, in the dimensionless form:

$$\frac{\sigma_x}{2k} = \frac{p_x}{2k} - 1. \quad (13)$$

Then the current longitudinal internal force will be equal to:

$$Q_{xvn} = (p_x - 2k)(h_1 + R\varphi^2)b = 2kRb\left(\frac{p_x}{2k}-1\right)\left(\frac{h_1}{R}+\varphi^2\right), \quad (14)$$

or, in the dimensionless form:

$$Q^*_{xvn} = \frac{Q_{xvn}}{2kRb} = \left(\frac{p_x}{2k}-1\right)\left(\frac{h_1}{R}+\varphi^2\right). \quad (15)$$

Distribution diagrams of current longitudinal stresses $\sigma_x/2k$ and force Q^*_{xvn} are shown in Fig. 3, *c, d*.

This force under these conditions for rolling is, along the entire length of the deformation site, compressing and directed against the strip motion (Fig. 4), at the contact zone boundaries it is zero, which corresponds to the process stationarity.

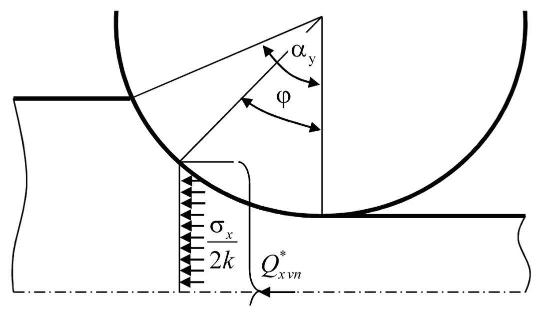


Fig. 4. Direction of action of internal stresses and a longitudinal force

As reported in many studies into internal flow of metal, under the influence of longitudinal forces at a certain part of the deformation site there emerges the «backward» movement of the metal. If one draws vertical lines at the surface of a strip (Fig. 5, line 1), then, when entering the deformation site, they bend in the direction opposite to the movement of the strip (Fig. 5, curves 2).

It should be emphasized that the character of distribution of force Q^*_{xvn} changes depending on the conditions for rolling. Thus, when rolling at $\alpha_y > f_y$, stresses σ_x change the sign at a certain part of the deformation site and become stretching (Fig. 6).

A change in the pattern of stressed state in a plastically deformed metal leads to a qualitative change in the diagram of force Q^*_{xvn}. At one side of the deformation site, it is compressing and directed opposite to the movement of the strip, and at another – stretching with the respective forwd direction of rolling. An increase in gripping angle α_y leads to an increase in the area at the deformation site where force Q^*_{xvn} is stretching, the result being that there is the case when the areas of compressing and stretching values for a given force become equal. At a further increase in α_y the area of stretching forces Q^*_{xvn} would exceed the area where these forces are compressing.

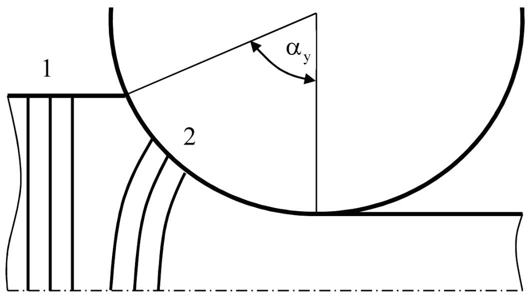
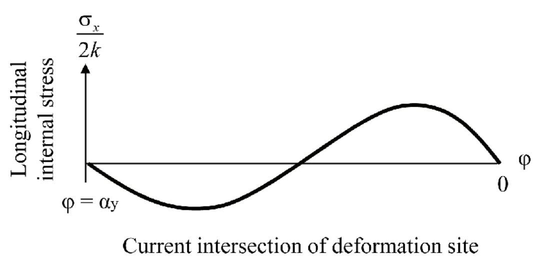
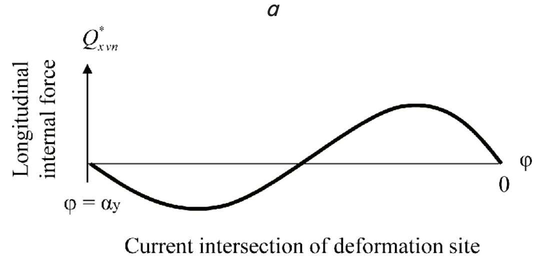


Fig. 5. Plastic flow of metal under a stable rolling process: 1 – drawn initial lines; 2 – curved lines at the deformation site



a



b

Fig. 6. Distribution across a deformation site (under condition $\alpha_y > f_y$) of: *a* – longitudinal internal stresses; *b* – longitudinal internal force

those forces that act in a contact between the metal and rollers.

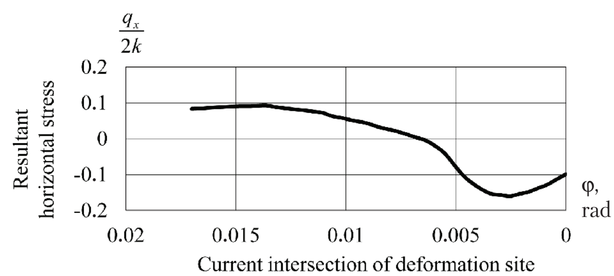
Analyze the diagrams of contact stresses that are shown in Fig. 3, *a*, *b*. A change in the resulting horizontal contact stress lengthwise the deformation site q_x is calculated from formula:

$$q_x = -p_x \sin\varphi + t_x \cos\varphi, \tag{16}$$

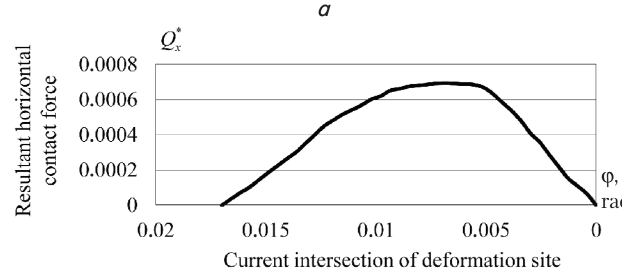
or, in the dimensionless form:

$$\frac{q_x}{2k} = -\frac{p_x}{2k}\sin\varphi + \frac{t_x}{2k}\cos\varphi. \tag{17}$$

Fig. 7, *a* shows a diagram of change in this stress. Their action is graphically illustrated in Fig. 8.



a



b

Fig. 7. Diagrams of distribution across a deformation site (R=300 mm, h_0=0.3 mm, α_y=0.017 rad, f_y=0.044): *a* – resultant horizontal contact stress; *b* – resultant horizontal contact force

Thus, at a simple process of rolling, one can derive from the distribution diagrams of contact stresses across a deformation site the charts of change in the longitudinal internal force, which is caused by the plastic deformation of a metal. The magnitude and direction of this force strongly depend on the conditions for rolling and, consequently, it must be taken into consideration when deriving a condition for gripping the metal with rollers.

5. 2. Theoretical analysis of the resultant horizontal contact force

Because the rolling process is stationary, and it is incorporated into boundary conditions for solving the Karman equation, then all forces that act at a deformation site must be balanced. It is obvious that internal forces can be balanced only by

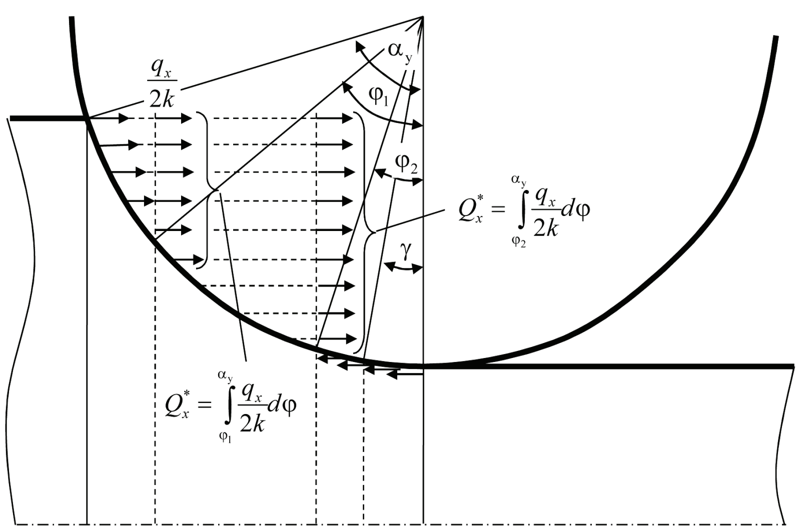


Fig. 8. Contact interaction between metal and rollers at a deformation site at rolling

In accordance with the distribution diagram $q_x/2k$, one can determine a change in the resultant horizontal contact force as follows:

$$Q_x = Rb\int_{\varphi}^{\alpha_y} q_x \mathrm{d}\varphi, \quad (18)$$

or, in the dimensionless form:

$$Q_x^* = \frac{Q_x}{2kRb} = \int_{\varphi}^{\alpha_y} \frac{q_x}{2k}\mathrm{d}\varphi. \quad (19)$$

As already noted, force Q_x^* is active as is directed in the direction of the strip motion (Fig. 8), it ensures the rolling process. The diagram of its distribution lengthwise a deformation site is shown in Fig. 7, *b*. One can see that Q_x^* at the contact zone boundaries is equal to zero. Hence, it follows that the velocity of the strip at the input to the deformation site and when exiting it remain unchanged over time, that is, as mentioned above, the process of deformation is stationary.

Comparing the diagrams Q_{xvn}^* (Fig. 3, *d*) and Q_x^* (Fig. 7, *b*), note that at any intersection of the deformation site the current values for these forces are interrelated as follows:

$$2Q_x^* = Q_{xvn}^*. \quad (20)$$

That also follows from an analysis of the Karman differential equation, given above, and matches equality (9).

Given that in a contact the horizontal forces Q_x^* act from the side of the upper and lower rolls, one can understand the presence of coefficient 2 in expression (20). Force Q_x^*, depending on the conditions for rolling, changes similarly to the force Q_{xvn}^*.

An important conclusion follows from the derived equality on that the retracting horizontal contact forces at a deformation site are used not only to overcome the ejecting ones, but to balance the longitudinal internal forces as well. Then the gripping capacity of rollers in a stable process of rolling should decrease. This issue is examined below.

Hereafter, we shall analyze only contact forces Q_x^*, because they are active, contributing to the process of deformation, and their action coincides with the direction of rolling.

5. 3. Definition and theoretical analysis of stability criterion in the process of thin sheet rolling

Consider the values acquired by force Q_x^* at different parameters for rolling. As an example, we shall consider a solution to the Karman equation taking into account the replacement of the arc of a circle with a parabola and a directly-proportional dependence between contact stresses ($t_x = f_y p_x$). Suppose that the rolling is performed for the following parameters: the diameter of rolls D=200 mm, the initial thickness of strip h_0=1 mm, gripping angle α_y=0.06 rad., and reduction angle Δh=0.36 mm. A variable parameter is the coefficient of friction f_y, which is equal to 0.06; 0.042, and 0.03. We shall construct theoretical diagrams of the contact, the resulting horizontal stresses, and the resultant horizontal contact force for these conditions of deformation. Results of the calculation are shown in Fig. 9.

One can see that a change in force Q_x^* strongly depends on parameter α_y/f_y. Under a given ratio equal to unity (Fig. 9, curve 1), at each intersection of a deformation site the force Q_x^* is positive (retracting). At $\alpha_y/f_y = 1.43$ (Fig. 9, curve 2), the resultant horizontal contact force Q_x^* at one part of the deformation site is positive, at another – negative (ejecting); in this case, the corresponding areas are equal. At ratio $\alpha_y/f_y = 2$, force Q_x^* at each intersection of the contact zone is less than zero.

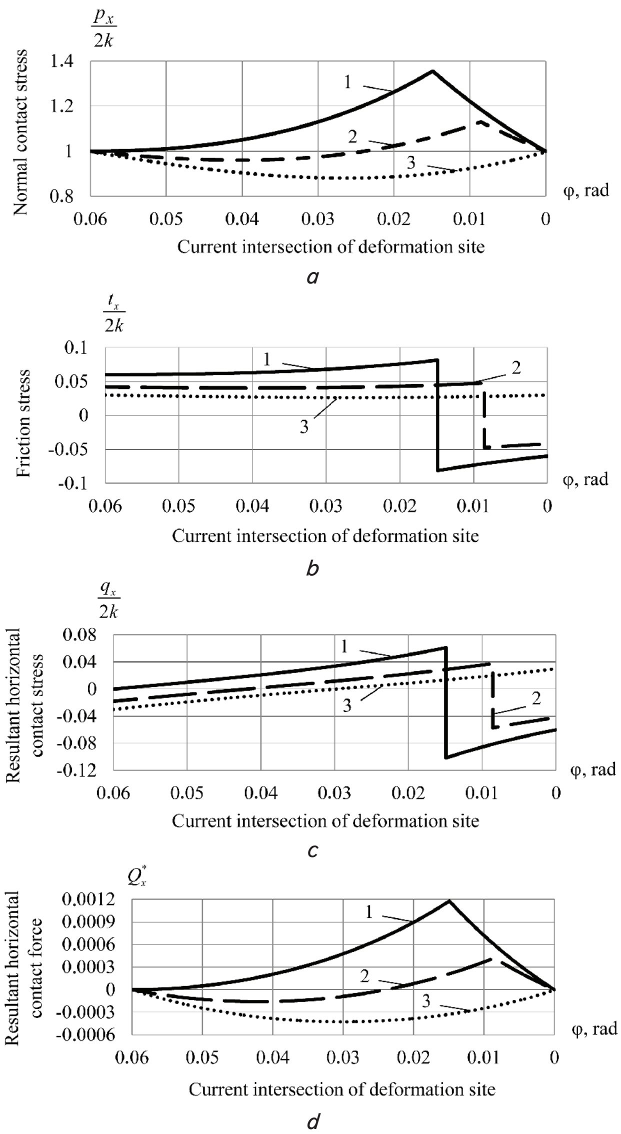


Fig. 9. Diagrams of distribution across a deformation site at friction model $t_x = f_y p_x$: *a* – normal contact stresses; *b* – friction stresses; *c* – resulting horizontal contact stresses; *d* – resultant horizontal contact force; 1 – f_y=0.06, 2 – f_y=0.042, 3 – f_y=0.03

Thus, if one introduces a dimensionless criterion or indicator:

$$K_{ct} = \frac{1}{\alpha_y}\int_{0}^{\alpha_y} Q_x^* \mathrm{d}\varphi, \quad (21)$$

then, for the first case (Fig. 9, curve 1), it will be positive, for the second (curve 2) – equal to zero, and in the third (curve 3) – negative. That is, this criterion describes which forces (retracting or ejecting) dominate a deformation site.

Thus, we note that forces Q_x^* and Q_{xvn}^* have not been given sufficient attention to in the theory of rolling up to now, while their magnitude is significantly affected, as shown above, by the conditions of deformation. These forces can also be calculated based on the experimental diagrams of contact stresses [20].

Consider some results from a theoretical research into determining an indicator K_{ct} under different conditions of rolling and compare them with the experimental data provided above.

Thus, in the experiments involving the rolling of lead samples to the resulting thickness of 1.0–1.1 mm the loss of strip equilibrium in the rolls occurred at gripping angle $\alpha_y^{\max} = 0.35$ rad.

Given such conditions of deformation, as well as different gripping angles, we performed a theoretical calculation of the contact, the resulting horizontal stresses, force Q_x^* and indicator K_{ct}. The law of friction for a deformation site was the model constructed in work [21].

Based on the results from calculations, we constructed the generalized theoretical dependencies for criterion K_{ct} and forward creep S depending on a gripping angle (Fig. 10).

Comparing the results from experiments (Fig. 1, *a*) with theoretical calculations (Fig. 10), we can conclude that the strip slipping in rolls occurs when criterion K_{ct} becomes zero (0.36 rad. – derived from theoretical calculations; 0.35 rad. – based on the results of experiment), regardless of the magnitude for a forward creep. At a positive value for K_{ct}, there is a stable process of deformation, and at $K_{ct} < 0$, the rolling process is impossible.

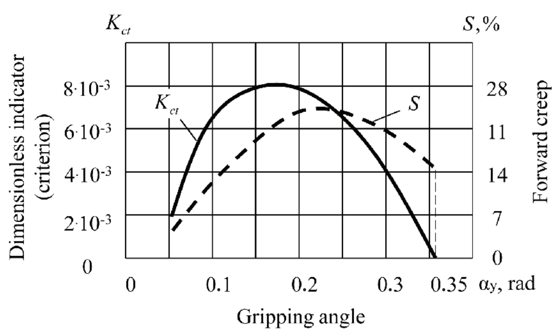
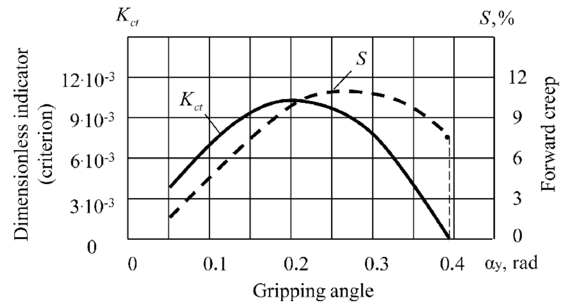


Fig. 10. Dependence of criterion K_{ct} and forward creep S on gripping angle α_y, calculated for experiments where the resulting thickness of samples is 1.0–1.1 mm

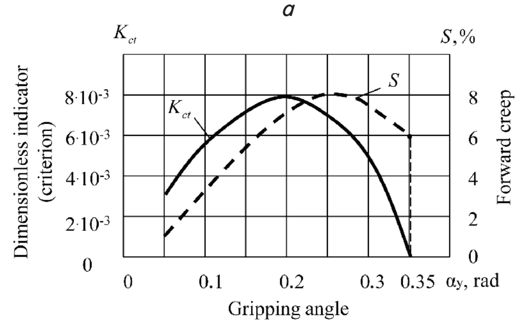
Similar research and comparison of theoretical results with experimental was also conducted for experiments where the resulting thicknesses of samples are 3.75 mm and 4.2 mm. The generalized theoretical dependences of K_{ct} and S depending on α_y are shown in Fig. 11.

By comparing the theoretical calculation with the results from experimental data, we again can conclude that the strip slipping in rolls occurs when the value for criterion K_{ct} accepts a null value. Under the conditions of experiments when the resulting thickness of samples is 3.75 mm (Fig. 1, *b*), inaccuracy at determining the maximum gripping angle is approximately 5 % (0.39 rad. – based on theoretical calculations; 0.37 rad. – according to the results from experiment), and under the conditions of our own experiments when the resulting thickness of samples is 4.2 mm (Table 1) it is 3 % (0.35 rad. – according to theoretical calculations; 0.362 rad. – according to the results from experiment).

Thus, our theoretical analysis has revealed that regardless of the magnitude for the forward creep the boundary conditions of rolling occur when the value for a dimensionless criterion K_{ct} equals zero. Note that in all examined cases the ratio of the maximum gripping angle to a friction coefficient $\alpha_y^{\max}/f_y$ is significantly less than two. This is due, as already noted, to the fact that contact retracting forces must overcome the ejecting forces and balance the internal ones.



a



b

Fig. 11. Dependence of criterion K_{ct} and forward creep S on gripping angle α_y, calculated for experiments: *a* – when the resulting thickness of samples is 3.75 mm; *b* – when the resulting thickness of samples is 4.2 mm

5. 4. Algorithm of the force method for estimating stability of the thin sheet rolling process

Thus, we have constructed a force method for estimating the longitudinal stability of the thin sheet rolling process considering the effect of contact and internal forces, which implies the following:

1. Acquire the theoretical distribution diagrams of contact stresses ($p_x/2k$ and $t_x/2k$) across a deformation site when solving a differential equation of equilibrium by Karman, applying the model of friction at a deformation site, or experimental (p_x and t_x), employing different methods (point load-cell strain gauge, force-measuring anvil, polarization-optical method).
2. Based on the contact stresses diagrams, determine the curves of change in the resultant horizontal stresses and forces across a deformation site.
3. Based on the distribution diagram for the latter, compute a dimensionless criterion K_{ct} (or dimensional – for the case of analysis of experimental diagrams of contact stresses), which characterizes the stability of the deformation process in the longitudinal direction.
4. Assess the stability of the rolling process: If $K_{ct}>0$, the deformation will proceed under a steady mode, while at a negative value for the criterion, a stable process is impossible, and for the case $K_{ct}=0$, the boundary conditions for deformation occur.

5. 5. Theoretical study of the maximal gripping angle under a stable mode of the simple process of thin sheet rolling

By employing the above algorithm, we performed a theoretical study into determining the maximal gripping angle under a stable mode at the simple process of thin sheet rolling.

Variable parameters included the thickness of a strip (1–5 mm), the coefficient of friction (0.05–0.2), the diameter of rolls (100–400 mm). The results of calculations have shown that the ratio of the maximum gripping angle to the coefficient of friction at a deformation site $\alpha_y^{\max}/f_y$ is in the range of 1.43–1.44 and almost does not depend on the above-mentioned parameters. This theoretical conclusion is confirmed in the experimental studies by several authors that were analyzed at the beginning of this paper. A similar conclusion has been drawn in paper [22].

Such a discrepancy between the data obtained and the provisions from theory can be explained by that in theoretical calculation the maximum gripping angle affects the equilibrium at a deformation site not only of contact forces, as is accepted in the theory of rolling, but the internal forces as well, which leads to a decrease in the gripping capacity of rolls.

6. Discussion of results of constructing a force method for estimating stability of a simple process of thin sheet rolling

The result of a theoretical analysis of the interaction between a metal and the rolls during thin sheet rolling and the application of the Karman equation of equilibrium is the construction of a force method for estimating stability of a simple rolling process. This method differs from known dependences in the theory of rolling by considering, when analyzing the equilibrium at a deformation site, not the contact forces only, but the internal ones as well, which arise as a result of the plastic deformation of a metal. This is the main advantage of the devised method. The physical meaning of the action of these forces is explained in Fig. 4, 5.

Determining the longitudinal stability of the rolling process is carried out based on a criterion of stability, which is derived from the distribution diagrams of normal contact stresses due to stresses of friction. It is a well-grounded approach, because they act in the zone of contact between the metal and rollers.

The reliability of determining the boundary conditions for rolling based on a given criterion was tested by comparing theoretical calculations with the results from experiments. An error of the theoretical evaluation of determining the stability of the process does not exceed 5 %, indicating a satisfactory convergence between the method constructed and experimental data.

Therefore, the interruption of a simple process of rolling under actual conditions in the presence of forward creep (for example, results from the experiments are shown in Fig. 1) occurs as a result of failing to take into consideration, in the theoretical dependences, specifically the condition of gripping (1), the internal forces of the plastically deformed metal.

Thus, the constructed force method is a new improved approach in assessing the stability of a simple process of thin sheet rolling.

It should be noted that a given force method can only be used for the estimation of the longitudinal stability of the process of thin sheet (hot, cold) rolling, that is, under conditions of the flat deformation when a Karman equilibrium equation holds.

The disadvantage of the proposed approach is that we did not consider, when solving a differential equation of equilibrium, the front and back tensions. This could be achieved by introducing appropriate coefficients at the boundaries of a deformation site, which depend on tensions. Resolving this issue will be the area of further research by Authors.

The application of the devised method for a simple process of deformation of thin and thick sheet is predetermined in this work by comparison with experimental data, which were obtained for the rolling without tensions.

In advancing a given approach together with the models, procedures, and systems, analyzed above, the force method could be used for the development and improvement of deformation modes at thin sheet rolling mills in order to maintain a stable process of deformation at lower energy costs. One of the areas to improve energy efficiency of the thin sheet rolling process is to increase the tension of strips. This issue was analyzed in detail in paper [23].

7. Conclusions

1. We have performed a theoretical analysis of the force interaction between a metal and rollers at thin sheet rolling. In contrast to the generally accepted theoretical concepts, this analysis took into consideration not only the contact forces, but the internal forces as well. The existence of internal forces at a deformation site has been proven in practice, for example, in experiments when bending the vertical lines drawn at the surface of a strip (Fig. 5). The contribution of these forces to equilibrium is also due to the Karman equation. The sequence for determining the horizontal contact and internal forces has been described in detail. Initial data are the distribution diagrams of contact stresses in a contact zone between a metal and rollers. It should be noted that determining the horizontal contact and internal forces has not been paid attention to in the theory of rolling. So, the novelty is the shape of distribution charts for these forces across a deformation site, based on which one can assess the longitudinal stability of the rolling process. We have substantiated a given approach based on the results from theoretical and experimental studies.

2. We have devised a force method for estimating the longitudinal stability of the process of thin sheet rolling, which implies determining a stability criterion. For the case of its positive values, the process proceeds in a stable fashion; at a negative value, the stable process is impossible and at a zero value of the indicator, there is the boundary case of deformation. Such an analysis can be performed using both the theoretical diagrams of contact stresses and the experimental ones. The application of a given criterion for rolling with tensions is possible as a result of the introduction of coefficients at the boundaries of a contact zone between the strip and rollers, which depend on the front and back tensions.

3. Based on the processing of theoretical diagrams of contact stresses and by comparing their results with experimental data, it has been shown that an error in determining the boundary conditions for thin sheet rolling does not exceed 5 %. Therefore, application of the devised approach makes it possible to deal with as-yet-unresolved issues related to the interruption of a simple process of rolling in the presence of forward creep.

4. We have theoretically investigated and refined a commonly accepted condition for gripping a metal by rollers under a stable mode of rolling without tensions. It is shown that the ratio of the maximum gripping angle to the coefficient of friction at a deformation site equals to 1.43–1.44, which is confirmed by results from experiments by several researchers.

Acknowledgements

The team of authors expresses sincere gratitude to the employees at the Department of Processing of Metals by

Pressure from Dnipro State Technical University (Ukraine) for their help in conducting the experiments. We especially grateful to the Head of Department, Doctor of Sciences, Professor O. P. Maksimenko – a scientific supervisor of the thesis by R. Ya. Romaniuk; PhD, Associate Professors E. V. Halytsky, V. P. Kapelyushy, V. M. Samokhval, M. K. Izmailova, who participated in the theoretical and experimental study and substantiated their results.

We also express gratitude for the professional advice and ongoing scientific support to the Head of the Laboratory of Problems of Structure Formation and Properties of Ferrous Metals at IChM named after Z. I. Nekrasov (Ukraine), PhD, Professor G. V. Levchenko, PhD, Professor of NMetAU (Ukraine) Ya. D. Vasilev, Director of NVO «Dniprofmash» (Kamenske, Ukraine), PhD, Professor M. E. Nekhayev.

References

1. Grudev A. P. Zahvatyvayushchaya sposobnost' prokatnyh valkov. Moscow: «SP Intermet Inzhiniring», 1998. 283 p.
2. Maksimenko O. P., Romanyuk R. Ya. Estimation of Rolling Process Stability by Contact-Stress Diagrams // Metallurgical and Mining Industry. 2010. Vol. 2, Issue 2. P. 122–129.
3. Vasylev Ya. D., Minaiev O. A. Teoriya pozdovzhnoi prokatky: pidr. Donetsk: UNITEKh, 2009. 488 p.
4. Romaniuk R. Ya. Analiz metodiv otsinky pozdovzhnoi stiykosti protsesu tonkolystovoi prokatky // Obrabotka materyalov davlenyem. 2014. Issue 1. P. 173–177.
5. Upravlenie kachestvom tonkolistovogo prokata: monografiya / Mazur V. L. et. al. Kyiv: Tekhnika, 1997. 384 p.
6. Steel sheet surface quality improvement on the basis of new solutions in cold rolling theory / Garber E. A., Pavlov S. I., Kozhevnikova I. A., Timofeeva M. A., Kuznetsov V. V. // Vestnik Cherepoveckogo gosudarstvennogo universiteta. 2010. Issue 2. P. 116–125.
7. Issledovanie nestabil'nosti poperechnogo profilya polos na nepreryvnyh shirokopolosnyh stanah goryachey prokatki / Garber E. A. et. al. // Proizvodstvo prokata. 2010. Issue 2. P. 2–7.
8. Stabilizaciya tekhnologicheskih rezhimov shirokopolosnyh stanov dlya uluchsheniya kachestva poperechnogo profilya goryachekatanyh polos / Garber E. A. et. al. // Stal'. 2010. Issue 8. P. 56–61.
9. Garber E. A., Bolobanova N. L. Modelirovanie uprugih deformaciy v valkah kletey i opredelenie ih konstruktivnyh parametrov, obespechivayushchih povyshenie tochnosti holodnokatanyh polos // Proizvodstvo prokata. 2012. Issue 1. P. 17–18.
10. Sistema avtomaticheskogo regulirovaniya ploskostnosti polos s ispol'zovaniem beskontaktnyh metodov izmereniya ploskostnosti i temperatury / Prihod'ko I. Yu. et. al. // Stal'. 2009. Issue 3. P. 41–45.
11. Coupled approach for flatness prediction in cold rolling of thin strip / Abdelkhalek S., Montmitonnet P., Legrand N., Buessler P. // International Journal of Mechanical Sciences. 2011. Vol. 53, Issue 9. P. 661–675. doi: https://doi.org/10.1016/j.ijmecsci.2011.04.001
12. Li G.-H. EVC-Plus technique of transverse thickness deviation control of non-oriented silicon steel during cold rolling // IRON AND STEEL. 2018. Vol. 53, Issue 8. P. 68–72. doi: https://doi.org/10.13228/j.boyuan.issn0449-749x.20180174
13. Ji J. Stability of the coupled vibrations of work roll and strip in cold rolling process // Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture. 2017. Vol. 231, Issue 7. P. 1169–1181. doi: https://doi.org/10.1177/0954405415585376
14. Experimental Investigations and ALE Finite Element Method Analysis of Chatter in Cold Strip Rolling / Niroomand M. R., Forouzan M. R., Salimi M., Shojaei H. // ISIJ International. 2012. Vol. 52, Issue 12. P. 2245–2253. doi: https://doi.org/10.2355/isijinternational.52.2245
15. Experimental study on transverse displacement of the metal during cold thin strip rolling / Ren Z., Xiao H., Yu C., Wang J. // Procedia Engineering. 2017. Vol. 207. P. 1326–1331. doi: https://doi.org/10.1016/j.proeng.2017.10.891
16. Kozhevnikov A., Kozhevnikova I., Bolobanova N. Dynamic model of cold strip rolling // Metalurgija. 2018. Vol. 57, Issue 1-2. P. 99–102.
17. Romaniuk R. Ya., Levchuk K. O. Osoblyvosti sylovoi vzaiemodiyi shtaby z valkamy pry tonkolystoviy prokattsi // Zbirnyk naukovykh prats Dniprovskoho derzhavnoho tekhnichnoho universytetu (tekhnichni nauky). 2017. Issue 2 (31). P. 51–55.
18. Osakada K. History of plasticity and metal forming analysis // Journal of Materials Processing Technology. 2010. Vol. 210, Issue 11. P. 1436–1454. doi: https://doi.org/10.1016/j.jmatprotec.2010.04.001
19. Opredelenie neytral'nogo ugla pri holodnoy prokatke s ispol'zovaniem utochnennoy modeli napryazheniy treniya / Vasilev Ya. D. et. al. // Obrabotka metallov davleniem. 2013. Issue 3 (36). P. 81–85.
20. Romaniuk R. Ya. Stalist protsesu prokatky na osnovi doslidnykh epiur kontaktnykh napruzhen // Zbirnyk naukovykh prats Dniprodzerzhynskoho derzhavnoho tekhnichnoho universytetu. Tekhnichni nauky. 2014. Issue 1 (24). P. 32–38.
21. Romaniuk R. Y., Levchuk K. O., Hasylo Y. A. Forecasting of theoretical orthographic epures of contact voltages at thin sheet rolling // Bulletin of National Technical University «KhPI»: coll. sci. papers. Ser.: Innovative technologies and equipment handling materials in mechanical engineering and metallurgy. 2017. Issue 37. P. 71–76.
22. Maksimenko O. P., Kachan O. O. Vliyanie rezul'tiruyushchey prodol'nyh sil na ugol neytral'nogo secheniya // Zbirnyk naukovykh prats Dniprodzerzhynskoho derzhavnoho tekhnichnoho universytetu. Tekhnichni nauky. 2016. Issue 2 (29). P. 38–42.
23. Vasilyev Y. D. Theoretical study of the infl uence of tension on the energy effi ciency of cold-rolled strip // Izvestiya Visshikh Uchebnykh Zavedenii. Chernaya Metallurgiya = Izvestiya. Ferrous Metallurgy. 2012. Vol. 55, Issue 6. P. 3–5. doi: https://doi.org/10.17073/0368-0797-2012-6-3-5