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ANALYTICAL STUDY OF MULTIFRACTAL INVARIANT ATTRIBUTES OF TRAFFIC FLOWS

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Автотранспортний комплекс формується множиною автотранспортних потоків та автодорожньою мережею. Перехід на новий рівень функціонування автотранспортного комплексу вимагає розробки нових методів формалізації колективної взаємодії всіх учасників дорожнього руху. Це пов'язано зі збільшенням частки автономних автотранспортних засобів у сумісному трафіку. Встановлено, що транспортно-технологічна самоорганізація автотранспортних потоків є мультифрактальною структурою. Така структура достатньо достовірно описується регулярними ієрархічними α-множинами Кантора стосовно параметра динамічного габариту кожного окремого автотранспортного засобу. Доведено, що основними мультифрактальними ознаками автотранспортних потоків є їх параметр фрагментації та фрактальна розмірність. Наведені ознаки функціонально визначаються інтенсивністю, швидкістю, щільністю трафіка та інтервалом руху автотранспортних засобів. Відповідно, вирізнено три основні режими руху автотранспортних засобів. Відсутність взаємних перешкод між автотранспортними засобами, незначна швидкість та мала інтенсивність трафіку характеризує вільний рух. Цей рух визначає межу колективного та синхронізованого потоків. Колективному руху притаманна вища щільність автотранспортного потоку, а швидкість обмежується можливостями автодороги. Якщо визначального значення набувають показники технічного та експлуатаційного стану автомобільної дороги отримуємо насичений (синхронізований) потік. Аналітичними дослідженнями встановлено логарифмічно-показникову функціональну залежність між параметром фрагментації автотранспортного потоку та фрактальною розмірністю. З'ясовано, що сукупність декількох автотранспортних потоків при багатосмуговій організації трафіків визначає динаміку зміни основних мультифрактальних ознак множини автотранспортних засобів. При цьому, збільшення кількості смуг руху автомобільної дороги призводить до зростання параметра фрагментації та зменшення фрактальної розмірності сукупності автотранспортних потоків. Розглянута можливість створення відповідних навігаційних алгоритмів варіативної оптимізації мультифрактальних ознак автотранспортних потоків. В такому випадку забезпечуються безпечні транспортно-технологічні режими функціонування автотранспортного комплексу. Це ж стосується й умови зростання частки автономних роботизованих безпілотних автотранспортних засобів в складі автотранспортних потоків

Ключові слова: автотранспортний потік, безпілотний автотранспортний засіб, α-множина Кантора, мультифрактальність, параметр фрагментації

1. Introduction

The acceleration of the rapid evolutionary development of the motor transport complex in the late XX – early XXI century is predetermined by many factors. This is associated with the transition of mankind from the industrial society to the information and communication system of its organization. In this case, one of the defining means of communication is the motor transport complex. The motor transport complex combines both a variety of motor vehicles and a set of motorway network infrastructure. From a topographical point of view – it is a zone of reserve technology lanes along highways within a specific natural and anthropogenic geo-ecosystem [1].

Here the definition of "evolutionary development" concerning the characteristics of the motor transport development complex is deliberately used. With the improvement of motor vehicles and the improvement of the highway network infrastructure, up to now the principle of gradual improvement of some of its components properties has been used. This was manifested by increasing engine capacity, adjusting the chemical composition of fuel, improving the comfort of the car interior, using more modern materials for the construction of road pavements, etc.

However, not a single qualitative "leap", the transition to a fundamentally new level of functioning of the motor transport complex occurred. Indeed, according to the principle of work, and the nature of the functions performed, the modern car practically does not differ from the car of the beginning of the twentieth century. The road, in most cases, has only become wider and tougher as compared with the time when it was used for horse carriages.

Only at present the idea of creating completely autonomous robotic vehicles is being developed. That is why the transition to a qualitatively new level of the entire motor transport functioning complex should take place [2]. In the long run, the massive use of unmanned vehicles is inevitable. In this case, their functioning is expected in streams that are saturated and are possibly completely composed of a set of autonomous objects [3]. Therefore, there is a need to develop forms and methods that are fundamentally different from those existing in order to organize their collective interaction in the composition of motor transport flows.

There is an issue on the development of new methods for formalizing the collective interaction of all road users without exception. First of all, it concerns a specific autonomous vehicle. It should be considered in interaction with similar autonomous objects, as well as with a set of various objects of the road network infrastructure [4].

2. Literature review and problem statement

As noted at [5], in the general sense the motor traffic is the motion of motor vehicles organized by the road network. The characteristic features of the motor transport flow, which are presented in [6], are its intensity, velocity, and density, as well as certain specific parameters that are presented in [7]. As established in [8], specific parameters determine the specific characteristics of streams of vehicles, their mutual influence and redistribution in space and time. However, the above characteristics of the motor transport are purely empirical and do not make it possible to analytically formalize the flow parameters to study their dynamic properties

In [9], authors made an attempt of topological formalization of the motor transport flow as a set (C) of vehicles, consisting of oppositely directed subsets (C_1 and C_2). The combined movement along some section of the highway is characterized by such physical features as dimension (G), dynamic gauge (DG), interval of movement (I) of vehicles. In this case, as noted in paper [10], the movement of a vehicle in the composition of motor traffic is significantly different from the movement of an isolated vehicle. This causes a change in the load-speed modes of engines, fuel consumption and emissions of harmful substances, as proved in work [11]. The main factors influencing the transport flows to the environment, as defined in [7], are: composition and flow structure, speed, loading mode, intensity and density of motion, technical condition and operational properties of individual physical units of flow, chemical composition of fuel, etc.

In [11], authors determined that in addition to purely technical factors of the motor transport, the characteristics of the motor transport network are the determining factor. These characteristics include the indicator of the motor capacity of the natural and man-made geo-ecosystem territory. The landscape features of the trace, the structural parameters of the highway and the damage to the road surface (technical condition) are also significant. In terms of performance indicators, determining the correspondence of the traffic intensity level and the specific load on road surfaces is decisive. The availability of gas and dust protection infrastructure within the reserve technology strip is important. In addition, as stated in [8], it is necessary to take into account the local variation of the established functional state of the motor transport in sections of the highway with special movement conditions. These are crossroads and nodes of the motor network, downhill-climbs, serpentines, tunnels, bridge transitions, eco-design, etc.

However, from the point of view of the above definition, the motor transport flow can be considered only a set of vehicles [12]. The specified set for a definite meaningful argument (arguments) is a continuous function of a certain trajectory, which is outlined by the road of the highway [13]. In the general case, the continuity of the function $f(x)$ is determined by three following conditions [8]:

– function $f(x)$ must be necessarily defined at the point x_0;

– there is a limit $\lim_{x\to x_0} f(x)$;

– $\lim_{x\to x_0} f(x)=f(x_0)$.

If at least one of the three conditions above is violated at any point, then the function $f(x)$ has a gap at that point. At the same time, the gap of function $f(x)$ can be both removable (due to the operation for definition of the function to the continuous one) and non-removable gap. This is the so-called break point (leap) that arises for $f(x)$ if

$$\lim_{x\to(a-0)} f(x)\neq\lim_{x\to(a+0)} f(x)\ f(x),$$

as defined in [8].

Thus, there is a need to formalize the intensity of motor transport flows influence on the equilibrium of a natural-technogenic geo-ecosystem. In this case, the lower boundary, which determines the collective movement of motor vehicles as an automobile flow, must be some plane in the coordinate system $q\sim\rho\sim f(q,\rho)$. Therefore, it is necessary to establish the minimum values of q and ρ, for which the function $f(q,\rho)$ acquires the signs of continuity.

3. The aim and objectives of the study

The aim of this study is to synchronize the modes of traffic flow of motor transport and to ensure the coherence of robotic unmanned vehicles by optimizing their fractally invariant features.

To achieve the goal, the following tasks were set:

– to establish the influence of the motor transport complex on the ordering of traffic on the characteristic attributes of vehicles movement;

– to determine the dynamics of multifractal attributes of motor traffic with multi-band traffic change organization;

– to substantiate navigational algorithms parameters of safe transport-technological modes of motor transport functioning complex provided there are autonomous robotized objects.

4. Materials and methods to study motor transport flows

The research was carried out based on classical methods of fractal calculus and the Cantor method for determining fractal dimensionalities. At the same time, the hierarchy of the Cantorian α- sets was taken into account.

Consider a fragment of the motor transport flow (Fig. 1), as a directed totality (a set) of motor vehicles on the road (along the axis x-x). In this case, subsets C_1 and C_2 move in opposite directions. We assume that the contribution of subsets C_1 and C_2 is equal to the number of their shares in length, which fall on the section A_x–B_x of the motor road. If this quantity is denoted by $R(t, \chi_{A-B})$, then $R(t, \chi_{A-B})$ is continuously smooth with respect to t, and therefore has piecewise continuous derivatives of the first and second order [8].

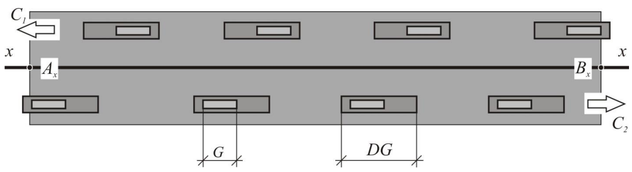

Fig. 1. A fragment of the motor transport flow of subset (C_1 *and* C_2) moving along x-x

If velocity V of the motor traffic is a known function with its density ρ, we obtain the function of the motor transport flow state in the form [8]:

$$\frac{\partial R}{\partial t}(t,\chi)=\frac{\partial R}{\partial t}(t,\chi)\cdot f\left[\rho(t,\chi)\right], \qquad (1)$$

where χ is the scale (step) of measurement of the A_x–B_x section of the highway, which is *a priori* selected to be less than the length of the dynamic gauge (DG) of the vehicle [5]; $\rho(t,\chi)$ is the vehicle traffic density, which is determined by the number of motor vehicles in the A_x–B_x section of the highway during the period t of overcoming A_x–B_x by a single vehicle; $f\left[\rho(t,\chi)\right]$ is the function that determines the continuity of traffic vehicles and must satisfy the conditions of traffic density vehicles.

According to (1), it is necessary to distinguish between three main modes of motor vehicles movement [7]:

– free movement – is characterized by small intensities, the absence of mutual obstacles between individual vehicles and the corresponding speed V_c. Insignificant density of vehicles causes a weak correlation between V_c and ρ;

– collective movement (collective flow) is determined by increasing density ρ of the motor transport, the collective velocity V_V of motor vehicles is determined by the design capacity of the highway. Correlation bond V_c and ρ are sufficiently dense;

– saturated (synchronized) flow – characterized by significant interference of individual vehicles, the speed of the vehicle traffic V_t is closely correlated with the intensity q and density ρ. A characteristic feature of the synchronized motor transport flow is a significant variation in the value of the average flow velocity. The technical and operational condition of the highway becomes determinative.

The categories of the road are determined by some point-based index K, which acquires values K=1a; 1b; 2; 3; 4; 5 (DBN of Ukraine, V.2.3-4: 2007, "Transportation Facilities. Roads", "AASHTO. A Policy on the Geometric Design of Highways and Streets. – Washington D.C.: American Association of State Highway and Transportation Officials", "FHWA. Flexibility in Highway Design. – Washington, D.C.: Federal Highway Administration" etc.). The results of analysis of function (1), taking into account its piecewise-linear nature, are presented in Fig. 2 [8].

Graphical analysis of the ascending left-side trend of the values q and ρ (Fig. 2) in the direction of the inverse growth of the coordinate K clearly defines the coordinates of the characteristic plane Φ. Plane Φ separates the mode of free movement of motor vehicles from collective and synchronized motor traffic flows. The coordinates of this plane are given by the values q and ρ [8]:

$$\begin{cases} q=0{,}035, \\ \rho=0{,}0021. \end{cases} \qquad (2)$$

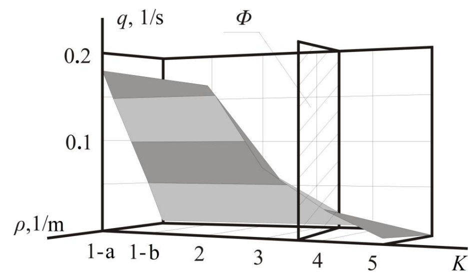

Fig. 2. The surface of the functional attributes of the state of motor traffic in coordinates: the category of the road (K); motor traffic density (ρ); the intensity of the motor traffic flow (q)

The values of q (s^{-1}) and densities ρ (m^{-1}) of motor transport given in (2) determine the minimum speed V f motor vehicles. At such speed, there is a transition from free movement to collective and synchronized flows, m/s:

$$V=\frac{q}{\rho}=\frac{0{,}035}{0{,}002}=17{,}5. \qquad (3)$$

Thus, [12] the minimum values of V, q and ρ are established, in which the character of the motor vehicle acquires the attributes of collective movement followed by the synchronization of the motor transport flow (Table 1).

Table 1

Minimum values of density ρ, intensity q and velocity V, in which the motor transport acquires characteristic features [8]

Parameter characterizing the collective movement of vehicles	Minimum parameter value
Intensity of motor transport flow, car/day	3.024
Density of motor transport flow, car/km	2.1
Speed of motor transport flow, km/h	63.0

According to the results of specific intensity calculations q_i According to the results of specific intensity calculations q_i of the motor transport, [14] the fractal nature of the motor flows structural organization with Hausdorff-Besikovich dimension D=0.620 was established. With some approximation, this corresponds to the fractal dimension of the "classical" empty undifferentiated Cantorian set with a fractal dimensionality D_k=0.631 (Table 2 [14]).

The establishment of fractal features of motor transport streams organization [14] allows us determine the basic principles of their synchronization in terms of coherence [8]. This also applies to the conditions of saturation of these streams by autonomous robotic vehicles. At the same time, it is necessary to mention the wide limits of the variation of the intensity indices q, density ρ and velocity V of motor transport flows. This also determines the significant limits of the variation of their fractal dimensions D. In this connection, it is envisaged to study their fractal-invariant traits for the entire hierarchical range of α-sets.

Table 2

Results of determining the value for a road network fractal dimensionality by the indicator of a highway category [8]

Road category	1-a	1-b	2	3	4	5
Estimated rate of traffic intensity, q_m, [1/m]	$\frac{7000}{30}=500$	$\frac{7000}{20}=350$	$\frac{2000}{8}=250$	$\frac{1000}{7{,}5}=133$	$\frac{500}{7}=71$	$\frac{200}{4{,}5}=44$
Estimated dimensionless complex, D		0.700	0.714	0.532	0.534	0.620
The average value of a dimensionless complex, $\overline{D}$		0.620				
Fractal dimensionality, D_k		0.631				
Model coefficient of traffic intensity, q_m, [1/m]	504	318	200	126	79	50

5. Results of research into multifractal attributes of a motor transport flow

Let us consider the order of the graph-analytic construction of a hierarchical graphic model α-set (Fig. 3).

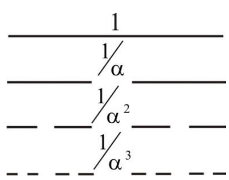

Fig. 3. The first three steps of the hierarchical construction of Cantor α-set

For a single segment, to measure its length $L_0=1$, it is sufficient to single scale $\chi_0=1$ to apply once $N_0=1$. So, for a zero measurement we have [5]:

$$\begin{cases} L_0 = 1, \\ \chi_0 = 1, \\ N_0 = 1. \end{cases} \qquad (4)$$

We leave at both ends of the segment two parts in length 1/α, where, by definition α>2 and the remainder (in the middle of the unit segment) is removed. In this case, the value of α can be considered a parameter of fragmentation. We select a scale equal to 1/α and, by applying it twice, we measure the length of the resulting segments, more precisely, we have for the first measurement:

$$\begin{cases} L_1 = \dfrac{2}{\alpha}, \\ \chi_1 = \dfrac{1}{\alpha}, \\ N_1 = 2. \end{cases} \qquad (5)$$

For each of the resulting segments, repeat the above procedure, as shown in Fig. 3.

The real graphical model of a hierarchical Cantorian *α*-set is formed after an infinite number of iterations. That is, the hierarchical construction of such a model assumes that N>>1. According to [5, 8, 14], we obtain:

$$D_1 = 1 + \frac{\log \frac{2}{\alpha}}{\log \alpha}, \qquad (6)$$

or considering the properties of the logarithm:

$$D_1 = \frac{\log 2}{\log \alpha}. \qquad (7)$$

In this case, if α>2, then $D_1<1$.

Now, by specifying the fragmentation parameter α of the motor transport, you can always determine its fractal dimensionality.

The real nature of the transport operation on the certain road section is determined by the linear size R. The size R consists of at least two oppositely directed motorways with arbitrary fragmentation parameters α_1 and α_2As a result, we have different fractal dimensions D_1 and D_2. That is, the so-called multifractality [5] with a new fractal dimension D_x. As was noted above, the scale of measurement χ is convenient to choose:

$$\chi = \frac{1}{R}. \qquad (8)$$

Substituting expression (8) into Mandelbrot-Richardson's formula [5, 14] in the form:

$$\eta \cdot L = C \cdot (\eta \cdot \chi)^{1-D}, \qquad (9)$$

where η is the scale factor [5], we obtain:

$$\frac{1}{R} \cdot L = C \cdot \left(\eta \cdot \frac{1}{R}\right)^{1-D}. \qquad (10)$$

Since the measurement scale is defined, the brackets in (10) can be uncovered. After the corresponding reductions we get a relation that connects the dimensions of the L fractal object (in this case, the motor traffic flow) and the linear size of the R section of the highway:

$$L \sim R^D. \qquad (11)$$

The value of R for each case retains its meaning – the linear size of the road section where the traffic flow is with the fractal organization of its structure. It is obvious that for each case, under fractal dimension it is necessary to understand D_1, D_2, D_3 and so on.

Let us consider two oppositely directed fractal motor flows with different dimensions, which are due to their fragmentation parameters α_1 and α_2. When measuring each fractal object separately, according to (11) in section R, we have:

$$\begin{cases} L_1 = R^{D_1}, \\ L_2 = R^{D_2}. \end{cases} \qquad (12)$$

Since the total length of fractal objects (motorways) can be written as:

$$L_\Sigma=L_1+L_2, \qquad (13)$$

then

$$R^{D_1}+R^{D_2}=R^{D_x}. \qquad (14)$$

At the same time, we assume that the uncertain scale multiplier N (1) for each term in (14) is the same.

Thus, for the case of two oppositely directed motor transport with arbitrary fragmentation parameters α_1 and α_2 we obtain:

$$R^{\frac{\log 2}{\log\alpha_1}}+R^{\frac{\log 2}{\log\alpha_2}}=(2\cdot R)^{D_x}. \qquad (15)$$

Apply logarithm to the right and left sides (15):

$$\log\left[R^{\frac{\log 2}{\log\alpha_1}}+R^{\frac{\log 2}{\log\alpha_2}}\right]=D_x\cdot\log 2R. \qquad (16)$$

Hence:

$$D_x=\frac{\log\left[R^{\frac{\log 2}{\log\alpha_1}}+R^{\frac{\log 2}{\log\alpha_2}}\right]}{\log 2R} \qquad (17)$$

or

$$D_x=\frac{\log\left[R^{\frac{\log 2}{\log\alpha_1}}+R^{\frac{\log 2}{\log\alpha_2}}\right]}{\log 2+\log R}. \qquad (18)$$

A more complex case for determining the fractal characteristics of motor traffic flows is the case of multi-band traffic organization on motor roads. The number of m lanes in this case, as a rule, can acquire the values of 2, 3 (on separate sections of highways), 4, 6, 8.

Consider the case of m-rank traffic. We will assume that the fragmentation parameter α_1 is the same for each streams of vehicles moving on separate lanes of the highway. According to (14) we can determine the total fractal dimensionality of D_x traffic as:

$$(m\cdot R)^{\frac{\log 2}{\log\alpha_1}}=(m\cdot R)^{D_x};\ (m\neq 0). \qquad (19)$$

As in the previous case, let us establish the dependence:

$$D_x=\varphi(\alpha_1). \qquad (20)$$

Divide (19) by $m\cdot R^{D_x}$, we obtain:

$$\frac{m\cdot R^{\frac{\log 2}{\log\alpha_1}}}{m\cdot R^{D_x}}=\frac{m^{D_x}\cdot R^{D_x}}{m\cdot R^{D_x}} \qquad (21)$$

or

$$R^{\frac{\log 2}{\log\alpha_1}-D_x}=m^{D_x-1}. \qquad (22)$$

Appy logarithm (22):

$$\left[\frac{\log 2}{\log\alpha_1}-D\right]\cdot\log R=(D_x-1)\cdot\log m. \qquad (23)$$

Divide (23) by $(-1)\cdot\log R$, herewith: $D_x\neq 1$, $R\neq 1$:

$$\frac{\log 2-D_x\cdot\log\alpha_1}{(D_x-1)\cdot\log\alpha_1}=\frac{\log m}{\log R}. \qquad (24)$$

By denoting

$$\frac{\log m}{\log R}=K,$$

we obtain:

$$\frac{\log 2-D_x\cdot\log\alpha_1}{(D_x-1)\cdot\log\alpha_1}=K. \qquad (25)$$

Hence, after the corresponding transformations, we find:

$$\log 2-D_x\cdot\log\alpha_1=K\cdot D_x\cdot\log\alpha_1-K\cdot\log\alpha_1, \qquad (26)$$

$$-\log\alpha_1-K\cdot D_x\cdot\log\alpha_1=-K\cdot\log\alpha_1-\log 2, \qquad (27)$$

$$D_x\cdot[\log\alpha_1+K\cdot\log\alpha_1]=K\cdot\log\alpha_1+\log 2, \qquad (28)$$

$$D_x=\frac{K\cdot\log\alpha_1+\log 2}{(K+1)\cdot\log\alpha_1}=$$
$$=\frac{K\cdot\log\alpha_1}{(K+1)\cdot\log\alpha_1}+\frac{\log 2}{(K+1)\cdot\log\alpha_1}. \qquad (29)$$

We finally receive:

$$D_x=\frac{\log 2}{(K+1)\cdot\log\alpha_1}+\frac{K}{K+1}. \qquad (30)$$

According to the results of the fractal dimensionality D and the fragmentation parameter α of the motor transport flow interaction analysis (7), the logarithmic-index character of their functional dependence is established (Fig. 4). Moreover, the simple summation of several Cantorian α-sets leads to a decrease in the "total" multifractal dimensionality D_Σ of both traffic aggregate (17). This is true in cases of traffic flows along several lanes of the highway.

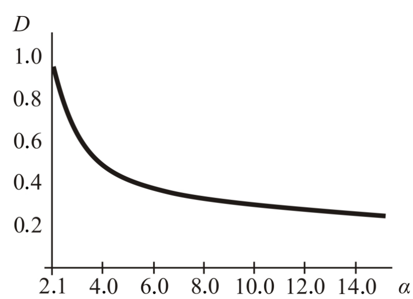

Fig. 4. Dependence of fractal dimensionality D on the fragmentation parameter α of the motor flow

Fig. 5 shows the simplest case of "summing up" two Cantorian α-sets for oppositely directed traffic.

Given the presence of identical fragmentation parameters α=3 (Fig. 5), according to equation (14) we obtain:

$$\left(\frac{1}{\chi}\right)^{n\cdot D_1}+\left(\frac{1}{\chi}\right)^{n\cdot D_1}=\left(2\cdot\frac{1}{\chi}\right)^{D_\Sigma}, \qquad (31)$$

or $\alpha^{n \cdot D_1} + \alpha^{n \cdot D_1} = (2 \cdot \alpha)^{n \cdot D_\Sigma}$, or $2 \cdot (3^{n \cdot 0{,}631}) = 6^{n \cdot D_\Sigma}$,

where D_Σ is the multifractal dimensionality of a set of two oppositely directed traffic flows.

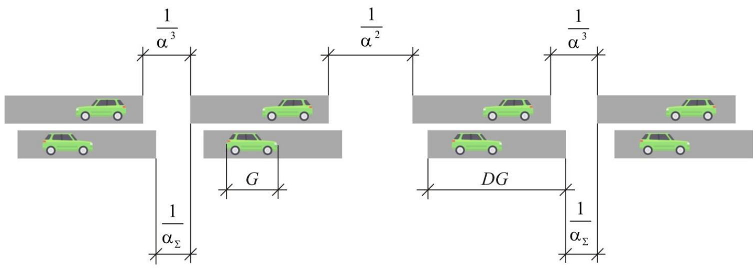

Fig. 5. The set of two oppositely directed traffic flows, which is defined by the third step of the hierarchical construction of the Cantorian α-set

Numerical solution (31) allows us to set the value of the multifractal dimensionality D_Σ of two oppositely directed traffic set. The multifractal dimensionality D_Σ is always lower than the fractal dimensionality D_1 of each separately considered motor transport stream. In addition, D_Σ decreases for each next step of the hierarchical construction of the Cantorian α-set:

$$\begin{cases} D_{\Sigma_1} = 0{,}583, \\ D_{\Sigma_2} = 0{,}516, \\ D_{\Sigma_3} = 0{,}485, \\ D_{\Sigma_4} = 0{,}464, \\ \quad \ldots\ldots\ldots \end{cases} \tag{32}$$

Therefore, the "total" fragmentation parameter α_Σ for a given case is $\alpha_\Sigma > 3$. Moreover, the "total" fragmentation parameter α_Σ increases with each subsequent step of the hierarchical construction of the Cantorian α-set:

$$\begin{cases} D_{\Sigma_1} = 3{,}281, \\ D_{\Sigma_2} = 3{,}831, \\ D_{\Sigma_3} = 4{,}179, \\ D_{\Sigma_4} = 4{,}557, \\ \quad \ldots\ldots\ldots \end{cases} \tag{33}$$

When modeling the structure of motor flows using Cantorian α-sets, the fragmentation parameter α is a function of intensity q, velocity V, density ρ of traffic and traffic interval I of motor vehicles:

$$\alpha = \varphi(q, p, V, I). \tag{34}$$

Similar results can be obtained for case (30) of multiband (m-band) organization of traffic on highways (Fig. 6).

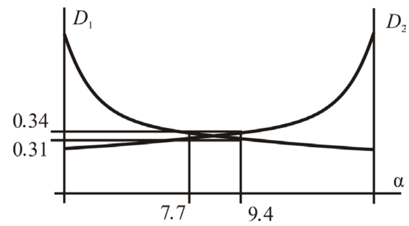

Fig. 6. Results of numerical simulation of two traffic on adjacent lanes of the highway interaction, as hierarchical Cantor α-sets

The stabilization of two adjacent traffic coherence flows takes into account the gradients of their movement direction and the conditional time for the adoption of operational navigational solutions. This is the time of the "speed" of the navigation and computing complex and the control mechanisms of the vehicle. The above applies to the case of saturation of traffic by autonomous robotic vehicles. Therefore, the following values of the fragmentation parameter α_Σ for the totality of traffic and their multifractal dimensionality must be observed D_Σ:

$$\begin{cases} \alpha_\Sigma \gg 9{,}4, \\ D_\Sigma \ll 0{,}31. \end{cases} \tag{35}$$

An option of visualizing the simulation results for a four-lane section of the highway with number m=2 flows in each direction is shown at Fig. 7.

The analytical dependences of structural multifractal organization properties of motor vehicles traffic are obtained. These dependences are a sound basis for the further development of specific algorithms for implementing safe modes of the motor transport operation complex. This is relevant in the context of increasing the share of autonomous robotic vehicles in the composition of motor transport flows.

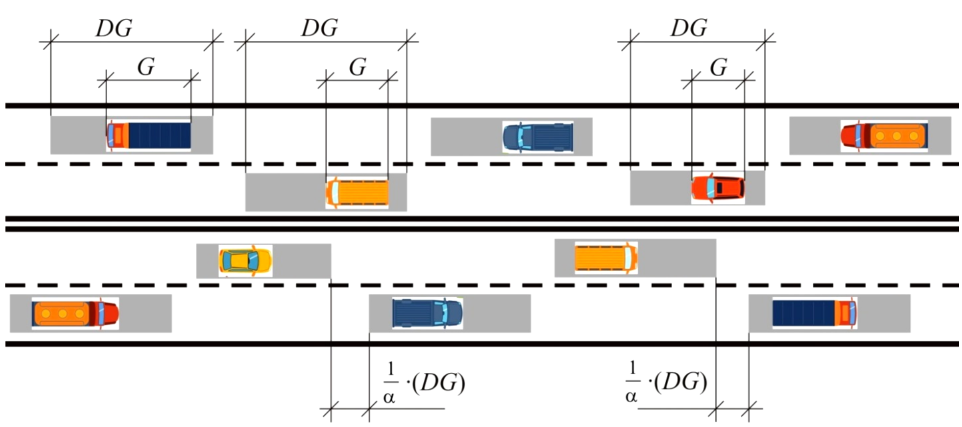

Fig. 7. Variant of the motor transport structural organization model along the four-lane highway section

6. Discussion of results of studying the parameters for safe traffic of motor vehicles

According to the research objectives stated above, the motor traffic is a plurality of (C) vehicles that form a certain shared traffic. The dynamics of traffic is functionally determined by a set of physical and topological features, namely: speed V, intensity q and density ρ of vehicle traffic, their gauge (G), dynamic gauge (DG) and traffic interval (I). On the basis of the specific intensity analysis of traffic q_i the fractal nature of the structural organization of motor flows with the Hausdorff-Besikovich dimension D=0.620, is established, which is dimensionally proportional to the fractal dimensionality D_k=0.631 of the "classical" Cantor set (Table 2). However, the wide limits of the variation of the intensity, density, and velocity of motor flows determine the significant limits of the variation of fractal dimensionalities, which leads to the need to study fractally invariant traits for the entire hierarchical range of Cantor α-sets (Fig. 3) with α>2. Thus, a dynamic multifractality of traffic with a variable fractal dimension D_x is formed. The parameter of the motor transport fragmentation α and the fractal dimension D_x are related by the logarithmic index functional dependence (6), (7).

Analytical modeling of traffic dynamics using the fragmentation parameter α allows us to investigate the traffic structure not only for the specific three modes (phases) of the motor transport – free movement, collective movement, synchronized flow, but also transitional structures in the transition of traffic from one phase to another.

Minimum values of density ρ, intensity q and velocity V of free traffic determine the boundary of the modes start of collective and synchronized flows. At the same time, the highest level of motor vehicles mutual influence at speed V, intensity q and density ρ is inherent in the saturated (synchronized) flow.

An analysis of motor transport flows structural organization graphical models (Fig. 6, 7) determines the necessity of further development of the transport and operational characteristics of the traffic structure study not only taking into account the fragmentation parameter α of the Cantor's Discontinuum, but on such features as the number of "filled" segments in this discontinuity and "Stage" (step) hierarchical construction of a fractal set of vehicles.

Any motorway consists of at least two opposing traffic streams. Each stream is endowed with arbitrary fragmentation parameters α, and therefore has different fractal dimensionalities D. In this connection, multifractality is also formed with its fractal dimensionality Dx. Thus, in the case of motor traffic flows on several lanes of the motorway, the "total" multifractal dimensionality Dx decreases for the aggregate of both traffic. For each subsequent step of the hierarchical construction of the Cantorian α-set, the "total" parameter of the fragmentation α_Σ increases. The obtained results of the multifractal multiband organization of traffic structure study allow us to formalize modes of movement of motor transport with different values of velocity V, intensity q and density ρ of vehicles.

Observance of a traffic set fragmentation and their multifractal dimensionalities rational parameters ensures the stabilization of the coherence of several adjacent motor transport streams (including multiple-directional ones). The obtained analytical dependencies determine the properties of the structural multifractal organization of traffic vehicles. In the applied aspect, the research results can be used to develop specific algorithms for safe modes of operation of the motor transport complex. This is especially important when increasing the share of autonomous robotic vehicles in the composition of motor transport flows.

7. Conclusions

1. We established that the multifractal structure of motor transport streams is reliably described by the regular hierarchical Cantor sets in relation to the parameter of the dynamic size of each individual motor vehicle. We proved that the main multifractal attributes of motor transport are the fragmentation parameter and fractal dimension, which are functionally determined by the intensity, speed and density of vehicle traffic. The upper boundary of the free movement of vehicles with an intensity of 0.035 s^{-1}, density 0.0021 m^{-1} and a flow rate of 17,5 m/s is established. Higher values of indicators characterize the collective and synchronized flow.

2. We proved that the combination of several motor flows, including those directed oppositely, with multi-band traffic influences the dynamics of the main multifractal attributes' variation of vehicles set. So, with an increase in the number of the motorway lanes, the parameter of traffic fragmentation increases, and the fractal dimension of traffic flows aggregate decreases. The value of the fragmentation parameter at the level of 9.4 and the multifractal dimensionality – 0.31 as the necessary and sufficient condition for achievement of the coherence of two adjacent motor transport streams is substantiated.

3. The obtained analytical dependences and the results of numerical simulation of the interaction of a several traffic combination on adjacent lanes of the highway as a hierarchical cantor α-sets are the basis for the development of the basic parameters of navigation algorithms that would ensure safe transport and technological modes of the motor transport operation complex provided that traffic is saturated autonomous robotic objects.

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