Проведеними дослідженнями процесу транспортування газоконденсатної суміші від вибою свердловини до сепараційної установки промислу встановлені особливості ізотермічної і неізотермічної течії флюїду. Доведено, що при неізотермічній течії на гідравлічні втрати в продуктопроводі істотно впливає дросель-ефект і ефект акомодації енергії. Проаналізовано вплив швидкості та об'ємної витрати газорідинної суміші на гідравлічний опір і перепад тиску на ділянці продуктопроводу з урахуванням неізотермічності течії. Встановлено, що оцінка гідравлічного опору і падіння тиску за запропонованими залежностями на 95 % збігається зі стандартизованими. Результат отримано на базі розробленої системи рівнянь математичної моделі неізотермічного нестаціонарного одновимірного руху газорідинної суміші вуглеводнів в трубопроводі. Запропонована система вигідно відрізняється від відомих урахуванням внутрішнього конвективного теплообміну, оціненого по інтегральному ефекту Джоуля-Томсона.

Відмінною особливістю вдосконаленої методики розрахунку стало введення температурної поправки і коефіцієнта акомодації в розрахунках гідравлічного опору трубопроводу як системи з розподіленими параметрами. Завдяки цьому стало можливим вдосконалення методики розрахунку неізотермічного транспортування гомогенної газоконденсатної суміші. На основі аналізу розрахункових кривих за відомими методиками (формули Колбрука, Лейбензона і ВНІІГАЗу) для ізотермічних і неізотермічних процесів і пропонованою методикою показані раціональні області їх застосування. Всі обчислення зроблені при швидкостях газорідинного потоку в діапазоні 0–50 м/с, шорсткості труб 0.01–0.05 мм і їх діаметрі 100–300 мм, використані дані реальних промислових трубопроводів Новотроїцького нафтогазоконденсатного родовища. Порівняння теоретичного і промислового експериментів показало достатню для інженерної практики точність розрахунку падіння тиску на ділянках нафтогазових шлейфів і дозволяє рекомендувати розроблені аналітичні залежності для впровадження в промисловий інженерії

Ключові слова: неізотермічна течія, транспортування трубопроводом, газорідинна суміш вуглеводнів, гідравлічні втрати, коефіцієнт гідравлічного опору

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IMPROVEMENT OF AN ENGINEERING PROCEDURE FOR CALCULATING THE NON-ISOTHERMAL TRANSPORTATION OF A GAS-LIQUID MIXTURE

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1. Introduction

Engineering practice in the oil and gas industry solves the problem of the evaluation of hydraulic resistance of a pipeline for transportation of hydrocarbons. In addition, for their mixture, this problem is greatly complicated by different state of hydrocarbons, constantly changing geometry of their distribution in the pipeline cavity, different ratios of specific volumes of transported substances.

The general structure of oil and gas extraction and transportation, as a rule, involves the transportation of petroleum fluids from productive strata by conditionally vertical wells and further along conditionally horizontal industrial and main pipelines. For engineering calculations, it is important to take into consideration hydraulic losses on all sections of transportation of raw, prepared, as well as commercial hydrocarbons. In particular, in oil and gas extraction, depending on hydraulic losses, diameters of vertical oil well tubing (OWT) of horizontal well lead lines and inter-field flow lines [1–3] are selected.

At mainline multi-pipe transportation of oil and gas, it is also appropriate to change thermo-hydraulic operation modes in the networks, transform the topology and schemes of transportation [4–7].

Transportation of hydrocarbon mixtures is characterized by conditionally isothermal or substantially non-isothermal flow mode. The former may include underground main pipelines of oil and gas transportation, the latter – industrial transport networks of hydrocarbons from well bottom to cutting-in units in the main pipeline, as well as pipelines with heat transfer agents. The procedures for calculation of conditionally isothermal flows are well developed [4]. When calculating non-isothermal flows, as a rule, the entire main pipeline is divided into the sections with conditionally constant temperature mode. Distribution of thermo-baric parameters in the network is not take into consideration. That is, non-isothermal networks are considered in first approximation as a set of conventionally isothermal, which results in calculation errors. In addition, to calculate non-isothermal flows, the standardized procedures, which apply only to fixed flow modes and do not work under transient modes, are used. In general, these shortcomings of the known procedures lead to overstatement of designing diameters of pipelines by 10–20 % [5]. This causes the relevance of improvement of engineering procedure for calculating non-isothermal transportation of the gas-liquid mixture. This mostly applies to well flows of fluids with elevated temperatures and often changing flow modes. It is expected to reach the accuracy of calculation of the non-isothermal transportation of the gas-liquid mixture by taking into consideration the throttle-effect and energy accommodation effect.

2. Literature review and problem statement

In numerous papers, for example [5–8], well-known simplified empirical Darcy formulas that describe isothermal motion of fluids and gases are used in hydraulic calculation. Thus, papers [7–8] contain the examples of using the Darcy formula for gases, and papers [5, 6] – for mixtures of gases and fluids. However, when using these dependences in non-isothermal transportation systems, this leads to significant errors in the calculation of pressure losses and overrating design diameters of pipe flow areas [9]. The reason for this is the failure to take into consideration the effects of compressibility-incompressibility of the transported fluid and their varying viscosity. Thus, during the transportation of liquid hydrocarbons, a change in the fluid velocity due to heat exchange along a transportation section due to little compressibility of fluids is relatively small. Another pattern is observed during the transportation of the compressible gas-liquid mixture, which necessitates taking into consideration the impact of heat exchange processes on a change in hydraulic losses during its transportation. The valiant of overcoming related difficulties could be the development and improvement of the procedure for calculating of non-isothermal transportation of the gas-liquid mixture. This is the approach used in [10], where the modified Leibenson formulas are used for calculation of non-isothermal flows. However, their applicability is limited to fixed empirical coefficients found for certain conditions. All this makes it possible to argue that it is appropriate to continue the search for more sophisticated approaches to the calculation of non-isothermal fluid flows under conditions of their extraction and industrial transportation.

The mathematical model of non-isothermal non-stationary one-dimensional motion of the gaseous mixture of hydrocarbons in the pipeline, taking into consideration heat exchange through a cylindrical wall is known in the oil and gas industry [4, 7]:

$$\frac{\partial(\rho\cdot w)}{\partial t}+\frac{\partial\left(\rho\cdot w^2\right)}{\partial x}=-\frac{\partial P}{\partial x}-\frac{\lambda\cdot\rho\cdot w^2}{2\cdot D}-\rho\cdot g\frac{dH}{dx}$$

is equation of motion;

$$c^2\cdot\frac{\partial(\rho\cdot w)}{\partial x}=-\frac{\partial P}{\partial t} \qquad (1)$$

is equation of flow continuity;

$$\rho\cdot w\cdot S\cdot\frac{dJ}{dx}=\pi\cdot D\cdot q_n$$

is equation of thermal balance,

$P=\rho\cdot z\cdot R\cdot T$ – equation of state,

where P, T, ρ, w, z are the mean values of pressure, temperature, density, velocity and coefficient of compressibility of the gas-liquid flow in the pipe cross-section, respectively; R is the gas constant; D, S, H are the technical characteristics of a pipeline: inner diameter, cross-section area, geometrical height of center of gravity of the element of volume of carbohydrates mixture, respectively; x is the coordinate by the length, t is the time; c is the sound velocity in gas flow; J is the enthalpy of the transported flow; q_n is the thermal flow passing through the unit of the pipeline surface area in a unit of time.

In the system of equations (1) equation of flow continuity and equation of thermal balance are described in more detail in the following expanded form [2, 9]:

$$\frac{\partial(\rho\cdot w)}{\partial t}+\frac{\partial}{\partial x}\left(P+\rho\cdot w^2\right)=-\frac{\lambda(\mathrm{Re},\mathrm{k_e})\rho\cdot w^2}{2\cdot D}-\rho\cdot g\cdot\sin\alpha,$$

$$\frac{\partial(\rho w)}{\partial x}+\frac{\partial\rho}{\partial t}=0, \qquad (2)$$

$$\rho\left(\frac{\partial e}{\partial t}+w\frac{\partial e}{\partial x}\right)=-\frac{4\cdot K_t}{D}(T-T_o)-\rho\frac{\partial w}{\partial x}+\frac{\lambda(\mathrm{Re},\mathrm{k_e})\cdot\rho\cdot w^3}{2\cdot D},$$

$$P=\rho\cdot z\cdot R\cdot T,$$

where α is the angle of inclination of a pipeline section to horizon; e is the internal energy of a weight unit of the mixture, $e_{\text{BH}}=C_vT+\text{const}$; C_v is the thermal capacity at constant volume; K_t is the full coefficient of heat transfer from the hydrocarbon mixture to the environment; T_o is the ambient temperature; $\lambda(\mathrm{Re},\ k_e)$ is the coefficient of hydraulic resistance, the function of Reynolds number Re and relative surface of the surface of pipes k_e.

It was noticed in a series of experiments that for liquid hydrocarbons, hydraulic resistance changes at the transition from the isometric to non-isothermal transportation. Thus, for example, to determine coefficient of hydraulic resistance at non-isothermal oil transportation λ_g, in [10], the expression was obtained based on the experimental data:

$$\lambda_g=a_h\cdot\left(\frac{8\cdot\mathrm{Re}}{N+2\left(1+\sqrt{9+N}\right)}\right)^{b_h}\cdot\left(\frac{\mu_w}{\mu_f}\right)^{0.62}, \qquad (3)$$

where $a_h=2.9\cdot\mathrm{He}^{-0.403}$; $b_h=1.26\cdot\mathrm{He}^{-0.265}$; He is the Headstream number; μ_w is the dynamic viscosity on a pipeline wall; μ_f is the average dynamic viscosity of the near-axial part of the flow; N is the parameter of considering dimensionalities.

By analogy with (3), for the mixture of oil and gas, one should also expect a change in pipeline hydraulic resistance, depending on temperature difference on the transportation section. In papers [11–13], approaches to solving systems of equations (1) and (2) using of the finite-different-axial methods (with dynamic adaptation of grid dimensionality) for the gas-liquid mixture were proposed. However, the accuracy of obtained results is strongly influenced by the structural forms of the gas-liquid flow [12]. In particular, these methods almost do not work at cork and separate-wave patterns of the gas-liquid flow.

In practice, for the calculation of hydraulic resistance under isothermal modes of pipelines, the Colebrooke formula [14–20] and VNIIgas formula [21] are used most often. The Darcy-Weisbach formula or the Leibenson formula family are used for calculation of pressure losses under the same conditions [6].

Under non-isothermal operation modes, the modified formulas of Leibenson of the form of $\lambda = A/\mathrm{Re}^m$, where A and m are the empiric coefficients [20], are used in practice to calculate hydraulic resistance of a pipeline. According to data from [22],

$$A = 0.11(68 + \Delta\mathrm{Re})^{0,25};\ m = (1 - 560D/(\Delta\mathrm{Re}).$$

The Darcy-Weisbach formula with the correction by Shukhov is used to calculate pressure losses [21]. In some calculations, the Liebenson formula is specified by analogy with (3) to take into consideration the viscosity difference (the ratios of viscosities and Prandtl numbers) in the near-wall and near-axial zones. However, the Liebenson formula was obtained and verified in practice for $\mathrm{Re} \leq 60{,}000$. The Leibenson formula also gives great errors in the mixed friction area. This is a big drawback, as the mixed friction area covers a wide interval of Reynolds numbers, at which oils with low viscosity and light petrochemicals are usually pumped [23]. Darcy-Weisbach formula with the Shukhov correction was adapted for conditions of flow cooling along the flow line. Currently, in some countries, the Leibenson formula is a basis for the production guidance document RD 39-30-1061-84.

The reverse case (flow heating) is not presented in the special literature on oil and gas profile.

When transporting gas-liquid hydrocarbon mixtures through pipelines, determining thermo-hydrodynamic parameters depends on heat exchange processes, as well as on throttle effect in adiabatic expansion of fluid. Not taking into consideration the throttle effect in the industry standards used in practice leads to errors of determining the temperature drops up to tens of degrees. In particular, during the evacuation of the gas-liquid mixture from a well which is 3,000 m deep and has pressure drop of 10 MPa, additional temperature drop from throttling will make up 40 degrees Kelvin [24]. The absence of the correction for throttle effect and inter-pipe accommodation of energy causes an error of determining hydraulic resistance and the forecasting errors of gas-condensate fields development. The accumulation of errors increases at successive calculation of lifting and subsequent commercial transportation to separation systems. However, solving problems of taking into consideration the throttle effect and accommodation when calculating non-isothermal flow in flow lines is a complex iterative algorithmic problem and demands modern computing and software support.

That is why there is an interest of the academic community to further search for the methods of calculation of hydraulic resistance and pressure losses at non-isothermal modes of transportation of different-phase homogeneous hydrocarbon mixtures.

3. The aim and objectives of the study

The aim of the current research is to develop a mathematical model for non-isothermal quasi-stationary one-dimensional motion of the gas-liquid mixture of hydrocarbons in a pipeline. In this case, we should take into consideration the throttle effect during adiabatic expansion of fluid along the inner cavity of the field pipeline and energy accommodation.

To achieve the aim, the following tasks were set:

– to develop the system of equations of the mathematical model for non-isometric quasi-stationary one-dimensional motion of the gas-liquid mixture of hydrocarbons in a pipeline;

– based on the obtained model, to develop the calculation formulas for determining the hydraulic resistance of a pipeline and pressure losses;

– to perform a comparative analysis of the results of calculation of hydraulic resistance of a pipeline and pressure losses by the obtained calculation formulas and formulas of Leibenson, VNIIgas, Colebrooke, as well as experimental data.

4. Materials and methods for studying the non-isothermal quasi-stationary one-dimensional motion of homogeneous gas-liquid hydrocarbon mixture in a pipeline

4. 1. Studied materials and equipment used in the experiment

The flow of a homogeneous oil-gas condensate mixture in tubing strings and field lead lines of Novotroitsk oil-gas condensate field of Ukraine was explored.

Manometers MTI, CDV, IMO; thermometers TSMU, TSPU, TB-2; flowmeter PT878GC were used to measure pressure, temperature and mass flow rate of fluid in the pipelines.

4. 2. Procedure for studying the non-isothermal motion of a homogeneous gas-liquid mixture of hydrocarbons in the pipeline

To develop a mathematical model for non-isothermal one-dimensional motion of the gas-liquid mixtures of hydrocarbons in the pipeline, the classical analytical- empirical method was applied. In this case, the known analytical equations of heat and mass transfer and empirical equations of boundary conditions were used. Development of mathematical model of non-isothermal transportation of the gas mixture was based on the method of synthesis of analytical relations of a pipeline hydraulic resistance with ambient temperatures, longitudinal changes of temperature during heat transfer and throttling.

The study was methodologically divided into three stages.

At the first stage, we analyzed the assumptions of the transition from the known system of differential equations in particular derivatives for non-isothermal non-stationary process for transportation of the gas-liquid mixture to the system of nonlinear equations of quasi-stationary non-isothermal process.

At the second stage, the analytical expressions for pressure drop and hydraulic resistance of the moving mixture of hydrocarbons on the transportation section were found. Based on the obtained system of nonlinear equations of quasi-stationary non-isothermal process, the expressions for pressure drop of the moving mixture of hydrocarbons on vertical and horizontal sections were found by means of mathematical transformations. The calculation dependence for hydraulic resistance was differentiated depending on Reynolds number.

At the third stage, hydraulic resistances and pressure drops of the moving mixture of hydrocarbons were calculated using the equations obtained at the second stage for the conditions of wells and lead fields of Novotroitsk oil-gas condensate field. In parallel, pressure, temperature and mass flow rate of fluid on the section “well bottom – lead line” were recorded. The calculation and empirical data were compared.

The method for numerical solution of a system of nonlinear equations of a mathematical model is based on the modified Lax-Vendroff scheme and the Rank-Kutt algorithm of the 4th order. The latter turned out to be more effective and consistent for realization of the considered problem in the Mathcad program in comparison with purely implicit scheme or a scheme with splitting by physical processes.

5. Results of studying the non-isothermal quasi-stationary one-dimensional motion of a gas-liquid mixture of hydrocarbons in a pipeline

5. 1. Development of the system of equations for a mathematical model

To develop the problem of the research into the system of equations, we will take into consideration the inter-pipe convective heat transfer in the system of equations (2). To do this, we will introduce an additional component using the Joule-Thomson coefficient *Dj* and divide the energy equation from system (2) by density of mixture ρ and isobaric heat capacity of mixture *Cp*. After transformations, we will obtain the general system of equations of the mathematical model of non-isothermal non-stationary one-dimensional motion of the gas-liquid mixture in the pipeline during heat transfer through a cylindrical wall in the form:

$$\frac{\partial P}{\partial x}+\frac{\partial(\rho w)}{\partial t}+(1+\beta)\frac{\partial(\rho w^2)}{\partial x}+\frac{\rho g\partial h}{\partial x}+\frac{\lambda\rho w^2}{2D}=0;$$

$$\frac{\partial\rho}{\partial t}+w\frac{\partial\rho}{\partial x}+\rho\frac{\partial w}{\partial x}=0; \qquad (4)$$

$$P=\rho zRT;$$

$$w\frac{\partial T}{\partial x}+\frac{\partial T}{\partial t}-D_j\frac{w\partial P}{\partial x}+\frac{4K_t\left(T-T_0\right)}{\rho DC_p}-K_\lambda\frac{\lambda}{D}\frac{w^3}{C_p}=0;$$

$$w=\frac{\partial x}{\partial t};$$

$$M_q=wS\rho,$$

where *P* is the pressure of the mixture; *T* is the temperature of the mixture; *w* is the flow velocity averaged in cross-section; ρ is the density of the mixture; *D* is the diameter of a pipeline; *x* is the longitudinal coordinate of a pipeline; *t* is the time; T_o is the temperature of soil and rocks near a pipeline (environment); K_t is the coefficient of thermal conductivity from the mixture to the environment of the system “mixture – pipeline – environment”; D_j is the Joule-Thomson coefficient; K_λ is the coefficient of convective heat transfer in the mixture flow; C_p is the isobaric thermal capacity of the mixture; *z* is the coefficient of compressibility of the mixture; λ is the coefficient of hydraulic resistance of a pipeline; *g* is the gravity acceleration; β is the Coriolis coefficient; M_q is the mass flow rate of the mixture, *L* is the pipeline length.

The resulting system of equations (4) differs from known ones (1), (2) by taking into consideration convective heat transfer in the internal cavity of the pipeline and compressibility of the gas-liquid mixture.

5. 2. Development of calculation formulas for determining hydraulic resistance of a pipe and pressure losses

To obtain the calculation data, the system of equations (4) should be brought to the conditions of the quasi-stationary mode of fluid flow. To do this, we will consider the mass flow rate of the mixture M_q (*x*, *t*)=const. We will add up the right and left parts of the equations of forces balance, continuity and state (the first three in system (4)), reduce the terms and perform the integration within the range of the pressure change on the considered section P_1, P_2. After performing the transformation for M_q in the quasi-stationary process, we have:

$$M_q=S\cdot\left(\frac{\left(P_1^2\cdot e^{-bL}-P_2^{\,2}\right)Db}{\lambda\cdot z\cdot R\cdot T_o\cdot\phi_t\cdot(1-e^{-bL})}\right)^{0.5},$$

where coefficient

$$b=\frac{2\cdot g\cdot\Delta h}{z\cdot R\cdot T_o\cdot\phi_t\cdot L},$$

$$\phi_t=\left(1+\frac{T_1-T_2}{T_o\cdot\ln\left(\dfrac{T_1-T_o}{T_2-T_o}\right)}\right).$$

Here Δ*h* is the geodetic difference of the heights between the beginning and the end of the pipeline, ϕ_t is the temperature correction.

In this case, equation of state *P*=ρ·*z*·*R*·*T;* reduced to the conditions of quasi-stationary mode will be written down as:

$$\frac{2}{3}\cdot\left(P_1+\frac{P_2^{\,2}}{P_1+P_2}\right)=\rho\cdot z\cdot R\cdot T_o\cdot\phi_t.$$

At constant M_q, the equation of balance of energies (fourth in system (4)) for the quasi-stationary process, taking into consideration the throttle effect is converted into the form:

$$M_q\cdot C_p\cdot((T_2-T_1)-D_j\cdot(P_2-P_1))=$$
$$=K_t\cdot\pi\cdot D\cdot\int_0^L(T_0-T_x(x))\mathrm{d}x,$$

where $T_x(x)$ is the dependence of temperature on longitudinal coordinate in the pipeline; T_1, T_2 are the temperatures at the beginning and at the end of the considered pipeline section.

We will note that coefficient of thermal conductivity $K_t=Nu\cdot\lambda_T/D$, where at turbulent flow $Nu=k_o\mathrm{Re}^{0.8}Pr^n$ [17], where k_o is the coefficient of energy accommodation. At the transition from longitudinal cooling to transversal heating of the flow the power at Prandtl number changes within $n=0.3–0.45$ [17, 25]. Coefficient k_o characterizes energy interaction of molecules of the transported medium with the surface of a solid and, evidently, depends on the electro-kinetic characteristics of the walls of a pipeline and molecules of the medium [26].

Thus, after making the transformation, we will obtain the system of equations of a mathematical model of the non-isothermal quasi-stationary one-dimensional motion of the homogeneous gas-liquid mixture in a pipeline with heat exchange through a cylindrical wall in the form:

$$M_q = S\cdot\left(\frac{\left(P_1^2\cdot e^{-bL}-P_2^2\right)Db}{\lambda\cdot z\cdot R\cdot T_o\cdot\phi_t\cdot\left(1-e^{-bL}\right)}\right)^{0.5};$$

$$\begin{aligned}&M_q\cdot C_p\cdot((T_2-T_1)-D_j\cdot(P_2-P_1))=\\&=K_t\cdot\pi\cdot D\cdot\int_0^L(T_0-T_x(x))\mathrm{d}x;\end{aligned}\tag{5}$$

$$\frac{2}{3}\cdot\left(P_1+\frac{P_2^2}{P_1+P_2}\right)=\rho\cdot z\cdot R\cdot T_o\cdot\phi_t;$$

$$K_t\cdot D/\lambda_T=k_o\cdot\mathrm{Re}^{0.8}\cdot Pr^n.$$

System (5) contains equations of the balance of mechanical and thermal energy, equation of the state and accommodation of energy.

At the second stage of the research, by mathematical transformations, we get the expression for pressure drop and hydraulic resistance of the moving mixture of hydrocarbons on the transportation section from the obtained system of nonlinear equations (5).

We will note that determining the pressure drop ΔP during the transportation of the gas-liquid mixture in vertical and horizontal sections has its own specifics. First, we will determine pressure drop ΔP during the transportation of the gas-liquid mixture on the horizontal area. Taking into consideration the introduced temperature correction for the non-isothermal process ϕ_t, we have from the equation of balance of mechanical and thermal energy of system (5):

$$\Delta P=\frac{8M_q^2 zRT_o L\phi_t\lambda}{P\pi^2 D^5}.\tag{6}$$

Dependence (6) is universal both for liquid and for gas, since the gas-liquid mixture is accepted as the one described correctly by the Darcy equation. Parameters z and R represent mean values for the two-phase mixture.

Analyzing equation (6), we see that hydraulic losses ΔP are directly proportional to ϕ_t, the transportation length L and ambient temperature T_o. For the vertical section $\Delta P=P_2–P_1$. Taking into consideration that in this case $\Delta h=L$ and coefficient $b=2\cdot g/(z\cdot R\cdot T_o\cdot\phi_t)$, dependence between P_2 and P_1 in the implicit form takes the form:

$$P_2=\sqrt{P_1^2 e^{-\frac{2gL}{ZRTo\phi t}}-\frac{8M_q{}^2\lambda z^2R^2T_o{}^2\phi_t{}^2}{D^5\pi^2 g}\left(1-e^{-\frac{2gL}{ZRTo\phi t}}\right)},\tag{7}$$

where P, T, M_q, λ are the pressure, temperature, mass flow rate and coefficient of hydraulic resistance of the vertical pipeline; z, R are the coefficient of compressibility and gas constant of the mixture of hydrocarbons; $L=H$ is the depth (heights difference), D is the diameter of the vertical pipeline.

In the calculations of hydraulic losses (pressure drops) in vertical oil-gas pipelines, the known formulas by Adamov and Krylov [1, 2, 8] corresponding to the resulting formula (7), are used.

For hydraulic resistance λ, the authors proposed to write down the formula in a general form for 7 ranges of changing Reynolds number (respectively, empirical dependences of Poiseuille, Nikuradze, Shifrinson, Altshul, Blasius, Stokes, Haaland). Mathematical calculations were performed using Boolean algebra:

$$\begin{aligned}\lambda=&\left(\mathrm{Re}\le 2320\right)\otimes\frac{64}{\mathrm{Re}}+\\&+\left(2320\prec\mathrm{Re}\le 10^4\right)\otimes\left(\frac{64(e^{-0.2(\mathrm{Re}-2320)})}{\mathrm{Re}}+\frac{0.3164(e^{-0.2(\mathrm{Re}-2320})}{\sqrt[4]{\mathrm{Re}}}\right)+\\&+\left(10^4\prec\mathrm{Re}\le 10^5\right)\otimes\frac{0.3164}{\sqrt[4]{\mathrm{Re}}}+\\&+\left(10^5\prec\mathrm{Re}\le\frac{27}{(\frac{k_e}{D})^{1.143}}\right)\otimes\left(0.0032+\frac{0.221}{\mathrm{Re}^{0.237}}\right)+\\&+\left(\frac{27}{(\frac{k_e}{D})^{1.143}}\prec\mathrm{Re}\le\frac{500D}{k_e}\right)\otimes 0.11\left(\frac{k_e}{D}+\frac{68}{\mathrm{Re}}\right)^{0,25}+\\&+\left(\frac{500D}{k_e}\prec\mathrm{Re}\le 10^7\right)\otimes 0,11\left(\frac{k_e}{D}\right)^{0.25}+\\&+\left(10^7\prec\mathrm{Re}\right)\otimes\frac{0.25}{\left(\log\frac{k_e}{3.7D}\right)^2}+\lambda_s,\end{aligned}\tag{8}$$

where λ_t takes into consideration friction of rolling; λ_s is the additional value to λ_t (takes into consideration friction of slipping) in geometrically complicated different-phase flows. According to formula of Sakharov-Volovodov, Mokhov [19]: $\lambda=\lambda_t+\lambda_s$.

Taking into consideration temperature correction ϕ_t for λ in the equation of Darcy-Weisbach the authors received the final form of formula:

$$\lambda=(\lambda_t+\lambda_s)\,\phi_t\,(T/T_o).\tag{9}$$

We will note that calculation formulas previously proposed in scientific literature [4, 10, 14] took into consideration 4–5 ranges of change of Reynolds number. In particular, the algorithm of calculation of Liebenson hydraulic resistance coefficient λ considers 4 ranges (for four fixed values $m=0$; 0.123; 0.25; 1). A more detailed approach to the calculation of λ through taking into consideration transition ranges from the laminar to the turbulent flow $2{,}300<\mathrm{Re}<10^4$ and transition from the turbulent to the developed turbulent flow $500D/k_e<\mathrm{Re}<10^7$ was proposed. In the latter case, a more accurate description is given by the Haaland formula [25], and the central section of particular extremum of the curve at medium rates [19] can be described more precisely by Shifrinson and Altshul formulas). Thus, formulas (8) and (9) take into consideration 7 ranges of change in the Reynolds number.

At the third stage, to determine hydraulic resistance and pressure drops of the mixture of hydrocarbons, wells and lead lines of Novotroitsk oil-gas condensate field were accepted as model object. Calculations were performed according to formulas (5)–(9).

Viscosity μ, compressibility *z* and resistance λ depending on them were calculated for three different temperatures (initial, final and intermediate) according to formulas of Starling-Ellington and Latonov-Gurevich [15]:

$$z(P,T)=\frac{0.1\cdot P}{P_{pk}}+\left[0.4\times\log\left(\frac{T}{T_{pk}}\right)+0.73\right]^{\frac{P}{P_{pk}}}. \tag{10}$$

$$\mu(P,T,M)=\frac{(9.41+0.02\cdot M)\cdot(1.8\cdot T)^{1.5}}{(209+19\cdot M+1.8\cdot T)\cdot 10^{7}}\times$$
$$\times EXP\left(\left(3.5+\frac{547.8}{1.8\cdot T}+0.01\cdot M\right)\cdot\left(\frac{P\cdot 10^{3}}{z(P,T)\cdot\frac{8314.3}{M}\cdot T}\right)^{\left(2.4-0.2\left(3.5+\frac{547.8}{1.8T}+0.01\cdot M\right)\right)}\right), \tag{11}$$

where μ(*P, T, M*) is the dynamic viscosity (as the function of three parameters); *M* is the molar weight of the mixture; P_{pk}, T_{pk} are the pseudo-critical pressure and temperature of hydrocarbon mixture.

When calculating the parameters of the transportation of liquid petroleum products, the authors of [17] recommend considering correction of viscosity μ according to molar weight of the mixture, and compressibility *z* constant only for large values of density.

In practical calculations, there arises a complexity of determining molar weight *M* and pseudo-critical parameters P_{pk}, T_{pk} for the multi-component hydrocarbon mixture. That is why compressibility *z*(*P, T*) of the transported mixture and dynamic viscosity μ is taken for one middle point of mode parameters or as the average by calculations at extreme points.

It seems more convenient to introduce temperature correction to adjust λ:

$$\phi_t(T,\Delta T,T_o)=\left(1-\frac{\Delta T}{T_o\cdot\ln\left(\frac{\mathrm{T}+\Delta\mathrm{T}/3-T_o}{\mathrm{T}-2\Delta\mathrm{T}/3-T_o}\right)}\right), \tag{12}$$

where Δ*T* is the temperature drop on the section of a field pipeline.

In this case, temperature correction is determined based on initial temperature and temperature drop on the section.

According to the examples of calculations in [4], coefficients of hydraulic resistance and heat transfer of the product pipeline, viscosity and density of the hydrocarbon mixture, as well as other gas dynamic parameters change along the piping route (in this case, from well bottom to wellhead). Hence, it is appropriate to perform iterative calculations to improve accuracy after setting the transportation mode with maximum detailing the parameters on the intervals. But for calculations of engineering precision (up to 5–10 %), it is advisable to use the concepts of medium and edge parameters. That is, in approximate engineering calculations it seems appropriate to use only three control points of real parameters and, respectively, two operating sections [8].

As it is seen from equations (8) and (9), determining plays a key role in assessing the pressure drop and remains in these formulas without scaling or attraction of correction coefficients.

5. 3. Comparative analysis of the results of calculation of hydraulic resistance of a pipeline and pressure losses by the obtained calculation formulas and well-known formulas

The calculation example is performed for pipelines of different roughness (0.01–0.03 mm) and diameters (100–500 mm) of the length of 1000 m. We assigned the mixture of hydrocarbons of associated Novotroitsk oil-gas condensate field before entering the liquid separator in the range of actual mode parameters of well lead lines (*P*~3–5 MPa, *T*~300–330 K, *Q*~0.1–1 m^3/s).

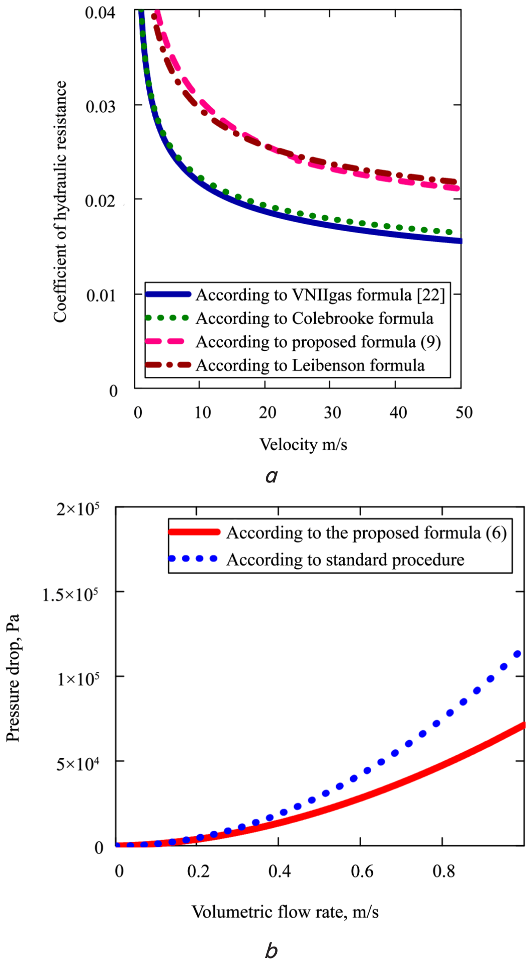
Based on the original data of the example for pipelines and mode parameters of mixtures transportation, the Leibenson formulas with parameters *A* and *m* were synthesized. In particular, for the transition range from laminar to turbulent flow mode, *A*=0.1358 and *m*=0.13, i. e. $\lambda=A/\mathrm{Re}^{m}=0.1358/\mathrm{Re}^{0.13}$. Temperature drop ΔT before the substitution in calculation formulas (9) and subsequently in (5) was corrected taking into consideration the influence of Joule-Thomson effect.

Fig. 1–4 show the obtained calculation dependences of coefficient of hydraulic resistance on velocity and pressure losses on volumetric gas flow rate of the gas-liquid mixture by the dependences obtained by the authors. It also shows dependences, obtained by the standard procedure, based on the Leibenson formula. In particular, Fig. 1 – for the cooled pipe of roughness of 0.01 mm and diameter of 100 mm. In Fig. 2 – for the cooled pipe of roughness of 0.05 mm and diameter of 100 mm. In Fig. 3 – for the heated pipe of roughness of 0.01 mm and diameter of 300 mm. In Fig. 4 – for the heated pipe of roughness of 0.05 mm and diameter of 300 mm.

To provide a possibility to compare the results of calculations of λ and Δ*P* between the isothermal and non-isothermal modes, a part of the curves in Fig. 1–4 is shown for isothermal calculations (cooling and heating is not considered). These are the curves plotted by the formulas of VNIIgas and by Thomas Colebrooke.

In parallel to the conducted modeling research, pressure, temperature and mass flow rate of fluid on the section "well bottom – lead line" of Novotroitsk oil and gas condensate field were recorded. Comparison of calculation and empirical data is presented in Table 1.

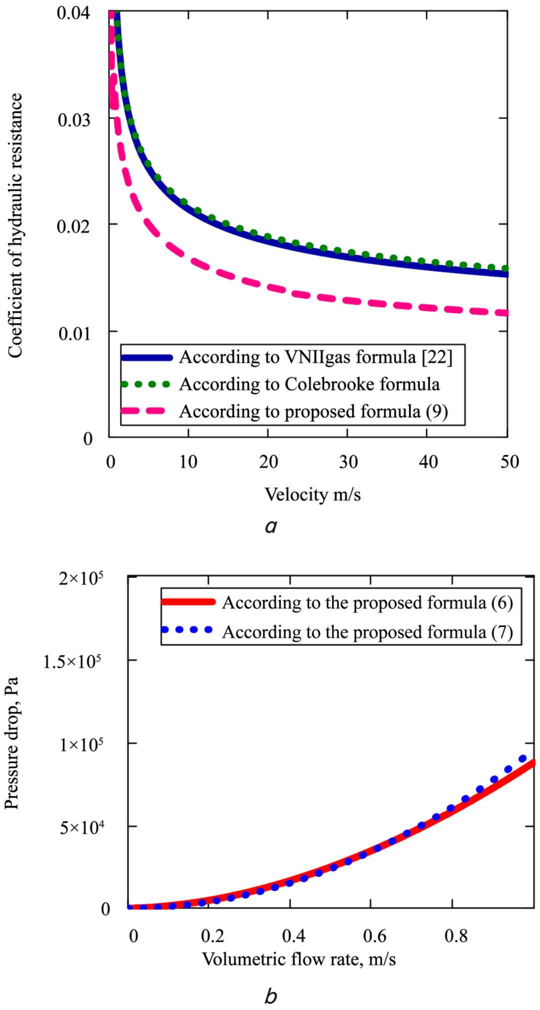
Geometric and mode parameters of pipelines: diameter is 100–300 mm; pipe roughness is 0.01–0.05 mm; volumetric flow rate of mixture of hydrocarbons 0.5–1 m^3/s. Average integral temperature is 305 K, temperature drop on the transportation section is 40 K.



a

b

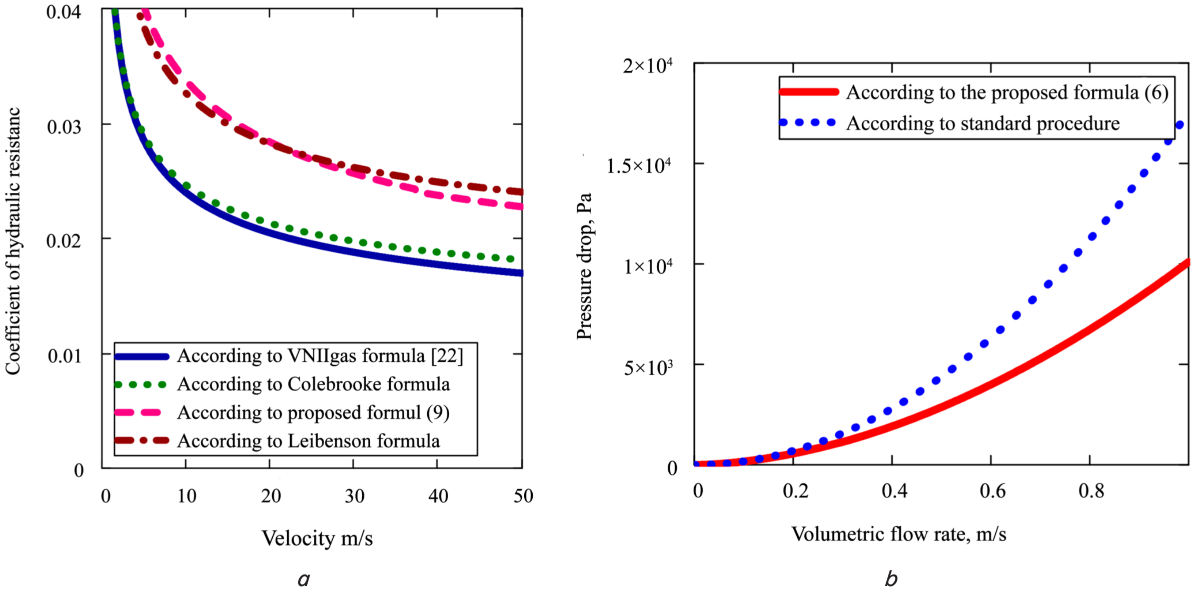
Fig. 1. Calculation dependences of coefficient of hydraulic resistance on flow velocity and of pressure losses on volumetric flow rate of the cooled gas-liquid mixture along the pipeline of diameter of 100 mm at roughness of pipes of 0.01 mm: *a* — dependence $\lambda = F(w)$; *b* — dependence $\Delta P = F(Q)$



a

b

Fig. 3. Calculation dependences of coefficient of hydraulic resistance on velocity and of pressure losses on volumetric flow rate of the heated mixture in the pipeline of diameter of 300 mm at roughness of pipes of 0.01 mm: *a* — dependence $\lambda = F(w)$; *b* — dependence $\Delta P = F(Q)$



a *b*

Fig. 2. Calculation dependences of coefficient of hydraulic resistance on velocity and of pressure losses on volumetric flow rate of the cooled gas-liquid mixture by the pipeline of diameter of 100 mm at roughness of pipes of 0.05 mm: *a* — dependence $\lambda = F(w)$; *b* — dependence $\Delta P = F(Q)$

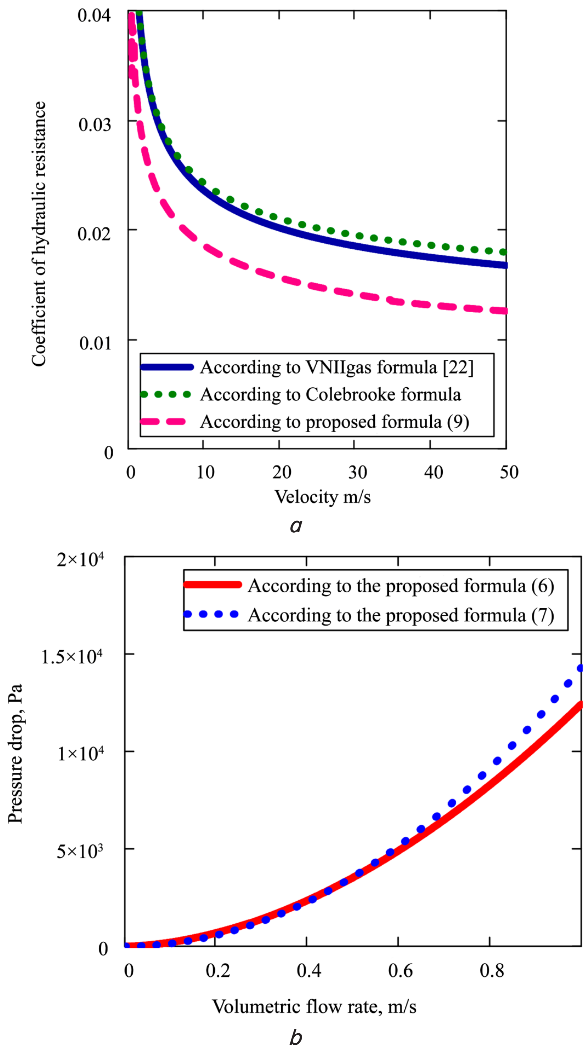


Fig. 4. Calculation dependences of coefficient of hydraulic resistance on velocity and of pressure losses on volumetric flow rate of the heated gas-liquid mixture in a pipeline of diameter of 300 mm at roughness of pipes 0.05 mm: *a* – dependence $\lambda=F(w)$; *b* – dependence $\Delta P=F(Q)$

6. Discussion of results of modeling and numerical studies

Analysis of the obtained data shows that in the range of velocities of 0–50 m/s for coefficient of hydraulic resistance of the isothermal (formula of Thomas Colebrooke and of VNIIgas) and non-isothermal (Leibenson formula and the formula proposed by the authors) flow of hydrocarbons show that they differ by 16–31 %. Calculations were performed for pipes of diameter of 100–300 mm with roughness of 0.01–0.05 mm (Fig. 1, 2). This is a significant discrepancy, indicating unsuitability of the formulas of Thomas Colebrooke and VNIIgas for the calculation of non-isothermal modes of flows (without the use of temperature corrections).

At velocities of up to 3–5 m/s, calculation data of hydraulic resistance coefficient for all the considered formulas converge (Fig. 1–4), which is explained by the laminar mode of flow of the hydrocarbons mixture.

The maximum difference in the calculation of hydraulic resistance coefficient according to Liebenson formula and the one proposed by the authors (Fig. 1, *a*, 2, *a*) is 5 %. It is observed in the transition zone of the flow (from laminar to turbulent flow – velocity of up to 7–8 m/s) and at high velocities – 40–50 m/s) and, consequently, intense turbulence. In all cases, dependence of $\lambda=F(w)$ has a descending character, which is caused by the larger autohesive friction of fluid layers at laminar flow and by smaller – at turbulent.

Calculation values of pressure losses from flow rate of the cooled gas-liquid mixture (Fig. 1, *b*, 2, *b*) in the region of volumetric flow rate of up to 0.5 m^3/s according the procedure of the standard and the proposed formula are close. In the region of volumetric flow rate of up to 0.5–1.0 m^3/s they differ by 0.15–0.5 atm, which is 0.5–1.7 per cent of the operating pressure in the system at roughness of pipes of 0.01 mm. At roughness of 0.05 mm, this difference increases accordingly up to 0.2–0.75 atm. (0.7–2.5 %). Thus, the difference of calculation data for pressure losses along the pipeline by the standard procedure and the proposed formulas are within an engineering error.

Fig. 3, 4 do not show calculation curves with the use of Leibenson equation with the temperature correction by Shukhov, because this equation operates only for cooled flows.

Table 1

Results of modeling and field experiment for product pipelines of Novotroitsk oil-gas condensate field

No. of experiment	Type of experiment	Roughness of pipes, mm	Diameter of pipeline, mm	Volumetric flow rate of mixture, m^3/s	Pressure drop on section, MPa
1	Field	0.01	100	0.5	3.2
1	Model	0.01	100	0.5	3.381
2	Field	0.05	100	0.5	4.9
2	Model	0.05	100	0.5	5.056
3	Field	0.01	300	0.5	0.011
3	Model	0.01	300	0.5	0.0105
4	Field	0.05	300	0.5	0.016
4	Model	0.05	300	0.5	0.0158
5	Field	0.01	100	1	13.47
5	Model	0.01	100	1	13.52
6	Field	0.05	100	1	20.3
6	Model	0.05	100	1	20.22
7	Field	0.01	300	1	0.05
7	Model	0.01	300	1	0.0422
8	Field	0.05	300	1	0.07
8	Model	0.05	300	1	0.0632

It should be noted that the trend of curves $\Delta P=F(Q)$ for all examined cases is the same, specifically, ΔP increases nonlinearly at an increase in volumetric flow rate Q, which is explained by directly proportional dependence ΔP on squared flow velocity. In this case, the influence of velocity prevails over the influence of hydraulic resistance.

In calculations of coefficient of hydraulic resistance during hydrotransportation of the heated gas-liquid mixture, the curves (Fig. 3, *a*, 4, *a*) for isothermal process in the range of velocities of 0–50 m/s are located above the curve for the non-isothermal process. This is explained by the influence of viscosity factor on calculation results. In the isothermal process ceteris paribus the viscosity of the fluid is the same. In non-isothermal process, the viscosity of the heated gas-liquid mixture decreases, which is taken into consideration in the proposed formula and explains the described location of curves $\lambda=F(w)$.

Calculation values of pressure losses on flow rate of the heated gas-liquid mixture (Fig. 3, *b*, 4, *b*) in the region of volumetric flow of up to 0–1 m^3/s by the proposed formulas (5) and (10) are close.

The conducted analysis reveals the fact that in the non-isothermal mode at roughness of 0.05 mm, a change in pipe diameter from 100 to 300 mm causes an increase in pressure losses at volumetric flow rate of 1.0 m^3/s by 30 %. At the same time, at roughness of 0.01 mm, this effect is virtually non-existent.

The obtained calculation data correspond well to the experimental results in Table 1. An increase in the diameter of a pipeline, a decrease in its roughness and volumetric flow rate leads to pressure drop in the fluid transportation section.

Thus, the proposed improved engineering procedure for calculating non-isothermal transportation of the mixture of hydrocarbons more fully reflects the physical processes and effects during the transportation of homogeneous gas-liquid mixtures. This causes greater accuracy of calculation of thermal-hydraulic mode parameters. Simultaneous taking into consideration the throttle effect and effect of energy accommodation for the homogeneous mixture of hydrocarbons with a specific value of its molar mass, put into the developed mathematical model, allows making it possible to obtain more accurate calculation value of the product pipeline diameter.

The developed procedure has limitations in use, similar to the above-mentioned standard methods (RD-30 and RD-50). Additional limitations include the use of expressions (10) and (11) only for hydrocarbon gas-condensate mixtures [16].

Undoubtedly, the performed studies and their results need checking for resistance (reliability and unambiguity) of calculation algorithms for solving the developed system of nonlinear equations. In particular, it should be done in a more extended range of operating parameters.

In addition, under actual working conditions, the refined procedure will allow not only determining the need for replacement of a pipeline, but also its more precise selection from the standard range of diameters in correspondence to the changes of current operating parameters.

This could be considered the prospect for further research.

7. Conclusions

1. We developed the system of equations for a mathematical model of the non-isothermal non-stationary one-dimensional motion of a gas-liquid mixture of hydrocarbons in the pipeline, which differs from the known ones by simultaneous taking into consideration the effect of Joule-Thomson and energy accommodation.

2. Based on the obtained model, the calculation formula for determining hydraulic resistance of a pipeline and pressure losses during transportation of the gas-liquid mixture under conditions of the actual non-isothermal process were developed. In addition, the temperature correction and accommodation coefficient in the calculation of hydraulic resistance on a pipeline as a system with distributed parameters was introduced.

3. Analysis of the curves obtained for the developed model and corresponding calculation formulas shows:

– practical unsuitability of the formulas by Thomas Colebrooke and VNIIgas for calculation of coefficient of hydraulic resistance of pipelines with non-isothermal modes of flow of gas-liquid mixtures;

– convergence of results of calculation of hydraulic resistance coefficient using the Leibenson formula and that proposed by the authors at the level of 95 % for the cooled flow of gas-liquid mixtures at velocities in the range of 5–50 m/s. Under the same conditions, the discrepancy of calculation data for pressure losses along the pipeline according to the standard procedure and the proposed formulas are within an engineering error;

– in calculations of hydraulic resistance coefficient at hydrotransportation of the heated gas-liquid mixture, the specific feature is that curves for the isothermal process throughout all the range of velocities of 0–50 m/s are located above the curve for the non-isothermal process. In this case, the calculation values of pressure losses from flow rate of heated gas-liquid mixtures in the region of volumetric flow rate of up to 0–1 m^3/s according to the formulas proposed for vertical and horizontal sections of the product pipeline are close;

– in the non-isothermal mode at roughness of 0.05 mm, a change in pipe diameter from 100 to 300 mm causes an increase in pressure losses at the volumetric flow rate of 1.0 m^3/s by 30 %. At the same time, at roughness of 0.01 mm, this effect is virtually non-existent.

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