Умови роботи асфальтобетонних шарів на цементобетонних плитах значно відрізняється від інших конструктивних рішень. В практиці дорожнього будівництва розроблені не досить досконалі методи розрахунку міцності асфальтобетонних покриттів жорстких дорожніх одягів, оскільки нормують лише окремі критерії міцності асфальтобетонного шару. Існуючі методи розрахунку не враховують особливостей напружено-деформованого стану асфальтобетонного шару на цементобетонних плитах при різних умовах прикладення навантаження, таких як середина плити, край плити та кут плити. Не відповідність умов розрахунку фактичному напружено-деформованому стану конструкції зумовлює передчасне руйнування асфальтобетонного шару і, як наслідок, зниження міжремонтного строку та додаткових витрат на передчасні ремонтні роботи.

Проведено моделювання напружено-деформованого стану конструкції дорожнього одягу методом скінчених елементів за допомогою програмного комплексу ANSYS з трьома варіантами розміщення транспортного навантаження, а саме в центрі плити, на краю плити та в куті плити.

Для прийнятих варіантів розміщення транспортного навантаження визначено та наведено значення напружень по Мізесу, головних напружень, горизонтальних та максимальних дотичних напружень. Значення напружень визначено на поверхні асфальтобетонного шару, на контакті асфальтобетонного шару і цементобетонної плити та на контакті цементобетонної плити і основи.

Виконано порівняння визначених напружень в шарах дорожнього одягу при різних варіантах прикладення транспортного навантаження та співставлення отриманих результатів з відомими рішеннями.

Це дозволило встановити, що для асфальтобетонного шару розміщення навантаження в куті плити є найбільш небезпечним, як за дотичними зсуваючими напруженнями, так і за напруженнями по Мізесу. Напруження, які виникають при розташуванні навантаження в куті плити, приблизно на 10 % перевищують напруження при розташуванні навантаження на краю плити та приблизно на 20 % перевищують напруження при розташуванні навантаження в центрі плити

Ключові слова: асфальтобетонний шар, модуль пружності, напружено-деформований стан, розташування навантаження, цементобетонна плита

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DETERMINING THE MOST DANGEROUS LOADING APPLICATION POINT FOR ASPHALT-CONCRETE LAYERS ON A RIGID BASE

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1. Introduction

The construction of motor roads using a cement-concrete coating is widely used in all countries. In the world, more than 5 % of the automobile network is covered with a cement-concrete coating; its share is constantly growing [1]. Cement-concrete coatings are durable enough, but there comes a time when they need repair. For the most part, cement-concrete slabs retain their monolithic character and carrying capacity and thus need only the improvement of transporting-operating indicators. Cement-concrete coatings are characterized by complexity in carrying out repair work, which is why their repair and new construction involves the laying of coating made from an asphalt-concrete mixture [2]. Such a structural solution makes it possible to improve the coating evenness, as well as the grip between a wheel and the coating. An asphalt-concrete layer acts as a protective layer or a wear layer on cement-concrete slabs. For such structures, it is necessary, first of all, to ensure the strength of an asphalt-concrete coating layer, because it directly affects the condition of a road bed.

Operating conditions of an asphalt-concrete layer on cement-concrete slabs in the form of a rigid base differ from the conditions of work for a conventional structure of a non-rigid road bed because cement-concrete slabs are arranged with the temperature joints [3]. In this case, there may be different variants for the application of a transport loading relative to the edges of a cement-concrete slab. Given this, it is a relevant task to undertake a research aimed at identifying which variant of the application of a transport loading relative to the edges of a slab would be the most dangerous for the strength of an asphalt-concrete layer.

2. Literature review and problem statement

Paper [1] reports results from observing the destruction of asphalt-concrete coatings on a rigid base. It is noted that there are several structural solutions to prevent the occurrence of cracks in an asphalt-concrete layer above the joints between cement-concrete slabs. One of the most successful methods is the construction of expansion joints in an asphalt-concrete coating over the seams of cement-concrete slabs. Such an approach to eliminate the emergence of cracks was also substantiated in [3]. Thus, a road bed is cut into combined slabs that work separately from each other. However, paper [3] did not consider the strength of an asphalt-concrete coating for different variants in the application of a transport loading relative to the edges of the slab. All solutions were developed for the condition when a transport loading is applied at the center of a slab.

Study [4] found that the performance of combined slabs, which are composed of asphalt-concrete layers on cement-concrete slabs, differs significantly from other structural solutions. According to [5], the practice of road construction applies the insufficiently developed methods for calculating the durability of asphalt-concrete coatings over rigid road beds. The reason is the lack of detailed studies into the stressed-strained state of asphalt-concrete layers on a rigid base. Moreover, only separate criteria for the strength of an asphalt-concrete layer are standardized, which do not fully correspond to the actual operating conditions.

A study into the stressed-strained state of asphalt coatings on a road bridge is reported in [6], but the unresolved issue was the patterns in the stressed-strained state of an asphalt-concrete layer on cement-concrete slabs for different variants of load application. According to paper [7], the application of a loading in the middle of a slab, at the edge of a slab, and at the corner of a slab significantly alters the stressed-deformed state of a structure. Therefore, the durability of an asphalt-concrete coating is significantly affected by the load application point relative to a combined slab.

It has been currently recognized that the most productive method for calculating engineering structures are the calculations based on the criteria for strength [8]. According to the results reported in [9], there is a reason to believe that the lack of detailed studies into the stressed-strained state of an asphalt-concrete layer on a rigid base under various conditions of load application leads to a mismatch between existing criteria for strength and the actual operating real conditions. Similar conclusions were drawn in paper [10], which addressed operating conditions of thin asphalt-concrete layers on a rigid base, as well as determining the relevant criteria for strength.

As a result, given the insufficient investigation of the stressed-strained state of a structure, it becomes impossible to match the criteria assigned for strength to the real operating conditions of structural layers of road beds. The lack of research into the stressed-strained state of structures for different conditions of load application leads to shorter inter-maintenance duration of a road bed service and additional costs to unplanned repair operations. All this allows us to assert that it is expedient to undertake a study to analyze the stressed-strained state of an asphalt-concrete coating layer on a rigid base and to define which variant of load application is the most dangerous.

3. The aim and objectives of the study

The aim of this work is to study the stressed-strained state of an asphalt-concrete layer on a rigid base for different variants of a transport loading application that would make it possible to establish the most dangerous variant a loading application point and to substantiate the need to devise recommendations in order to improve a method for calculating asphalt-concrete layers in a coating on a rigid base.

To accomplish the aim, the following tasks have been set:

– to define parameters of an estimated model for modelling the stresses-strained state of an asphalt-concrete coating on a rigid base;

– to simulate the stressed-strained state of a road bed structure using a finite element method when a loading is applied in the middle of a slab, at the edge of a slab, and at the corner of a slab;

– to analyze the results from modeling the stressed-strained state and to compare them with known results.

4. Modeling the stressed-strained state of a road bed structure

4. 1. Parameters for the estimated model

The rigid base accepted here is a monolithic cement-concrete slab with temperature joints, which is characterized by that its modulus of elasticity and strength almost does not depend on temperature, humidity, and the duration of load exposure. The asphalt-concrete layer is understood as a layer with the properties of asphalt concrete, arranged atop a cement-concrete slab.

To prevent the shifts in an asphalt-concrete coating followed by its quick destruction, it is necessary to ensure the condition for the joint work of an asphalt-concrete coating and a rigid base [3, 5, 7]. The further modeling employs the condition for the ensured grip between the asphalt-concrete layer in a coating and a rigid base.

To prevent the appearance of cracks in the asphalt-concrete layer over the seams of cement-concrete slabs, the estimated model implies the arrangement of expansion joints in the asphalt-concrete layers over the seams of cement-concrete slabs. Thus, the structure of a road bed under consideration that is the combined slabs. A combined slab is composed of a cement-concrete slab on an elastic half-space, atop which is an asphalt-concrete layer under condition for adhesion between the asphalt-concrete layer and the cement-concrete slab. We consider the stressed-strained state of a combined slab whose dimensions are limited by temperature joints. The validity of this approach is substantiated in paper [10].

To consider the estimated scheme and to select a model of materials, we shall apply general methods for solving the problems on mechanics of a deformed solid body. The material of cement concrete has the structure that consists of large particles (gravel) and fine particles (sand) of the filler, joined by cement stone. The concept of a body as a continuous medium simplifies the possibility for a mathematical notation of its performance. The material of cement concrete is accepted as an elastic, solid, homogeneous, and isotropic body [11–13].

The rigid base is characterized by:

– the dimensions of cement-concrete slab in the plan;

– thickness;

– the module of elasticity;

– the temperature coefficient of linear expansion;

– the total (equivalent) module of elasticity of the base under cement-concrete slabs.

To solve the set problems, according to [14–16], an asphalt-concrete layer is accepted as an elastic body, that is, it deforms without residual deformations. For the elastic deformation, accepted in the current work, we have completely rejected such a property of the body model as creep and visco-elasticity [14]. Such an approach is possible because the duration of a transport loading is less than 0.1 s, and, over this period, asphalt concrete has no time to demonstrate the visco-elastic or plastic properties of the material.

Asphalt concrete refers to solid bodies, which are characterized by their own shape, defined volume, and the capability to resist the influence of external load [17]. The arrangement of individual structured elements within the volume of an asphalt-concrete material is random in character. The dimensions of the bitumen-mineral conglomerates (elements of the mineral skeleton and the surrounding asphalt binder) are «rather small» in comparison with the dimensions of the asphalt-concrete coating of a road bed. That is why, on order to calculate the layers of asphalt concrete, it is possible to idealize the actual environment at which the material of asphalt concrete is considered to be quasi-homogeneous [18].

The structural elements of asphalt concrete are arranged within a solid body in a random fashion. The microdefects are distributed within the volume of the material in the same manner. As shown by studies [14, 18, 19], a body that has macrodefects can be considered to be a quasi-homogeneous system within its base – the matrix.

Based on modern ideas about the effect of structure and texture on the stressed-strained state of a rigid body, in the study of asphalt layers the material is taken as a quasi-homogeneous, a quasi-isotropic body. In this case, the idealization of the actual environment relative to its uniformity, and continuity of isotropy, does not lead to fundamental errors in calculations based on the general solutions from elasticity theory [14, 18, 20].

The parameters of the estimated models that are used to model the stressed-strained state of a three-layered structure of a road bed are shown in Fig. 1.

The first layer of asphalt concrete has a thickness of 4 cm, the second layer is a cement-concrete slab with a thickness of 28 cm, the third layer is the elastic base. The size of the slab is 3 m to 6 m. It is implied that the asphalt-concrete layer has the expansion joints lengthwise the entire thickness of the layer, located above the seams in cement-concrete slabs. Thus, the coating is divided into slabs that work independent of each other.

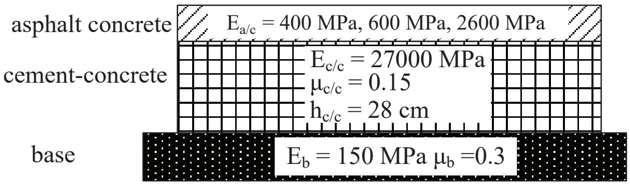


Fig. 1. Structure of a road bed accepted for modeling the stressed-strained state

The general equivalent modulus of elasticity at the surface of the base $E=150$ MPa, a Poisson's coefficient $\mu=0.3$. The modulus of elasticity of a cement-concrete slab: $E_{c/c}=27{,}000$ MPa, Poisson's ratio $\mu=0.15$. We accounted for the temperature by changing the modulus of elasticity of the asphalt-concrete coating and the Poisson's coefficient (the calculation is for $E_{a/c}=400$ MPa at 55 °C, 600 MPa at a temperature of 40 °C, 2,600 MPa at a temperature of 10 °C).

4. 2. Modeling the stressed-strained state of a road bed structure using a finite element method

Modeling the stressed-strained state is performed using the method of finite elements in the programming environment ANSYS.

The calculation was performed for the following conditions:

– the coupled contact between layers;

– load $P=0.8$ MPa;

– the stamp is round, hard, a diameter of 34.5 cm.

The calculation is performed for 3 variants of stamp location, which is shown in Fig. 2.

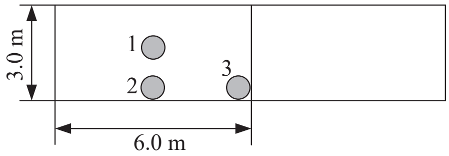
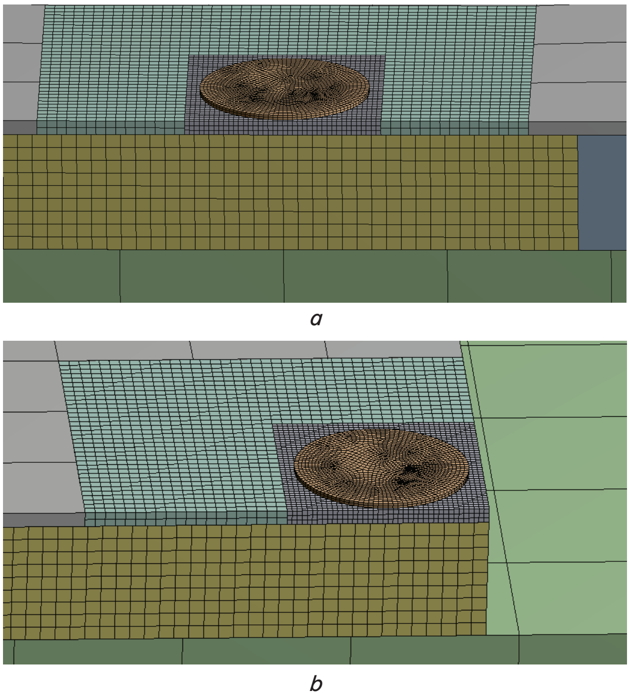


Fig. 2. Variants for the application point of the hard stamp that simulates a car wheel: 1 – loading in the center of the slab; 2 – loading at the edge of the slab (in the middle); 3 – loading at the corner of the slab

The grid for the model is built using 49,884 elements, predominantly hexagonal in shape. The elements' shape was chosen in line with studies [6, 14, 20]. The grid is thick near the stamp. The examples of estimated models are shown in Fig. 3.


a

b

Fig. 3. Estimated model for load application: *a* – loading the edge; *b* – loading the corner

Boundary conditions:

– for calculation at the center of the slab, movement is restricted to four sides;

– for calculation at the edge of the slab, movement is restricted to three sides, the estimated side is subject to the condition of free movement;

– for calculation at the corner of the slab, movement is restricted to two sides, the two sides of the corner are subject to the condition of free movement.

5. Results of modeling the stressed-strained state

The result of simulating the stressed-strained state are the defined:

– von Mises stresses σ_{Miz};

– principal stress $\sigma_{\max Pr}$;

– horizontal stresses σ_h;

– maximum shear stresses ($\tau_{\max}$).

The results of calculations are given in Tables 1–3.

For the further analysis of the stressed-strained state of a road bed structure, depending on the loading application point relative to the edges of the combined slab, we shall consider the following cross-sections:

– at the surface of the asphalt-concrete layer;

– at the bottom surface of the asphalt-concrete layer;

– at the top surface of the cement-concrete slab;

– at the bottom surface of the cement-concrete slab.

Table 1

Stresses for the case of stamp location in the middle of the slab

$E_{a/c}$, MPa/$\mu_{a/c}$	$E_{c/c}$=27,000 MPa, $\mu_{c/c}$=0.15, $h_{c/c}$=28 cm			
	σ_{Miz}, MPa	$\sigma_{\max Pr}$, MPa	σ_h, MPa	$\tau_{\max}$, MPa
Upper boundary of asphalt-concrete layer				
400/0.40	0.452	–0.566	–0.594	0.231
600/0.32	0.564	–0.463	–0.531	0.286
2,600/0.22	0.602	–0.421	–0.521	0.306
Lower boundary of asphalt-concrete layer				
400/0.40	0.537	–0.534	–0.519	0.304
600/0.32	0.574	–0.390	–0.368	0.317
2,600/0.22	0.623	–0.298	–0.214	0.338
Upper boundary of cement-concrete slab				
400/0.40	0.815	–0.786	–1.208	0.468
600/0.32	0.828	–0.793	–1.256	0.477
2,600/0.22	0.818	–0.819	–1.270	0.472
Lower boundary of cement-concrete slab				
400/0.40	1.189	1.190	1.141	0.606
600/0.32	1.206	1.208	1.158	0.615
2,600/0.22	1.211	1.213	1.163	0.617

Table 2

Stresses for the case of stamp location at the edge of the slab

$E_{a/c}$, MPa/$\mu_{a/c}$	$E_{c/c}$=27,000 MPa, $\mu_{c/c}$=0.15, $h_{c/c}$=28 cm			
	σ_{Miz}, MPa	$\sigma_{\max Pr}$, MPa	σ_h, MPa	$\tau_{\max}$, MPa
Upper boundary of asphalt-concrete layer				
400/0.40	0.510	–0.581	–0.590	0.278
600/0.32	0.572	–0.479	–0.516	0.301
2,600/0.22	0.638	–0.436	–0.441	0.352
Lower boundary of asphalt-concrete layer				
400/0.40	0.586	–0.519	–0.535	0.334
600/0.32	0.616	–0.368	–0.391	0.347
2,600/0.22	0.688	–0.214	–0.298	0.392
Upper boundary of cement-concrete slab				
400/0.40	2.039	0.576	–0.230	1.177
600/0.32	1.956	0.575	–0.295	1.124
2,600/0.22	1.824	0.562	–0.368	1.039
Lower boundary of cement-concrete slab				
400/0.40	2.030	2.012	0.268	1.049
600/0.32	1.979	1.967	0.288	1.053
2,600/0.22	2.026	2.009	0.307	1.047

Table 3

Stresses for the case of stamp location at the corner of the slab

$E_{a/c}$, MPa/$\mu_{a/c}$	$E_{c/c}$=27,000 MPa, $\mu_{c/c}$=0.15, $h_{c/c}$=28 cm			
	σ_{Miz}, MPa	$\sigma_{\max Pr}$, MPa	σ_h, MPa	$\tau_{\max}$, MPa
Upper boundary of asphalt-concrete layer				
400/0.40	0.529	–0.563	–0.584	0.283
600/0.32	0.596	–0.453	–0.509	0.309
2,600/0.22	0.711	–0.332	–0.425	0.387
Lower boundary of asphalt-concrete layer				
400/0.40	0.590	–0.510	–0.517	0.334
600/0.32	0.625	–0.355	–0.366	0.347
2,600/0.22	0.718	–0.177	–0.214	0.392
Upper boundary of cement-concrete slab				
400/0.40	1.661	1.349	0.979	0.955
600/0.32	1.683	1.351	0.976	0.970
2,600/0.22	1.673	1.359	0.943	0.964
Lower boundary of cement-concrete slab				
400/0.40	1.590	1.035	–0.875	0.912
600/0.32	1.597	1.055	–0.874	0.916
2,600/0.22	1.604	1.072	–0.876	0.921

6. Analysis of modelling results and determining the most dangerous loading application point relative to a combined slab

Analysis and comparison of modeling results were performed for three variants of stamp location, for asphalt concrete with a module of elasticity E=2,600 MPa, Fig. 4–7.

During analysis, we used the results from calculating the von Mises stresses (σ_{Miz}), since these stresses reflect the volumetric stressed state of a structure. The results of calculation show that for a cement-concrete slab:

– the stresses caused by loading applied at the edge of the slab are larger than the stresses caused by loading applied in the center of the slab by 60 %;

– the stresses caused by loading applied at the corner of the slab are larger than the stresses caused by loading applied at the center of the slab by 35 %.

A given conclusion is confirmed by the results of calculations given in papers [21, 22].

Study [21] reports the results of calculation at load Q=42.41 kN, distributed over the area of a circle with radius a=0.15 m in the slab with thickness h=0.2 m, the slab's module of elasticity E=28,000 MPa, the coefficient of the slab's transverse deformation v=0.15, on the base with a bed factor k=60 MPa/m. Similar solutions underlie the calculation of stresses for a structure shown in Fig. 1 in the absence of an asphalt-concrete layer. The vertical load (Q=65 kN) was distributed over the area of a circle with radius a=0.1725 m on the slab of thickness h=0.28 m, with a module of elasticity E=27,000 MPa, a coefficient of transverse deformation v=0.15.

Comparison of the calculation results is given in Table 4 and shown in Fig. 8.

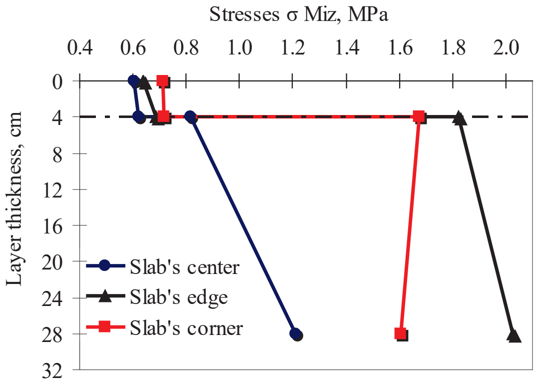


Fig. 4. Comparison of results from calculating the stresses σ_{Miz} for three variants of stamp location

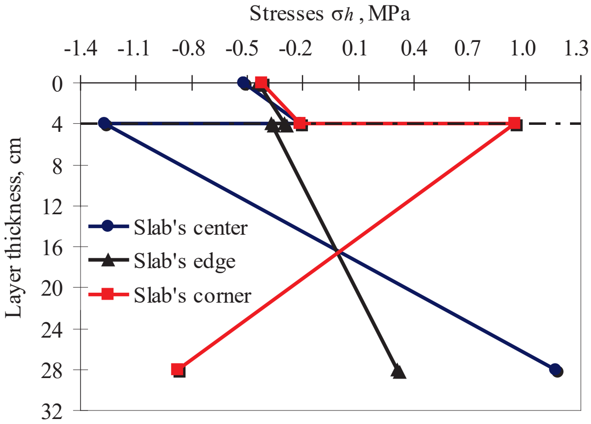


Fig. 6. Comparison of results from calculating the stresses σ_h for three variants of stamp location

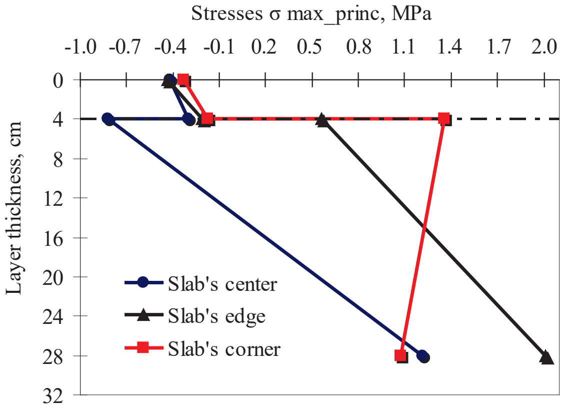


Fig. 5. Comparison of results from calculating the stresses σ_{maxPr} for three variants of stamp location

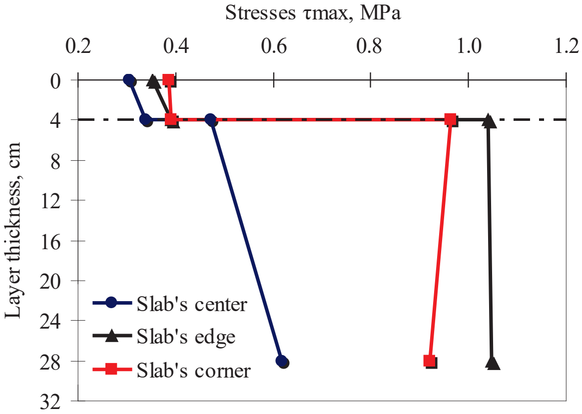


Fig. 7. Comparison of results from calculating the stresses τ_{max} for three variants of stamp location

Table 4

Results from calculating the stresses in a cement-concrete slab for different variants of a loading application point

$E_{c/c}$, MPa	E_b, MPa	Q, kN	$\nu_{c/c}$	ν_b	h of slab, m	k of road base, MPa/m	l	Area radius a, m	σ, MPa
Loading applied in the center of the slab; stress determined from a Vestergard solution at the lower surface of the slab									
28,000	80	42.41	0.15	0.25	0.2	61.2817	0.74038	0.15	1.28
27,000	150	65	0.15	0.3	0.25	115.3686	0.75404	0.1725	1.19
Loading applied in the center of the slab; stress determined from a Gorbunov-Posadov solution at the bottom surface of the slab									
28,000	80	42.41	0.15	0.25	0.2	60.54329	0.74941	0.15	1.31
27,000	150	65	0.15	0.3	0.25	116.7757	0.74496	0.1725	1.20
Loading applied at the edge of the slab; stress determined from a Vestergard solution at the lower surface of the slab									
28,000	80	42.41	0.15	0.25	0.2	60.54329	0.74941	0.15	2.46
27,000	150	65	0.15	0.3	0.25	116.7757	0.74496	0.1725	2.22
Loading applied at the corner of the slab; stress determined from a Vestergard solution at the surface of the slab									
28,000	80	42.41	0.15	0.25	0.2	60.54329	0.74941	0.15	1.69
27,000	150	65	0.15	0.3	0.25	116.7757	0.74496	0.1725	1.52

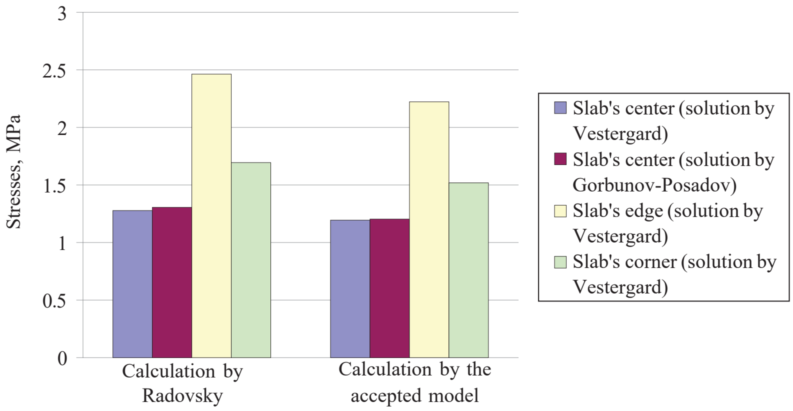


Fig. 8. Comparison of calculation results

The results of the performed calculations are confirmed by the calculation results given in papers [21, 22]:

– when load is applied at the corner of the slab, the maximum stretching stress is larger than that when the load is applied in the center of the slab by 20–35 % at the thickness of a cement-concrete coating from 15 cm to 36 cm, respectively [21, 22]. At the corner of the slab, the maximum stretching stress occurs at the top plane of the slab [21, 22], which corresponds to the results from our modeling. In Fig. 6, the top plane of the slab is exposed to the maximum stretching stresses;

– the maximum stretching stress near the edge of the slab exceeds the stress at stamp location in the center of the slab by 45–55 % at the thickness of a cement-concrete coating from 15 cm to 36 cm, respectively [21, 22]. At the edge of the slab, the maximum stretching stresses occur at the bottom of the slab [21, 22], which matches simulation results. In Fig. 6, the bottom of the slab is exposed to the maximum stretching stresses;

– in the center of the slab, the maximal stretching stresses act along the bottom of the slab [21, 22], which also corresponds to the results of our calculations. In Fig. 6, the bottom of the slab is exposed to the maximum stretching stresses.

7. Discussion of results of modeling the stressed-strained state of a road bed structure

It follows from the obtained results (Tables 1–3 and Fig. 4–7) that for the asphalt-concrete layer the application of loading at the corner of the slab is the most dangerous both in terms of the tangent shear stresses ($\tau_{\max}$) and the von Mises stresses (σ_{Miz}). This is predetermined by the fact that when the loading is applied in the center of the slab, the movement of an asphalt-concrete layer is limited from all four sides. When the loading is applied at the edge of the slab, the movement is limited from three sides, and the movement from the estimated side is free. When the loading is applied at the corner of the slab, the movement is limited from two sides, and the movement from the two sides near the corner is free.

The conclusions from the current study can be considered feasible because they make it possible to substantiate the mismatch between an estimated loading application point when using existing calculation methods and the most dangerous loading application point under actual conditions. Therefore, it is necessary to improve the existing methods for calculating the strength of an asphalt-concrete coating on a rigid base, since the estimated point of load application at present is the center of the slab, while the maximum strains occur when the loading is applied at the corner of the slab. Our conclusions are confirmed by observing the examined sections along the motor roads Kharkiv-Pereschepino-Krasnograd, bypassing the city of Kharkiv, Kyiv-Znamyanka, and others, where it was registered that the destruction of the asphalt-concrete coating in the form of tearing off a cement-concrete slab and shift in the coating most often occurs at the corners of the slab. In the further development of the estimated scheme for the asphalt-concrete coating performance on cement-concrete slabs and when assigning the boundary conditions and criteria of strength, it is necessary to take into consideration a possibility to apply external loading at the corner of the slab.

However, one should note that the derived conclusions could apply only for cement-concrete slabs without the reinforcement, by pins, of the transverse and longitudinal joints for a partial transfer of load from one slab to another. Such an assumption imposes certain limitations to the application of the results obtained, which can be regarded to be a shortcoming of the current study. The impossibility to eliminate the above assumption within the framework of this study could lead to a potentially interesting area for the further research. The current study could be advanced by addressing the reinforcement of seams by pins, taking into consideration the joint effect from stresses caused by a transport loading and stresses caused by temperature deformation.

8. Conclusions

1. We have defined parameters for an estimated three-layered model of a road bed structure with an asphalt-concrete coating on a cement-concrete slab, which is used for modeling the stressed-strained state using a finite element method. Dimensions of the combined slab were taken to be 3 m by 6 m. Based on modern understanding of the effect of structure and texture on the stressed-strained state of a rigid body, in the current study the material of the layers was considered to be quasi-homogeneous, quasi-isotropic. The general equivalent modulus of elasticity and a Poisson’s ratio at the surface of the base do not depend on fluctuations of temperature and are accepted bo be constant $E=150$ MPa and $\mu=0.3$, respectively. The modulus of elasticity and the coefficient of a cement-concrete slab also do not depend on fluctuations of temperature and are considered to be constant, modulus of elasticity $E_{c/c}=27{,}000$ MPa, Poisson’s ratio $\mu=0.15$. The possibility to change the parameters for an asphalt-concrete coating depending on temperature fluctuations was accounted for by changing the module of elasticity and Poisson’s ratio (calculation is given for $E_{a/c}=400$ MPa and $\mu=0.4$, $E_{a/c}=600$ MPa and $\mu=0.32$, $E_{a/c}=2{,}600$ MPa and $\mu=0.22$).

2. By using a method of finite elements in the programming environment ANSYS, we have established the stressed-strained state of a combined slab caused by external load of magnitude 0.8 MPa distributed over a circle of radius 34.5 cm. External loading was applied in the middle of the slab, at the edge of the slab, and at the corner of the slab. According to the von Mises stresses (σ_{Miz}), which reflect the volumetric stressed state of a cement-concrete slab:

– the stresses caused by loading applied at the edge of the slab are larger than the stresses caused by loading applied in the center of the slab by 60 %. The increase in stresses comes at the expense of the «edge effect» and the increased strain. This is explained by the fact that when the loading is applied in the center of the slab the movement is limited from all four sides of the slab, when the loading is applied at the edge of the slab the movement is limited from three sides, while the movement from the estimated side is free.

– the stresses caused by loading applied at the corner of the slab are larger than the stresses caused by loading applied in the center of the slab by 35 %. This is due to the fact that when the loading is applied in the center of the slab the movement is limited from all four sides, when the loading is applied at the corner of the slab the movement is limited from both sides of the slab, while the movement from the two sides near the corner is free. This conclusion is confirmed and is consistent with the results reported in papers [21, 22].

Based on the results of simulating the stressed-strained state of a combined slab from an asphalt-concrete layer on a rigid base, it was established that the edge of the slab is the most dangerous place both in terms of the stretching and shear stresses for a cement-concrete slab. According to the existing methods for calculating the strength of cement-concrete slabs, the estimated loading application point is exactly the edge of the slab, which is why the existing methods for the calculation of durability of cement-concrete slabs are perfectly acceptable and do not require improvement.

3. It was found based on the results from modeling the stressed-strained state of a combined slab from an asphalt-concrete layer on a rigid base:

– the maximum von Mises stresses (σ_{Miz}), which reflect the volumetric stressed state, occur when the loading is applied at the corner of the slab and they exceed the stresses that occur when the loading is applied at the edge of the slab and in the center of the slab;

– the maximum shear stresses ($\tau_{\max}$) occur when the loading is applied at the corner of the slab and exceed the stresses that occur when the loading is applied at the edge of the slab and in the center of the slab;

For the asphalt-concrete layer, the application of loading at the corner of the slab is the most dangerous both in terms of the tangent shear stresses ($\tau_{\max}$) and the von Mises stresses (σ_{Miz}). At the same time, in accordance with the existing methods for calculating the strength of an asphalt-concrete coating on a rigid base, the estimated load application point is the center of the slab.

References

1. Radovskiy B. S. Opyt ispol'zovaniya starogo cementobetona kak osnovaniya pod asfal'tobetonnoe pokrytie v SShA // Dorozhnaya tekhnika. 2010. Issue 1. P. 20–32.
2. Hameliak I. P., Koretskyi A. S., Koretskyi S. S. Pro neobkhidnist budivnytstva v Ukraini avtomobilnykh dorih iz tsementobetonnym pokryttiam // Avtoshliakhovyk Ukrainy. 2013. Issue 5. P. 24–31.
3. Korochkin A. V., Ahmetov S. A. Zavisimost' sostoyaniya pokrytiya ot tolschiny asfal'tobetonnyh sloev zhestkoy dorozhnoy odezhdy // Avtomobil'nye dorogi. 2009. Issue 12. P. 27–29.
4. Nevinhlovskyi V. F. Teoretychni aspekty rozrakhunku zalyshkovoho resursu asfaltobetonnoho pokryttia na zalizobetonnykh avtodorozhnikh mostakh // Avtomobilni dorohy i dorozhnie budivnytstvo. 2013. Issue 89. P. 225–234.
5. Dorozhko E., Ryapuhin V., Makovyey R. Design Procedure by Strength Criteria of Asphalt Layers on a Rigid Base Taking into Account the Simultaneous Action of External Loads and Thermal Stresses // Procedia Engineering. 2016. Vol. 134. P. 101–108. doi: https://doi.org/10.1016/j.proeng.2016.01.045
6. Onischenko A., Aksenov S., Nevynhlovskyy V. Numerical Simulation of Stress-Strain State of Asphalt Concrete Pavement on the Carriageway of the South Bridge in Kiev // Procedia Engineering. 2016. Vol. 134. P. 322–329. doi: https://doi.org/10.1016/j.proeng.2016.01.014
7. Dorozhko Ye. V. Vrakhuvannia sumisnoi diyi zovnishnoho navantazhennia ta temperatury pry rozrakhunkakh tonkykh asfaltobetonnykh shariv na zhorstkiy osnovi // Naukovyi visnyk budivnytstva. 2015. Issue 4. P. 132–136.
8. Determining the Contribution of Different Structural Layers of Asphalt Pavement System to Rutting Using Transverse Profile Analysis / Hussan S., Kamal M. A., Khan M. B., Irfan M., Hafeez I. // American Journal of Civil Engineering and Architecture. 2013. Vol. 1, Issue 6. P. 174–180. doi: https://doi.org/10.12691/ajcea-1-6-7
9. Stress–strain state of an elastic soft functionally–graded coating subjected to indentation by a spherical punch / Volkov S. S., Vasiliev A. S., Aizikovich S. M., Seleznev N. M., Leontieva A. V. // PNRPU Mechanics Bulletin. 2016. Issue 4. P. 20–34. doi: https://doi.org/10.15593/perm.mech/2016.4.02
10. Agallkov Yu. A., Smirnov A. V. Raschet tolschiny asfal'tobetonnyh pokrytiy na zhestkom osnovanii po usloviyam sdviga // Nauka i tekhnika v dorozhnoy otrasli. 1997. Issue 5. P. 7–8.
11. Chen X., Wu S., Zhou J. Analysis of mechanical properties of concrete cores using statistical approach // Magazine of Concrete Research. 2013. Vol. 65, Issue 24. P. 1463–1471. doi: https://doi.org/10.1680/macr.13.00113
12. Shurgaya A. G., Gamelyak I. P., Yakimenko Ya. N. Vysokoprochnyy cementobeton s kompleksnymi dobavkami dlya dorozhnogo i aerodromnogo stroitel'stva // Avtomobil'nye dorogi i mosty. 2015. Issue 2 (16). P. 34–42.
13. Mathematical models of the properties of high–strength cement – concrete for road construction / Gameliak I. P., Shurgaya A. G., Jakymenko J. M., Chyzhenko N. P. // Zbirnyk naukovykh prats Ukrainskoho derzhavnoho universytetu zaliznychnoho transportu. 2017. Issue 169. P. 103–110.
14. Riapukhin V. M., Nechytailo N. O. Modeliuvannia napruzheno-deformovanoho stanu asfaltobetonnykh shariv // Mistobuduvannia ta terytorialne planuvannia. 2011. Issue 40. P. 258–263.
15. Golchin B., Hamzah M. O., Hasan M. R. M. Optimization in producing warm mix asphalt with polymer modified binder and surfactant-wax additive // Construction and Building Materials. 2017. Vol. 141. P. 578–588. doi: https://doi.org/10.1016/j.conbuildmat.2017.02.123

16. Bennert T. Implementation of Performance–Based HMA Specialty Mixtures in New Jersey // Journal of the Association of Asphalt Paving Technologists. 2011. Vol. 80. P. 719–740.
17. Ladygin B. I. Prochnost' i dolgovechnost' asfal'tobetona. Minsk, 1970. 288 p.
18. Ryapuhin V., Nechytaylo N., Burlachka V. Criteria for flexible pavement calculation // TRANSBALTICA 2011: Proceedings of the 7th International Scientific Conference. Vilnius, 2011. P. 252–256.
19. Pisarenko G. S. Deformirovanie i prochnost' materialov pri slozhnom napryazhennom sostoyanii. Kyiv, 1976. 415 p.
20. Merzlikin A. E. Kapustnikov N. V. Pogreshnosti, voznikayuschie pri raschete dorozhnyh odezhd s pomosch'yu metoda konechnyh elementov // Stroitel'nye materialy. 2010. Issue 10. P. 26–29.
21. Radovskiy B. S. Prochnost' i raschet betonnyh pokrytiy v SShA // Beton i zhelezobeton. 2011. Issue 2 (5). P. 118–131.
22. Radovskiy B. S. Cementobetonnye pokrytiya v SShA // Dorozhnaya tekhnika. 2009. Issue 1. P. 50–58.

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A PROCEDURE OF STUDYING STATIONARY MOTIONS OF A ROTOR WITH ATTACHED BODIES (AUTO-BALANCER) USING A FLAT MODEL AS AN EXAMPLE

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Модернізований енергетичний метод дослідження динаміки роторів. Метод застосовний для роторів на ізотропних пружно-в'язких опорах, коли до ротора приєднані тіла, на які при відносному русі діють пружні і в'язкі сили. Метод призначений для пошуку, визначення умов існування і оцінки стійкості стаціонарних рухів роторної системи. На стаціонарних рухах відносні руху приєднаних тіл припиняються, і система обертається як одне ціле навколо осі обертання, утвореної опорами.

Ефективність методу проілюстрована на прикладі плоскої моделі ротора з автобалансиром з багатьма вантажами у вигляді куль, роликів і маятників.

Встановлено, що як при наявності, так і відсутності демпфірування в опорах, при достатній балансувальній ємності автобалансира система має сім'ю основних рухів (на них ротор збалансований).

При відсутності демпфірування в опорах система має:

– при наявності неврівноваженості ротора – ізольовані побічні рухи (на них ротор незбалансований), в яких центри мас вантажів відхилені в бік неврівноваженості або в протилежний бік;

– при відсутності неврівноваженості ротора – однопараметричні сім'ї побічних рухів, в яких центри мас вантажів лежать на одній прямій.

При наявності демпфірування в опорах:

– при наявності неврівноваженості ротора система має ізольовані побічні рухи, в яких центри мас вантажів лежать на одній прямій, і пряма утворює з вектором неврівноваженості кут, що залежить від швидкості обертання ротора;

– при відсутності неврівноваженості ротора побічних рухів не існує.

При відсутності демпфірування в опорах побічні рухи і області їх існування не залежать від кутової швидкості обертання ротора, а при наявності – залежать.

Як при наявності, так і при відсутності демпфірування в опорах:

– на дорезонансних швидкостях обертання ротора стійким може бути тільки той побічний рух, на якому сумарна неврівноваженість ротора і вантажів найбільша;

– на зарезонансних швидкостях обертання ротора може бути стійка тільки сім'я основних рухів

Ключові слова: ротор, ізотропна опора, автобалансир, стаціонарний рух, стійкість руху, рівняння усталеного руху

1. Introduction

Passive auto-balancers are used to balance high-speed rotors [1–3]. The motion of such systems sets in over time. Loads (balls, rollers, pendulums, etc.) balance the rotor at the so-called main steady motions but not at the secondary ones. From a mathematical point of view, for the auto-balancer to be operable, it is necessary and sufficient that the main