Наведено спосіб побудови резонансних траєкторій руху вантажу хитної пружини. Хитною пружиною (swinging spring) називають різновид математичного маятника, який складається з точкового вантажу, приєднаного до невагомої пружини. Другий кінець пружини фіксується нерухомо. Розглядаються маятникоподібні коливання пружини у вертикальній площині за умови збереження прямолінійності її осі. Розрахунки виконано на базі розв'язків системи диференціальних рівнянь, з компонентами, у які входять значення частот вертикальних і горизонтальних переміщень точки на пружині.

Актуальність теми визначається необхідністю дослідження технологічних процесів динамічних систем, коли нелінійно зв'язані коливальні компоненти системи обмінюються енергією між собою. За допомогою феномена хитної пружини ілюструється обмін енергіями між поперечними (маятниковими) і поздовжніми (пружинними) коливаннями. При цьому також враховується вплив початкових умов ініціювання коливань. Особливе значення має дослідження стану резонансу хитної пружини - коли частота поздовжніх коливань відрізняється в кратну кількість разів від частоти поперечних коливань. Крім розповсюдженого «класичного» випадку (резонансу 2:1) є необхідність розв'язувати задачі з іншими значеннями відношення частот. В результаті було знайдено геометричні форми траєкторії руху вантажу хитної пружини, які відповідають особливостям стану її резонансу.

Одержані результати дозволяють за допомогою комп'ютера синтезувати траєкторію руху вантажу хитної пружини, яка відповідатиме заданому відношенню частот поздовжніх і поперечних коливань. Для цього, крім основних параметрів (маси вантажу, жорсткості пружини та її довжини в ненавантаженому стані), ще залучаються початкові значення параметрів ініціювання коливань. А саме, «стартові» координати положення вантажу, та початкові швидкості рухів вантажу в напрямку координатних осей. Розглянуто приклади побудови траєкторій руху вантажу для випадків резонансів типу 2:1, 7:3, 9:4 і 11:2. Одержані результати проілюстровано комп'ютерними анімаціями коливань відповідних хитних пружин для різних випадків резонансу.

Результати можна використати як парадигму для вивчення нелінійних зв'язаних систем, а також при розрахунках варіантів механічних пристроїв, де пружини впливають на коливання їх елементів. А також у випадках, коли у технологіях використання механічних пристроїв необхідно відмежуватися від хаотичних рухів вантажів і забезпечити періодичні траєкторії їх переміщень

Ключові слова: хитна пружина, резонанс хитної пружини, маятникові коливання, траєкторії руху вантажу

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MODELING THE RESONANCE OF A SWINGING SPRING BASED ON THE SYNTHESIS OF A MOTION TRAJECTORY OF ITS LOAD

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1. Introduction

This is continuation of the research reported in papers [1, 2]. We have considered an approach to solving the class of problems when, within a certain dynamic system, its nonlinearly connected oscillatory components could exchange energy. Studies [3–6] give many examples of such problems. In this case, the authors address the issues about dependence of the sharing energy activity on the parameters of a system's control. The task is to determine the total energy of the system and to properly estimate the energy magnitudes over time, as well as their relation to each of the components.

To illustrate this approach, they use a *two-dimensional spring pendulum* as a mechanical model to study multiple nonlinearly connected systems. The two-dimensional spring pendulum in a perfect form consists of the «point» load of mass *m,* attached to the end of a weightless spring of rigidity *k* and length *h* in a no-load state. The other end of the spring is fixed stationary. The oscillatory system, formed in such a manner, should move in a vertical plane only, while *maintaining the spring axis straight*. The point load simultaneously participates in two types of oscillations: spring-like – when moving along the straight spring axis, and pendulum-like – when it executes oscillations jointly with the axis. This kind of an oscillatory system is denoted in the scientific literature as a swinging spring [7].

Using a swinging spring visualizes the exchange of energies among the transverse (pendulum) and longitudinal (spring) oscillations. In this case, one should take into consideration the influence of the initial conditions for oscillation initiation. Of particular importance is to study the condition for the emergence of resonance state of the swinging spring. That is, when the frequency of longitudinal oscillations will differ by the multiple number of times from the frequency of transverse oscillations. In addition to a common «classic» case (resonance 2:1), it is advisable to solve problems with different values for the frequency ratio. For example, there is a need [8] to build the motion trajectories of a load for the cases of such resonances as: 2:1, 7:3, 9:4, 11:2, and others. The would-be geometrical shapes of the motion trajectory of a swinging spring load under assigned parameters could help define the characteristics for the solution to the selected problem.

Papers [3–6] report many possible implementations based on the application of the idea about a swinging spring's oscillations. Large part of this list is directly related to the breach in stability and controllability of aircraft or high-speed vessels in the process of their movement. When calculating the displacement of a dynamic system in space (ship or aircraft), one must take into consideration the exchange of energy between the transverse and path (longitudinal) oscillations as components of the system. In most cases, the frequency of these oscillations is accepted to be 2:1.

However, for a detailed research, it is advisable to consider other frequency ratios. This is especially true when studying the dynamics of aircraft oscillations of the type «Dutch roll» [9]. Such oscillations occur in the case of a large transverse stability of the aircraft compared with a low directional stability. Then the lateral movement of an aircraft is characterized by interdependent oscillations due to heel and sliding. Moreover, the oscillations with sliding lag the phase of oscillations due to heel, which is associated with weak directional and excessive transverse stability. The heel of an airplane is the cause of slip of the plane, eliminating which occurs with a delay due to weak directional stability. The accompanying slip prompts the need for an emergency heel of the aircraft in the opposite direction due to the increased lateral stability, and the process is repeated. To damp oscillations, aircraft employ yaw dampers, whose calculation should be performed by using the concept of energy transfer of a swinging spring in the state of resonance.

It is clear that the state of *resonance* of a swinging spring should occur at a certain combination of values for the parameters of a swinging spring. In a trivial case, when a period of vertical oscillations would be about twice less than that over the period of horizontal oscillations:

$$2T_Y = T_X,$$

where

$$T_X = 2\pi\sqrt{\frac{m}{k}},\ \ T_Y = 2\pi\sqrt{\frac{h}{g}},$$

where *m* is the mass of a load, *k* is the rigidity of a spring, *h* is the length of a spring under a no-load state, g is the acceleration of the Earth's gravity. Or – as an ambiguous statement – when the frequency of vertical oscillations $\omega_Y = \sqrt{k/m}$ is approximately twice the frequency of horizontal oscillations $\omega_X = \sqrt{g/h} : 2\omega_X = \omega_Y$.

However, the resonance state of a swinging spring must be additionally affected by the initial values for the parameters of oscillation initiation. This could be tested if the state of resonance is to be interpreted by using the motion trajectory of a swinging spring load. Note: sometimes, periodic, found among the possible motion trajectories [10]. To find it, it is necessary to devise a universal technique to synthesize a set of trajectories depending on the swinging spring parameters, and, importantly, on the parameters for its oscillation initiation. And attention should be focused on cases where trajectories are represented by periodic curves.

Given the above, it is a relevant task to undertake a research aimed at geometric modeling of motion trajectories of a swinging spring load, which would meet all the conditions for a given type of resonance. That is, conditions when the frequency of vertical oscillations of a «point» mass on a swinging spring would be multiple times larger than the frequency of horizontal oscillations, and would account for a maximum number of parameters for the swinging spring oscillation.

2. Literature review and problem statement

Papers [1, 2] reviewed those studies that addressed the subject of swinging springs – taking into consideration the state of their resonance. Thus, we shall cite here those articles that enlarge the concept of resonance. Note that the movement of a swinging spring load is noticeably more complicated compared to the load of a mathematical pendulum, which is why the effect of using a swinging spring as a mechanical interpretation will be expectedly more pronounced. For example, the conducted laboratory experiments involving a swinging spring provide for a new understanding of the motion of planetary waves in the Earth's atmosphere [11, 12].

Consider an oscillatory system of the type «swinging spring» in a vertically located plane with the Cartesian coordinates *Oxy*. The system includes a weightless spring, to which at one end a load of mass *m* is attached, and the other end is fixed at point *O* in the coordinate origin. It is believed that during the pendulum-like oscillation the axis of the spring maintains its straightness. Rigidity of the spring is denoted by *k*, *h* denotes the length of the spring without a load, and *H* is the length of the spring with a load under the equilibrium (vertical) state.

The equation of a swinging spring motion takes the following form:

$$m\ddot{x}(t) = -T\sin\nu;$$

$$m\ddot{y}(t) = -T\cos\nu - mg, \qquad (1)$$

where T is the pull of the spring, v is the angle of spring deviation from the vertical, g is the acceleration of the Earth's gravity. Then the variable length of the spring is a function of time with obvious physical interpretation:

$$u(t) = \sqrt{x(t)^2 + [H - y(t)]^2}. \qquad (2)$$

Considering:

$$T = k[u(t) - h]; \quad k[H - h] = mg; \qquad (3)$$

$$\sin\nu = \frac{x(t)}{y(t)}; \quad \cos\nu = \frac{H - y(t)}{u(t)},$$

we obtain the equation of a swinging spring motion in the form [13]:

$$\ddot{x}(t) + \omega_X^2 x(t) = \lambda x(t) y(t), \qquad (4)$$

$$\ddot{y}(t) + \omega_Y^2 y(t) = \lambda x(t)^2 / 2,$$

where

$$\omega_X = \frac{g}{H}; \quad \omega_Y = \frac{k}{m}; \quad \lambda = k\frac{h}{H^2}.$$

Frequency ω_X defines the oscillations of a mathematical «linear pendulum», and value w_Y describes the frequency at which the «point» mass on a spring oscillates vertically. If $\lambda=0$, then equation (4) could be solved independently, but the pendulum and spring movements will be linked through the non-linear conditions [13]. A detailed research into all possible movements of a swinging spring is reported in [14] in terms of parameter μ determined from

$$0 \le \mu^2 = \frac{\omega_X}{\omega_Y} \le 1. \qquad (5)$$

Intermediate cases for μ were studied using Poincaré sections and bifurcation diagrams. Paper [14] provided descriptions for all combinations of possible movements of a swinging spring.

Referring to work [13], consider the case when a «spring» motion of a swinging spring dominates over its «pendulum-like» movement. That is, when $x(t) \ll y(t)$. In this case,

$$\ddot{x}(t) + \omega_X^2 x(t) = \lambda x(t) y(t); \qquad (6)$$

$$\ddot{y}(t) + \omega_Y^2 y(t) \approx 0,$$

hence:

$$\ddot{x}(t) + \omega_X^2 x(t) = \lambda x(t) y(t); \qquad (7)$$

$$y(t) = y(0)\cos(\omega_Y t).$$

The result is:

$$\ddot{x}(t) + \left[\omega_X^2 - \lambda y(0)\cos(\omega_Y t)\right] x(t) = 0. \qquad (8)$$

Equation (40) that is known as a Mathieu equation [15], as well as its solutions, are unstable in cases when:

$$\frac{\omega_Y}{\omega_X} = \frac{2}{n}, \quad n = 1, 2, 3\ldots \qquad (9)$$

Ratio (9) yields the sequence of possible conditions for the occurrence of resonance of a swinging spring and explains the need to analyze the ratios of oscillation frequencies of «spring» ω_Y and «pendulum» ω_X for determining the resonance state of a swinging spring.

Paper [13] gives a variant to determining the resonance state of a swinging spring using a correlation between parameters h and H:

$$\frac{H}{H - h} = \frac{4}{n^2} \text{ or } H = \frac{H}{H - n^2} h. \qquad (10)$$

In the trivial cases, the instability of a swinging spring motion could be observed in laboratory experiments at $n=1$, that is, $\omega_Y=2\omega_X$ and $H=4h/3$. A detailed summary of the theory of a swinging spring from the standpoint of laboratory experiments for the specified cases could be found in [16]. In papers [11, 17], the issue on the oscillations of a swinging spring was studied for the case of a three-dimensional space with coordinates x, y, z. Article [18] shows that the derived mathematical expressions could be used to describe the large-scale movements of layers in the Earth's atmosphere, as well as for the case of forced damping of a swinging spring. This means, in particular, that the laboratory experiments with a swinging spring could provide for a new understanding of the dynamics of primary resonance clusters in random nonlinear wave systems, which are characterized by three-wave resonances.

An interesting problem related to a swinging spring is outlined in a research into the Wilberforce pendulum [19] aimed at verifying whether its equations of motion are identical to those that describe the dynamics of certain common resonance clusters [13]. A Wilberforce pendulum consists of a massive load, suspended from a long spring, which can freely rotate relative to its vertical axis. Such a pendulum, in contrast to a swinging spring, does not execute pendulum-like oscillations. At appropriate setting, a Wilberforce pendulum demonstrates the process of energy transfer between the mode of «vertical» oscillations of a load (up and down) and the mode of «revolving» oscillations of a load around the axis of the spring. An analysis of normal modes for a Wilberforce pendulum could be found in [20] along with a detailed description of possible laboratory experiments and examples of numerical simulation.

The authors of [21] investigated at the analytical level a condition for resonance between the two modes of oscillations, which is determined from equality:

$$\omega_{angle} = \omega_{vertical}. \qquad (11)$$

It was experimentally demonstrated that in the case of meeting a condition for resonance (11), «neither the length nor the diameter of the wire, nor the step of coiling the spring, nor the number of its revolutions would affect the state of resonance» [21]. This observation is extremely important since a Wilberforce pendulum can be described mathematically much closer to the «physical» Wilberforce pendulum, compared, for example, with a description of the mathematical pendulum relative to the physical linear pendulum. Paper [13] considers possible implementations of this approach in the field of a nonlinear resonance analysis of wave turbulent systems, performed in the laboratory.

Paper [22] illustrated, based on the created program, the parametric resonance of a swinging spring, which manifests itself in the transfer of energy from the vertical oscillations of a load to horizontal, and vice versa. It was shown that the speed and amplitude of energy transfer strongly depend on initial conditions. Study [23] illustrated the «flow» of energy between longitudinal and transverse oscillations of a point on a swinging spring. However, the study lacks software implementation of this effect. In a cycle of papers [24–28], the authors reported a study of resonance 2:1 in a swinging spring and its connection to the motion trajectory of a swinging spring load. However, the authors confined themselves to only one variant of the resonance.

Papers [29–31] provide computer animations of the oscillations of appropriate swinging springs that illustrate the technique under consideration.

By summing up, we note that known techniques to study the resonance of a swinging spring are based on the following type of equality:

$$\mu\omega_X = \omega_Y,$$

where $\omega_Y = \sqrt{k/m}$ is the frequency of vertical oscillations, and $\omega_X = \sqrt{g/h}$ is the frequency of horizontal oscillations of some point on a spring. Hereinafter, m [kg] is the mass of a load, k [N/m] is the spring rigidity, h [m] is the length of the spring under a no-load state, g [m/s^2] is the acceleration of the Earth's gravity. In formula $\mu\omega_X = \omega_Y$, m is the coefficient of proportionality between frequencies that actually determines the type of resonance.

However, the above works do not consider two more parameters for a swinging spring that significantly affect its oscillations. Specifically, parameters in the form of initial distances of a point load on a swinging spring from the coordinate axes of the chosen coordinate system, as well as the initial velocities of change in the position of a load in the direction of coordinate axes. It is convenient to control the effect of these parameters on the resonance state by using the graphical component of oscillations – the motion trajectory of a swinging spring load. To implement them, it is necessary to define values for the parameters that would ensure the periodic motion trajectory. The periodic trajectory should be used when implementing a swinging spring. This is the field of research that has not been addressed so far.

It follows from the above analysis that it is necessary to devise a universal technique to synthesize the displacement trajectory of swinging springs' load depending on patterns in the state of its resonance. Specifically, taking into consideration not only the basic parameters for a swinging spring (rigidity, length under a no-load state, and a load mass), but the initial conditions for oscillation initiation as well.

3. The aim and objectives of the study

The aim of this study is to geometrically model the resonance of a swinging spring based on the construction of motion trajectory of its load taking into consideration not only the basic parameters for a swinging spring, but the initial conditions for the occurrence of oscillations.

To accomplish the aim, the following tasks have been set:

– to describe the motion process of a swinging spring considering the flow of horizontal oscillations in vertical, and vice versa, which is typical for the resonance state of a swinging spring;

– to describe the motion trajectory of a swinging spring load by using a system of differential equations with components, which include values for the frequencies of vertical and horizontal displacements of a point on a spring;

– to determine the set of motion trajectories of a swinging spring load, which would match the assigned ratios of vertical and horizontal frequencies of load oscillations (for example – 2:1, 7:3, 9:4, and 11:2); – to define values for the variable parameters at which the motion trajectory of a swinging spring load would take the form of periodic.

4. Determining the motion trajectories of a swinging spring load under condition for its resonance

4. 1. The phenomenon of energy «flow» during oscillations of a swinging spring under the state of its resonance of the type 2:1

Consider a swinging spring loaded with a mass m, rigidity k and length L_0 under a no-load state. In the field of gravity with a free fall acceleration of g=9.81 the period of oscillation of such a spring is $T_y = 2\pi\sqrt{m/k}$. The position of the load equilibrium in this case will shift down to a height of $\Delta L=mg/k$. At such a lengthening of the spring the power elasticity compensates for the effect of gravity. However, the period of vertical oscillations relative to the new position of equilibrium with a stretched spring will remain the same. A period of horizontal oscillations of the stretched swinging spring is expressed through the acceleration of free fall g and its full length $L=L_0+\Delta L$, that is $T_x = 2\pi\sqrt{L/g}$. An analysis of the additional stretching of a spring in the field of gravity makes it possible to find out [23] the ratio between the periods:

$$\frac{T_x}{T_y} = \sqrt{\frac{L_0+\Delta L}{g}\frac{k}{m}} = \sqrt{1+\frac{L_0}{\Delta L}} > 1. \qquad (12)$$

Thus, for a swinging spring the period of horizontal oscillations is always greater than the period of vertical ones: $T_x > T_y$. A swinging spring is the simplest example of the implementation of the Fermi resonance. This resonance occurs when the periods of vertical and horizontal oscillations are linked via an approximated ratio $T_x=2T_y$. Or, when using frequencies of independent oscillations along the vertical ω_Y and the horizontal ω_X, via relation $\omega_Y=2\omega_X$. That is, the horizontal and vertical oscillations begin to seemingly flow one into another. The energy of oscillations will transfer from vertical oscillations into the horizontal, and vice versa. In this case, which is very interesting, strictly vertical oscillations turn out to be unstable.

The emergence of relation $T_x:T_y=2:1$ could be explained as follows from [23]. We shall denote through $X(t)$ and $Y(t)$ in a Cartesian plane Oxy the coordinates of time-dependent deviation t of a load from the equilibrium position. At such a deviation, the potential energy grows by magnitude:

$$\Delta U = \frac{k}{2}\left(\left(\sqrt{(L-Y)^2+X^2}-L_0\right)^2-(\Delta L)^2\right)+mgY. \qquad (13)$$

If one chooses coordinates X and Y to be substantially smaller than L, then expression (2) is approximately equal to:

$$\Delta U \approx \frac{k}{2}Y^2 + \frac{mg}{2L}X^2 - \frac{L_0}{L^2}X^2Y = U_Y + U_X + U_{XY}, \qquad (14)$$

while adding other components, which are characterized by the higher degrees of deviations. The magnitudes U_Y and U_X are the potential energies that occur as a result of vertical and horizontal oscillations. And the magnitude:

$$U_{XY} = -\frac{L_0}{L^2}X^2Y$$

is a special addition that generates the interaction between the specified oscillations. Owing to such an interaction, the vertical oscillations will affect the horizontal oscillations, and vice versa.

We shall introduce new designations for convenience:

$$\omega_X^2 = \frac{g}{L};\quad \omega_Y^2 = \frac{k}{m};\quad c = \frac{L_0}{2L^2}\omega_Y^2$$

and consider the system of equations of oscillations horizontally and vertically:

$$\ddot{X} + \omega_X^2 X = 2cXY;$$

$$\ddot{Y} + \omega_Y^2 Y = cX^2. \qquad (15)$$

Without the right-hand sides, equations (15) yield descriptions of independent oscillations vertically and horizontally at frequencies ω_Y and ω_X. Additions in the right-hand sides point to the existence of a force whose action leads to additional swing in oscillations. If frequencies ω_Y and ω_X are arbitrary, this force is small and does not manifest itself in any substantial effect. However, if the relation $\omega_Y=2\omega_X$ holds, then the state of resonance occurs. The force that drives oscillations contains the component, for both types of oscillations, with the same frequency as the oscillation itself. As a result, this force would contribute to executing one type of oscillations and to damping another type [23]. That is exactly the way that the horizontal and vertical oscillations transfer from one to another.

To confirm the «flow» of energy between the longitudinal and transverse oscillations of a point on a swinging spring, we developed a computer program. Here are three examples that illustrate the specified effect. To this end, we select the parameters values for a swinging spring, which would approximately match the state of its resonance. For example, $m=0.35$; $k=150$; $L_0=0.1$. Then:

$$\omega_Y = \sqrt{\frac{k}{m}} = 20.702;\quad \omega_X = \sqrt{\frac{g}{L_0}} = 9.905,$$

hence $\omega_Y/\omega_X=2.09$. That is, the condition for the occurrence of resonance of the type 2:1 more or less holds. The full length of a spring is to be equal to $L=0.5$. A system of differential equations (15) should be solves numerically using a Runge-Kutta method. The chards are built based on 5,000 derived points.

Example 1. System of equations (15) is to be solved at the following initial conditions: $x_0=0.1$; $Dx_0=0$; $y_0=-0.2$; $Dy_0=0$. That is, for a point with initial motion coordinates $X(0)=0.1$; $Y(0)=-0.2$ and in the absence of assigning their speed along the directions of co-ordinate axes.

Fig. 1 shows the motion trajectory of a point with coordinates $X(t)$, $Y(t)$ over time $T=35$. Fig. 2 shows charts of a load's deviations for respective coordinates. Fig. 3 shows charts of potential energies for oscillations along respective coordinates, as well as chart of addition that generates interaction between the specified oscillations.

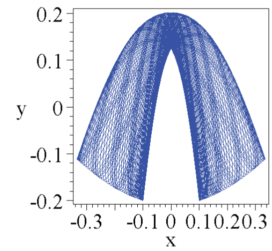
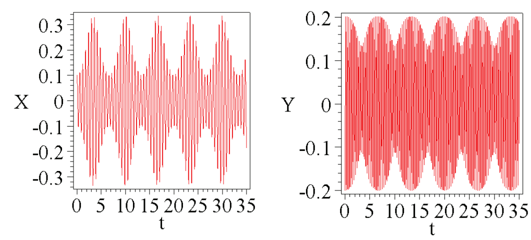
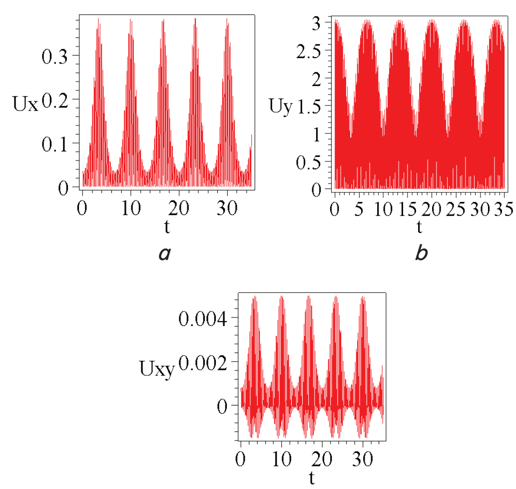


Fig. 1. Motion trajectory of a point for example 1



a *b*

Fig. 2. Deviation charts of a load:
a – for coordinate *X*; *b* – for coordinate *Y*



c

Fig. 3. Charts of potential energies for:
a – oscillations along coordinate *X*; *b* – oscillations along coordinate *Y*; *c* – addition that generates interaction between the specified oscillations

Example 2. System of equations (4) is to be solved at the following initial conditions: $x_0=0.001$; $Dx_0=0$; $y_0=-0.2$; $Dy_0=0$. That is, for a point that prior to the onset of motion is located «almost» along the Oy axis.

Fig. 4 shows the motion trajectory of a point with coordinates $X(t)$, $Y(t)$ over time $T=35$. Fig. 5 shows the deviation charts of a load for respective coordinates. Fig. 6 shows the charts of potential energies for oscillations along the respective coordinates, as well as the chart of addition that generates interaction between the specified oscillations.

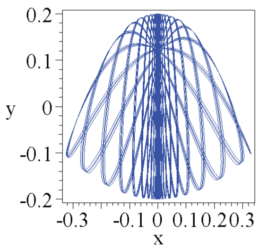


Fig. 4. Motion trajectory of a point for example 2

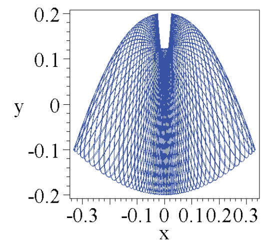
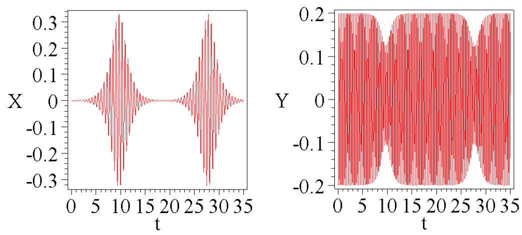
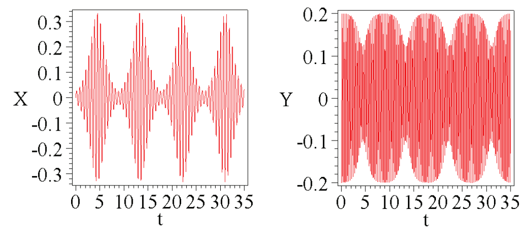


Fig. 7. Motion trajectory of a point for example 3



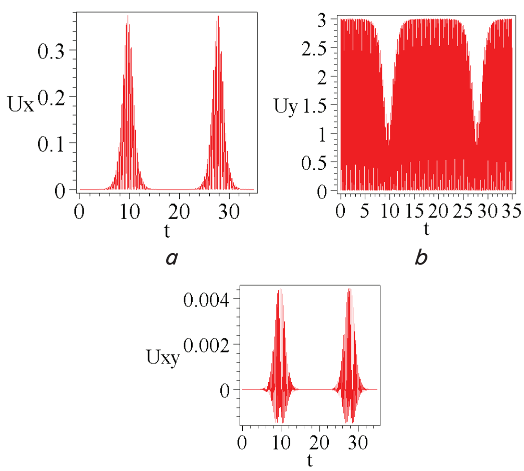
a *b*

Fig. 5. Deviation charts of a load:
a – for coordinate *X*; *b* – for coordinate *Y*



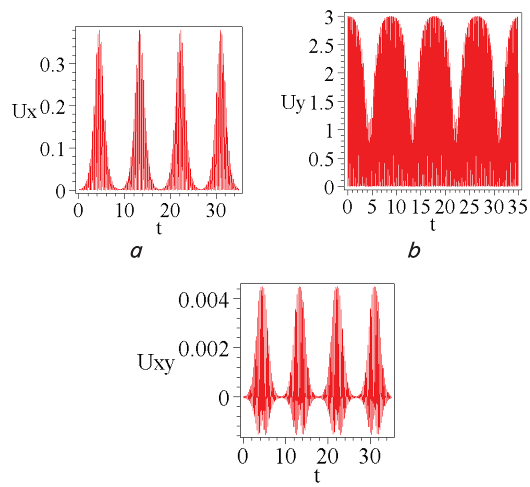
a *b*

Fig. 8. Deviation charts of a load:
a – for coordinate *X*; *b* – for coordinate *Y*



a *b* *c*

Fig. 6. Charts of potential energies for:
a – oscillations along coordinate *X*; *b* – oscillations along coordinate *Y*; *c* – addition that generates interaction between the specified oscillations



a *b* *c*

Fig. 9. Charts of potential energies for:
a – oscillations along coordinate *X*; *b* – oscillations along coordinate *Y*; *c* – addition that generates interaction between the specified oscillations

Example 3. System of equations (4) is to be solved at the following initial conditions: $x_0=0$; $Dx_0=0.25$; $y_0=-0.2$; $Dy_0=0$. That is, for a point, which is located along the *Oy* axis, and which was set into motion by the pulse of magnitude 0.25 conditional units.

Fig. 7 shows the motion trajectory of a point with coordinates $X(t)$, $Y(t)$ over time $T=35$. Fig. 8 shows the deviation charts of a load for respective coordinates.

Fig. 9 shows the charts of potential energies for oscillations along respective coordinates, as well as the chart of addition that generates interaction between the specified oscillations.

Note that when the choise of the parameters for a swinging spring is arbitrary, the motion trajectory of its load might not obey any law. A motion trajectory could even fill some area in a plane. Next, we shall consider the technique for finding specific periodic motion trajectories of a load. In this case, the sequence of observations is as follows.

At the first stage, we define, for the chosen type of resonance $\mu\omega_X=\omega_Y$, by using the program developed, the values for parameters *m*, *h*, and *k*, which could ensure a given resonance. Let a point load have coordinates (X_0, Y_0). Then the criterion for the existence of resonance is based on the representation of two charts of functions $X(t)$ and $Y(t)$ that describe the distance of the point load to the corresponding coordinate axes. The plotted charts should possess the following pattern: maximum values for one chart should be achieved at minimum values for another chart.

At the second stage, we select one of the parameters (for example, Y_0 or DY_0) as a variable magnitude. Next, we build, in the form of computer animation [1, 2], the set of a load motion trajectories dependent on parameter Y_0. The result is the selected values for parameter Y_0, which correspond to the periodic motion trajectories of a swinging spring load, shown on the corresponding frames in the animation.

4. 2. The set of motion trajectories of a swinging spring load for the resonance of oscillations of the type 2:1

When constructing patterns of a swinging spring trajectory, we shall employ papers [1, 2]. Intermediate results shall be omitted; only the obtained results are shown. The motion trajectory of a swinging spring load along the vertical plane *Oxy* is to be determined form the system of differential equations (15) depending on the mass of a load *m*, the original length of a spring under a no-load state *h*, the rigidity of a spring *k*, and the initial conditions for oscillation initiation. The full length of a spring is denoted via *L*.

It is believed that for all the problems the main condition for the idealization of a load motion holds: the energy dissipation process is slow compared to the characteristic scale of time (that is, an oscillatory system is conservative).

Note that $\omega_Y = \sqrt{k/m}$ denotes the frequency of longitudinal oscillations of a point on a non-deviated spring, and $\omega_X = \sqrt{g/h}$ indicates the frequency of lateral oscillations of a point along the axis of the spring as a mathematical pendulum (g=9.81).

The set of trajectories is built under the condition for a change in the initial values x_0, Dx_0, y_0, Dy_0 for the position of a point onto plane $0xy$. Hereafter, a system of differential equations (15) is to be solved numerically using the method of Runge-Kutta. The charts are built based on 1,500 derived points.

Select the values for a swinging spring's parameters: m=1.91; h=0.5; k=150: L=1.5. Then $\omega_Y / \omega_X = 2$. That is, the condition for resonance of the type 2:1 is satisfied.

Case 1. Assume $x0=0.6$; $Dx0=0$; $Dy0=0$ and the parameter $y0$ changes within $0.1<y0<0.6$. Fig. 10 shows the shape of the point's deviations charts from the respective axes for $y0=0.6$. In the case of randomly selected parameters, a swinging spring load's trajectory could fill some area in the plane. Choosing a value for the parameter $y0$ could help for the trajectory to take the form of a periodic curve. The choice of $y0$ affects the ratio of amplitudes in deviation charts. Fig. 11 shows the found variants of solutions when the motion trajectory of a swinging spring load is periodic. In paper [31], this is illustrated by using the developed computer animation.

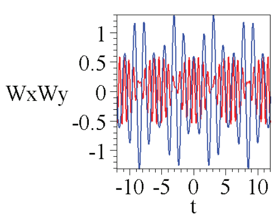
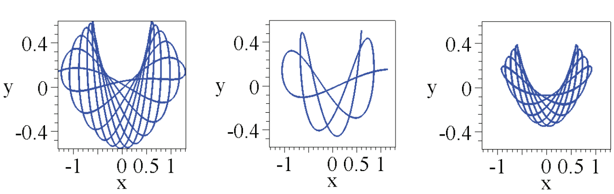


Fig. 10. Charts of the point's deviation from respective axes for $y0=0.6$ (case 1)



a *b* *c*

Fig. 11. Variants for the periodic trajectories of a swinging spring load's motion for case 1: *a* – $y0=0.6$; *b* – $y0=0.5$; *c* – $y0=0.39$

Case 2. For comparison, we give a solution for parameters $x0=0.1$; $Dx0=0$; $Dy0=0$, and the parameter $y0$ changes within $0.1<y0<0.6$. Fig. 12 shows the shape of deviation charts of a point for respective points for $y0=0.6$. Fig. 13 shows the variants of solutions when the motion trajectory of a swinging spring load is periodic.

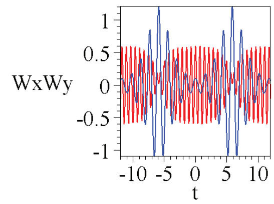
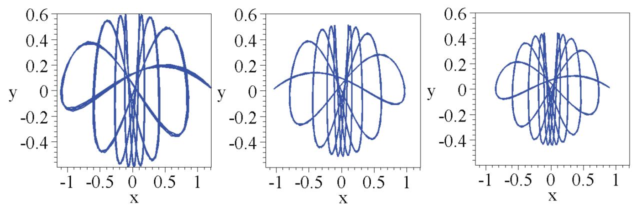


Fig. 12. Charts of the point's deviation from respective axes for $y0=0.6$ (case 2)



a *b* *c*

Fig. 13. Variants for the periodic trajectories of a swinging spring load's motion for case 2: *a* – $y0=0.6$; *b* – $y0=0.52$; *c* – $y0=0.445$

Case 3. Assume a variable magnitude is the initial velocity along the axis Oy, which changes within $0.1<Dy0<6$. We give a solution for parameters $x0=1$; $Dx0=0$; $y0=0$. Fig. 14 shows the shape of charts of the point's deviation from respective axes for $Dy0=4.112$. Fig. 15 shows the variants of solutions when the motion trajectory of a swinging spring load is periodic.

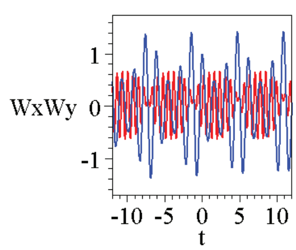
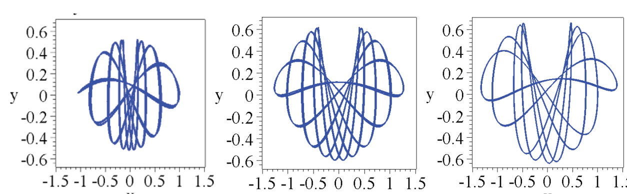


Fig. 14. Charts of the point's deviation from respective axes for $Dy0=4.112$ (case 3)



a *b* *c*

Fig. 15. Variants for the periodic motion trajectories of a swinging spring load for case 3: *a* – $Dy0=1.428$; *b* – $Dy0=3.463$; *c* – $Dy0=4.122$

Case 4. Assume a variable magnitude is also the initial velocity along the axis *Oy*, which changes within $0.1<Dy0<6$.

We give a solution for parameters $x0=0.25$; $Dx0=0$; $y0=0$. Fig. 16 shows the shape of Charts of the point's deviation from respective axes for $y0=4.17$. Fig. 17 shows the variants of solutions when the motion trajectory of a swinging spring load is periodic.

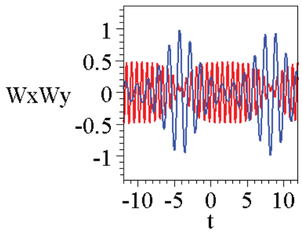


Fig. 16. Charts of the point's deviation from respective axes for $Dy0=4.17$ (case 4)

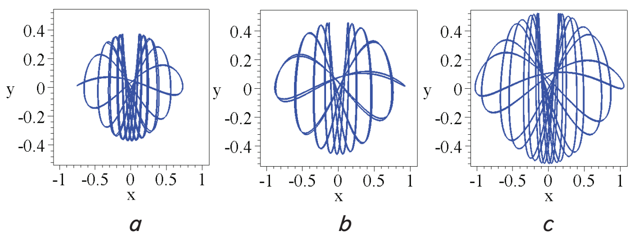


Fig. 17. Variants for the periodic motion trajectories of a swinging spring load for case 4: *a* – $Dy0=3.139$; *b* – $Dy0=3.935$; *c* – $Dy0=4.525$

Thus, the shape of the derived periodic trajectories can be used to easily compare features of the same type of resonance.

4. 3. The set of motion trajectories of a swinging spring load for the resonance of oscillations 7:3

The set of trajectories is built under the condition of change in the initial values $x0$, $Dx0$, $y0$, $Dy0$ for the position of a point in plane $0xy$. Select the values for a swinging spring's parameters: $m=0.28$; $h=0.1$; $k=150$; $g=9.81$. Then the frequency of vertical oscillations is:

$$\omega_Y = \sqrt{\frac{k}{m}} = 23.11;$$

and the frequency of horizontal oscillations is:

$$\omega_X = \sqrt{\frac{g}{h}} = 9.9.$$

Hence $\omega_Y/\omega_X = 7/3$. That is, the condition for resonance of the type 7:3 will be satisfied.

Case 1. Assume $x0=0.1$; $Dx0=0$; $Dy0=0$; $L=0.5$, and the parameter $y0$ changes within $0.2<y0<0.6$. Fig. 18 shows the shape of charts of the point's deviation from respective axes for $y0=0.6$. Fig. 19 shows the derived variants of solutions when the motion trajectory of a swinging spring load is periodic.

Case 2. Assume $x0=1$; $Dx0=0$; $y0=0$; $L=1.3$, and the parameter $Dy0$ varies within $0.2<Dy0<6.5$. Fig. 20 shows the shape of charts of the point's deviation from respective axes for $Dy0=6.5$. Fig. 21 shows the derived variants of solutions when the motion trajectory of a swinging spring load is periodic.

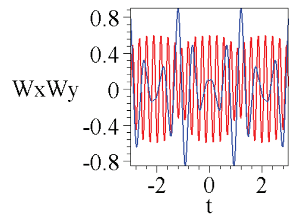


Fig. 18. Charts of the point's deviation from respective axes for $y0=0.6$ (case 1)

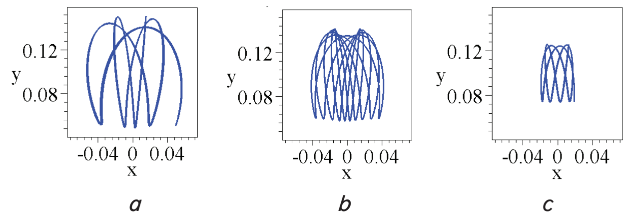


Fig. 19. Variants for the periodic motion trajectories of a swinging spring load for case 1: *a* – $y0=0.516$; *b* – $y0=0.426$; *c* – $y0=0.264$

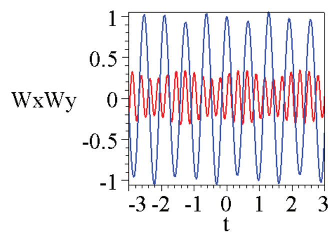


Fig. 20. Charts of the point's deviation from respective axes for $Dy0=6.5$ (case 2)

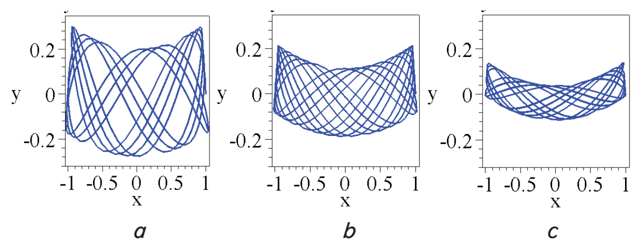


Fig. 21. Variants for the periodic motion trajectories of a swinging spring load for case 2: *a* – $Dy0=5.332$; *b* – $Dy0=3.223$; *c* – $Dy0=1.2431$

Paper [8] argues on the feasibility of constructing the motion trajectories of a load for resonance 7:3.

4. 4. The set of motion trajectories of a swinging spring load for the resonance of oscillations 9:4

Select the values for a swinging spring's parameters: $m=4.893$; $h=9$; $k=27$; $g=9.81$. Then the frequency of vertical oscillations is:

$$\omega_Y = \sqrt{\frac{k}{m}} = 2.35;$$

and the frequency of horizontal oscillations is:

$$\omega_X = \sqrt{\frac{g}{h}} = 1.04.$$

Hence $\omega_Y/\omega_X = 9/4$. That is, the condition for resonance of the type 9:4 will be satisfied.

Case 1. Assume $x0=0.2$; $Dx0=0$; $y0=0$; $L=5$, and the parameter $Dy0$ varies within $0.1<Dy0<1.5$. Fig. 22 shows the shape of deviation charts of a point from respective axes for $Dy0=1.42$. Fig. 23 shows the derived variants of solutions when the motion trajectory of a swinging spring load is periodic.

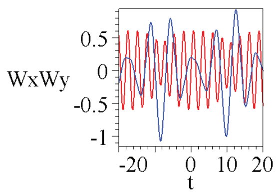
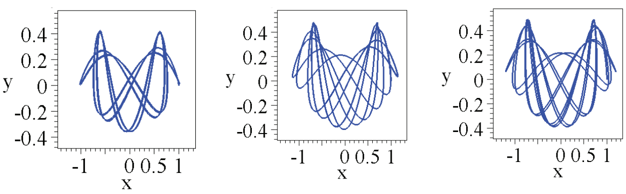


Fig. 22. Charts of the point's deviation from respective axes for $Dy0=1.42$ (case 1)



a *b* *c*

Fig. 23. Variants for the periodic motion trajectories of a swinging spring load for case 1: *a* — $Dy0=0.142$; *b* — $Dy0=0.513$; *c* — $Dy0=0.569$

Case 2. Assume $x0=0.1$; $Dx0=0$; $Dy0=0$; $L=5$, and the parameter $y0$ changes within $0.1<y0<1.52$. Fig. 24 shows the shape of charts of the point's deviation from respective axes for $y0=0.52$. Fig. 25 shows the derived variants of solutions when the motion trajectory of a swinging spring load is periodic.

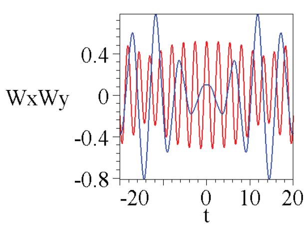
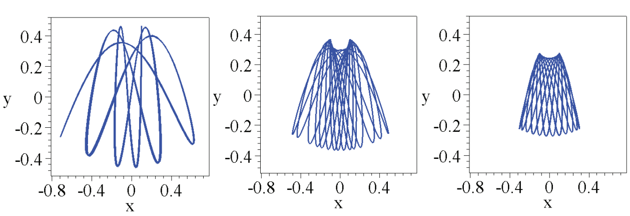


Fig. 24. Charts of the point's deviation from respective axes for $y0=0.52$ (case 2)



a *b* *c*

Fig. 25. Variants for the periodic motion trajectories of a swinging spring load for case 2: *a* — $y0=0.46$; *b* — $y0=0.37$; *c* — $y0=0.27$

Paper [8] argues on the feasibility of constructing the motion trajectories of a load for resonance 9:4.

4. 5. The set of motion trajectories of a swinging spring load for the resonance of oscillations 11:2

Select the values for a swinging spring's parameters: $m=37.51$; $h=9$; $k=207$; $g=9.81$. Then the frequency of vertical oscillations is:

$$\omega_Y=\sqrt{\frac{k}{m}}=5.45$$

and the frequency of horizontal oscillations is:

$$\omega_X=\sqrt{\frac{g}{h}}=0.99.$$

Hence $\omega_Y/\omega_X=11/2$. That is, the condition for resonance of the type 11:2 will be satisfied.

Case 1. Assume $x0=0.1$; $Dx0=0$; $Dy0=0$; $L=5$, and the parameter $y0$ changes within $0.1<y0<0.52$. Fig. 26 shows the shape of charts of the point's deviation from respective axes for $y0=0.52$. Fig. 27 shows the derived variants of solutions when the motion trajectory of a swinging spring load is periodic.

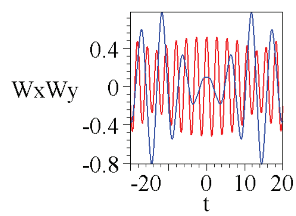
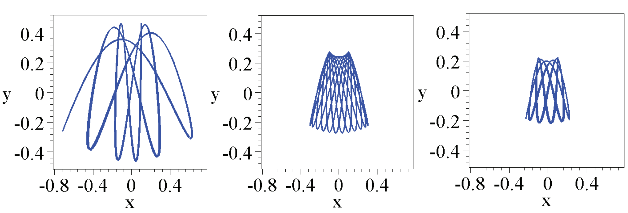


Fig. 26. Charts of the point's deviation from respective axes for $y0=0.52$ (case 1)



a *b* *c*

Fig. 27. Variants for the periodic motion trajectories of a swinging spring load for case 1: *a* — $y0=0.457$; *b* — $y0=0.3898$; *v* — $y0=0.226$

Case 2. Assume $x0=0.1$; $Dx0=0$; $y0=0$; $L=5$, and the parameter $Dy0$ varies within $0.1<Dy0<1.65$. Fig. 28 shows the shape of charts of the point's deviation from respective axes for $Dy0=1.65$. Fig. 29 shows the derived variants of solutions when the motion trajectory of a swinging spring load is periodic.

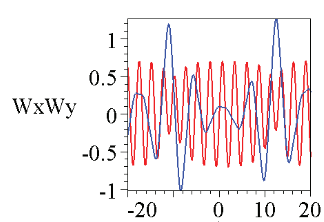


Fig. 28. Charts of the point's deviation from respective axes for $Dy0=1.65$ (case 2)

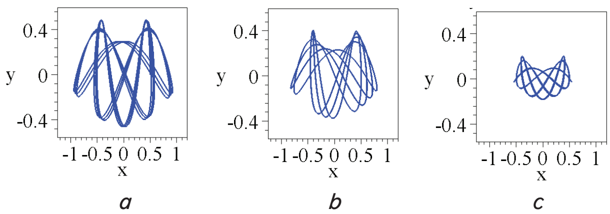


Fig. 29. Variants for the periodic motion trajectories of a swinging spring load for case 1: *a* – *Dy*0=1.0455; *b* – *Dy*0=0.8285; *c* – *Dy*0=0.2628

Note that the shape of the derived periodic trajectories makes it possible to compare features of the same type of resonance. Paper [8] argues on the feasibility of constructing the motion trajectories of a load for resonance 11:2. The website [31] hosts computer animations that illustrate the obtained results.

5. Discussion of the geometrical modeling of resonance of a swinging spring based on the construction of the motion trajectory of its load

The idea of using swinging springs as a mechanical model is expedient in order to analyze modern technological processes as the dynamic systems. These systems may consist of nonlinearly connected oscillatory components that exchange energy. Oscillations of a swinging spring should be considered in a combination with the geometric component – the motion trajectory of its load. The result is the possibility to characterize the resonance of oscillations of a swinging spring using periodic trajectories, selected from possible movements during oscillations of a swinging spring load. Moreover, in order to synthesize a trajectory, it is necessary to use not only the basic parameters for a swinging spring, but also the parameters for the initial conditions of oscillation initiation. After all, this case most effectively demonstrates the angular swing of a swinging spring at the expense of energy of this spring. The development of a random transversal perturbation would proceed until reaching a fixed value for the amplitude, because the reserves of energy in a spring are exhaustive. Upon reaching such an amplitude, during oscillations of a swinging spring there would again occur the stretching (or compaction) of the spring.

Paper [32] reports a phenomenological technique to construct the contour of a vertical cross-section of a liquid's surface in a tank that oscillates due to the movement of this tank. The specified contours are called the Faraday waves. The technique is based on the mechanical «pendulum» analogy of the process of fluid oscillation. Specifically, the Faraday waves are interpreted as the motion trajectories of a mathematical pendulum's load (not a spring), suspended from a movable trolley. The authors considered the issues on constructing the formulae that would approximately relate the parameters for a liquid to the parameters for a pendulum under the trolley. Based on this example, we shall consider the following.We shall state the precondition for using the model of a swinging spring as a hypothesis. To find a solution to the considered class of problems, it is necessary, when stating it, to define two (as an example) nonlinearly connected oscillatory components that exchange energy. Next, one should determine the basic parameters for the system (that significantly affect the solution), and match them against the parameters for a swinging spring – its rigidity, the length under a no-load state, and the mass of a load. As well as the parameters that define the initial conditions for the oscillations of a swinging spring – the initial angle of spring deviation and the speed of its shift. Then the desired solution to the problem could be related to the periodic motion trajectory of a swinging spring load. And it is necessary to look for a trajectory of the smallest length among the set of the periodic motion trajectories. It is possible to compare the features of resonance trajectories based on a condition for the density of pixels that make up the image of a certain trajectory. According to the general principle of «minimum energy», it is logical to assume that it is the case with the shortest periodic trajectory (or rather, with its one period) that might prove interesting in the course of a particular implementation. The derived periodic motion trajectory of a load can always be represented in a digital form as the sequence of coordinates of the points that compose it.

Given the specified positions, it will be interesting to investigate the nonlinearly connected systems the involve interacting subsystems using examples of problems related to engineering. An important role in the construction mechanics belongs to the modified model of a swinging spring – a model of the flexible thread. After all, the flexible thread is a kind of spring that acts on stretching only. In a typical 2D model, the flexible thread could simultaneously perform transverse oscillations in its plane (similar to angular oscillations in a swinging spring with a load) and pendulum oscillations connecting support fixings (similar to vertical oscillations). A field of the possible research could involve the wires of high-voltage lines whose condition is affected by wind gusts. At the ratio of frequencies of the specified oscillations of 1:2, there is a loss of dynamic stability, followed by the transverse oscillations of a thread, whose amplitude could reach rather large values. Note that a possibility of the occurrence of such phenomena must be considered when calculating various structures for construction mechanics. For example, hanging bridges, cable-beam systems, cable roads, various antenna for rope system for maintaining facilities, flexible hoses, etc.

Difficulties in the advancement of research in this field might be associated with an effort to determine the resonance state while studying oscillations in a spatial swinging spring.

6. Conclusions

1. The phenomenon of the «flow» of energy between the longitudinal and transverse oscillations of a swinging spring is shown using the example with the following parameters: $m=0.35$; $k=150$; $h=0.1$; $L=0.5$. Then:

$$\omega_Y=\sqrt{\frac{k}{m}}=20.702;\quad \omega_X=\sqrt{\frac{g}{h}}=9.905,$$

hence it follows that the approximated resonance condition $\omega_Y/\omega_X=2.09$ is satisfied.

2. For a swinging spring with parameters $m=1.91$; $h=0.5$; $k=150$; $L=1.5$; $x0=1.2$; $Dx0=0$; $Dy0=0$ for the case of resonance $\omega_Y/\omega_X=2$, we have found, by using the derived charts, values $y0=0.6$; $y0=0.5$; $y0=0.39$, which correspond to the periodic motion trajectories of a load.

3. For specific resonances of the types 7:3, 9:4, and 11:2, by applying computerized animation, we have derived values for the parameters of a swinging spring, which correspond to the periodic trajectories of its load motion.

4. We have derived the values for parameters when the motion trajectory of a swinging spring load takes the form of periodic for the case of resonance, for example, for:

a) 2:1 – $m=1.9113$; $h=0.5$; $k=150$: $L=1.5$; $x0=1.2$; $y0=0.5$; $Dx0=0$; $Dy0=0$;

b) 7:3 – $m=0.2808$; $h=0.1$; $k=150$; $L=0.5$; $x0=0.2$; $y0=0.516$; $Dx0=0$; $Dy0=0$;

c) 9:4 – $m=4.893$; $h=9$; $k=27$; $L=5$; $x0=1$; $y0=0$; $Dx0=0$; $Dy0=0.142$;

d) 11:2 – $m=37.5127$; $h=9$; $k=207$; $L=5$; $x0=0.1$; $y0=0.52$; $Dx0=0$; $Dy0=0$.

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OPTIMIZATION OF ANGULAR VELOCITY OF DRUM MIXERS

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Барабанні змішувачі можуть забезпечити заданий рівень рівномірності змішування, в тому числі компонентів кормових добавок. Однак питання теоретичного та експериментального обґрунтування конструкційно-кінематичних параметрів барабанних змішувачів недостатньо науково досліджене. Метою роботи є підвищення ефективності виробництва кормових сумішей шляхом забезпечення оптимальної кутової швидкості обертання барабанного змішувача.

Для визначення радіальної швидкості руху частки по лопатці барабана був використаний розв'язок однорідного диференційного рівняння. Визначення числового значення кутової швидкості реалізовано методом комп'ютерного моделювання. Проведення експериментальних досліджень рівномірності перерозподілу кормових компонентів у суміші комбікорму здійснювалося за допомогою розробленого експериментального барабанного змішувача. Змішувач складався з камери, прямокутної рамки, опорної рами та приводу. Камера змішування мала завантажувально-розвантажувальне вікно закрите кришкою. Всередині камери по всій її довжині і рівномірно по периметру були встановленої радіальні лопатки.

Досліди проводилися із використанням барабанного змішувача із радіусом барабана 0,17 м, який мав радіальні лопатки шириною 25 мм та при коефіцієнті заповнення камери 0,5. Встановлено, що барабанний змішувач забезпечує максимальне розсіювання часточок матеріалу по поверхні робочого сегменту при кутовій швидкості обертання барабану 9,69 рад/с.

У результаті експериментальних досліджень встановлено, що при частоті обертання барабана лабораторної установки 9,42 рад/с рівномірність сумішки становить 92,5–93 %, що відповідає існуючим зоотехнічним вимогам для всіх видів комбікормів. При цьому максимальне відхилення теоретичних та експериментальних даних становило біля 9 %. Отримані результати дають змогу стверджувати про можливість визначення числового значення кутової швидкості барабанних змішувачів запропонованим методом комп'ютерного моделювання

Ключові слова: лопатка, радіальна швидкість, коефіцієнт заповнення, час змішування, кормові добавки, контрольний компонент, суміш, радіус барабана

1. Introduction

Among animal feed, a leading role belongs to mixed fodder, which is a concentrated source of nutrients and a means to balance rations in accordance with standards of animal nutrition. Over recent time, production of mixed fodder is moved from specialized mixed fodder factories directly to agricultural manufacturing. Engineers designed and produced a significant number of mixed fodder units whose operation is based on using own grain raw materials and the commercially