*Поставлена задача розроблення методу оптимального керування роботою нагнітачів природного газу за критерієм, який не тільки мінімізує витрату паливного газу, але й враховує потужність викидів оксиду азоту в атмосферу. При цьому взяті до уваги технічні стани газоперекачувальних агрегатів, обмеження на технологічні параметри та вимога до забезпечення планового показника на перекачку газу групою паралельно працюючих нагнітачів.*

*Технічний стан кожного агрегату або вузла оцінюється за певними ознаками. Якщо вести спостереження на протязі певного періоду часу за такими ознаками, то отримаємо множину ознак. З використанням штучної нейронної мережі типу Кохонена множина ознак (образів) розбита на три класи. Кожному класові присвоюється певна кількість балів, яка і характеризує його технічний стан. За набраною кількістю балів визначають коефіцієнт завантаження кожного нагнітача, який враховується в обмеженні на загальну продуктивність групи нагнітачів.*

*Формалізований запис задачі оптимального керування вміщує залежності, які апроксимуються поліномом заданої степені. У результаті отримують емпіричну модель, структуру якої визначають з використанням апарату генетичних алгоритмів.*

*З цілого ряду причин (похибки вимірювань технологічних параметрів, похибки методів вимірювань, дія зовнішніх впливів, обмежений обсяг експериментального матеріалу та ін.) ідентифікація значень параметрів емпіричних моделей ґрунтується на неточній інформації. Тому параметри емпіричних моделей трактуються як нечіткі величини. Виходячи із прийнятої концепції, отриманий формалізований запис задачі оптимального керування роботою нагнітачів природного газу.*

*Реалізація результатів досліджень дасть змогу отримати економію паливного газу та зменшити обсяги викидів оксиду азоту в навколишнє середовище*

*Ключові слова: нагнітач, природний газ, оксид азоту, технічний стан, штучний інтелект*

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# DEVELOPMENT OF A METHOD FOR OPTIMIZING OPERATION OF CENTRIFUGAL GAS SUPERCHARGERS UNDER CONDITIONS OF UNCERTAINTY

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## 1. Introduction

The gas transportation system (GTS) which transports natural gas from Russian Federation to countries of Central and Eastern Europe is one of the largest in the world. Part of this GTS passes through the territory of Ukraine where underground gas storage facilities are located. According to Ukrtransgaz JSC, Ukraine, total active capacity of underground gas storage facilities is 31 billion cubic meters.

In 2018, volume of gas transportation through the territory of Ukraine from Russian Federation to Moldova and European countries amounted to 86.8 billion cubic meters. In order to provide specified volumes of gas pumping, the Ukrainian GTS is equipped with compressor stations with centrifugal natural gas superchargers, mostly gas-turbine driven. Natural gas extracted directly from the GTS serves as energy resource for gas turbine engines. Ukrtransgaz JSC spends about 240 million cubic meters of gas monthly for this purpose.

One of the most promising ways to reduce natural gas consumption for own needs consists in ensuring optimal operation of gas pumping units (GPU) taking into account technological constraints. In addition, optimal GPU operation mode should not only save fuel gas but also take into consideration requirements to reduction of harmful emissions into atmosphere.

## 2. Literature review and problem statement

A concept of an intelligent system for controlling the process of natural gas compression in a form of a human-machine structure is proposed in [1]. Such a concept requires development of a whole set of methods and algorithms, such as determination of technical conditions of GPU, prevention of surging phenomena, optimal control of the GPU group, etc.

Effective operation of a GPU is largely dependent on its technical condition. Therefore, a series of publications [2–4] are devoted to diagnosing both individual assemblies and a unit in general. Author of [2] proposes installing additional sensors of electrical parameters for detecting malfunctions in centrifugal superchargers with electric drives. According to readings of the installed sensors and using the developed algorithms, it is possible to detect malfunctions in individual assemblies of the supercharger. Installing of additional sensors increases cost of the system and reduces its reliability. Assessment of impact of technical condition of the supercharger on its performance is an unresolved issue, which makes impossible use of the information obtained to effectively control the supercharger operation.

Using the theory of reliability, a method was developed in [3] to optimize periodicity of turbo-compressor maintenance. Total cost of maintenance and optimum time of unit maintenance are estimated using the proposed criterion. However, degradation of turbocharger assemblies is gradual over a period of time. Therefore, it is important to have a strategy for operating a unit during the period between "good" condition of the supercharger and the condition causing its shutdown and further scheduled maintenance.

Authors of [2, 3] indicate that change of technical condition significantly affects technical and fuel-and-energy indicators of the GPU operation.

Based on information about vibration of moving parts of rotor-type machines, entropy characteristics such as entropy of the singular spectrum, entropy of the power spectrum and approximate entropy are obtained in [4]. Using a probabilistic neural network, technical condition of the rotor-type machines is established. Disadvantage of this method consists in that it is impossible to diagnose with its help faults of GPU assemblies such as a combustion chamber, oil system, the supercharger setting, etc.

To predict operation of the centrifugal supercharger of natural gas, a wavelet neural network is used in [5] which, according to the author, is a simple algorithm, has structural stability and a high speed of convergence. However, the author does not specify how the obtained information on prediction of the supercharger efficiency can be used in the system of optimal control of the natural gas compression process.

Identification of impact of assessment of the GPU technical condition on solution of the problem of optimal control of the process of natural gas compression is an important scientific problem. Assessment of the GPU technical condition proceeding from the change of technological parameters and environment-related characteristics using the theory of fuzzy sets is proposed in [6]. This assessment results in obtaining of a coefficient of the GPU technical condition which is introduced into the formalized record of the problem of optimal GPU control. Disadvantage of this method consists in that in order to solve the problem it is necessary to involve experts. This introduces a subjective factor in determining optimal GPU operation mode.

To determine optimal GPU operation mode, criteria characterized by total energy costs [7] for compression of natural gas are used. In [8], such costs are defined as consumption of fuel gas to drive gas turbine engines.

In most papers, such as in [7], the problem of optimal control of the GPU operation is solved in a deterministic statement. This formalized approach is too simplistic since the GPU as an object of automatic control is subject to numerous interferences [1] which must be taken into account in the process of solving the optimization problem.

Gas turbine-driven compressor stations are major pollutants of environment [9]. Therefore, in the process of controlling the GPU operation, it is necessary to choose an operation mode that enables minimization of harmful emissions into atmosphere.

Thus, when formalizing the scientific problem of optimal control of GPU operation, the issues of consideration of external influences causing inaccuracies in creation of mathematical models remain unresolved.

Classification of technical conditions of superchargers without involvement of experts as well as taking into consideration the GPU effect on environment form another component of the problem.

## 3. The aim and objectives of the study

The study objective is development of a method and an algorithmic support for the system of optimal control of GPU operation in conditions of uncertainty taking into account GPU technical condition and power of nitric oxide emission into environment.

To achieve this goal, the following tasks were set:

– to formalize the problem of optimal control of operation of centrifugal natural gas superchargers;

– to develop a method of identification of technical condition of centrifugal natural gas superchargers using artificial neural networks;

– to develop an algorithmic support for the method of constructing boundary curves between the classes of attributes of the supercharger technical condition;

– to develop a method for minimizing the optimality criterion which is a function of fuel gas consumption and the rate of nitrogen oxide emissions into environment based on the problem formalism taking into account technical condition of the supercharger and limitation of technological parameters.

## 4. Formalization of the problem of optimization of natural gas superchargers

Let us assume that the compressor station has $L$ GPU groups. There are $m$ natural gas superchargers with gas turbine drive in each group. For driving each $i$-th supercharger, $G_i$ mass units of natural gas costing $c$ UAH are spent per a mass unit in a time unit.

When burning natural gas in the GPU combustion chamber, such harmful substances as nitrogen and carbon oxides and other compounds are emitted into atmosphere. Nitrogen oxide has a dominant influence on environment over other pollutants produced by fuel gas combustion. Nitric oxide not only adversely affect the human body and pollutes environment, it also reduces visibility and slows down photochemical reactions in atmosphere [10].

In this connection, the controlling authorities of the Ministry of Environment of Ukraine have tightened control over the limit emission volumes including that for nitrogen oxides. Also, an environmental tax on harmful emissions was introduced [11].

Thus, a problem arises when choosing such optimal mode of operation of the GPU which would take into account not only cost of the fuel gas but also penalty payments for emission of harmful substances into atmosphere subject to an environmental tax.

Let us solve the following problem. Determine the number of revolutions $n_i,\ i=\overline{1,m}$ of rotor of a centrifugal natural gas supercharger such that the total cost of its operation taking into consideration the environmental tax, would be minimal

$$R(\overline{n})=\sum_{i=1}^{m}\left(cG_i(\overline{n})+c_f E_{p,i}(\overline{n})\right), \tag{1}$$

where $c_f$ is the environmental tax value; $E_{p,i}(n_i)$ is the nitric oxide emission power.

In accordance with technological regulations for natural gas pumping, gas temperature $T_{out}^{(i)}$ at the outlet of the $i$-th supercharger and temperature $T_{tur}^{(i)}$ of combustion products at the outlet of the $i$-th low-pressure turbine are limited. To prevent phenomena of surge and overload of the gas turbine drive, the lower $n_{i,min}$ and upper $n_{i,\max}$ speeds of the rotor of the centrifugal natural gas supercharger are limited [6]

$$T_{out}^{(i)}\le T_{out,\max}^{(i)}, \tag{2}$$

$$T_{tur}^{(i)}\le T_{tur,\max}^{(i)}, \tag{3}$$

$$n_{i,\min}\le n_i\le n_{i,\max}, \tag{4}$$

where $T_{out,\max}^{(i)}$ are the maximum permissible values of temperatures $T_{out}^{(i)}$ and $T_{tur}^{(i)}$.

Each group of parallel-operated superchargers must provide gas pumping in specified volumes $Q_0$

$$\sum_{i=1}^{m}k_i q_i(n_i)=Q_0, \tag{5}$$

where $k_i$ and $q_i$ are the utilization factor and efficiency, respectively, of the $i$-th natural gas supercharger.

The utilization factor $k_i$ is a characteristic of the technical condition of the $i$-th natural gas supercharger.

To determine value of $k_i$, a universal nonlinear approximator based on a fuzzy inference model is synthesized in [12] using a hierarchical knowledge base containing data on technological parameters that characterize technical condition of a centrifugal supercharger. The process of implementation of such an approximator requires construction of an expert knowledge matrix based on the information obtained from the experts' opinion survey.

Involvement of experts in the process of synthesis of the generalized approximator introduces some subjectivity in assessment of technical condition of the supercharger and complicates automatic determination of a generalized indicator of technical condition during the supercharger operation.

In order to exclude subjectivism from the process of assessing technical condition of the supercharger, an integral criterion of technical condition of the supercharger is proposed in [8]. It is based on the theory of pattern recognition.

In the process of GPA operation, degradation of its assemblies occurs. This leads to a change in its technical condition which manifests itself by the change in technological indicators characterizing operation of the GPA in its operating conditions. If operation of the GPA is monitored for a certain period of time while registering characteristics of technical condition of its assemblies, many images (signs) can be built in the attribute space. Each such image is represented by a vector whose components are indices of technical condition of individual assemblies. The task is to divide the resulting space of conditions into separate regions each of which corresponds to a specific technical condition of the selected GPA assembly.

Suppose that the space of attributes of the $j$-th assembly of the $i$-th GPU is divided into separate regions and a specific rating $r_i^{(j)}$ is assigned to each of them.

$$R_i=\sum_{j=1}^{N_q} r_i^{(j)}. \tag{6}$$

Calculate factor of technical condition of the $i$-th GPU as a ratio of the $i$-th GPU rating to the total rating of a GPU in a group

$$k_i=\frac{R_i}{\sum_{j=1}^{m}R_j}. \tag{7}$$

According to the method of determining value of $k_i$ for a GPU group with $m$ units, there is an obvious relation

$$\sum_{i=1}^{m}k_i=1.$$

To provide a specified balance of natural gas pumping, present formula (5) in the following form:

$$m\sum_{i=1}^{m}k_i q_i(n_i)=Q_0$$

or

$$\sum_{i=1}^{m}k_i q_i(n_i)=q_0, \tag{8}$$

where $q_0=Q_0/m$ is average efficiency of a centrifugal supercharger determined by the task of gas pumping.

## 5. Identification of technical conditions of centrifugal natural gas superchargers using artificial neural networks

The process of dividing the set of attributes of technical condition of a particular GPU assembly into separate subsets (classes) is carried out on the basis of methods of the pattern recognition theory.

Suppose that the space of attributes $\Omega$ and technical conditions of an object is a set of images $\omega_1, \omega_2, \ldots, \omega_n$. Attributing the image $\omega_j$ to a particular class $\Omega_k,\ k=\overline{1,N}$ (where $N$ is the number of classes of division) occurs by means of the indicator function $g(\omega)$ (of the recognition rule) [6] which is unknown to the observer. The problem consists in obtaining (by the results of observations) an estimate of the indicator function $\hat{g}(\omega)$, with the help of which the next image will be assigned to a particular class. In the general case, $g(\omega)\ne\hat{g}(\omega)$. In the case when the space of attributes $\Omega$ is characterized by a probabilistic measure, effectiveness of the dividing rule can be determined as follows:

$$\min: P\left\{\hat{g}(\omega)\ne g(\omega)\right\}.$$

Solution of the problem of recognizing technical condition of the GPU elements is based on availability of precedents that can be obtained by observing the object over time.

Depending on the precedent information, the pattern recognition methods are divided into two classes: training with a teacher and training without teacher. In the first case, correct classification of images divided into classes is known and it is necessary to attribute the new image to a particular class according to the results of observations. If proper classification of images is unknown, then a problem of dividing the set of attributes into classes by similarity of corresponding attributes arises. This problem is called clustering [13].

The methods used to solve image recognition problems can be divided into two groups.

The first group is based on statistical processing of observation results. This group includes the Bayesian method and the method of minimal average risk [14]. The Bayesian method requires a priori information about the law of distribution of attributes for each class and the method of minimum average risk gives unsatisfactory results with a small number of classes of division. For example, if the number of classes is two, then the probability of correct division is 0.5. As the number of classes increases, likelihood of dividing attributes into classes increases.

The clustering problem can be considered as a partial case of the classification problem when the number of classes is unknown a priori. Division of a set of attributes into classes is made by comparing the attributes according to a certain criterion with the attributes of different classes and selecting the closest one.

The second group of methods implements a neural network approach for dividing multiple attributes into classes. This approach is based on the Hebb associative rule and the mechanism of competition based on the generalized Kohonen rule [15].

For an example, consider division of the plane of attributes of technical condition of a setting in the centrifugal natural gas supercharger of the GPU using the Kohonen network. The degree of pressure increase $\varepsilon$ and the polytropic factor of efficiency $\eta$ were selected as attributes of this setting [16].

Technical condition of the centrifugal supercharger will be characterized by three states: "good", "acceptable" set and "permissible" [17].

Fig. 1 shows a set of images represented by vectors $\bar\omega(\eta,\ \varepsilon)$ divided into three classes using the Kohonen network. Fig. 1, "1" indicates the first class or "good", Fig. 1, "2" indicates the second class or "acceptable" and the Fig. 1, "3" indicate the third class or "permissible".

The experimental data were obtained as a result of observation of operation of the GPU of GT-750-6 type at the Lubenska compression station No. 7 of the Kyivtransgaz main gas line (Ukraine). The observation was conducted from 10.05.2018 to 24.09.2018. Polytropic efficiency $\eta$ and the degree of pressure increase are components of the $\bar x$ vector. They were converted into dimensionless units as follows:

$$x_i = \frac{X_i - X_{i,\max}}{X_{i,\min} - X_{i,\max}}, \quad i = 1,2,$$

where $X_1 = \eta$; $X_2 = \varepsilon$.

Each class of attributes of technical condition of the GPU is characterized by its centers of clustering which are marked by numbers "1", "2" and "3" (Fig. 1).

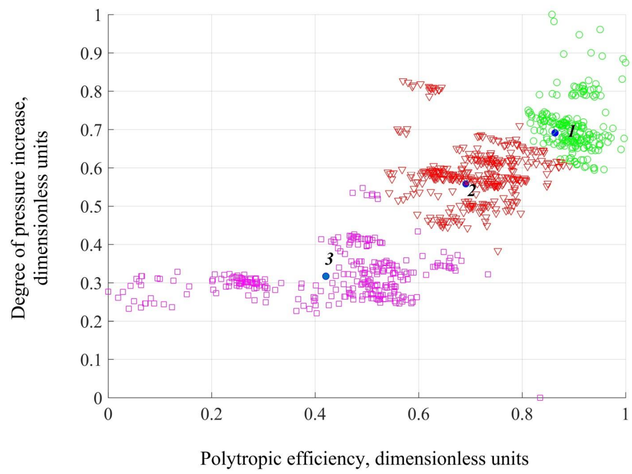


Fig. 1. Division of attribute plane into classes

## 6. Construction of boundary curves between classes

To attribute technical condition of the centrifugal natural gas supercharger, which is characterized at the time of observation by the vector of attributes of division $\bar\omega(\eta,\ \varepsilon)$ to one of the three classes, it is necessary to plot a boundary line.

Let us consider the more general case where the plane of attributes is divided into $z$ classes. Then the boundary line between $z_{i-1}$ and $z_i$ classes will be described by the function $f(\bar a, \bar x)$, where $\bar a$ is the vector of parameters of the boundary line; $\bar x$ is the vector of parameters of dimension control, $m_t$. Assume that the $z_{i-1}$ class contains $N_O$ attributes (images) and the $z_i$ class contains $N_W$ attributes. After selection of structure of the $f(\bar a, \bar x)$, function, it is necessary to identify its parameters which are components of the $\bar a$. vector. To solve the problem, components of the parameter vector $\bar a$ are proposed in [18] to be determined by minimizing the functional

$$F(\bar a) = \frac{1}{N_O}\sum_{i=1}^{N_O}\left(f\left(\bar x^{oi}, \bar a\right) - 1\right)^2 + \frac{1}{N_W}\sum_{j=1}^{N_W}\left(f\left(\bar x^{wj}, \bar a\right) + 1\right)^2, \quad (9)$$

where

$$f\left(\bar x^{oi}, \bar a\right) \approx 1,$$

if $\bar x^{oi}$ belongs to the $z_{i-1}$ class,

$$f\left(\bar x^{oi}, \bar a\right) \approx -1,$$

if it belongs to the $z_i$ class.

Select structure of the $f(\bar a, \bar x)$ function from the class of linear functions with respect to its parameters

$$\bar a = \left(a_0, a_1, \ldots, a_{m-1}\right)^{\mathrm{T}}.$$

Thus,

$$f(\bar a, \bar x) = \sum_{p=0}^{m-1} a_p \varphi_p(\bar x). \quad (10)$$

Present function (10) as a scalar product of two vectors: a vector-function $\bar\varphi(\bar x)$ and a vector of parameters $\bar a$

$$f(\bar a, \bar x) = \bar a^T \bar\varphi(\bar x). \quad (11)$$

Taking into account (11), the criterion for approximation (9) will be:

$$F(\bar{a})=\frac{1}{N_o}\sum_{i=1}^{N_o}\left(\bar{a}^T\bar{\varphi}\left(\bar{x}^{oi}\right)-1\right)^2+\frac{1}{N_w}\sum_{j=1}^{N_w}\left(\bar{a}^T\bar{\varphi}\left(\bar{x}^{wj}\right)+1\right)^2. \quad (12)$$

Minimization of the approximation criterion (12) by the vector-parameter $\bar{a}$ will give the following result:

$$\frac{1}{N_o}\sum_{i=1}^{N_o}\bar{a}^T\bar{\varphi}\left(\bar{x}^{oi}\right)\bar{\varphi}\left(\bar{x}^{oi}\right)+\frac{1}{N_w}\sum_{j=1}^{N_w}\bar{a}^T\bar{\varphi}\left(\bar{x}^{wj}\right)\bar{\varphi}\left(\bar{x}^{wj}\right)=\bar{b}, \quad (13)$$

where

$$\bar{b}=\frac{1}{N_o}\sum_{i=1}^{N_o}\bar{\varphi}\left(\bar{x}^{oi}\right)-\frac{1}{N_w}\sum_{j=1}^{N_w}\bar{\varphi}\left(\bar{x}^{wj}\right). \quad (14)$$

After multiplying the expressions below the sum signs in (13), the following is obtained

$$\bar{a}^T\bar{\varphi}\left(\bar{x}^{oi}\right)\bar{\varphi}\left(\bar{x}^{oi}\right)=\mathrm{A}\left(\bar{x}^{oi}\right)\bar{a}, \quad (15)$$

where

$$\mathrm{A}\left(\bar{x}^{oi}\right)=\begin{bmatrix}\alpha_{00}\left(\bar{x}^{oi}\right) & \alpha_{01}\left(\bar{x}^{oi}\right) & \cdots & \alpha_{0,m-1}\left(\bar{x}^{oi}\right)\\ \alpha_{10}\left(\bar{x}^{oi}\right) & \alpha_{11}\left(\bar{x}^{oi}\right) & & \alpha_{1,m-1}\left(\bar{x}^{oi}\right)\\ \cdots & \cdots & \cdots & \cdots\\ \alpha_{m-1,0}\left(\bar{x}^{oi}\right) & \alpha_{m-1,1}\left(\bar{x}^{oi}\right) & \cdots & \alpha_{m-1,m-1}\left(\bar{x}^{oi}\right)\end{bmatrix}$$

is a matrix with dimensions $m\times m$;

$$\alpha_{kl}\left(\bar{x}^{oi}\right)=\varphi_k\left(\bar{x}^{oi}\right)\varphi_l\left(\bar{x}^{oi}\right), \quad k,l=\overline{0,m-1}.$$

By analogy with (15), the following is obtained:

$$\bar{a}^T\bar{\varphi}\left(\bar{x}^{wj}\right)\bar{\varphi}\left(\bar{x}^{wj}\right)=\mathrm{A}\left(\bar{x}^{wj}\right)\bar{a}, \quad (16)$$

where

$$\mathrm{A}\left(\bar{x}^{wj}\right)=\begin{bmatrix}\alpha_{00}\left(\bar{x}^{wj}\right) & \alpha_{01}\left(\bar{x}^{wj}\right) & \cdots & \alpha_{0,m-1}\left(\bar{x}^{wj}\right)\\ \alpha_{10}\left(\bar{x}^{wj}\right) & \alpha_{11}\left(\bar{x}^{wj}\right) & \cdots & \alpha_{1,m-1}\left(\bar{x}^{wj}\right)\\ \cdots & \cdots & \cdots & \cdots\\ \alpha_{m-1,0}\left(\bar{x}^{wj}\right) & \alpha_{m-1,1}\left(\bar{x}^{wj}\right) & \cdots & \alpha_{m-1,m-1}\left(\bar{x}^{wj}\right)\end{bmatrix}$$

is a matrix with dimensions $m\times m$;

$$\alpha_{kl}\left(\bar{x}^{wi}\right)=\varphi_k\left(\bar{x}^{wi}\right)\varphi_l\left(\bar{x}^{wi}\right), \quad k,l=\overline{0,m-1}.$$

Taking into account the obtained relations (15) and (16), formula (13) will take the following form:

$$\mathrm{A}\bar{a}=\bar{b}, \quad (17)$$

where

$$\mathrm{A}=\frac{1}{N_o}\sum_{i=1}^{N_o}\mathrm{A}\left(\bar{x}^{oi}\right)+\frac{1}{N_w}\sum_{j=1}^{N_w}\mathrm{A}\left(\bar{x}^{wj}\right). \quad (18)$$

Take the equation of the boundary line as a polynomial

$$f(\bar{a},\bar{x})=\sum_{p=0}^{M-1}a_p\prod_{\nu=1}^{m_t}x_\nu^{s_{\nu p}}, \quad (19)$$

where $s_{\nu p}$ is powers at the arguments that must satisfy the constraint $\sum_{\nu=1}^{m_t}s_{\nu p}\le r$. Then equation of boundary line (19) will be a polynomial of order $r$.

The number of terms of polynomial (16) is determined from the following formula [12]:

$$M=\frac{(m_t+r)!}{m_t!\,r!}. \quad (20)$$

Comparison of expressions (10) and (19) allows us to conclude that

$$\varphi_p(\bar{x})=\prod_{\nu=1}^{m_t}x_\nu^{s_{\nu p}}. \quad (21)$$

Selection of a function in a form of formula (21) leads to the following result when calculating values:

$$\alpha_{kl}\left(\bar{x}^{oi}\right)=\prod_{\nu=1}^{m_t}x_\nu^{(oi)(s_{\nu k}+s_{\nu l})};$$

$$\alpha_{kl}\left(\bar{x}^{wj}\right)=\prod_{\nu=1}^{m_t}x_\nu^{(wj)(s_{\nu k}+s_{\nu l})}, \quad k,l=\overline{0,m-1}, \quad (22)$$

and the $\bar{b}$ vector components will be:

$$b_k=\frac{1}{N_o}\sum_{i=1}^{N_o}\prod_{\nu=1}^{m_t}x_\nu^{(oi)s_{\nu k}}-\frac{1}{N_w}\sum_{j=1}^{N_w}\prod_{\nu=1}^{m_t}x_\nu^{(wj)s_{\nu k}}, \quad k=\overline{0,m-1}. \quad (23)$$

To construct boundary lines (19) between the first and second and between the second and third classes (Fig. 1), choose the second-order polynomial ($r$=2). Since the number of attributes of technical condition of the centrifugal natural gas supercharger is two, the number of coefficients to be determined is calculated by the formula (20). Hence, $m$=6.

Equation of the boundary line for $r$=2 and $m_t$=2 will take the form:

$$a_0+a_1x_1+a_2x_2+a_3x_1^2+a_4x_1x_2+a_5x_2^2=0.$$

Parameters of the boundary line $a_i$, $i=\overline{0,5}$ are calculated as solution of the matrix equation (17) where elements of the matrices $\mathrm{A}\left(\bar{x}^{oi}\right)$ and $\mathrm{A}\left(\bar{x}^{wj}\right)$ were calculated from formulas (22) and the components of vector $\bar{b}$ from formula (23).

Software for the problem of calculating parameters of boundary curves was developed in the MATLAB environment and gave the following results:

– between the first and second classes:

$a_0$=–2.9189, $a_1$=–2.2208, $a_2$=8.4892,

$a_3$=3.9045, $a_4$=–0.5160, $a_5$=–4.0198;

– between the second and third classes

$a_0$=–2.2962, $a_1$=–19.6479, $a_2$=5.4134,

$a_3$=12.6974, $a_4$=9.1891, $a_5$=–7.5004.

Boundary lines between the first and second and between the second and third classes were plotted on the plane of the attributes of technical condition of the centrifugal natural gas supercharger (Fig. 2).

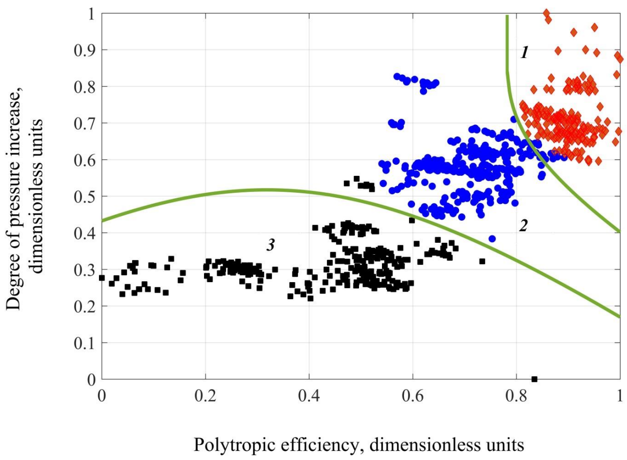


Fig. 2. The plane of technical conditions of the centrifugal natural gas supercharger with the boundary lines applied

Analysis of the obtained results (Fig. 2) shows that some of the attributes from the second class went into the first class, and a part of the attributes from the third class went to the second class.

It follows from Fig. 2 that 5 out of 572 objects from the third class were found in the second class which is 0.87 %; 7 of the 686 objects from the second class were assigned to the first class accounting for 1.02 %.

Efficiency of dividing attributes into classes for data organized into separate clusters is always higher than with blurred data [20]. Given that the data presented in Fig. 2 were blurred, the results of erroneous assigning the attributes to another class were small and did not exceed 1 %.

Let us give each of the three classes a certain number of points (rating). The first class: "good" or 60 points; the second class: "acceptable" or 40 points; the third class: "permissible" or 20 points.

If, for example, condition of a centrifugal natural gas supercharger is characterized by values (in dimensionless units) $x_1$=0.91 and $x_2$=0.7, then this condition will be "permissible" and have a rating of 60 points.

## 7. Optimal control of the GPU operation under conditions of uncertainty

Optimal control of the GPU operation involves determination of a series of revolutions of the centrifugal supercharger rotor so that the optimality criterion (1) becomes minimal when limitations (2) to (4) and (8) are satisfied.

The optimality criterion (1) includes quantities of gas flow rate $G$ and nitrogen oxide emission power $E_p$.

Fuel gas consumption is a function of such operation mode parameters [6] as temperature $T_{in}$ and pressure $P_{in}$ at the outlet of the supercharger, a degree of pressure increase ε, pressure $P_c$ and ambient temperature $T_c$

$$G_i = f_G^{(i)}\left(n_i, P_{in}, T_{in}, \varepsilon, P_c, T_c\right). \tag{24}$$

The authors of [21] have found that power of nitrogen oxide emissions depends on both technological and design parameters of the combustion chamber. During operation of the combustion chamber, its design parameters practically do not change. Therefore, the nitric oxide emission power is a function of such mode parameters as pressure $P_{ac,i}$ and temperature $T_{ac,i}$ at the outlet of the axial compressor, temperature $T_{ht,i}$ at the outlet of the high-pressure turbine and the fuel gas flow rate $G_i$

$$E_{p,i} = f_{E,i}\left(P_{ac,i}, T_{ac,i}, T_{ht,i}, G_i\right). \tag{25}$$

It was shown in [6] that conditions (2) and (3) correct constraint (4). To this end, the following dependences are determined:

$$T_{out}^{(i)} = f_{out}^{(i)}\left(n_i, P_{in}, T_{in}, \varepsilon, P_c, T_c\right), \tag{26}$$

$$T_{tur}^{(i)} = f_{tur}^{(i)}\left(n_i, P_{in}, \varepsilon, P_c, T_c\right). \tag{27}$$

Functional dependence of the fuel gas flow rate on the mode factors will take the form:

$$q_i = f_i^{(i)}\left(n_i, P_{in}, T_{in}, \varepsilon, P_c, T_c\right). \tag{28}$$

The degree of pressure growth is determined from the following formula [22]:

$$\varepsilon = \frac{P_{out}}{P_{in}},$$

where $P_{in}$, $P_{out}$ are absolute pressures.

Since the centrifugal superchargers operate in parallel for the common manifold, pressure $P_{in}$ and temperature $T_{in}$ at the supercharger inlet, pressure $P_{out}$ at the supercharger outlet, pressure $P_c$ and ambient temperature $T_c$ are assumed to be the same.

The dependences (24) and (25) to (28) will be approximated by a polynomial of order $r$ which in its structure coincides with polynomial (19).

To select optimal structure of the polynomial, a genetic approach is applied [3]. Suppose that the polynomial order (19) is selected. A part of coefficients of the empirical model (19) may have zero values and the other part is non-zero. An ordered sequence of units and zeros of length $M$ (20) is formed. If the $i$-th coefficient of polynomial (19) is non-zero, then a unit will be at the $i$-th position of the ordered sequence and if the $j$-th coefficient of polynomial (19) is zero, then zero will be at the $j$-th position in the ordered sequence. Such an ordered sequence is called a chromosome in the theory of genetic algorithms. A set of chromosomes forms a population [23]. The most "adapted" chromosome should be selected from the formed population by the method of natural selection. Such selection is carried out by means of an adaptation function using the operators of crossing and mutation [6]. To select the best chromosome, a set of all possible experimental values used to construct the empirical models (24) and (26) to (28) is divided into two parts: training and checking. The training part of the total set is used to calculate the model parameters. The model structure is determined by the chromosome selected from the formed population using a certain algorithm. The checking part of the total set serves to select the best chromosome from the population by means of an adaptation function which is a criterion for regularity or shift [24]. The chromosome selected by one of two criteria determines structure of the empirical model of optimal complexity.

Once the empirical model structure is selected, its parameters are recalculated over the entire set of experimental data.

Analysis of dependence (25) shows that power of nitrogen oxide emissions, besides other factors, depends on the fuel gas flow rate. On the other hand, as dependence (24) shows, fuel gas flow rate is a function (besides other factors) of the rotor speed. Therefore, write relation (25) in the following form:

$$E_{p,i} = f_{E,i}\left(n_i, P_{ac,i}, T_{ac,i}, T_{ht,i}\right),\quad i = \overline{1,m}. \tag{29}$$

Empirical dependence (29) shall be searched for in the form of polynomial (19) of order $r$. To determine optimal structure of the empirical model (19), the method [25] based on the idea of genetic algorithms should be used. Each factor of the model (19) shall be set in correspondence to "1" or "0". These measures result in an ordered sequence which is called a chromosome in the theory of genetic algorithms. Such a chromosome of length $M$ (20) determines structure of the empirical model. In order to select optimal structure of the model (19), a set of chromosomes is randomly generated. This set forms a population. A new population is formed from the initial population using better "adapted" chromosomes by the use of cross-breeding and mutation operators that mimic natural selection. Chromosomes for a new population are selected by one of the criteria: regularity or shift [26]. To select the best chromosome, the set of experimental data is divided into two parts: training and checking in a certain proportion [24]. The training part is used to determine parameters of model (19) whose structure depends on the next chromosome. Adaptation criterion is calculated in the checking part using the model obtained. The process of forming new populations from the best chromosomes is repeated cyclically until the minimum value of one of the adaptation criteria is reached. The chromosome that minimizes the function of adaptation determines structure of empirical model (19) of optimal complexity.

The experimental data obtained at the Lyubny compressor station of the Kyivtransgaz main gas line by observing operation of the GPU of GT-750-6 type were approximated by polynomial (19) of order $r$. Arguments of the functional dependence (19) are given in dimensionless form by the following formula:

$$x_i = \frac{X_i - X_{i,\max}}{X_{i,\max} - X_{i,\min}},\quad i = 1\ldots4, \tag{30}$$

where $X_1 = P_{ac}$; $X_2 = T_{ac}$; $X_3 = T_{ht}$; $X_4 = n$.

The following parameters of the algorithm of synthesis of empirical models of optimal complexity were selected:

– the highest polynomial order: $r=3$;

– the number of chromosomes in the population: $t_s=20$;

– the number of observation points: $N_{po}=12$;

– the number of chromosomes in a subgroup (selection method: tournament [23]): $N_g=4$;

– the probability of crossing: $P_c=0.9$;

– the probability of mutation: $P_m=0.1$;

– the training set of experimental data: $N_A=[0.7N_{po}]$;

– the checking set of experimental data: $N_B=N_{po}-[0.7N_{po}]$;

– the selection criterion: regularity criterion.

The following structure of dependence (28) was obtained and chosen as a polynomial (19):

$$\begin{aligned} E_{p,i} &= a_1 + a_2x_1 + a_3x_2 + a_4x_4 + a_5x_2^2 + \\ &+ a_6x_1x_3 + a_7x_2x_3 + a_8x_1x_4 + a_9x_2x_4 + a_{10}x_3x_4 + \\ &+ a_{11}x_1^3 + a_{12}x_1x_2^2 + a_{13}x_2^3 + a_{14}x_1^2x_3 + \\ &+ a_{15}x_2^2x_3 + a_{16}x_2x_3x_4 + a_{17}x_3^2x_4 + a_{18}x_1x_4^2, \end{aligned} \tag{31}$$

where

$a_1=1.2583$; $a_2=2.7995$; $a_3=7.8450$;

$a_4=-13.9777$; $a_5=34.7126$; $a_6=-26.1026$;

$a_7=-76.1143$; $a_8=65.5680$; $a_9=-16.9500$;

$a_{10}=34.4158$; $a_{11}=-84.8591$; $a_{12}=168.0457$;

$a_{13}=-25.1591$; $a_{14}=27.4411$; $a_{15}=-13.8948$;

$a_{16}=-169.4052$; $a_{17}=99.1393$; $a_{18}=-18.5229$.

The result of work of software of the problem of synthesis of models of optimal complexity is reflected in Fig. 3, where "+" denotes experimental data and "o" denotes the data obtained by calculation using formula (31).

Analysis of Fig. 3 shows that there is a fairly high convergence of experimental data and data obtained by calculation with the help of empirical model (31). The coefficient of correlation between these data is close to one.

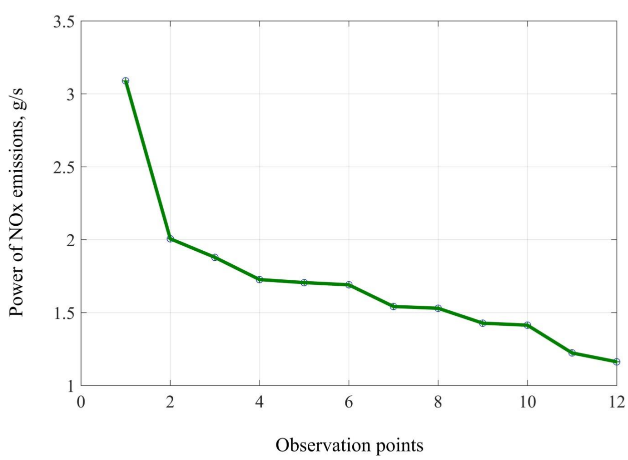


Fig. 3. Dependence of nitric oxide emission power on technological factors

The empirical models (24) and (26) to (28) were constructed using the same algorithm as model (29). As an example, Fig. 4 shows a graph of dependence of temperature at the outlet of the natural gas supercharger on the technological factors that are arguments of function (26).

Adequacy of the empirical model obtained was verified by constructing confidence intervals (Fig. 5) and calculating the coefficient of correlation [27] between the experimental data and the data obtained in calculations using formula (19). The calculated value of the correlation coefficient $K_y=0.9896$. Analysis of Fig. 5 and the value of the correlation coefficient close to one indicate adequacy of the empirical model obtained.

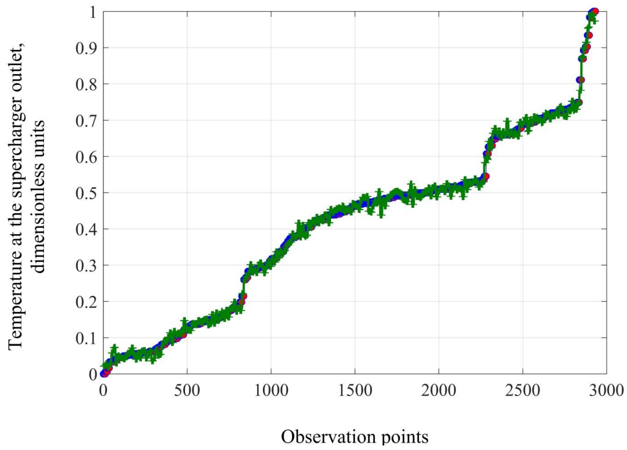


Fig. 4. Dependence of gas temperature at the outlet of the supercharger on technological factors

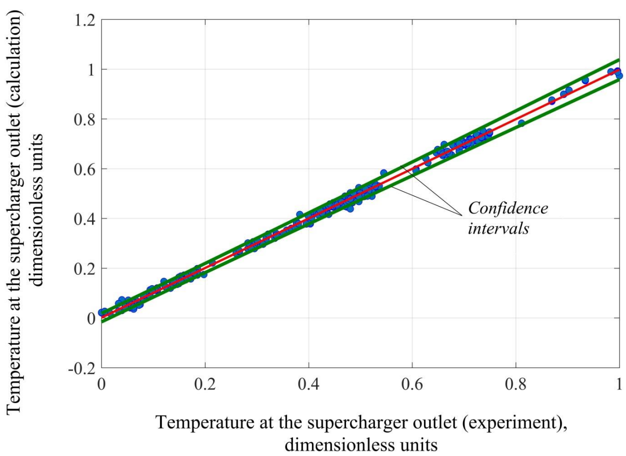


Fig. 5. Confidence intervals for functional dependence (26)

The formula of conversion of dimensionless units into physical units of temperature is simply derived from expression (30)

$$T_{out} = \Delta_{out} x_{out} + T_{out,\min}, \tag{32}$$

where $\Delta_{out}=T_{out,\max}-T_{out,\min}$; $x_{out}$ is the value of temperature at the supercharger outlet in dimensionless units; $T_{out,\max}$, $T_{out,\min}$ are minimum and maximum values of temperature at the supercharger outlet in the generated array of experimental data.

Constraints (2) and (3) must be taken into account to solve the problem of optimal control of operation of centrifugal natural gas superchargers. Constraints (2), (3) are taken into consideration by the procedure from [28]. For a certain technological mode, the values that are arguments of functional dependences (26), (27) will acquire concrete values. Their conversion to the dimensionless form by formula (30) and substitution of the obtained values in the corresponding empirical models of optimal complexity give the following polynomial dependences:

$$y_j = \sum_{k\in\Re_j} a_k^{(j)} x_{1,j}^{s_{kz}}, \quad j = 1, 2, \tag{33}$$

where $y_1$ and $y_2$ are the natural gas temperature at the supercharger outlet and the combustion product temperature at the outlet of the low pressure turbine; $a_k^{(j)}$ is non-zero coefficients of polynomial (33); $\Re_j$ is the set of nonzero coefficients of dependence (33); $s_{kz}$ is elements of the matrix of polynomial exponents (19); $z$ is the number of column of the matrix of polynomial exponents with which the $n_i$ value is associated.

The matrix of polynomial exponents (19) takes the following form:

$$S = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1m_t} \\ s_{21} & s_{22} & \cdots & s_{2m_t} \\ \cdots & \cdots & \cdots & \cdots \\ s_{M1} & s_{M2} & \cdots & s_{Mm_t} \end{bmatrix}.$$

Obviously, the sum of elements of each row of the matrix, $S$, satisfies the following condition:

$$\sum_{v=1}^{m_t} s_{kv} \le r, \quad v = \overline{1, m_t}.$$

Since the value of $y_1$ is dimensionless, transition to the dimensional $T_{out}$ is carried out by formula (32). The same conversion of the dimensionless value of $y_2$ is used. As a result, dimensional value of $T_{tur}$ is obtained.

Write formula (32) in the general form

$$T_j = \Delta_j y_j + T_{j,\min}, \tag{34}$$

where

$$T_1 = T_{out}; \quad T_2 = T_{tur};$$

$$\Delta_j = T_{j,\max} - T_{j,\min}; \quad j = 1, 2.$$

Substitute the value of $y_i$ from (33) in (34). As a result, the following is obtained:

$$T_j = \Delta_j \sum_{k\in\Re_i} a_k^{(j)} x_{1,j}^{s_{kz}} + T_{j,\min}. \tag{35}$$

Let us denote maximum values of temperatures defined by relations (2) and (3) as $\tilde{T}_{j,\max}$, where

$$\tilde{T}_{1,\max} = \tilde{T}_{out,\max};$$

$$\tilde{T}_{2,\max} = \tilde{T}_{tur,\max}.$$

Note that in the general case, $\tilde{T}_{j,\max}$ will differ from the values of $T_{out,\max}$ and $T_{tur,\max}$ which are fixed as the corresponding maximum values in the temperature arrays.

When substituting the value $\tilde{T}_{j,\max}$ into formula (35), the following equation is obtained:

$$\Delta_j \sum_{k\in\Re_i} a_k^{(j)} x_{1,j}^{s_{kz}} + T_{j,\min} - \tilde{T}_{j,\max} = 0. \tag{36}$$

Here is an example of application of the developed method of taking into account constraint (2). According to the results of observations of the supercharger operation, an empirical model of dependence (26) in a form of a polynomial of third order is obtained on the basis of genetic algorithms (Fig. 4):

$$y_1 = a_1 + a_2x_1 + a_3x_3 + a_4x_1^2 + a_5x_2x_3 + \\ +a_6x_3^2 + a_7x_1x_4 + a_8x_3x_4 + a_9x_1x_5 + \\ +a_{10}x_3x_5 + a_{11}x_1x_6 + a_{12}x_2x_6 + a_{13}x_4x_6 + \\ +a_{14}x_2^3 + a_{15}x_1^2x_3 + a_{16}x_1x_2x_3 + a_{17}x_2^2x_3 + \\ a_{18}x_3^3 + a_{19}x_1^2x_4 + a_{20}x_2^2x_4 + a_{22}x_1x_3x_4 + \\ +a_{23}x_1x_3x_5 + a_{24}x_3^2x_5 + a_{25}x_2x_4x_5 + \\ +a_{26}x_4^2x_5 + a_{27}x_4x_5^2 + a_{28}x_2^2x_6 + a_{29}x_3^2x_6 + \\ +a_{30}x_1x_5 + a_{31}x_3x_4x_6 + a_{32}x_2x_4x_6 + a_{33}x_5^2x_6 + \\ +a_{34}x_1x_6^2 + a_{35}x_3x_6^2 + a_{36}x_6^3. \quad (37)$$

Note that according to formula (20), a complete polynomial of six variables and of third order would have 84 terms. Application of the developed method of constructing empirical models of optimal complexity has made it possible to reduce number of terms of empirical model (19) from 84 to 36.

For a given mode of operation of the centrifugal supercharger, the technological parameters are converted to dimensionless units. These are parameters such as pressure $P_{in}$ and gas temperature $T_{in}$ at the supercharger inlet, the degree of natural gas pressure increase ε, ambient pressure $P_c$ and temperature $T_c$. Substitution of values of dimensionless parameters $x_2$, $x_3$, $x_4$, $x_5$ and $x_6$ in the empirical model (37) gives the following algebraic equation with respect to variable $x_1$:

$$A_0x_1^2 + A_1x_1 + A_2 = 0, \quad (38)$$

where

$$A_0 = (a_4 + a_{15}x_3 + a_{19}x_4)x_1^2;$$

$$A_1 = \begin{pmatrix} a_2 + a_7x_4 + a_9x_5 + a_{11}x_6 + a_{16}x_2x_3 + \\ +a_{22}x_3x_4 + a_{23}x_3x_5 + a_{30}x_5 + a_{34}x_6^2 \end{pmatrix} x_1;$$

$$A_2 = a_1 + a_3x_3 + a_5x_2x_3 + a_6x_3^2 + a_8x_3x_4 + \\ +a_{10}x_3x_5 + a_{12}x_2x_6 + a_{13}x_4x_6 + a_{14}x_2^3 + \\ +a_{17}x_2^2x_3 + a_{18}x_3^3 + a_{20}x_2^2x_4 + a_{24}x_3^2x_5 + \\ +a_{25}x_2x_4x_5 + a_{26}x_4^2x_5 + a_{27}x_4x_5^2 + a_{28}x_2^2x_6 + \\ +a_{29}x_3^2x_6 + a_{31}x_3x_4x_6 + a_{32}x_2x_4x_6 + a_{33}x_5^2x_6 + \\ +a_{35}x_3x_6^2 + a_{36}x_6^3 + (T_{1,\min} - \tilde{T}_{1,\max}) / \Delta_1.$$

Solution of the square trinomial (38) and extraction of its positive root gives the variable $x_1$. The variable $x_1$ found in this way corresponds to a certain number of revolutions of the supercharger rotor which should be converted to a dimensional form by the formula similar to (32):

$$\tilde{n}_1 = \Delta_n x_1 + n_{1,\min}, \quad (39)$$

where $\Delta_n = n_{1,\max} - n_{1,\min}$.

The found value of $\tilde{n}_1$ guarantees meeting the inequality (2).

A similar procedure of $\tilde{n}_2$ determination will guarantee satisfaction of condition (3). Then the upper limit of maximum speed of the supercharger rotor will be determined from the following relation [8]:

$$\tilde{n}_{\max} = \min:(n_1, n_2, n_{\max}). \quad (40)$$

Since there are no analytical expressions for functions (24) and (28), the dependences (24) and (28) will be approximated by polynomials of (19) type whose structure is determined on the basis of genetic algorithms.

Dependences (24), (28) and (29) included in the criterion of optimality (1) and the constraint (5) will be just functions of the variable $n_i$ for the specified technological parameters $P_{in}$, $T_{in}$, ε, $P_c$, $T_c$, $P_{ac,i}$, $T_{ac,i}$ and $T_{ht,i.}$

Assume that $z=1$ in formula (33) and the corresponding variables $P_{in}$, $T_{in}$, ε, $P_c$, $T_c$, $P_{ac,i}$, $T_{ac,i}$ and $T_{ht,i}$ are converted to a dimensionless form. Fuel gas flow rate and nitrogen oxide emission power will be measured in $m^3/s$ and kg/s, respectively. Then the optimality criterion (1) and the constraint (8) will depend on the functions that are polynomials of certain orders. The following is obtained:

$$R(\overline{x}^{(1)}) = c\sum_{j=1}^{m}\left(\sum_{k\in\Re_{g,j}} a_{g,k}^{(j)} x_{1,j}^{s_{k1}} + r_c \sum_{k\in\Re_{E,j}} a_{E,k}^{(j)} x_{1,j}^{s_{k1}}\right), \quad (41)$$

$$\sum_{j=1}^{m} k_j \left(\sum_{k\in\Re_{q,j}} a_{q,k}^{(j)} x_{1,j}^{s_{k1}}\right) = q_0, \quad (42)$$

where $r_c = \frac{c_f}{c}$; $\Re_{g,j}$, $\Re_{E,j}$, $\Re_{q,j}$ is the set of nonzero coefficients at the orders of the corresponding polynomials.

In view of condition (40), the restriction (4) on the supercharger rotor speed will take the following form:

$$n_{j,\min} \le n_j \le \tilde{n}_{j,\max}, \quad j = \overline{1,m}. \quad (43)$$

Write condition (43) in dimensionless units. To do this, use formula (40) in the following form:

$$n_j = \Delta_s x_{1,j} + n_{s,\min}, \quad j = \overline{1,m}. \quad (44)$$

If the value of $n_j$ determined by formula (44) is taken into account, the following is obtained:

$$\alpha_j \le x_{1,j} \le \beta_j, \quad j = \overline{1,m}, \quad (45)$$

where

$$\alpha_j = \frac{n_{j,\min} - n_{s,\min}}{\Delta_s}; \ \beta_j = \frac{\tilde{n}_{j,\max} - n_{s,\min}}{\Delta_s}, \quad j = \overline{1,m}.$$

In a general case, the same set of observations is used to construct empirical models (24) and (28) and another set of observations can be taken to construct empirical model (25). In this case, $\Delta_s \in \{\Delta_{gq}, \Delta_E\}$ and $n_{s,\min} \in \{n_{gq,\min}, n_{E,\min}\}$.

In accordance with RD 52.04-186-89 normative document, nitric oxide concentrations are measured 4 times a day. To construct an empirical dependence (29), it is necessary to use an interpolation procedure, for example, by interpolation splines [29] for obtaining intermediate points in the observation interval, Therefore, values of the polynomial coefficients $a_{E,k}^{(j)}$ will be inaccurate but belong to some interval.

In order to measure flow rate of natural gas being transported by a group of superchargers, a confuser is installed on the inlet line to measure volumetric flow rate of natural gas according to the given pressure differential [22].

Studies carried out by the author of [30] have shown that measurement of flow rate of natural gas by means of a pressure differential in the confuser gives an error of up to 20 %.

Thus, coefficients $a_{E,k}^{(j)}$ and $a_{q,k}^{(j)}$ should be considered fuzzy numbers with a triangular membership function [6] described by the following relation:

$$\mu(\chi)=\begin{cases}-\dfrac{2}{\Delta}(\chi_0-\chi)+1, \chi\in[\chi_0-\Delta/2;\chi_0],\\ \dfrac{2}{\Delta}(\chi_0-\chi)+1, \chi\in[\chi_0;\chi_0+\Delta/2],\end{cases} \quad (46)$$

where $\chi_0$ is the model value of the membership function.

The triangular membership function $\mu(\chi)$ is an isosceles triangle with a height of 1 $(\mu(\chi_0)=1)$ and a base width of $\Delta$. Since the function $\mu(\chi)$ is piecewise continuous and there is no $\mu(\chi)$ derivative at points $\mu(\chi_0-1)$, $\mu(\chi_0)$ and $\mu(\chi_0+1)$, this function is inconvenient for practical use. Therefore, it is proposed in [6] to approximate the function $\mu(\chi)$ by a Gaussian function

$$\mu(\chi)=\exp\left(-\frac{(\chi-\chi_0)^2}{2\sigma_\mu^2}\right), \quad (47)$$

where $\sigma_\mu$ is the ratio of concentration of the fuzzy quantity $\chi$.

As shown in [25], choice of the parameter σμ provides a minimal error of approximation of the triangular membership function (46) by the Gaussian function if that membership function (47) passes through the point with coordinates $\left(\chi_0-\frac{\Delta}{4};\frac{1}{2}\right)$.

Then

$$\sigma_\mu=k_\sigma\Delta, \quad (48)$$

where $k_\sigma=(32\ln 2)^{-1/2}$.

Thus, it is supposed that the fuzzy quantities $a_{E,k}^{(j)}$ and $a_{q,k}^{(j)}$ have a membership function similar to (47) where the concentration ratio is calculated by formula (48).

Proceeding from the fact that $a_{E,k}^{(j)}$ is a fuzzy quantity, the other addend of the optimality criterion will also be a fuzzy quantity (41). Denote it as $O_N$, that is

$$O_N=r_c\sum_{k\in\Re_{E,j}}a_{E,k}^{(j)}x_{1,j}^{s_{k1}}. \quad (49)$$

Let us find the membership function $\mu(O_N)$ of the fuzzy quantity $O_N$. In the case where there is a sum of fuzzy quantities with a Gaussian membership function and each of them is multiplied by a constant exact quantity, a fuzzy quantity whose membership function is also Gaussian [31] is obtained. Thus,

$$\mu(O_N)=\exp\left(-\frac{(O_N-m_O)^2}{2\sigma_O^2}\right). \quad (50)$$

To find parameters of the membership function, use the rules of fuzzy algebra [31] which determines arithmetic actions over fuzzy quantities of ($L$–$R$) type in a Gaussian basis. It can be shown [32] that

$$m_O=r_c\sum_{k\in\Re_{E,j}}a_{E_0,k}^{(j)}x_{1,j}^{s_{k1}}, \quad (51)$$

$$\sigma_O^2=r_c^2\sum_{k\in\Re_{E,j}}\sigma_{\mu,k}^2x_{1,j}^{2s_{k1}}, \quad (52)$$

where $a_{E_0,k}^{(j)}$ is the modal value of fuzzy quantity $a_{E,k}^{(j)}$.

Let us consider that $\sigma_{\mu,k}^2$ is determined from formula (48). Then relation (52) takes the following form:

$$\sigma_O^2=r_c^2k_\sigma^2\sum_{k\in\Re_{E,j}}\left(\Delta_{E,k}^{(j)}\right)^2x_{1,j}^{2s_{k1}}, \quad j=\overline{1,m}. \quad (53)$$

Let us take a γ-slice for the membership function (49). The following equation is obtained:

$$\exp\left(-\frac{(O_N-m_O)^2}{2\sigma_O^2}\right)=\gamma.$$

Solution of the last equation relative to $O_N$ gives

$$O_N=\sigma_O\sqrt{\ln\frac{1}{\gamma^2}}+m_O, \quad (54)$$

where $0<\gamma\le 1$.

Determine some quantity $0<\delta_{k,j}\le 1$ for each factor $a_{E,k}^{(j)}$ included in formula (49) so that $\Delta_{E,k}^{(j)}=\delta_{E,k}^{(j)}a_{E_0,k}^{(j)}$. Taking into account the last relation, the following is obtained

$$\sigma_O^2=r_c^2k_\sigma^2\sum_{k\in\Re_{E,j}}\left(\delta_{E,k}^{(j)}a_{E_0,k}^{(j)}\right)^2x_{1,j}^{2s_{k1}}. \quad (55)$$

Taking into consideration the $m_O$ and $\sigma_O$ values determined from formulas (51) and (55), the following is obtained:

$$O_N=K_E\left(\sum_{k\in\Re_{E,j}}\left(\delta_{E,k}^{(j)}a_{E_0,k}^{(j)}\right)^2x_{1,j}^{2s_{k1}}\right)^{1/2}+r_c\sum_{k\in\Re_{E,j}}a_{E_0,k}^{(j)}x_{1,j}^{s_{k1}}, \quad (56)$$

where

$$K_E=r_ck_\sigma\sqrt{\ln\frac{1}{\gamma^2}}.$$

After taking into account the $O_N$ value given by expression (56), the optimality criterion (41) becomes:

$$R\left(\bar{x}^{(1)}\right)=c\sum_{j=1}^{m}\left(\begin{array}{l}\displaystyle\sum_{k\in\Re_{g,j}}a_{g,k}^{(j)}x_{1,j}^{s_{k1}}+r_c\sum_{k\in\Re_{E,j}}a_{E_0,k}^{(j)}x_{1,j}^{s_{k1}}+\\ \displaystyle+K_E\left(\sum_{k\in\Re_{E,j}}\left(\delta_{E,k}^{(j)}a_{E_0,k}^{(j)}\right)^2x_{1,j}^{2s_{k1}}\right)^{1/2}\end{array}\right). \quad (57)$$

In contrast to (41), the optimality criterion (57) contains an additional component which is a kind of "pay" for the inability to know precisely the dependence (49).

Since the coefficients $a_{q,k}^{(j)}$, $j=\overline{1,m}$, are fuzzy quantities with the triangular membership function to be approximated by the Gaussian function (47), then, repeating the calculations made earlier, the following conclusion is obtained:

$$\sum_{j=1}^{m}k_j\left(\begin{array}{l}\displaystyle\sum_{k\in\Re_{q,j}}a_{q_0,k}^{(j)}x_{1,j}^{s_{k1}}+\\ \displaystyle+K_q\left(\sum_{k\in\Re_{q,j}}\left(\delta_{q,k}^{(j)}a_{q_0,k}^{(j)}\right)^2x_{1,j}^{2s_{k1}}\right)^{1/2}\end{array}\right)=q_0, \quad (58)$$

where

$$K_q = k_\sigma \sqrt{\ln \frac{1}{\gamma^2}}; \;\; 0 < \delta^{(j)}_{q,\;k} \le 1; \;\; a^{(j)}_{q_0,k}$$

are modal values of coefficients of the empirical model.

Now, formulate the problem of optimal control of operation of centrifugal natural gas superchargers as follows. At the compressor station, there are $m$ parallel-working gas pumping units including centrifugal superchargers with gas turbine drive. For the current technological mode, it is necessary to determine values of the rotor speed for each supercharger so that the optimality criterion (57) acquires a minimum value on condition that constraints (45) and (58) are met.

Optimality criterion (57) together with constraints (45) and (58) form a problem of nonlinear programming. Its solution can be found by one of numerical methods [33].

Solution of this problem will make it possible to determine value of the rotor speed $x_{1,j}$, $j = \overline{1, m}$ in dimensionless units. In order to turn from dimensionless values $x_{1,j}$ to values $n_j$, $j = \overline{1, m}$ in dimensional units, formula (44) should be used.

## 8. Discussion of results obtained in the study of optimal control of operation of centrifugal natural gas superchargers

There are two aspects to be learned from the study findings. Firstly, it is assessment of technical condition of the supercharger executed by dividing the set of attributes into three classes and constructing boundary curves between classes (Fig. 2). This approach, unlike the method proposed in [12], eliminates subjective opinion of experts in assessment of technical condition of the supercharger. When implementing the proposed method, there may be some difficulties in a case when the next image characterizing the current state of the supercharger falls into the region close to the boundary curve. In this case, a decision is made on the principle of a "lesser evil".

Secondly, the optimality criterion has two components. The first one describes the fuel gas flow rate to drive the turbine engine. The second component is power of nitrogen oxide emissions into atmosphere.

Formalized statement of the problem of optimal control of operation of centrifugal natural gas superchargers requires construction of empirical models. The process of constructing such models is realized using the developed method which is based on the theory of genetic algorithms. Application of the developed method has shown that empirical models have optimal complexity and are adequate to the experimental data.

It should be noted that the developed algorithms of evaluation of technical condition of superchargers and construction of empirical models based on genetic algorithms are adequate experimental data, which confirms their efficiency and convergence.

The authors will continue their study towards development of algorithmic and software support of the computer system for optimal control of operation of centrifugal natural gas superchargers.

## 9. Conclusions

1. The problem of optimal control of operation of natural gas superchargers was formalized. This problem includes the criterion of optimality, restrictions on technological parameters and the condition of fulfilling the planned tasks of gas transportation by a group of parallel-operating superchargers. A criterion for optimal control of operation of centrifugal natural gas superchargers was developed. Unlike the known ones, it takes into account both the fuel gas flow rates for driving superchargers and emissions of harmful substances into atmosphere. Availability of such a criterion in the formalized formulation of the problem of optimal control of operation of natural gas superchargers will make it possible to reduce both gas consumption and nitrogen oxide emissions into atmosphere.

2. To identify technical condition of natural gas superchargers, attributes of technical condition of individual assemblies forming a set of images have been found. With the help of an artificial neural network, a set of images is divided into three classes, each characterizing one of technical conditions of the supercharger: "good", "acceptable" and "permissible". A definite rating was assigned to each class which has made it possible to determine utilization of the parallel-operating superchargers.

3. One class was separated from another by means of boundary lines constructed with the use of a functional whose value is determined by the boundary line equation and image coordinates. Algorithmic support of the problem has been developed. It enables determination of parameters of the boundary line equation according to the chosen order of polynomial. Parameters of the selected polynomial were determined by finding solution of the matrix equation obtained by minimizing the criterion of separation of two classes.

4. The problem of optimal control of operation of centrifugal natural gas superchargers includes a criterion of optimality that takes into account both energy costs and the power of nitrogen oxide emissions into atmosphere and constraints on technological parameters. Uncertainty in operation of the superchargers manifests itself by inability of error-free determination of values of the regression equation coefficients which are interpreted as fuzzy quantities. Using the rules of fuzzy arithmetic, values of the optimality criterion and the limitation of the number of revolutions of the supercharger rotor and technological parameters, such as temperature at the supercharger outlet and temperature of the combustion products at the outlet of the low-pressure turbine, which take into account uncertainty conditions were obtained. As a result, a nonlinear programming problem was obtained. It can be solved by one of numerical methods.

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