Розглядаються проблеми перепрофілювання віртуальних виробничих підприємств, зокрема перспективи розвитку віртуального приладобудівного підприємства (ВПП). Такі підприємства функціонують за рахунок зовнішніх територіально розподілених ресурсів та мають змогу бути гнучкими у виробництві продукції відповідно до кон'юнктури ринку. Запропоновано елементи інформаційної технології (ІТ) для підтримки прийняття рішень щодо організації роботи ВПП. Елементи ІТ включають в себе моделі та структури для прийняття рішень, в тому числі для раціонального використання ресурсів при функціонуванні ВПП. Призначенням запропонованої ІТ є вирішення задачі підтримки прийняття рішень щодо перепрофілювання виробництва на віртуальних приладобудівних підприємствах, відповідно до кон'юнктури ринку. Необхідність у перепрофілюванні виробництва викликана складними умовами ринкової економіки для невеликих приладобудівних підприємств та підприємств середньої ланки. Виробнича програма формується виходячи з існуючого на даний момент попиту на різні категорії виробів, що входять у номенклатуру товарів підприємства або відповідають напрямку функціонування ВПП. Елементами ІТ є моделі для інформаційно-аналітичного порталу (ІАП), а також структурна схема розгортання та підтримки єдиного простору знань при гнучкому перепрофілюванні виробництва відповідно до кон'юнктури ринку. До складу ІАП входять також експертні системи (ЕС). Розроблена ІТ для підтримки прийняття рішень керівника ВПП дозволить виготовляти той продукт, який буде мати попит на ринку та буде рентабельним у виготовленні. ІТ забезпечить визначення кількості та видів необхідних ресурсів за заданими критеріями, їх розподіленості, враховуючи напрямок перепрофілювання виробництва

Ключові слова: ресурси, віртуальне приладобудівне підприємство, моделі прийняття рішень, кон'юнктура ринку, перепрофілювання, математичні моделі

DEVELOPMENT OF INFORMATION TECHNOLOGY ELEMENTS FOR DECISION-MAKING SUPPORT AIMED AT RE-STRUCTURING PRODUCTION AT VIRTUAL INSTRUMENT-MAKING ENTERPRISES

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1. Introduction

A virtual instrument-making enterprise is an enterprise consisting of the community of geographically distributed subcontractors that interact during the production process and operate through telecommunication means [1].

From a marketing point of view, the goal of creating a VIE is to generate profit due to the maximum satisfaction of consumers by goods and services, by combining the resources of different partners into a single system. At the same time, the efficiency of distributing resources among production links could be greatly improved by providing them with traditional marketing functions [1, 2]. Thus, VIE activities are aimed not at satisfying the needs of an "average" buyer or a market segment, but at meeting individual demands by specific consumers.

Characteristic features of VIE are:
– flexible change of products range;
– warehouse-free operation;
– absence of fixed assets;
– minimum number of employees;
– priority to horizontal connections;
– relative autonomy and narrow specialization of enterprise's participants;
– high status of informational and human resources for integration;
– implementation of the principle of systemic use of resources;
– flexible and adaptive organizational structure;
– minimal starting capital;
– operation under conditions of uncertainty in demand.

Attracting significant resources is required for the traditional enterprise to develop and release new goods. In contrast to a conventional enterprise, in this case the process of functioning involves the search for new partners, possessing appropriate market-driven needs, resources, knowledge and capabilities, for joint organization and implementation of

activities. At the same time, the enterprise's partners (organizations, individual collectives, people) that possess the key resources for achieving a competitive advantage in the market are elected. Thus, VIE has a competitive advantage, because traditional instrument-making enterprises, especially at the medium level, under such circumstances, have difficulties gaining the market taking into consideration a large tax "burden". Therefore, such a condition affects the search for new ways to organize production. And one of the new approaches is setting up a virtual production. On the other hand, there are no detailed theoretical studies into this field while there are not enough existing industries to analyze the experience. Thus, it is a relevant task to develop new technologies for the organization of virtual production.

2. Literature review and problem statement

Paper [3] reports results from studying actual problems of small and medium-sized industrial enterprises. It was shown that medium-sized industrial enterprises have certain difficulties in competing with industrial giants. One of the possible reasons is that large industrial enterprises have established a turnover of resources and finished products. The popularity is also affected by a registered trademark. In addition, large enterprises can make their products cheaper at the expense of large production volumes. An option for small and medium-sized enterprises to overcome related difficulties is to establish a clear strategy for the organization of production. This very approach was considered in work [3]. The authors proposed a toolset for decision making in the organization of workers for a production process according to their competences. The cited article also suggested a methodology, based on the advantage index method, for decision-making at the stage of design of the life cycle of an industrial system. The methodology helps in choosing a suitable alternative from a large number of available options for solving industrial problems based on decision support systems. However, the advantage index method underlying the methodology ignores any relative importance among the attributes. The advantage index method is based on statistical calculations, which requires the development of software that could reduce the time of computations.

There are also other ways to organize human resources for innovative industrial projects, which are presented in work [4].

Instrument-making enterprises have certain difficulties in determining and efficient distribution of both human resources and production resources necessary for the enterprise. Resources could include industrial equipment and raw materials. At present, development of communication technologies allows the virtual and temporary cooperation in the chains that supply resources to gain mutual benefits. The advantages of such cooperation are efficiency when exchanging resources and information. The model of such a cooperation is described in paper [5]. A variant to solve tasks related to the supply and distribution of industrial resources is to define a strategy for creating virtual enterprises. Activities of virtual instrument-making enterprises are aimed at improving their competitiveness and optimizing processes on the utilization of resources.

Article [6] analyzed production enterprises in general, and considered the logistic processes at an enterprise, as well as supply chains of resources. The analysis results re-

vealed the need to search for scientific solutions on forming a production strategy within the framework of Industry 4.0 that could enable control over a network of resource supply. A production strategy should include flexibility in the organization of an industrial enterprise.

The results from studying adaptive distributed production systems are reported in paper [7]. It was shown that the organization and implementation of virtual enterprises increases the level of flexibility and adaptability of production to dynamic market conditions. A virtual enterprise provides new possibilities when an industrial system is not known in advance. A production system must be structured and optimized for manufacturing the types of products that are demanded at a given moment. However, the unresolved issue is the automation of information flows at virtual enterprises.

Research in the field of information technology to manage virtual enterprises is reported in papers [8, 9]. The articles give results from a study mainly on the distribution of human resources of an organization. However, there are unresolved issues on decision-making on determining an expediency of making a particular type of product and defining maximum permissible production volumes.

Methods and decision-making models have previously been used in production.

The growing dynamics of uncertainty in the external environment requires the search for new management techniques and decision-making methods, which would ensure the very existence of an enterprise (organization). Unforeseen changes are a prerequisite of modern business, which should be taken into consideration in the management process.

It is possible to improve stability of an organizations by converting dynamic changes to the planned ones based on the use of effective forecasting methods and decision-making support along with studying a life cycle theory. A methodology of strategic planning in the context of uncertainty in the external environment, proposed in [10], could be applied at industrial enterprises.

Research in [11] aims to develop a decision support framework on the life-cycle stability in order to rank ways to produce a clean energy carrier (hydrogen) by combining the assessment method for life-cycle stability and the method of interval multi-criterial decisions. Owing to the decision-making support method, the authors performed a sensitivity analysis to investigate the impact of performance weights on stability estimation. The proposed multi-criteria method for decision-making can take into account uncertainties.

The subject of research is the process of planning investment projects for the development of organizations. The purpose of article [12] is to construct an algorithm and develop software for a decision support system to choose the method of attracting funds into an investment project. The article studied a model of integrated planning and implemented an investment project. Among the resolved tasks of the cited article was to develop a method for choosing the optimal variant of investment project realization and to design elements for a decision support system in order to choose an investment technique. The research results showed that the effectiveness of an investment process is related to the assessment and selection of the most attractive investment projects from a series of alternatives that would ensure maximum profit in future. The authors also defined strategic tasks for investing in the development of organizations

and enterprises and considered basic finances that could be involved in the implementation of an investment project. Application of the proposed method and a decision support system at the stage of an investment project planning would make it possible to take justified decisions on the choice of an investment method depending on the main factors of the project and the investment object.

Virtual enterprises face complex problems in the organization of production. In particular, there are problems related to the re-structuring of production, given the competitive conditions of market economy and consumers' needs. Based on the analyzed literary sources, we can conclude that there are no theoretical studies and practical implementation of virtual production. Therefore, there is a need to create information technologies to organize activities of virtual instrument-making enterprises with flexible structure and the capability to adapt to market conditions, determining and distributing the resources required for production.

3. The aim and objectives of the study

The aim of this study is to create information technology elements to support decision-making process on re-structuring VIE production, taking into consideration market conditions. The information technology should take into account the rational use of resources in the operation of VIE, which would enable the automation of decision-making procedures aimed at re-structuring of VIE and the rational distribution of its resources.

To accomplish the aim, the following tasks have been set:
– to investigate patterns in the decision making on re-structuring production and planning a program for releasing each type of products from a product range of VIE;

– to substantiate the allocation of resources for VIE production, in order to gain a competitive advantage in the market.

4. Models and structures of elements in information technology

4. 1. Decision-making on the re-structuring of production and on planning a program for a virtual instrument-making enterprise output

Under current conditions, the process of promoting goods and services to the market, which already has a large number of competitors, is expensive, long and complex for many companies. In order to promote products in modern markets, marketing departments use different methods for marketing communications in their everyday life.

Marketing communications imply constant control over the promotion of activities to consumers and customers with the aim of:

1. Inform prospective consumers about their product, services, terms of sale.

2. Convince prospective consumers to give preference to the proposed goods and services.

3. Motivate prospective consumers to ensure that they act without postponing the order for the future.

Modern tools to promote goods include the Internet environment. Its main purpose is to gain the maximum effect from the potential audience at internet resources. At the same time, trends in the development of modern information technologies (IT) result in a constant growth in their complexity.

One of the components of the designed IT is an informational-analytical portal (IAP) of a VIE, which collects and processes information about market conditions. A model of the IAP processes is constructed using the graphics language IDESF0 and is shown in Fig. 1.

The model makes it possible for a customer to place orders that are consequently transmitted for production. When the required conditions for the feasibility (profitability) of producing the i-th type of a product are met, the order is added to the portfolio of a product range.

The informational model of IAP is the data that represent significant parameters and variables, as well as connections among them, inputs and outputs. Such a notation makes it possible to simulate a model's possible states by sending the information on changes in the input quantities to the model [13, 14].

The input data for IAP includes:
– information on market conditions;
– statistics on visiting the sites of informational sputniks and the main VIE site;
– the number of concluded contracts for product supply;
– volumes and terms of supply for each type of a manufactured product;
– places and ways to deliver products, linked to a particular type of a product (Fig. 2).
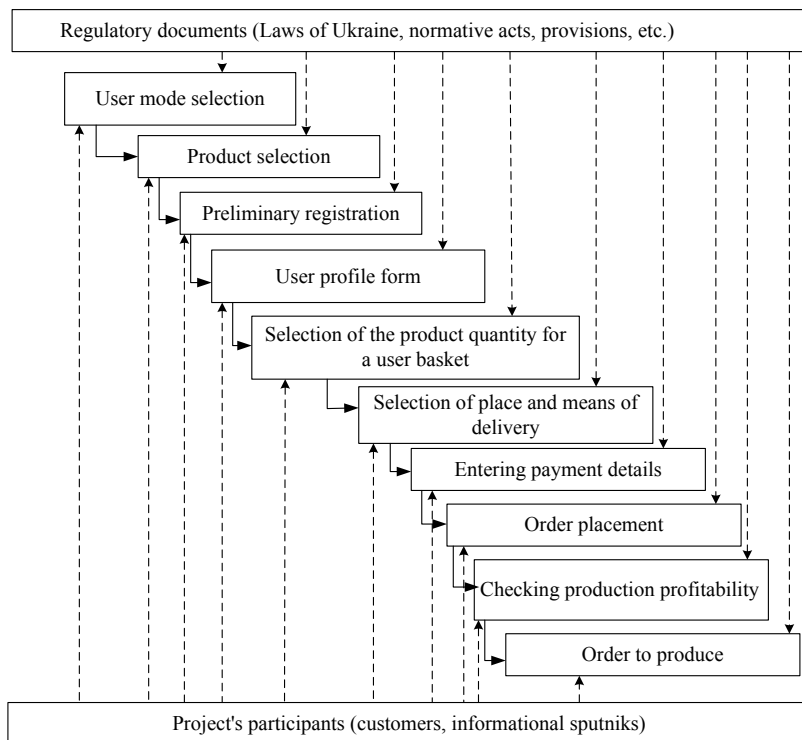

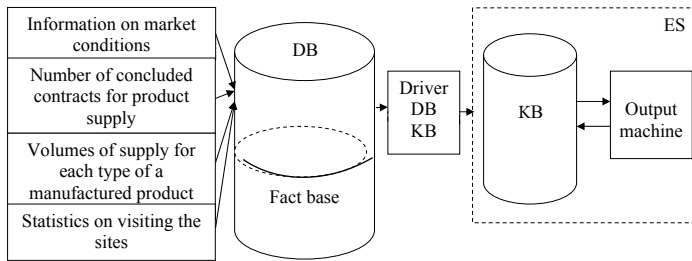Fig. 1. Model of processes at an informational-analytical portal

Fig. 2. IAP informational model

Based on the informational model, we formed a structural scheme for the construction, deployment, and support of a unified knowledge space at the flexible re-structuring of production, in accordance with market conditions (Fig. 3).

In Fig. 3, the following designations are used:

$\bar{X}_1 = \left\{ x_1^1, \ldots x_n^1 \right\}$ – a set of potential clients (Web-customers);

$\left\{ x_{1(i)}^1 + 1 \ldots x_{n(i)}^1 + n \right\}$ – a set of potential orders; n is a natural number, i – a type of product;

$N_{i\,min}$ – the minimally permissible number of orders for the i-th type of product sufficient to place the order for production;

EC – expert system;

UPP – unit for placing orders for production.

The essence of the developed IT is to determine the minimum batch of each type of a product, which could ensure the profitability of production and is justified for a VIE.

Information sputniks are also called the reflexive agents. Reflexive agents represent the single-page sites that find search engines based on the qualitative characteristics of products proposed for production. IT takes into consideration the number of queries to such sites, which is the basis for determining the significance and priority of a product's characteristics when creating an order portfolio.

IT implies successive implementation of three main stages – cumulative, analytical, and directive.
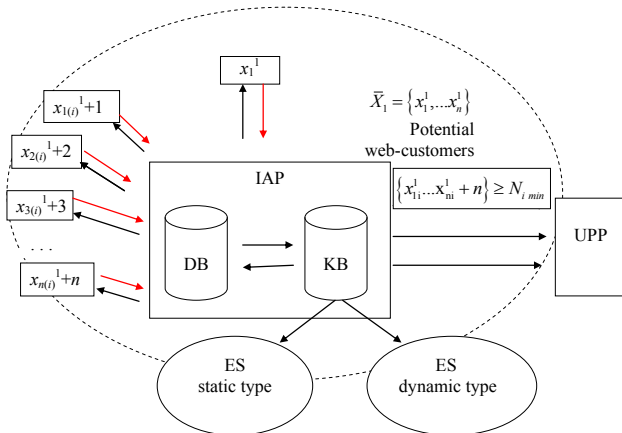

Fig. 3. Block diagram of deployment and support of a unified knowledge space at the flexible re-structuring of production according to market conditions

Stage 1 (cumulative). It requires that each IAP user should perform the following:

– Select User mode.
– Choose VIE product category.
– Select a particular type of product (products).
– Preliminary user registration.
– User profile form.

– Form consumer basket, indicating the volume of supplies for each type of product.
– Choice of place, method, and term of delivery of products to the consumer.
– Enter consumer's payment details.
– Check the profitability of manufacturing.
– Place an order.

Stage 2 (analytical). At this stage, an analysis is carried out in the environment of ES over a certain period (month, quarter) based on the information about the concluded contracts for the supply of each type of product, supply volumes, place, method and timing of delivery to a customer.

Stage 3 (directive). Based on the analysis of market conditions from the preceding stage, one forms decisions in the environment of ES about production re-structuring; a program of product output is determined for each type of a product from the range of VIE products. The information obtained at this stage enables the head of VIE to create reasonable order for production.

Condition $\left\{ x_{1(i)}^1 + 1 \ldots x_{n(i)}^1 + n \right\} \geq N_{i\,min}$ – the number of orders needed to achieve profitability of producing the i-th product; in this case:

$$N_{i\,min} \to min$$

$$\left\{ x_{1(i)}^1 + 1 \ldots x_{n(i)}^1 + n \right\} \to max. \tag{1}$$

The modeling subsystem for calculating the required permissible production volume of the i-th type of product:

$$N_{i\,min} = x_n^1 i \cdot n, \tag{2}$$

where $x_n^1 i$ is the number of orders for the i-th type of product; n is the quantity of the i-th product.

The corresponding finite-differential equation for the permissible volume of production of the i-th type of product takes the form:

$$N_{i\,min+n} = x_n^1 i \cdot n \text{ (initial value } n=1). \tag{3}$$

The born IT provides flexibility in the organization of production at VIE by accounting for the dynamics of change in market conditions.

Thus, it is possible to achieve the stable flow of orders and the informativeness of the market object by applying primitive reflexive agents.

4. 2. Mathematical model of allocating resources for manufacture of products at a virtual instrument-making enterprise

There is a resource in the amount R. Raw materials, equipment, time, etc. can be used as a resource. There are N users of the resource, each of which is assigned with function $\Psi(r_j)$, the effect achieved by j consumers when the amount of the utilized resource is one. It is necessary to split the cash resource among consumers so as to maximize the total effect, that is, one needs to find

$$\max \sum_{j=1}^{N} \psi_j \left(r_j \right) \tag{4}$$

under condition

$$\sum_{j=1}^{N} r_j \le R\left(r_j \ge 0, j=1,2\ldots,N\right).$$

Numerous meaningful interpretations of this task are well known. There are other statements of a resource allocation task, among which only two are taken into consideration here.

It is required to find min max $\Psi_j(r_j)$ under condition

$$\sum_{j=1}^{N} r_j \le R\left(r_j \ge 0, j=1,2\ldots,N\right). \tag{5}$$

The specified statement of the problem arises, for example, in the distribution of equipment over a certain period in a multichannel control system with time division of channels or in a pulse control system over N objects through a single communication channel.

For the case of control over a system with the time division of channels, R is the period of switching channels; rj is the time during which a communication channel operates in the j-th control circuit. The allocation of time to control objects over a period should ensure that the error in the worst channel is minimized.

In the following variant, it is required to find

$$\max \prod_{j=1}^{N} \psi_j(r_j), \tag{6}$$

under condition

$$\sum_{j=1}^{N} r_j \le R\left(r_j \ge 0, j=1,2\ldots,N\right).$$

One of IT's tasks is to allocate resources to different components of VIE production, which provides for the maximum probability of finding a correct solution to the task set to an enterprise. For example, if R is the total time to execute a task by an enterprise, and r_j is the time allocated to execute the j-th subtask.

If the graph of function $\Psi_j(r_j)$ is convex, that is if

$$d^2\Psi_j(r_j)/dr^2 < 0 \ (j=1, 2,\ldots, N),$$

the solution to problem (3) is then unique, and it follows from the conditions for a saddle point in a Lagrange function that the desired resource allocation is the solution to a system of equations

$$d^2\Psi_j(r_j)/dr^2 < 0 \ (j=1, 2,\ldots, N), \tag{7}$$

$$\sum_{j=1}^{N} r_j - R = 0.$$

There may be two cases here:
1) a solution to system (7) does exist and then, due to the concave function, it is unique;
2) a solution to system (7) does not exist. The latter means the existence of dominant and recessive objects.

An object is called dominant (recessive) if a derivative from the benefit function of this object over the entire range of the change is greater (less) than the derivatives from the benefit functions of other objects. In the case of dominant objects, the entire resource is distributed among them. Recessive objects do not receive a resource at all. Below we

shall consider a case of solution within the area of permissible values of the distributed resource. You will see that the proposed techniques for organizing collective behavior ensure the attainment of optimal distribution even for the case when a solution is boundary.

Similar to analyzing the allocation, if functions $\Psi_j(r_j)$ are descending, the solution to problem (5) is unique and the desired allocation of the resource is determined from a solution to the system of equations

$$\begin{cases} \psi_j(r_j) - \lambda = 0 \ (j=1,2,\ldots,N) \\ \sum_{j=1}^{N} r_j - R = 0 \end{cases}. \tag{8}$$

Remarks on dominant and recessive objects hold for this case as well.

When comparing (7) and (8), it follows that

$$\max \sum_{j=0}^{N} \int_0^{r_j} \psi_j(x)\,dx; \tag{9}$$

ensures

$$\min \ \max \ \psi_j(r_j), \tag{10}$$

thus, techniques to solve task (4) clearly apply to the solution to problem (5). Therefore, we shall hereafter consider only the solution to problem (4).

Solving a task on resource allocation has two aspects: computational and managerial. In the case when functions $\Psi_j(r_j)$ are known, there is a computational problem of nonlinear programming, a series of ways to solve which are well known. On the other hand, if functions $\Psi_j(r_j)$ are *a priori* unknown, and we know only their current values, and functions themselves, as it often happens in practice, change over time, then there is a problem on the operative redistribution of resource during the functioning of the system, that is, a control task.

Given the fact that the equations from system (7) are partial derivatives from a Lagrange function, we can behave in the decentralized fashion when setting a resource allocation task, which is based on the gradient method for solving this task:

Such an approach is equivalent to the approach when every consumer of a resource maximizes his local utility function in the form:

$$r_j = \begin{cases} x_j & \text{at} \quad \sum_{j=1}^{N} x_j \le R \\ \dfrac{R_{xj}}{\sum_{j=1}^{N} x_j} & \text{at} \quad \sum_{j=1}^{N} x_j \succ R \end{cases}, \tag{11}$$

where x_j is the request from the j-th consumer of the resource.

Now, why not consider the local rules of conduct that maximize the total effect. As follows from the above, the required resource allocation is determined from the solution to system (7), and the solution of system (7) corresponds to a maximum of local benefit functions (10), provided that the demand is equal to the proposal. Then it is only natural to change the magnitude of request at every step according to the following rules:

$$x_j(t+1) = x_j(t) + k1\frac{\Delta\psi_j(r_j(t))}{\Delta r_j(t)} - \lambda(t),$$

$$\lambda(t+1) = \lambda(t) + k2\left[\sum_{j=1}^{N} x_j - R\right], \qquad (12)$$

where the price of a resource is formed by IAP based on the difference between supply and demand.

It follows directly from (12) that the point of equilibrium within a system corresponds to the optimal resource allocation. This raises the question of stability and attainability of the equilibrium point. Difficulties associated with the analysis of stability of such a system are in that the point of equilibrium rests on hyperplane

$$\sum_{j}^{N} r_j - R = 0,$$

at whose opposite sides the resource is distributed by different rules (11), which leads to a rupture in derivatives on the hyperplane of constraints.

We can solve this complexity by changing the rules of resource allocation among consumers. If the query amount does not exceed the existing resource, then all requests would still be met in full. If the query amount exceeds the amount of the resource, then not the entire resource is distributed, and our share of the resource equal to ε remains unallocated, that is

$$r_j = \begin{cases} x_j & \text{at } \sum_{j=1}^{N} x_j \le R \\ \dfrac{(R-\varepsilon)x_j}{\sum_{j=1}^{N} x_j} & \text{at } \sum_{j=1}^{N} x_j > R \end{cases}. \qquad (13)$$

In this case, the optimal allocation would be not found not for constraint

$$\sum_{j=1}^{N} r_j - R = 0,$$

but for constraint

$$\sum_{j=1}^{N} r_j - R = -\varepsilon.$$

In this case, the equilibrium point lies under the hyperplane

$$\sum_{j=1}^{N} r_j - R = 0,$$

inside the region where we seduce consumers. The amount of the resource coincides with a demand for it, and the behavior of system of equations (12) coincides with the behavior of a difference scheme that implements the gradient method. Then, the stability within small system (12) holds for all the results in a small gradient method in this task on nonlinear programming. System stability in general requires, in addition to the stability of a pure gradient method, the following characteristic for the system trajectories in the parameter space x_j. A different trajectory after the finite number of hyperplane resettlements would remain in the parameter domain that includes the equilibrium point.

Consider the stability of a system of differential equations equivalent to the systems of difference equations (12). In this case, we denote through $\varphi_i(x_j)$ the function $\dfrac{d\psi_j(x_j)}{dx_j}$. The ε parameter can be considered to be the accuracy of allocation, or as a certain reserve, which should be kept at the optimum allocation (in this case, when attaining the optimal distribution one can use the entire resource, including the reserve ε. We would also assume that price λ can accept values from interval $[\lambda_1, \lambda_2]$, where $\lambda_1 \le \lambda_1^*, \lambda_2 \le \lambda_2^*$, while λ_1^* and λ_2^* can be derived from *a priori* estimates of function $\varphi i\,(r_j)$. Thus, consider the system

$$\begin{cases} x_i = k_1 \oint_i (x_1, x_2, ..., x_{n+1}) \ (i = 1,2,...,n) \\ x_{n+1} = -k_2 \oint_{n+1} (x_1 x_2, ..., x_n), \end{cases}$$

$$f_{n+1}(x_1, x_2, ..., x_n) = \begin{cases} 0, & \text{if } x_{n+1} = \lambda_1 \sum_{j=1}^{N} x_j - 1 + \varepsilon \ge 0, \\ 0, & \text{if } x_{n+1} = \lambda_2 \sum_{j=1}^{N} x_j - 1 + \varepsilon \le 0, \\ \sum_{j=1}^{N} x_j - 1 + \varepsilon & \text{in other cases,} \end{cases}$$

where $\varepsilon, \lambda_1, \lambda_2, k_1, k_2$ – some constants $(\varepsilon \in (0,1); k_1, k_2 > 0)$

$$f_i(x_1, x_2, ..., x_n) = \begin{cases} 0, & \text{if } x_i = 0 \text{ and } x_{n+1} - \mu_i \le 0 \\ 0, & \text{if } x_i = 0 \text{ and } x_{n+1} - \mu_i \ge 0 \\ x_{n+1} - \mu_i(x_1, x_2, ..., x_n) & \text{in other cases} \end{cases}$$

in this case

$$\mu_i(x_1, x_2, ..., x_n) = \begin{cases} \phi_i(x_1), & \text{if } \sum_{j=1}^{N} x_j, \\ \phi_i\left(x_1 \left| \sum_{j=1}^{N} x_j \right.\right), & \text{if } \sum_{j=1}^{N} x_j > 1. \end{cases} \qquad (14)$$

As regards numbers λ_1, λ_2 and function φ_i, we shall assume:

1) $\lambda_1 < \lambda_2, \varphi_1(0) = \varphi_2(0) = ... = \varphi_n(0),$ there is $h>0$ so that for any $i = 1,2,...,n$;

$$\lambda_1 - \phi_i(0) < -h, \lambda_2 - \phi_i(0) - a_i^{(2)};$$

2) there are $a_i^{(1)} > 0, \ a_i^{(2)} > 0 (1 \le i \le n)$ so that for any

$$\lambda_1 - \phi_i(0) < -h, \lambda_2 \phi_i(0) - a_i^{(2)} > h;$$

3) there are $a_i^{(1)} > 0, a_i^{(1)} > 0 (1 \le i \le n)$ so that for any $\xi \in (0,1), \eta \in (\xi, 1-\xi)$

$$a_i^{(1)} \le \frac{\phi_i(\xi + \eta) - \phi_i(\xi)}{\eta} \le a_i^{(2)}.$$

Note 1. Condition 3 is met, specifically, if function ϕ_i is differentiated along (0,1).

Note 2. If conditions 2 and 3 are met, for any $\eta \in [0,1]$

$$\lambda_1 - \phi_i(\eta) < -h, \lambda_2 - \phi_i(\eta) > h,$$

$$\lambda^* = \max\{|\lambda_1|, |\lambda_2|\}, \quad \phi^* = \min_{1 \le i \le n}(-\phi_i(0) - a_i^{(1)}),$$

$$a = \min_{j=1,-n} a_j^{(1)}, \quad A_1 \sum_{i=1}^n a_i^{(2)}, \quad \Phi_1 = \sum_{i=1}^n \phi_j(1), \lambda = \lambda_2 \lambda_1.$$

Next, we shall assume that system (14) has a single equilibrium state $x^* = (x_1^*, x_2^*, ..., x_{n+1}^*)$ such that x^* is the inner point within region

$$G_1 = \{x | x_1 \in [0,1], ..., x_n \in [0,1];$$

$$x_{n+1} \in [\lambda_1, \lambda_2]; \sum_{j=1}^N x_j \leq 1\}, \tag{15}$$

that is

$$\sum_{j=1}^N x_j^* = 1 - \varepsilon < 1,$$

$$x_{n+1}^* = \phi_i(x_i^*) \in (\lambda_1, \lambda_2), x_i^* \in (0,1).$$

The form of system (9) indicates that any trajectory of the system, starting in region

$$X_1 = \{x | x_1 \in [0,1], ..., x_n \in [0,1]; x_{n+1} \in [\lambda_1, \lambda_2]\}$$

does not leave the set X with an increase in time, and point x^* is the only state of equilibrium in region

$$G = G_1 \cap X.$$

Let ε^*, γ be some constants, such that

$$\varepsilon^* \in (0, \varepsilon), \gamma \in \left(0, \frac{a\varepsilon^*(\varepsilon - \varepsilon^*)^2}{(n\lambda^* + \Phi_1)}\right). \tag{16}$$

Then, while fulfilling the proposal, the following theorem holds.

Theorem 1. If ε^*, γ are selected from (16) and the following inequality is satisfied for k_1 and k_2.

$$\frac{k_2}{k_1} > \max\left\{\frac{A_1|\lambda_1 - \phi^*|}{\varepsilon - \varepsilon^*}, \frac{A_1 n\lambda^2}{\varepsilon h\left(\frac{a\varepsilon(\varepsilon - \varepsilon^*)^2}{n\lambda^* + \Phi_1} - \gamma\right)}\right\}, \tag{17}$$

the solution to system $x = x^*$ (14) is then asymptomatically stable with the gravity domain X. The proof rests on the following lemmas (whose proof is omitted).

Lemma 1. Let $\delta > 0$ be a random number and $k_2 > \frac{n\lambda^2}{\varepsilon h \delta}$. Then no trajectory x(t) of system (9), such that

$$x(t^*) \in H = \left\{x \left| \sum_{j=1}^n x_j = 1\right.\right\} \cap X, \tag{18}$$

can lie in region X / G at all $t \in [t^*, t^* + \delta]$.

Lemma 2. Let $\varepsilon^* > 0$ be some number from interval $(0, \varepsilon)$,

$$\frac{k_2}{k_1} > \frac{A_{1|\lambda_1 - \phi^*|}}{\varepsilon - \varepsilon^*}, \tag{19}$$

functions ϕ_i have derivatives along $(0,1)$; ratio (15) is satisfied. Then any trajectory x(t) of system (14), passing at time t^*, at

$$t \in \Delta^* = \left[t^*, t^* + \frac{\varepsilon^*}{k_1(n\lambda^* + \Phi_1)}\right], \tag{20}$$

would be in layer

$$G_{\varepsilon^*} = \left\{x \left| \sum_{j=1}^n x_j \in [1 - \varepsilon^*, 1]\right|\right\} \cap G$$

and, therefore, does not leave region G.

Lemma 3. A time derivative from function

$$\upsilon(x) = \sum_{j=1}^n \left(x_j - x_j^*\right) + \frac{k_1}{k_2}\left(x_{n+1} - x_{n+1}^*\right)^2 \tag{21}$$

given the power of system (14), is integral, and in this case the set

$$M = \left\{x \left| \frac{d\upsilon(x)}{dt} = 0\right.\right\}, \tag{22}$$

does not contain the entire trajectories of system (14).

Lemma 4. Under the accepted assumptions for k_1 and k_2, any trajectory x(t) of system (14) cannot cross the hyperplane

$$\sum_{j=1}^n x_j = 1 - \varepsilon \text{ more than } N_{x,t} = \left[\frac{\upsilon(x(0))}{\gamma} + 1\right] + 1 \text{ times}$$

in this case, $|a|$ denotes an integer part of number α.

In conclusion, we note that if functions ϕ_i are such that $a_i^{(1)} < a_i^{(2)} < 0$, then, at $k_1 < 0$, $k_2 < 0$ there is a statement similar to the statement of the theorem.

Let there be ten consumers within a system with local functions

$$\begin{cases} \psi_1(r_1) = 2,125\ln(1+r_1), \\ \psi_2(r_2) = 2,125\ln(1+r_2), \\ \psi_3(r_3) = \sqrt{2r_4}, \\ \psi_4(r_4) = \sqrt{3r_4}, \\ \psi_5(r_5) = \frac{6}{\pi}\sin\frac{\pi}{3}r_s, \\ \psi_6(r_6) = 2\sin r_6, \\ \psi_7(r_7) = r_7(2,125 - r_7), \\ \psi_8(r_8) = \frac{1}{2}r_8(4,25 - r_8), \\ \psi_9(r_9) = 2,25(1 - e^{-r_9}), \\ \psi_{10}(r_{10}) = 2,25(1 - e^{-r_{10}}). \end{cases} \tag{23}$$

In the considered example, R=1.

Note that the resource consumers do not know the form of their function $\psi_j(r_j)$ and are guided only by its current values. Resource allocation quickly converges to the optimum.

The above-considered mathematical model has drawbacks caused by several reasons.

First, if this can be solved by a gradient method and there is a possibility for a local determination of partial derivatives from the function maximized (minimized), the organization of collective behavior is not very difficult. Studying such models, except overcoming a series of task-specific diffi-

culties, does not provide any substantial material for the advancement in the study of collective behavior.

Second, the use of Lagrange's multipliers is natural, as it allows for a convenient meaningful interpretation, for example, price, but determining the values for Lagrange multipliers requires that this task should be solved by IAP. When studying models of collective behavior, there is a natural desire to maximally simplify the IAP functions, by passing all the difficulties in solving a task to the common behavior of the members of a collective.

In complex systems, they often distinguish informational, energy, and material flows. It should be remembered in this case that the energy and material flows carry information about the location of at least the amount of transferred energy or materials. This principle underlies a humoral regulation in a heavenly body. To control such a complex organism as an anthill, the information contained in the fluxes of common food digestion is perhaps more important than sharing signals by individual ants. These considerations kickstarted us to try, in the organization of a collective behavior for a resource allocation task, to use the information contained in the amount of the resources coming from each consumer.

To assess the effectiveness of behavior of each consumer, one needs to know the amount of income, that is, the difference between the "production" $\psi_j(r_j)$ and the cost of the spent resource $\lambda r_j = C_j$.

It follows from the gradient method for solving a resource allocation task that a consumer must know a resource price, that is

$$\frac{C_j}{r_j} = \lambda. \tag{24}$$

The latter suggests using, as a parameter of interaction, not a request to smoke the required amount, but a certain amount of money C_j for which the j-th consumer asks to allocate a resource. Once again, we warn that all the economic terms used in the description of the organization of behavior are purely conditional. All applications for a resource in monetary terms are sent to the center where the resource is allocated proportionally to the money received, that is

$$x_j = \frac{RC_j}{\sum_{j=1}^{N} C_j}. \tag{25}$$

Then the price for a resource unit equals

$$\lambda = \frac{\sum_{j=1}^{N} C_j}{R}, \tag{26}$$

and is easily restored by the consumer based on the amount of the resulting resource $\lambda = \frac{C_j}{r_j}$. Hence, the obvious organization of behavior.

We find it easy to see that a solution to system

$$\begin{cases} \dfrac{d\Psi_j(r_j)}{dr_j} - \dfrac{C_j}{r_j} = 0, \\ r_j = \dfrac{RC_j}{\sum_{j=1}^{N} C_j}, \end{cases} \tag{27}$$

coincides with the solution to system (7), and $C_j = \lambda r_j$. Therefore, the method of "naove gradient", based on (27), must have an equilibrium point, it coincides with the optimum distribution of the resource.

To organize collective behavior that ensures optimal crack allocation, we would demand that every consumer should at each step submit a request to IAP for the desired amount of resource C_j. The IAP allocates the resource in proportion to applications, received in accordance with (26). In this case, the magnitude of the request at each step is formed by consumers as follows:

$$C_j(t+1) = C_j(t) + k\left[\frac{\Delta\Psi_j(r_j)}{\Delta r_j} - \frac{C_j}{r_j}\right]. \tag{28}$$

Consider the problem on allocating resource R among N consumers, ensuring $\max \sum_{j=1}^{N} \psi_j(r_j)$ under constraint

$$\Phi(r_1, r_2, ..., r_N) \le 0. \tag{29}$$

If functions $\psi_j(r_j)$ are convex

$$\left(\frac{d^2\psi_j(r_j)}{dr_j^2} < 0\right) \text{ and } \Phi(r_1, r_2, ..., r_N) \le 0$$

is an ascending function

$$r_j\left(\frac{\partial\Phi(r_1, r_2, ..., r_n)}{\partial r_j}\right) > 0, \quad (j = 1, 2, ..., N),$$

the optimal allocation is unique and is determined from solving a system of equations

$$\left.\begin{aligned} \frac{d\psi_j(r_i)}{dr_j} &= \lambda - \frac{\partial\Phi(r_1, r_2, ... r_N)}{dr_j} = 0, j = 1, 2, ..., N, \\ \Phi(r_1, r_2, ... r_N) &= 0. \end{aligned}\right\} \tag{30}$$

When a series of conditions are satisfied, this system can be solved by a gradient method:

$$\left.\begin{aligned} \dot{r}_j &= \frac{d\psi_j(r_i)}{dr_j} - \frac{\partial\Phi(r_1, r_2, ... r_N)}{dr_j}, \\ \dot{\lambda} &= \Phi(r_1, r_2, ... r_N). \end{aligned}\right\} \tag{31}$$

Similarly to the previous case, every consumer of the resource sends to IAP, at each cycle of system operation, a certain "amount of money" C_j and receives a certain piece of resource r_j. The consumer must form his/her request so as to maximize the local benefit function

$$S_j = d\psi_j(r_j) - Cj. \tag{32}$$

S_j achieves a maximum at a point that meets the condition

$$\frac{\partial S_{j0}}{dr_j} = \frac{d\psi(r_i)}{dr_j} - \frac{\partial C_j}{\partial r_j} = 0. \tag{33}$$

At fixed λ, it can be assumed that $C_j = \lambda_j r_j$, where λ_j is the price of resource of the j-th consumer. Note that

$$\frac{\partial s_j}{\partial c_j} = \frac{\partial s_j}{\partial r_j} \times \frac{\partial r_j}{\partial c_j}$$

and the conditions for an extremum of S_j for r_j and C_j coincide. Local price

$$\lambda_j = \frac{c_j}{r_j}$$

can be restored to the consumer. Thus, in order for the system to reach a conditional extremum, it is necessary that the resource at IAP should be distributed such that

$$\frac{C_j}{r_j} = \lambda \frac{\partial \Phi(r_{1,}r_{2,...,}r_N)}{dr_j}. \qquad (34)$$

Hence, it follows that parts of the resource that are allocated to consumers are determined at IAP by implementing the system of equations

$$\begin{cases} r_j = \frac{C_j}{\lambda} \cdot \frac{1}{\dfrac{\partial \Phi(r_{1,}r_{2,...,}r_N)}{dr_j}}, \\ \Phi(r_{1,}r_{2,...,}r_N) = \Phi^*(\lambda) = 0. \end{cases} \qquad (35)$$

and the local behavior rule, according to (31), takes the form

$$\Delta C_j(t) = k \left[\frac{\Delta\psi_j(r_j)}{\Delta r_j(t)} - \frac{C_j(t)}{r_j(t)} \right]. \qquad (36)$$

We find it easy to reveal that the situations of equilibrium (31) and (36), considering (35), coincide.

There is a question about the conditions under which a difference equation system (36) is stable. This issue, due to significant analytical difficulties, was not investigated. However, at a reasonable choice of coefficient k, a solution to system (36) converges with a satisfactory speed. Once again note that the results indicate only that system (36) is robust.

5. Discussion of results of studying the re-structuring of virtual instrument-making enterprises

We have designed an informational-analytical portal, which ensures the accumulation and processing of information about market conditions. In addition, the process model of the informational-analytical portal has been developed, which makes it possible for a client to place the orders, which are then transferred to production. When the necessary conditions of feasibility (profitability) of production of a specific type of product are met, the order is added to the product range in production. The information model of IAP has been proposed. Based on the information model, a structural scheme has been devised for the construction, deployment, and support of a unified knowledge space at the flexible re-structuring of production, according to market conditions.

We have constructed a mathematical model for the justification of resource allocation for VIE production. Determining the profitability of output is calculated according to a formula that includes a set of potential orders. The number of orders is determined using IAP. The finite-difference equation for the maximally permissible volume of production

of a particular type of products has been proposed. The obtained calculation results make it possible to solve the task on re-structuring production.

Our study has established that the development of information technologies in order to manage virtual enterprises makes it possible to allocate human resources in the organization. However, there are unresolved issues related to decision-making on determining the expediency of making a particular type of products and the maximally permissible production volumes, as well as the allocation of production resources. Through the creation of IT elements, we managed to adapt a virtual production to market conditions, thereby defining and allocating the required resources. To ensure regular operation of IAP, it is necessary to give us a modern server with wide possibilities of processing a significant number of information flows.

The disadvantage of the proposed models relates to that the analysis and decision support are formed solely based on statistical data acquired by a portal, a site, or other Internet resources. And it was not taken into consideration that there are generations and categories of people who for some reason cannot use Internet. Thus, the data transmitted to the system to calculate the maximally permissible amount of manufacture of the product are not accurate, and have a rather large error. Therefore, this issue is still open and requires deeper research, implying taking into consideration more information channels for collecting static data on specific properties of products that influence an enterprise's decision on further functioning.

The current study could be advanced by the development of additional IT elements, which could be based on methods or models to form a product's structure with new qualitative properties, required by market conditions. Further development of this study must deal with approaches for creating additional information channels to collect static data on the need in new properties of products. Probable approaches could include questionnaires and polling of potential customers.

Planning the further development of the current research may be aimed at improving IT for decision making on the re-structuring of VIE production. In particular, IT can be developed in the direction of forecasting the probable terms of each stage of making a product.

6. Conclusions

1. Using the proposed IT solves the task on the expedience of re-structuring production and on planning a program to release each type of product according to market conditions. Making a decision on re-structuring VIE production is based on:

– creation of a structural scheme for deploying and maintaining a unified knowledge space at IAP at the flexible re-structuring of production according to market conditions;

– determining the profitability of industrial output according to the number of requests from potential buyers about the properties of a product or the functions that the consumer wants to see in the new product;

– calculating, in the form of a formula, the minimally permissible volume of certain type of a product, determined based on its profitability;

– determining the expediency of producing possible types of VIE products.

2. We have substantiated the rational allocation of resources to make VIE products by creating a mathematical model, which is an element of IT. The aim of this model is to gain competitive advantage in the market for the proposed product and VIE in general.

The proposed elements of IT provide for the calculation of a certain quantity of production resources, their qualitative composition for the production of VIE products, which is a significant for re-structuring, as it is important not only to make what the market requires but to make a product on time. In this case, the product must be of high quality and certified. These moments lie on the shoulders of production resources. A decision by a VIE head on resource allocation is determined by the criteria: quality, timing, price, etc. The choice of a subcontractor is almost the most important and the most responsible moment in the implementation of a VIE product output program.

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