
#### Abstract

$\square$ Запропоновано в якості додаткового енергетичного параметра, окрім основних енергетичних показників, при визначенні режиму короткого замикання в тяговій електромережі використовувати втрати потужності. Наведено три способи визначення перехідних характеристик електричної мережі, які грунтуються на методах наближення, дискретної електротехніки та спектрального аналізу. Необхідність у створенні цих методів обумовлена нестаціонарністю та нелінійністю електричних параметрів тягових мереж, які можуть створювати невизначеність моменту виникнення короткого замикання.

Вхідними даними для визначення енергетичних характеристик і втрат слугують функцї перехідної напруги та перехідного струму, що виміряні на затискачах комутаційного апарату, до яких приєднуються провідники живлення тягової мережі. На основі експериментальних вимірів та наведених методів проведено обчислення основних енергетичних параметрів мережі та втрат при різних коротких замиканнях.

Результати розрахунків показують, що при коротких замиканнях в тяговій мережі кількісні значення енергетичних показників та непродуктивних втрат дозволяють однозначно оцінити режим роботи мережі. Також отримані результати дозволяють визначити фактичне навантаження на захисні комутаційні апарати при коротких замиканнях в тяговій мережі.

Запропоновані методи можуть бути застосовані при створенні комп'ютерних моделей тягових мереж для їх дослідження та виконання інженерних розрахунків. Вони також можуть бути застосовані при проектуванні нових або реконструкиіі діючих тягових мереж постійного струму та більш точного обтрунтування проектних рішень. Також запропоновані методи можуть бути застосовані при створенні нових або для удосконалення діючих зразків захисної комутаційної апаратури

Ключові слова: перехідний процес, коротке замикання, втрати енергї, енергетичні показники, спектральний аналіз


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# DEFINING ENERGY INDICATORS FOR DETECTING SHORT CIRCUITS IN A DC ELECTRIC TRACTION SYSTEM 

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## 1. Introduction

An urgent task for traction, autonomous, and industrial power networks is the estimation of their parameters under emergency states. The most dangerous and destructive emergency mode is short circuit (SC).

However, given the growing complexity of electrical systems due to the widespread introduction of power semiconductor equipment, the processes of generation, transmission, and transformation of electrical energy become increasingly complicated. This is especially important for electrical networks of electrified transport, characterized by the non-stationarity and significant nonlinearity of natural parameters. That makes it difficult to identify such energy indicators that reliably indicate the fact of an emergency state, specifically SC. Thus, it is a relevant task to theoretically and practically define additional parameters for electricity, which are more informative about the character of flowing and transformation of electricity in traction networks.

## 2. Literature review and problem statement

It was shown in paper [1] that scientific activities in the field of electrified transport mainly focuses on the improvement of energy characteristics. Little attention is paid to the problems of detecting emergency regimes in DC electric traction networks. Available studies are dominated by those related to new or promising projects. Thus, papers [2,3] examined methods for determining short circuits in the AC traction networks $2 \times 25 \mathrm{kV}$ for high-speed trains. Such networks are specific and used only in Asian countries. The reported methods could not be applied for DC traction networks, which are used in Ukraine. Paper [4] addresses the issue of detecting an SC mode in a DC traction network, but at the clamps of primary windings of transformers at a traction substation. The considered method is relatively simple, but requires consideration of the time constant of a power transformer. Determining a short-circuit in a DS traction network directly at a controlled section, shown in studies [5-7], is a non-trivial task.

The system of efficiency indicators could be applied to describe the power processes in electrical circuits of electric traction [8]. Such a system is based on a harmonious approach, which could lead to uncertainty and complications when the energy fluxes are studied.

An option could be the use of unproductive (additional) energy losses in the elements of an electric traction system [9, 10]. These papers report a procedure for finding additional losses based on measuring the shape of a phase current of the synchronous traction machine and the AC distribution network.

The above allows us to argue that it is advisable to conduct a research into the application of energy indicators and the magnitude of unproductive losses in a DC traction network as the attributes of short circuits in DC electric traction networks.

## 3. The aim and objectives of the study

The aim of this study is to form a theoretical basis for using energy indicators and additional energy losses in a traction electric network to recognize a mode of SC. Such a network is characterized by the presence of many power sources, the presence of harmonious components of power voltage, the distributed and non-stationary parameters of the line.

To achieve the set aim, the following tasks have been solved:

- to form an analytical description of additional energy losses in a traction electric network;
- to define methods for the calculation of energy indicators and additional losses of electric power for determining SC modes in traction electric networks;
- to establish additional losses and other energy indicators under SC modes and to estimate operation efficiency of controlled section of traction networks and protective switching equipment.


## 4. Analytical description of additional energy losses in a traction electric network

The classical system of energy indicators includes a coefficient of phase offset of the basic harmonic of non-sinusoidal current relative to the input sinusoid voltage $\cos \varphi_{(1)}$; a current shape distortion coefficient relative to the shape of voltage (v); a phase asymmetry load coefficient ( $K_{H C}$ ); a coefficient of uneven power consumption $\left(K_{H P}\right)$.

These coefficients are based on the ratios of relevant capacities: active $P$, reactive $Q_{(1)}$, and full $S$, distortion $D$, asymmetry $H_{C}$, non-uniformity $H_{P}$. In this case, $\cos \varphi_{(1)}, P$ and $Q_{(1)}$ were determined only for the first harmonic. For the case of non-sinusoidal voltage and current, the coefficient $\cos \varphi$ will be a phase shift coefficient between the curves of voltage and current, power $Q$ will be a reactive power by Budean $Q_{B}$, and power $P$ will be predetermined by the same-name harmonics of voltage and current.

Therefore, these coefficients are determined through the components of full power $S$, in the following way:

$$
\begin{align*}
& \cos \varphi=\frac{P}{\sqrt{P^{2}+Q_{B}^{2}}},  \tag{1}\\
& \nu=\frac{\sqrt{P^{2}+Q_{B}^{2}}}{\sqrt{P^{2}+Q_{B}^{2}+D^{2}}}, \tag{2}
\end{align*}
$$

$$
\begin{align*}
& K_{H C}=\frac{\sqrt{P^{2}+Q_{B}^{2}+D^{2}}}{\sqrt{P^{2}+Q_{B}^{2}+D^{2}+H_{C}^{2}}},  \tag{3}\\
& K_{H P}=\frac{\sqrt{P^{2}+Q_{B}^{2}+D^{2}+H_{C}^{2}}}{\sqrt{P^{2}+Q_{B}^{2}+D^{2}+H_{C}^{2}+H_{P}^{2}}}=\frac{\sqrt{P^{2}+Q_{B}^{2}+D^{2}+H_{C}^{2}}}{S} . \tag{4}
\end{align*}
$$

The general indicator in this system is a power factor $\lambda$, which integrates multiplicatively all local coefficients (1) to (4) by expression:

$$
\begin{equation*}
\lambda=\frac{P}{S}=(\cos \varphi) \cdot v \cdot K_{H C} \cdot K_{H P} . \tag{5}
\end{equation*}
$$

As regards an electric traction system in the classical sense, a power factor $\lambda$, first, determines the degree of utilization of full power $S$, applied to a traction substation in general or to electric rolling stock in particular. Second, the magnitude $\lambda$ also characterizes the losses of active power in the power traction circles of electric rolling stock and in a traction network of electricity supply.

It should be noted that the power factor $\cos \varphi$ is not always a sufficiently obvious energy indicator (for circuits and modes in which capacities $D \neq 0, H_{C} \neq 0, H_{P} \neq 0$ ), which is why in recent years the traction power supply, similarly to industrial, has employed, as a reporting magnitude, a reactive power factor:

$$
\begin{equation*}
\operatorname{tg} \varphi=\frac{Q}{P} . \tag{6}
\end{equation*}
$$

To derive formulae for additional energy losses, we shall apply the concept by S. Friese [11]. S. Friese suggested representing any consumer of current $i(t)$ of arbitrary shape as a parallel connection of resistive element $R$ and reactive element $X$. A first one reflects the active energy consumed. A second one characterizes the consumption of inactive components of power: reactive power and power of distortion. It is believed that the active component of current $i_{a}(t)$ flows in a branch with element $R$, which coincides in shape with voltage $u(t)$ applied to the consumer.

Reactive component $i_{p}(t)$ flows along element $X$, orthogonal to voltage $u(t)$. Then, for an arbitrary time interval $[0 \ldots \tau]$ for the pulse of transitional $i(t)$, it could be represented by the sum:

$$
\begin{equation*}
i(t)=i_{a}(t)+i_{p}(t) \tag{7}
\end{equation*}
$$

or

$$
\begin{align*}
& \frac{1}{\tau} \int_{0}^{\tau} i^{2}(t) \mathrm{d} t=\frac{1}{\tau} \int_{0}^{\tau} i_{a}^{2}(t) \mathrm{d} t+\frac{1}{\tau} \int_{0}^{\tau} i_{p}^{2}(t) \mathrm{d} t, \\
& \frac{1}{\tau} \int_{0}^{\tau} i_{p}^{2}(t) \mathrm{d} t=\frac{1}{\tau} \int_{0}^{\tau} i^{2}(t) \mathrm{d} t-\frac{1}{\tau} \int_{0}^{\tau} i_{a}^{2}(t) \mathrm{d} t . \tag{8}
\end{align*}
$$

By multiplying all components (8) by $\frac{1}{\tau} \int_{0}^{\tau} u^{2}(t) \mathrm{d} t$, we
Bain. obtain:

$$
\begin{align*}
& \frac{1}{\tau} \int_{0}^{\tau} u^{2}(t) \mathrm{d} t \cdot \frac{1}{\tau} \int_{0}^{\tau} i_{p}^{2}(t) \mathrm{d} t= \\
& =\frac{1}{\tau} \int_{0}^{\tau} u^{2}(t) \mathrm{d} t \cdot \frac{1}{\tau} \int_{0}^{\tau} i^{2}(t) \mathrm{d} t-\frac{1}{\tau} \int_{0}^{\tau} u^{2}(t) \mathrm{d} t \cdot \frac{1}{\tau} \int_{0}^{\tau} i_{a}^{2}(t) \mathrm{d} t . \tag{9}
\end{align*}
$$

The left-hand side of expression (9) characterizes the Friese reactive power $Q_{\phi \tau}$, consumed in the interval $[0 \ldots \tau]$, that is:

$$
\begin{equation*}
Q_{Q \mathrm{q} \tau}^{2}=\frac{1}{\tau} \int_{0}^{\tau} u^{2}(t) \mathrm{d} t \cdot \frac{1}{\tau} \int_{0}^{\tau} i_{p}^{2}(t) \mathrm{d} t . \tag{10}
\end{equation*}
$$

Then the unproductive (additional) losses of active electricity in the consumer $R$ will be determined from expression:

$$
\begin{equation*}
\Delta W_{\mathrm{d} \mathrm{\tau}}=\frac{1}{\tau} \int_{0}^{\tau} R \cdot i_{p}^{2}(t) \mathrm{d} t=\frac{Q_{\phi \tau}^{2} \cdot R}{\frac{1}{\tau} \int_{0}^{\tau} u^{2}(t) \mathrm{d} t}=\frac{P_{\tau}^{2} \cdot R}{\frac{1}{\tau} \int_{0}^{\tau} u^{2}(t) \mathrm{d} t} \cdot \operatorname{tg}^{2} \varphi, \tag{11}
\end{equation*}
$$

where $Q_{\Phi \tau}$ and $P_{\tau}$ are the reactive and active capacities consumed by loading $R$.

## 5. Methods for determining energy characteristics

It follows from expressions (1) to (6) and (11) that in order to determine capacities as energy characteristics and, consequently, the energy indicators themselves, it is necessary to know the time functions of transitional voltage $u(t)$ and current $i(t)$. These dependences could be obtained either from experimental research at existing energy sections of power grids or by modeling under laboratory conditions. The task of defining the components of full power based on non-sinusoidal curves $u(t)$ and $i(t)$ has already been solved by authors of papers [12, 13], but those were the studies for time dependences of continuous technological oscillations of rectifying voltage and current under steady modes for daily and monthly records. As evidenced by practical research, under transitional, and especially emergency, modes, dependences $u(t)$ and $i(t)$ are the single short, lasting $\tau$, nonperiodic pulses of arbitrary shape, for example, shown in Fig. 1. The following three methods are suggested to determine the energy characteristics of an electric traction system under modes with such pulses.


Fig. 1. Oscillographs of transitional feeder voltage $u(t)$ and current $i(t)$ when disabling a short circuit in a traction network with a high-speed switch the type of $A B-2 / 4$

A first one, referred to as an approximation method, implies the following. For the graphically (oscillographs) or tabular assigned transients $u(t), i(t)$ (we denote them with a single function $a(t)$ ) one finds the approximated analytical expressions, that is they are analytically approximated based on the convergence or interpolation criteria. The feature and importance of these mathematical operations when solving the set energy problems is that by approximating or inter-
polating these functions $f(t)$, they should most accurately reflect the transitional magnitudes $u(t)$, otherwise an important information would be lost that could lead to inaccurate results. Therefore, among all criteria of convergence known in mathematics, convergence at each point:

$$
a(t)=\lim _{k \rightarrow \infty} f_{k}(t) \text { in region }[0 \ldots \tau],
$$

uniform convergence:

$$
\lim _{k \rightarrow \infty}\left(\max \left|a(t)-f_{k}(t)\right|\right)=0
$$

convergence in mean square:

$$
\lim _{k \rightarrow \infty} \int\left|a(t)-f_{k}(t)\right|^{2} \mathrm{~d} t=0
$$

the most advisable for approximation is the first one, most rigid, criterion. The system of functions $f_{1}(t), f_{2}(t), \ldots, f_{K}(t), \ldots, f_{\infty}(t)$ in the analytical expressions of convergence criteria is the functions whose combination could represent a transition magnitude $a(t)$ (either voltage or current). When choosing a method of approximation for $a(t)$, complex in shape, preference should be given to approximation, since in this case the interpolation polynomial will be of very high order, which makes the calculations too cumbersome. Moreover, there is a possibility to build a polynomial in simple ways not to any transitional $a(t)$, which would ensure a uniform convergence to $a(t)$ (if necessary).

Consequently, the considered method makes it possible to determine energy indicators from expressions (1) to (6) and (11), in which capacities are assessed according to the formulae given below based on the approximated transitional $u(t), i(t)$, namely.

Instantaneous power in the interval [ $0 \ldots \tau$ ] of the existence of a transition magnitude pulse could be determined from:

$$
\begin{equation*}
p_{\tau}(t)=u(t) \cdot i(t) \tag{12}
\end{equation*}
$$

Active power over the same time interval could be determined from expression:

$$
\begin{equation*}
P_{\tau}=\frac{1}{\tau} \int_{0}^{\tau} p_{\tau} \mathrm{d} t=\frac{1}{\tau} \int_{0}^{\tau} u(t) \cdot i(t) \mathrm{d} t \tag{13}
\end{equation*}
$$

Full power is determined from:

$$
\begin{equation*}
S_{\tau}=U_{\tau} \cdot I_{\tau}, \tag{14}
\end{equation*}
$$

where $U_{\tau}, I_{\tau}$ are the actual values for voltage and current in the time interval [0... $\tau]$ :

$$
\begin{equation*}
U_{\tau}=\sqrt{\frac{1}{\tau} \int_{0}^{\tau} u^{2}(t) \mathrm{d} t} ; \quad I_{\tau}=\sqrt{\frac{1}{\tau} \int_{0}^{\tau} i^{2}(t) \mathrm{d} t} \tag{15}
\end{equation*}
$$

Determining a reactive power is possible through many methods and approaches, and this issue has remained debatable up to now. The most promising approach to determining this non-active component is the concept by S. Friese [11], which is aimed at maintaining the functional character of describing the energy properties of circles in sinusoidal and
non-sinusoidal processes. In this case, reactive power is determined by Friese from expression:

$$
\begin{equation*}
Q_{\phi \tau}=\sqrt{S_{\tau}^{2}-P_{\tau}^{2}} . \tag{16}
\end{equation*}
$$

In a second method, «discrete electrical engineering», the analog pulses of transient $u(t)$ and $i(t)$ (hereafter, same as $a(t)$ ), by sampling at a certain time interval $\Delta t$, are converted into pulses, which make up an array of $N$ values (counts) of the investigated magnitude $a(t)$ (Fig. 2) $a_{1}=a\left(t_{1}\right) ; a_{2}=a\left(t_{2}\right) ; \ldots, a_{n}=a\left(t_{n}\right) ; \ldots, a_{N}=a\left(t_{N}\right) ; n=1,2,3, \ldots, N$; $\Delta t=\tau / N$ is the sampling rate interval in real time, choosing which is a very important operation and it should be based on the character of change in the studied magnitude $a(t)$.

The latter may be obtained by analyzing the circuit of the examined device or system, the principle of its operation, requirements to $a(t)$, reciprocal relationships between the elements, etc. In the first approximation, the analog pulse-function $a(t)$ could be adequately represented by discrete counts, if its frequency does not exceed a half of the Nyquist frequency (sampling rate frequency). For accurate calculations, one could set the maximum (upper) $f_{B}$ and minimum (lower) $f_{H}$ frequencies in the spectrum of a transitional magnitude. This could be done based on your own experience or indirect prerequisites. After that, one could determine $\Delta t$ from the Kotelnikov theorem, according to which any function $a(t)$ whose spectrum does not contain any components with frequencies above a certain value $\Omega_{b}=2 \pi f_{b}$ could be, without losing information, represented by its discrete counts $a_{1}, a_{2}, \ldots a_{n}$, (Fig. 2), taken with interval $\Delta t$, which is determined from the following inequality:

$$
\begin{equation*}
\Delta t \leq \frac{1}{2 f_{B}}=\frac{\pi}{\omega_{B}} . \tag{17}
\end{equation*}
$$

Studies also show that the effectively correct result regarding $\Delta t$ in the analysis of changes in voltage and loading currents is produced by estimation formula:

$$
\Delta t=(0,1 \ldots 0,25) T_{1}
$$

where $T_{1}$ is the smallest average time period for a device or system operation.

Next, after determining $\Delta t$ sampling rate pulses of transitional voltage $u(t)$ and current $i(t)$, and determining their counts, according to the considered method and Fig. 2, the required capacities are calculated from [12].

Active power $P_{\tau}$, according to (13), is found as the arithmetic mean over interval [ $0 \ldots \tau$ ]:

$$
\begin{equation*}
P_{\tau}=\frac{\sum_{n=1}^{N} u_{n} \cdot i_{n}}{N} \text { or } P_{\tau}=\frac{\sum_{n=1}^{N} p_{n}}{N}, \tag{18}
\end{equation*}
$$

where $u_{n}, i_{n}, p_{n}$ are the $n$-th value of voltage, current or instantaneous power; $N$ - total number of sampling points over time [ $0 \ldots . . \tau$ ].

At first glance, the active power $P_{\tau}$ for the considered modes could be determined as a product of the voltage $U_{c p}$ and current $I_{c p}$, averaged for the time of energy consumption by the system:

$$
P_{c p}=U_{c p} \cdot I_{c p},
$$

where

$$
U_{c p}=\frac{1}{N} \sum_{n=1}^{N} u_{n}, \quad I_{c p}=\frac{1}{N} \sum_{n=1}^{N} i_{n} .
$$

However, the values for power $P_{\tau}$ will be less accurate in this case, because it is then determined only by the power of zero harmonic (assuming the possibility of expanding $u(t)$ and $i(t)$ into a Fourier series) because $U_{c p}=U_{(0)}, I_{c p}=I_{(0)}$.

To determine the full power $S_{\tau}$, formula (14) is also used, in which the actual values for $U_{\tau}$ and $I_{\tau}$ are found as mean squares over $\tau$ from discrete values (Fig. 2):

$$
\begin{equation*}
U_{\tau}=\sqrt{\frac{\sum_{n=1}^{N} u_{n}^{2}}{N}} \tag{19}
\end{equation*}
$$

$$
\begin{equation*}
I_{\tau}=\sqrt{\frac{\sum_{n=1}^{N} i_{n}^{2}}{N}} \tag{20}
\end{equation*}
$$



Fig. 2. Discretization of a continuous pulse of transition magnitude $a(t)$

The reactive power by Friese is determined from formula (16), and the energy indicators from expressions (1) to (6) and (11).

A third method, which could be referred to as the method of digital spectral analysis, is based on the harmonic analysis of pulses from transient voltage and current, performed with the help of a discrete Fourier transform. To do this, instead of an actually obtained pulse of the transition magnitude $a(t)$ (the type of Fig. 1, 2), we shall imaginary form the periodic ones - (with an arbitrary period $T$ ) the sequence of such pulses (Fig. 3).

Now, the non-sinusoid functionpulse $a(t)$ can be regarded not in the interval $[0 \ldots \tau]$ of its existence, but continued periodically beyond the interval $[0 \ldots \tau]$. That is, the non-periodic pulse-function $a(t)$ has been transformed
into a periodic with period $T$, for which an expansion into a Fourier series holds in a real classic form:

$$
\begin{equation*}
a(t)=A_{m(K)} \sin \left(k \omega t+\psi_{a(K)}\right), \tag{21}
\end{equation*}
$$

where $A_{m(K)}, \Psi_{a(K)}$ are the amplitude and initial phase of the $k$-th harmonic in the series. They are determined from a complex amplitude:

$$
\underline{A}_{m(K)}=A_{m(K)} e^{-j \psi_{q(K)}},
$$

derived from a known expression:

$$
\begin{equation*}
\underline{A}_{m(K)}=\frac{2}{T} \int_{0}^{T} a(t) e^{-j k o t} \mathrm{~d} t . \tag{22}
\end{equation*}
$$

However, the function $a(t)$ is non-sinusoidal and arbitrary, often very complex in shape, so using the classical Fourier analysis for its spectral analysis is practically complicated, which is why it is advisable to use a discrete Fourier transform.


Fig. 3. Formation of a periodic pulse-function $a(t)$
For this purpose, similar to a second method, we perform discretization, at interval $\Delta t$, of the pulse-function $a(t)$, as it was done in Fig. 2. Then the values $a_{n}=a(n \cdot \Delta t)$ are the counts of the already periodic (Fig. 3) analog function $a(t)$ in the form of a sequence of delta function, «weighted» by counts $a(n \cdot \Delta t)$ of the analog function $a(t)$ (Fig. 4):

$$
\begin{equation*}
a(t)=\sum_{n=1}^{N} a(n \cdot \Delta t) \delta(t-n \cdot \Delta t) \tag{23}
\end{equation*}
$$



Fig. 4. Discretized pulse-function $a(t)$ in the form of a sequence of delta functions

By substituting (23) in (22), we obtain:

$$
\begin{equation*}
\underline{A}_{m(K)}=\frac{2}{T} \int_{0}^{T} \sum_{n=1}^{N} a(n \cdot \Delta t) \delta(t-n \cdot \Delta t) \cdot e^{-j k o t} \mathrm{~d} t . \tag{24}
\end{equation*}
$$

Because $a(n \cdot \Delta t)$ are constants (independent of $t$ ), and the function $\Delta(t-n \cdot \Delta t)$ is zero at any $t$ except for $t=n \cdot \Delta t$, then (24) could be rewritten in the form:

$$
\begin{equation*}
\underline{A}_{m(K)}=\frac{2}{T} \sum_{n=1}^{N} a(n \cdot \Delta t) \int_{0}^{T} \sum_{n=1}^{N} \delta(n \cdot \Delta t) e^{-j k o t} \mathrm{~d} t . \tag{25}
\end{equation*}
$$

Take into consideration the filtering property of a delta function, which implies that if this function is present under the integral as a multiplier, the result from integration would be equal to the value of another sub-integral function (or expression) at the point (time) where a delta-function is concentrated, regardless of the boundary of integration. Then expression (25) takes the form:

$$
\begin{equation*}
\underline{A}_{m(K)}=\frac{2}{T} \sum_{n=1}^{N} a(n \cdot \Delta t) e^{-j k \operatorname{cost}} . \tag{26}
\end{equation*}
$$

In (26), the spectrum is discrete with a frequency spacing between harmonics, which, according to Fig. 4 equals:

$$
\begin{equation*}
\omega=\frac{2 \pi}{T}=\frac{2 n}{N \cdot \Delta t} . \tag{27}
\end{equation*}
$$

Considering (27) and that $a(n \cdot \Delta t)$ is the value of counts and, therefore, $a(n \cdot \Delta t)=a(n)$, expression (26) could be written in the form:

$$
\begin{equation*}
\underline{A}_{m(K)}=\frac{2}{N \cdot \Delta t} \sum_{n=1}^{N} a(n) e^{-j \frac{2 \pi v \mathrm{k}}{N}} . \tag{28}
\end{equation*}
$$

Thus, the complex amplitude of the discrete Fourier series represents a linear combination of counts $a(n)$ of the discretized pulse of the transition magnitude $a(t)$.

In expression (28), the real time scale appears only in the $1 / \Delta t$ multiplier before the addition operator. In the analysis of discrete sequences, one typically operates the numbers of counts (1, $2, \ldots, n, \ldots, N$, Fig. 2, 4) and spectral harmonics without reference to an actual scale, time, and frequency. Therefore, the multiplier $1 / \Delta t$ shall be removed from expression (28), that is, we consider the frequency of sampling rate equal to unity. And then the final expression for the complex amplitude of the «k»>-th harmonic will equal:

$$
\begin{align*}
& \underline{A}_{m(K)}=\frac{2}{N} \sum_{n=1}^{N} a(n) e^{-j \frac{2 \pi n k}{N}}= \\
& =A_{m(K)} \cdot e^{j \Psi_{q(K)}} \tag{29}
\end{align*}
$$

which is the expression of the discrete Fourier transform of the analog pulsefunction $a(t)$.

After determining $A_{m(K)}$, and $j \psi_{a(K)}$ based on t his expression, and upon
recording a Fourier series in line with expression (21), the capacities $P_{\tau}$ and $S_{\tau}$ are found from known classical formulae from the theory of non-sinusoidal current circuits, namely:

- active power:

$$
P_{\tau}=\sum_{K=0} U_{m(K)} \cdot I_{m(K)} \cdot \cos \varphi_{(K)}
$$

- full power:

$$
S_{\tau}=U_{\tau} \cdot I_{\tau},
$$

where

$$
\begin{aligned}
& U_{\tau}=\sqrt{U_{(0)}^{2}+U_{(1)}^{2}+U_{(2)}^{2}+\ldots+U_{(K)}^{2}+\ldots} ; \\
& I_{\tau}=\sqrt{I_{(0)}^{2}+I_{(1)}^{2}+I_{(2)}^{2}+\ldots+I_{(K)}^{2}+\ldots}
\end{aligned}
$$

The reactive power is determined by Friese from formula (16), and the energy indicators - also from expressions (1) to (6) and (11).


Fig. 5. Oscillograms of change in transitional electric magnitudes when disabling HSS $2 \times$ VAB-43 at close $\mathrm{SC}, ~ /=0.5 \mathrm{~km}$, the setpoint current $I_{y}=3,500 \mathrm{~A}$ : $a$ - time dependences of feeder current $i(t)$ and voltage $u(t)$; $b$ - voltage on the electric arc of a high-speed circuit breaker $u_{\text {arc }}(t)$


Fig. 6. Oscillograms of transitional electric magnitudes when disabling HSS $2 \times$ VAB- 49 at close SC, $I=0.5 \mathrm{~km}, I_{y}=3,500 \mathrm{~A}: a$ - time dependences of feeder current $i(t)$ and voltage $u(t) ; b$-voltage on the electric arc of a high-speed circuit breaker $u_{\text {arc }}(t)$


Fig. 7. Oscillograms of transitional electric magnitudes when disabling HSS VAB-206 at close SC, $I=0.5 \mathrm{~km}, I_{y}=4,000 \mathrm{~A}: a$ - time dependences of feeder current $i(t)$ and voltage $u(t) ; b$ - voltage on the electric arc of a high-speed circuit breaker $u_{\text {arc }}(t)$


Fig. 8. Oscillograms of transitional electric magnitudes when disabling HSS VAB-206 at close SC, $I=2 \mathrm{~km}, I_{y}=3,000 \mathrm{~A}$

The following three methods are used to determine the energy indicators for an electric traction system under emergency modes with their voltage and current pulses: an approximation method, a discrete electrical engineering method, a spectral analysis method.

By using the specified methods, numerical calculation of indicators for energy exchange processes of electricity in
a traction power system for the emergency SC modes has been performed. The best results that are given in Table 1 were produced by the second method. Table 2 gives the established power and energy indicators for the DC arcing chambers HSS. The source data are the oscillographs (Fig. 5-8), as well as the source and traction network parameters.

It follows from the analysis of Table 1 that increasing the distance from the point of SC to TP the power $S_{\tau}, P_{\tau}, Q_{(1)}$, $Q_{\Phi \tau}$ of short circuit decreases in almost all cases although certain irregularities manifest themselves. Parameters $D, \lambda$, $\operatorname{tg} \varphi$ also do not determine the SC mode unambiguously. Even though the distance to the place of SC is growing, and consequently increases the resistance magnitude $R_{\text {区 }}$, in all cases additional losses of electricity $\Delta W_{d \tau}$ are reduced. The reason for this phenomenon is that under an SC mode the distance is not decisive, because the magnitudes of transitional $u(t)$ and $i(t)$ characterize the instantaneous value of power.

It follows from the analysis of Table 2 that during the switching period, prior to opening the contacts of the protective device, all power of SC passes through the switching unit. At the initial moment of opening the HSS contacts, the voltage drop on the contacts themselves is infinitesimal relative to the current of SC, which creates the effect of the current phase advance. This predetermines the changing phenomenon of a significant flow of reactive power with a negative sign.

Table 1
Main energy indicators of the SC mode in a direct current traction network

| Type of switch (line) | SC type |  | Energy indicators |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $S_{\tau} \cdot 10^{6}, \mathrm{VA}$ | $P_{\tau} \cdot 10^{6}, \mathrm{~W}$ | $Q_{(1)} \cdot 10^{6}, \mathrm{VAr}$ | $Q_{\Phi \tau} \cdot 10^{6}$, VAr | $D \cdot 10^{6}, \mathrm{VA}$ | $\lambda$ | $\operatorname{tg} \varphi$ | $\Delta W_{d \tau} \cdot 10^{6}, \mathrm{~W}$ |
| VAB-43 (Line 1) | 1 | Close | 19.5 | 10.5 | -3.1 | 16.4 | 16.4 | 0.48 | 1.55 | 9.4 |
|  | 2 | Medium | 12.3 | 9.4 | -0.56 | 8.0 | 8.0 | 0.76 | 0.85 | 2.4 |
|  | 3 | Far | 10.4 | 8.4 | -0.5 | 6.2 | 6.2 | 0.803 | 0.74 | 1.4 |
| VAB-49 (Line 2) | 4 | Close | 16.4 | 9.6 | -2.1 | 13.3 | 13.3 | 0.47 | 1.39 | 6.6 |
|  | 5 | Medium | 9.4 | 7.8 | -0.15 | 5.2 | 5.2 | 0.83 | 0.67 | 1.7 |
|  | 6 | Far | 8.6 | 7.4 | -0.2 | 4.5 | 4.5 | 0.86 | 0.60 | 1.3 |
| VAB-206 (Goryainovo) | 7 | Close | 5.5 | 3.1 | 0.43 | 4.4 | 4.4 | 0.9 | 1.42 | 8.1 |
|  | 8 | Far | 9.0 | 4.9 | 3.09 | 7.6 | 7.6 | 0.55 | 1.53 | 5.7 |
| VAB-206 (ND Vuzol) | 9 | Close | 13.1 | 7.9 | -2.43 | 10.4 | 10.4 | 0.25 | 1.31 | 4.4 |

Main energy indicators of arcing chamber of HSS under SC mode in a direct current traction network

| Type of switch | SC type |  | Energy indicators |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $S_{\tau} \cdot 10^{6}$, VA | $P_{\tau} \cdot 10^{6}, \mathrm{~W}$ | $Q_{(1)} \cdot 10^{6}$, VAr | $Q_{\Phi \uparrow \tau} \cdot 10^{6}, \mathrm{VAr}$ | $D \cdot 10^{6}$, VA | $\lambda$ | $\operatorname{tg} \varphi$ |
| VAB-43 | 1 | Close | 19.5 | 10.5 | -2.9 | 20.2 | 20.2 | 0.36 | 2.6 |
|  | 2 | Medium | 12.3 | 9.4 | -5.9 | 15.6 | 15.6 | 0.42 | 2.2 |
|  | 3 | Far | 10.4 | 8.4 | -0.51 | 13.0 | 13.0 | 0.38 | 2.4 |
| VAB-49 | 4 | Close | 16.0 | 5.9 | -2.1 | 14.9 | 14.9 | 0.37 | 2.52 |
|  | 5 | Medium | 9.7 | 4.8 | -1.8 | 8.4 | 8.4 | 0.49 | 2.76 |
|  | 6 | Far | 11.1 | 5.2 | -0.21 | 9.8 | 9.8 | 0.47 | 1.9 |
| VAB-206 (Goryainovo) | 7 | Close | 18.4 | 1.8 | -0.5 | 16.6 | 16.6 | 0.44 | 2.06 |
|  | 8 | Far | 13.9 | 6.6 | -3.11 | 12.2 | 12.2 | 0.48 | 1.85 |

It is also evident that the active short-circuit power, which is eliminated by obsolete samples of feeder switches, is much higher in comparison with new high-speed devices. Thus, during the time of SC in a traction network, the contact wire would be warmer. Instead, a single arcing chamber of the new types of HSS scatters about the same amount of energy as the two chambers, for example, the automatic type of VAB-43. The reactive power of the SC process is the power of distortion.

## 7. Discussion of results of calculating energy indicators as signs of short circuit in a direct current traction network

The uncertain form of transitional electromagnetic magnitudes, not defined in advance, complicates the process of scientific and practical research, so we conducted a critical analysis of three methods for calculating such magnitudes. The most suitable method for the analysis of transient electromagnetic magnitudes of random form within this study was the so-called discrete method. A given method of discrete electrical engineering is applied to analyze emergency electromagnetic processes in a DC power network. In line with this method, analog electromagnetic magnitudes $a(t)$, of free form, are quantized; they are subsequently treated with methods of discrete electrical engineering. The key result of the current paper is formula (29), which makes it possible to decompose the transient electromagnetic magnitudes of current $i(t)$ and voltage $u(t)$ of arbitrary form into harmonic components. By using formulae from the theory of nonsinusoidal current circuits, we determined the active (18), full (14), reactive (16) capacities, as well as energy indicators (1) to (6) and additional losses of electricity (11) during a SC. The proposed procedure could be used not only in an electric traction network, but also for any electrical circuits.

The problem in detecting and recognizing short circuits is an unambiguous dependence between the emergency and standard modes of an electric traction network (for example, the movement of a train with a load on a steering slope), when power currents of electric rolling stock could even exceed values for short circuit currents. The results of calculations, given in Table 1, show that during the existence of a SC the magnitude of additional losses $\Delta W_{d \tau}$ has a definite dependence on distance $l$ to the point of short circuit. Other energy indicators from Table 1 do not produce such a uniqueness. Therefore, further work implies a deeper research into
comparison of additional losses under emergency and operational modes. This comparison would make it possible to design microprocessor protective equipment based on new principles.

As regards the first method for determining the transition function $a(t)$ or $i(t)$ and $u(t)$, its accuracy is significantly dependent on the selected mathematical techniques of approximation.

The disadvantage of the second method for determining transitional function $i(t)$ and $u(t)$ is the dependence of calculation accuracy on the sampling rate degree. In turn, increasing the degree of sampling will affect the computational power for time spent on electronic protective equipment.

For the third method, a double transformation of current $i(t)$ and voltage $u(t)$ in the sampling process and the subsequent discrete Fourier transform affects the calculation accuracy.

This study employed the simplest methods of approximation and discrete Fourier transform, which do not guarantee a high accuracy of calculations. They were used solely to test a possibility of using the proposed methods for determining the power of SC and its energy indicators.

## 8. Conclusions

1. We have derived an analytical representation of additional (unproductive) losses in a traction power grid based on the concept by Friese. The main difference in determining these losses is the use of the instantaneous values for voltage and the active and reactive currents of the network.
2. Three methods for determining network capacities under an SC mode have been proposed on the basis of transient functions of voltage and current as the basic parameters for determining energy indicators and additional losses.
3. It has been established that the indicator for additional power losses $\Delta W_{d \tau}$ could be used as a sign of the SC mode in the line. Other energy parameters do not uniquely determine the mode of traction network operation.

The process of disabling SC current in a DC traction network relative to the switching unit could be characterized as a changing process of enabling the capacitance loading of considerable power.

Based on experimental data, it has been determined that the active short circuit power, which is eliminated by obsolete samples of feeder switches, is considerably higher in comparison with new high-speed devices.

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