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SIMULATION OF THE DISCRETE TRAFFIC FLOW MOVEMENT THROUGH NETWORK NODE

1 Problem statement

Most of the models for traffic phenomena study have been made under the framework of the Lighthill-Whitham-Richards traffic flow model [1]. But these models are ill-adopted for description of the interrupted flow (e.g. movement through intersection) [2]. Thus it is obvious that distribution of gaps between flow elements has significant influence on intersection capacity [3]. At the same time complex simulation packages with powerful possibility for prediction of traffic flow movement need high performance computing and many parameters hard to obtain. In this paper transformation process of traffic flow during diverge is considered in the scope of the stochastic model. This model takes into consideration traffic flow characteristic distribution (namely, time gap distribution, where time gap is time interval between rear of the preceding element and front of the successive element). Stochastic models allow describe significant changes during movement through intersections without great loss of simplicity and are studied by many authors [4-7].

In work **[7]** diverge process with one input and two output traffic flows is considered and it is proved that if time gap distribution of input flow $\rho_0(t)$ has exponential distribution than time gap distribution of output flow $\rho_0(t)$ also has exponential distribution. But the analytical expressions for output flow distribution which are of practical interest can be obtained for limited case of input flow time gap distributions. That's why simulation is needed. In simulation and data processing, we are forced to move from continuous to discrete models. The question of property retention of continuous models during transition to its discrete representation is arisen. In this work it is shown that if exponential distribution is reproducible during diverge process, than corresponding discrete distribution, formed during sampling process is also reproducible. The results are implemented in traffic flow simulation system SFMS [7]. These results are basic for the analysis of flows in complex transportation systems.

2 Traffic flow characteristics transformation during diverge

The transportation systems consist of the traffic flow and the infrastructure: transport channel network and operational components. Each component of the transport network is characterized by set of rules that specify the traffic flow element interactions between themselves and with this component. Let's examine the node of the transport network with one input flow and two output flows in the case of diverge. In general in the process of flow diverging input flow φ_0 divides into k flows. As an output flow φ_1 one can select any of the k flows, other flows are considered as single output flow φ_2, which contains all elements of φ_0 not included in φ_1. Therefore, without loss of generality it is possible to consider the process of flow diverge model shown in Fig. 1

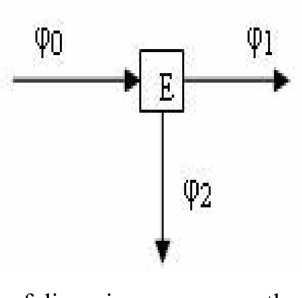

Fig.1 Model of diverging process on the intersection

Suppose that the elements during diverging process drop out from the input flow with a probability of w and there is no correlation between probabilities of input flow element elimination. The probability density function of time gap between traffic flow element in input flow φ_0 is $\rho_0(t)$ and in output flow φ_1 is $\rho_0(t)$, size of flow elements d=0. In this case the relationship between input flow and output flow time gap distributions can be expressed as (1). The description how to obtain this formula is presented in [7].

$$\rho_{\varphi_1}(t) = (1-w)\sum_{j=0}^{\infty} w^j \rho_{j+1}(t) \quad (1)$$

where $\rho_{j+1}(t)$ - time gap distribution determined as sum of j+1 variables distributed with $\rho_0(t)$

We can modify (1) taking into consideration the flow element size. Let's assume that size of elements in time dimension is constant value $d=d_1$ (it allows not to consider physical size and speed of flow elements) and obtain:

$$\rho_{\varphi_1}(t) = (1-w)\sum_{j=0}^{\infty} w^j \rho_{j+1}(t - jd_1) \qquad (2)$$

where $\rho_{j+1}(t)$ - time gap distribution determined as sum of $j+1$ variables distributed with $\rho_0(t)$ shifted on $j*d_1$ to the right.

It is not hard to obtain the dependence of the mean time gap of the output flow h_1 from element size $d=d_1$ and rules of the diverge process and mean gap of input flow h_0

$$h_1 = (h_0 + wd_1)/(1-w) \qquad (3)$$

The expression (3) can be used to obtain the influence of the element size on the output flow.

3 Property retention of continuous models during transition to its discrete representation

It is interesting to find parametric distribution that is not change its type during diverge process. In [6] May proposed to use the exponential distribution for approximation time gap distribution for low level flow rate. Also it is shown in [7] that the time gap distribution of φ_2 flow at large time intervals t has the asymptotic behavior, which can be well approximated by an exponential dependence. Let's consider the case when time gap distribution of input flow φ_0 has the exponential distribution with parameter $\lambda = 1/\theta$. Using expression (1) we can obtain a formula for output flow time gap distribution. The probability distribution function for time gap between elements of output flow is described by (4). The proof is presented in [7].

$$\rho_{\varphi_1}(t) = (1-w)\frac{1}{\theta}e^{-\frac{t}{\theta}(1-w)} \qquad (4)$$

Note that the result is valid only under the assumption that the elements of the input flow have zero size in time dimension.

In simulation and data processing, we are forced to move from continuous to discrete models. Let's make a histogram from exponential distribution with Δt - width of intervals and left boundary $k_0 = 0$ (Fig. 2).

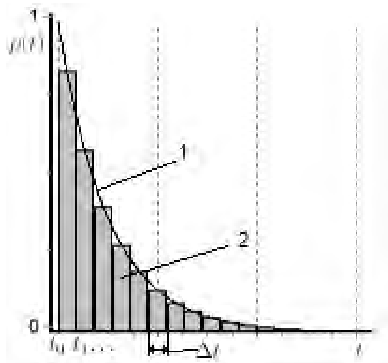

Fig.2 Probability density function of exponential distribution discretized into partitions of equal intervals Δt

The area under probability density function in i interval with k_i and k_{i+1} ($k_{i+1}=k_i+\Delta t$) boundaries can be expressed as:

$$p_i = e^{-\lambda k_i}(1-e^{-\lambda \Delta t}) \qquad (5)$$

As it not hard to obtain:

$$p_n = p_0 e^{-n\lambda\Delta t} \qquad (6)$$

Using that $p_0 = (1-e^{-\lambda\Delta t})$:

$$p_n = e^{-n\lambda\Delta t}(1-e^{-\lambda\Delta t}) \qquad (7)$$

where n - number of interval and Δt - its width. The (7) is a geometrical distribution with parameter $p=\exp(-\lambda\Delta t)$. So, any uniform partition of exponential distribution can be presented as geometrical distribution, we use this fact during following discussion.

Let's show that geometric distribution is reproducible during diverge process. It is known that if Y is geometrically distributed value with parameter p, then n sum of Y is distributed with negative binomial law with parameters *n* and *p*. Using (1) let's obtain formula for output flow distribution, when input flow has geometric distribution of time gaps:

$$\rho_1 = (1-w)(1-p)p^k \frac{1}{k!}\sum_{i=0}^{\infty}(i+1)(i+2)...(i+k)(w(1-p))^i \tag{8}$$

Using k-derivation of series of geometric progression (9)

$$\sum_{i=0}^{\infty} i(i-1)(i-2)...(i-k+1)q^{i-k} = \frac{k!}{(1-q)^{k+1}} \tag{9}$$

in (8) we can obtain (10)

$$\rho_1 = (1-\frac{p}{1-w(1-p)})(\frac{p}{1-w(1-p)})^k \tag{10}$$

So, the time gap distribution of output flow is geometrically distributed with parameter $\frac{p}{1-(w(1-p))}$. And that's why histogram with uniform sampling interval created from exponential distribution is reproducible during flow diverge process.

The analytical expressions for output flow distribution which are of practical interest can be obtained for limited case of input flow time gap distributions. Thus authors developed software SFMS (Stochastic Flow Modeling System) [7] which allows simulate traffic flow diverge process using Monte Carlo method in the scope of stochastic model. The basic algorithm for diverge flow does not depend on the time gap distribution of input flow; it is possible to vary input flow distribution type depending on the requirements of the problem. We are forced to move to discrete representation of the continuous distribution. On the Fig. 3 it is shown results of computer simulation of diverge process when input flow has exponential distribution with zero element size and corresponding analytical results (1 – generated exponential distribution for input flow and its corresponding analytical probability density function, 2 – output flow with and curve for function calculated using formula (4)).

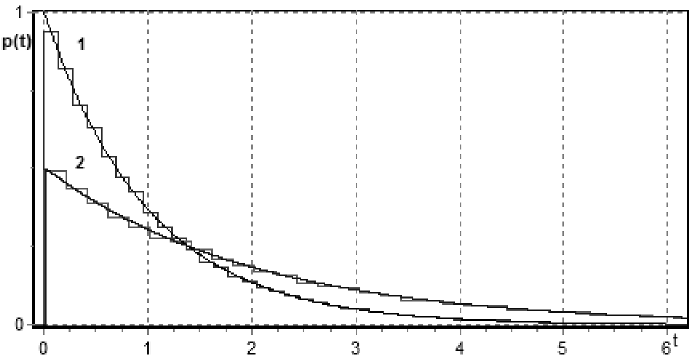

Fig. 3 Results of computer simulation of diverge process (1 – input flow, 2- output flow when w = 0.5) and its corresponding function calculated using (4)

One can see on Fig. 3 that analytical and simulation results are similar to each other.

4 Results and conclusions

In this paper model for diverging process of traffic flow is presented. Analytical expression for time gap distribution of output flow is obtained for system with finite element size. The impact of element size on time gap distribution of output flow is studied. It is shown that any histogram created from probability density function of exponential distribution with uniform width of intervals and left boundary k_0= 0 can be presented as geometric distribution. The reproducibility of geometric distribution on diverge process is proved. Computer simulation program which implements these results is presented. The comparison of mathematical analysis and simulation of the diverge process are presented. These results are basic for the analysis of flows in complex transportation systems for empiric data processing in the scope of stochastic model and discrete traffic flow simulation.

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