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SOME PROPERTIES OF DISCRETE SIGNALS OF CONTINUOUS PHASE FREQUENCY MODULATION

Discrete signals of continuous phase frequency modulation (CPFM) in comparison with known signals of digital modulation are characterized by the following advantages: constancy of envelope and absence of phase jumps of the modulated signal; absence of parasitic peak modulation when passing through narrow-band paths; compactness of a spectrum and small level of out-of-band emissions; high indicators of power and frequency efficiency. Due to these advantages CPFM signals are widely applied in systems of ground and satellite mobile communication.

Keywords: continuous phase frequency modulation, LRC, phase trellis, differential modulation.

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НЕКОТОРЫЕ СВОЙСТВА ДИСКРЕТНЫХ СИГНАЛОВ ЧАСТОТНОЙ МОДУЛЯЦИИ С НЕПРЕРЫВНОЙ ФАЗОЙ

Дискретные сигналы частотной модуляции с непрерывной фазой (ЧМНФ) по сравнению с известными сигналами цифровой модуляции характеризуются следующими преимуществами: постоянство огибающей сигнала и отсутствие скачков фазы модулированного сигнала; отсутствие паразитной амплитудной модуляции при прохождении через узкополосные тракты; компактность спектра и малый уровень внеполосных излучений; высокие показатели энергетической и частотной эффективности. Благодаря этим преимуществам сигналы ЧМНФ широко применяются в системах наземной и спутниковой мобильной связи.

Ключевые слова: частотная модуляция с непрерывной фазой, LRC, фазовая решётка, дифференциальная модуляция.

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ДЕЯКІ ВЛАСТИВОСТІ ДИСКРЕТНИХ СИГНАЛІВ ЧАСТОТНОЇ МОДУЛЯЦІЇ З НЕПЕРЕРВНОЮ ФАЗОЮ

Дискретні сигнали частотної модуляції з неперервною фазою (ЧМНФ) у порівнянні з відомими сигналами цифрової модуляції характеризуються наступними перевагами: сталість обвідної сигналу й відсутність стрибків фази модульованого сигналу; відсутність паразитної амплітудної модуляції при проходженні через вузькосмугові тракти; компактність спектра й малий рівень позасмугових випромінювань; високі показники енергетичної та частотної ефективності. Завдяки цим перевагам сигнали ЧМНФ широко застосовуються в системах наземного й супутникового мобільного зв'язку.

Ключові слова: частотна модуляція з неперервною фазою, LRC, фазова решітка, диференційна модуляція.

Problem statement

In the works listed below it is not paid attention to some important properties of CPFM signals, such as invariance and possibility of the mathematical description by differential model.

Analysis of published data

Temporal and spectral properties of CPFM signals with full and partial responses are presented in monographs [1, 2]. More detailed information can be found also in publications [3, 5].

Formulation of research objectives

The article task is statement of the important properties of CPFM signals.

Presentation of the main research

Mathematical description of CPFM signals

The discrete signal of CPFM looks like [2]:

$$S(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_0 t + \varphi(t) + \varphi_0\right), \qquad (1)$$

where the current phase is defined by the expression

$$\varphi(t) = 2\pi h \sum_{i=-\infty}^{\infty} (2\alpha_i - m + 1) g_{\varphi}(t - iT) . \quad (2)$$

Here E – energy of a signal with duration *of* T, f_0 both φ_0 – frequency and an initial phase; h – modulation index; m – the alphabet of modulating symbols $\alpha \in \{ 0..., m-1 \}$.

From expressions (1) and (2) follows that sequences of signals, S are not linear functions of sequences of modulating symbols α. The phase of a signal (2) during the discrete moments of time of $t_{k+1} = (k+1)T$ corresponding to the termination of k-th interval is possible to present as:

$$\varphi(t_{k+1}) = 2\pi h \sum_{i=-\infty}^{\infty} (2\alpha_i - m + 1) g_{\varphi}((k - i + 1)T) . \quad (3)$$

In the simplest case phase function $g_{\varphi}(t)$ of CPFM signal with a partial response looks like (fig. 1):

$$g_{\varphi}(t) = \begin{cases} 0 & , \quad t < 0, \\ \dfrac{t}{2LT} & , \quad 0 \le t \le LT, \\ \dfrac{1}{2} & , \quad t > LT. \end{cases} \quad (4)$$

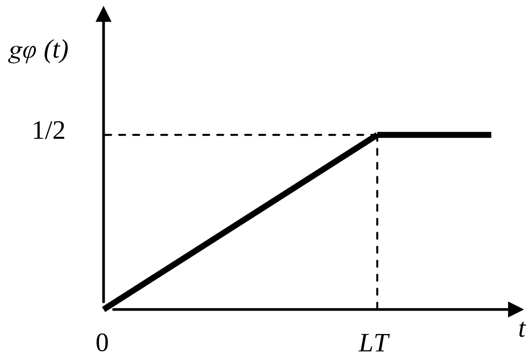

Fig. 1. Phase function of a CPFM signal

The type of the phase function used in a formula (4) will be

$$g_{\varphi}(k - i + 1)T = \begin{cases} 0 & , \quad i > k + 1, \\ \dfrac{k - i + 1}{2L} & , \quad k + 1 - L \le i \le k + 1. \\ \dfrac{1}{2} & , \quad i < k + 1 - L \end{cases} \quad (5)$$

For the description of CPFM signals it is usually set a form of a frequency impulse of $q_f(t)$ which is connected with a phase impulse by a known ratio

$$q_f(t) = \frac{d}{dt} g_{\phi}(t). \quad (6)$$

A classification of types of the frequency impulses defining names of the corresponding CPFM signals is given in table 1 below.

Fig. 2 shows the forms of frequency and phase impulses of CPFM signal with *L*RC (raised cosine pulse of length *L* symbol intervals) smoothing. It should be noted that the *L*RC smoothing of a frequency impulse is most often considered in theoretical researches and finds application in practice.

Typical forms of continuous phase and frequency of the *L*RC CPFM signal when transferring a sign-variable sequence (+1,–1,–1, +1,–1, +1,–1,–1,–1,–1) are shown on fig. 3.

Table 1

Frequency impulses of CPFM signals

Designation	Frequency impulse *qf*(*t*)
*L*RC – a raised cosine on *LT* interval	$q_f(t) = \frac{1}{2LT}[1-\cos(\frac{2\pi t}{LT})], 0 \le t \le LT$
GMSK – an FM with the minimum frequency shift and a Gaussian impulse smoothing	$q_f(t) = \frac{1}{2T}[Q(2\pi B_b \frac{t-\frac{T}{2}}{\sqrt{\ln 2}}) - Q(2\pi B_b \frac{t-\frac{T}{2}}{\sqrt{\ln 2}})],$ $Q(t) = \int_t^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{\tau^2}{2}} d\tau$
*L*REC – a rectangular frequency impulse on *LT* interval	formula (4)

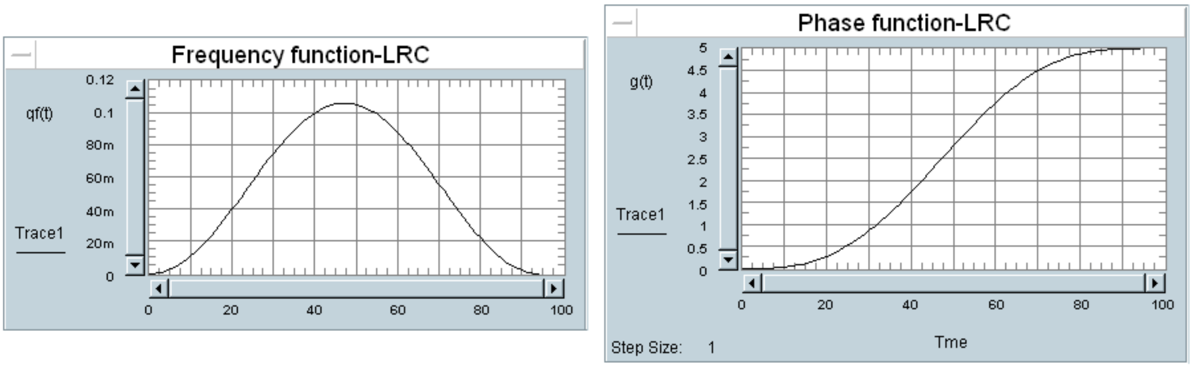

Fig. 2. *L*RC frequency and phase impulses

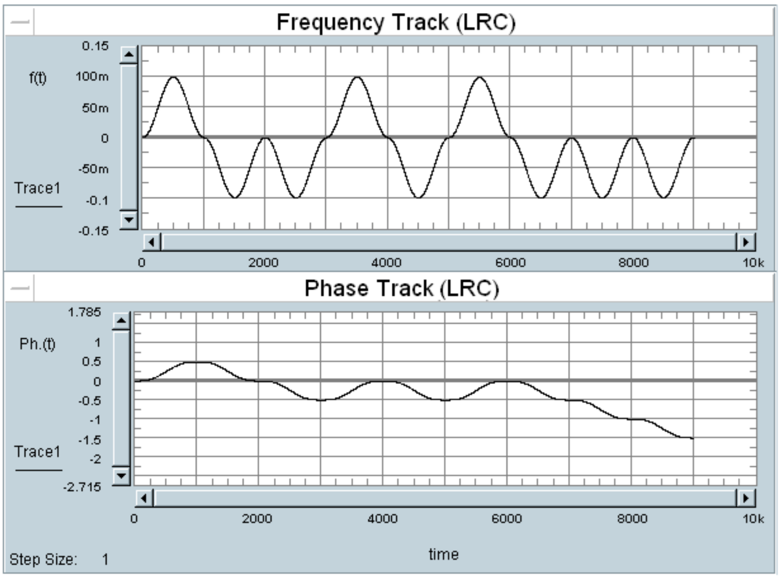

Fig. 3. Frequency and phase of CPFM signal with *L*RC impulse

Spectrum of a CPFM signal

The phase smoothing of a CPFM signal and absence of discontinuities of the phase function provide fast reduction in power-of-band energy spectrum. In [3] it is noted that if *p* – number of continuous derivatives of a phase function of a signal, then the energy spectrum of signal with a huge detuning is proportional to size f^{2p-4}. The spectrum of the minimum shift keying (MSK) modulation signals is most investigated. MSK energy spectrum looks like

$$G(f) = \frac{16PT}{\pi^2}\left(\frac{\cos 2\pi fT}{1-16f^2T^2}\right)^2, \qquad (7)$$

where P – average power of the signal.

Typical one-sided power spectrum of MSK is shown on fig. 4.

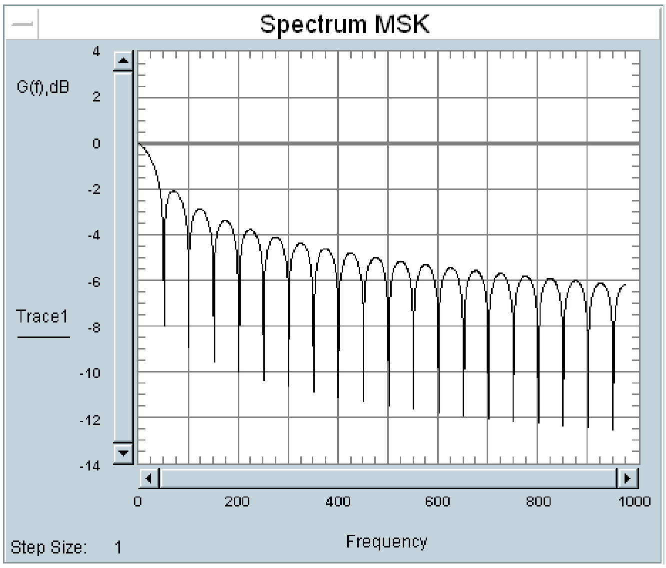

Fig. 4. One-sided power spectrum of MSK

Trellised structure of CPFM phase trajectories

The set of CPFM signal phase trajectories forms a phase trellis on the phase/time plane. An example of such trellis for LRC CPFM is given on fig. 5.

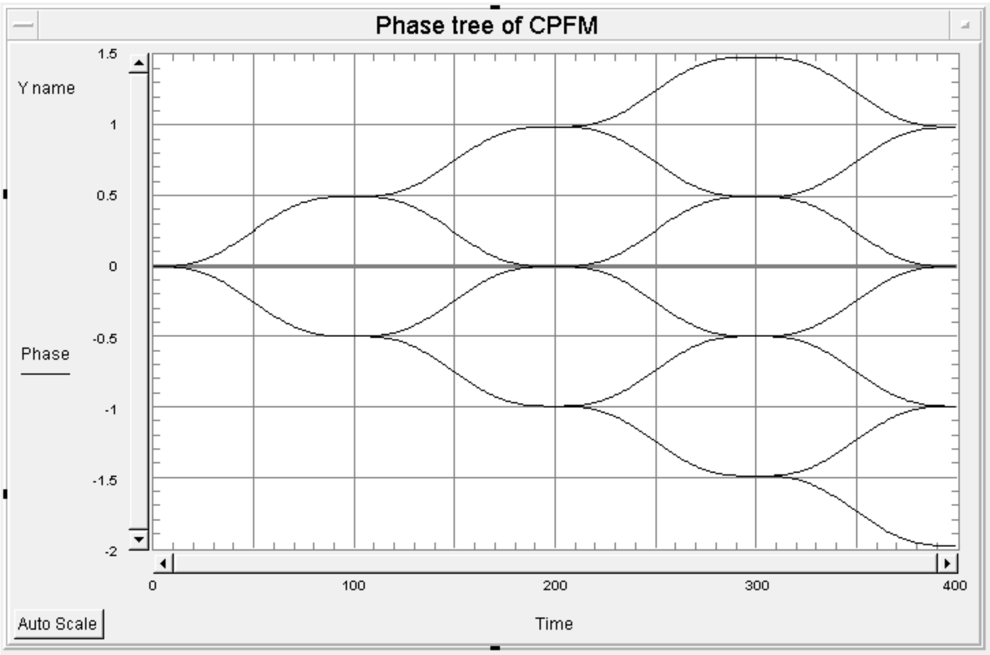

Fig. 5. Phase trellis of *L*RC impulses

Algebraic and structural models of CPFM signals

Taking into account expression (5) it is possible to present current phase of a signal in the form of four summands:

$$\varphi_1(\alpha_i) = 2\pi h \sum_{i=-\infty}^{k-L} \alpha_i ,$$

$$\varphi_2(\alpha_i) = 2\pi h \sum_{i=-\infty}^{k-L} \frac{1-m}{2} \quad ,$$

$$\varphi_3(\alpha_i) = 2\pi h \sum_{i=k+1-L}^{k+1} \frac{k+1-i}{L} \alpha_i \ ,$$

$$\varphi_4(i) = 2\pi h \sum_{i=k+1-L}^{k+1} \frac{k+1-i}{2L} (1-m) \, .$$

Let us note that summands $\varphi_1(\alpha_i)$ and $\varphi_3(\alpha_i)$ depend on information symbols and define the structure of a phase trellis.

Summands $\varphi_2(\alpha_i)$ and $\varphi_4(\alpha_i)$ do not depend on information symbols and define a regular increment of a phase on each tact.

When $L = 1$ we receive expression for the current phase of a full response CPFM signal:

$$\varphi(t_{k+1}) = 2\pi h \sum_{i=-\infty}^{k} \alpha_i + 2\pi h \sum_{i=-\infty}^{k} \frac{1-m}{2} \, . \quad (8)$$

CPFM signal as result of differential modulation

Continuous phase frequency modulation can be treated as a differential modulation method. It follows from expression (8). The first difference of signal phases is defined by a transferred information symbol

$$\overset{1}{\Delta}(\varphi(t_{k+1}) = (\varphi(t_{k+1} - \varphi(t_k)) = 2\pi h \alpha_i \, .$$

This property of CPFM signals defines the possibility of their use in channels with slow fading and also the possibility of application of earlier developed effective methods of the differential FM signal processing.

Invariance of CPFM of signals

It is easy to prove that CPFM signal (1) satisfies to an invariance condition. Indeed, suppose that we are given two sequences:

$$S'(t) = \sqrt{\frac{2LE}{LT}} \cos\left(2\pi f_0 t + 2\pi h \sum_{i=-\infty}^{\infty} (2\alpha'_i - m + 1) g_\varphi(t - iT)\right) ,$$

$$S''(t) = \sqrt{\frac{2LE}{LT}} \cos\left(2\pi f_0 t + 2\pi h \sum_{i=-\infty}^{\infty} (2\alpha''_i - m + 1) g_\varphi(t - iT)\right) . \quad (9)$$

Assuming that the modulating symbols do not coincide ($\alpha'_i \neq \alpha''_i$), when $i < 0$, we will determine a square of the Euclidean distance on an interval (0..., NT). If the narrowband condition of CPFM signal is satisfied ($f_0 T >> 2\pi$), we obtain

$$\delta_N^2(\bar{S}', \bar{S}'') = 2EN\left[1 - \frac{1}{NT} \int_0^{NT} \cos(2\pi h \sum_{i=-\infty}^{\infty} 2(\alpha'_i - \alpha''_i) g_\varphi(t - iT)) dt\right] . \quad (10)$$

Since transition to new information sequences $\overline{\alpha'^*} = \overline{\alpha}' + \overline{\alpha}^*$ and $\overline{\alpha''^*} = \overline{\alpha}'' + \overline{\alpha}^*$ ($\overline{\alpha}^*$ – any sequence) does not change distance (10), signals of CPFM of the form (9) belong to *invariant* category, and distance $\delta_N^2(\bar{S}', \bar{S}'')$ depends only on a difference of the phase trajectories defined by arguments of cosines in (9). Therefore, subsequently it is advisable to analyze the given phase function

$$\tilde{\varphi}(t_{k+1}) = \varphi_1(t_{k+1}) + \varphi_3(t_{k+1}) = \frac{2\pi h}{L}\left(\sum_{i=-\infty}^{k-L} L\alpha_i + \sum_{i=k-L+1}^{k+1} (k+1-i)\alpha_i\right) . \quad (11)$$

Since the expression for $\tilde{\varphi}(t_{k+1})$ is included in the cosine argument, we can proceed to modified phase function

$$\tilde{\tilde{\varphi}}(t_{k+1}) = \frac{2\pi h}{L}\left(\sum_{i=-\infty}^{k-L} L\alpha_i + \sum_{i=k-L+1}^{k+1} (k+1-i)\alpha_i\right) \mathrm{mod}(2\pi) \, . \quad (12)$$

For a modulation index $h = p/q$ where p and q are integers, the expression (12) can be written as

$$\tilde{\tilde{\varphi}}(t_{k+1}) = \left[p\left(L\sum_{i=-\infty}^{k-L} \alpha_i + \sum_{i=k-L+1}^{k+1} (k+1-i)\alpha_i\right)\right] \mathrm{mod}(Lq) \, . \quad (13)$$

To expression (13) there corresponds the finite state automaton (fig. 6) describing phase on the output of CPFM modulator. On fig. 6 and further, the symbols of addition and multiplication mean transactions of addition and multiplication modulo *Lq*.

Subsequent conversions are conveniently produced by a polynomial representation of the sequences in the form of polynomials of argument *D*, where the time delay *T* corresponds to operation of multiplication by *D*. Then, transfer function of the structure represented on fig. 6, will look like:

$$K(D)=\frac{\tilde{\tilde{\varphi}}(D)}{\alpha(D)}=\left[p\left(1+2D+3D^2+...+(L-1)D^{L-2}+L\frac{D^{L-1}}{1-D}\right)\right]\mathrm{mod}(Lq). \quad (14)$$

More compact structure of the automaton with the same transfer function is shown on fig. 7.

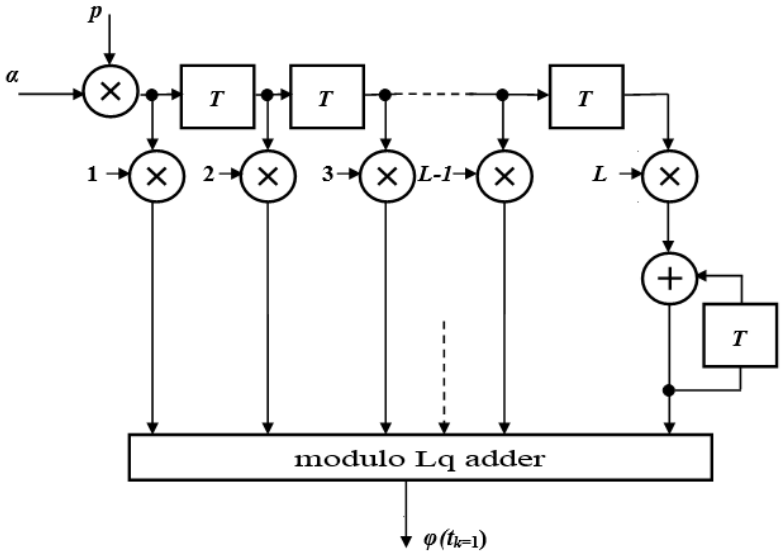

Fig. 6. Model of CPFM modulator in the form of the finite state automaton with memory

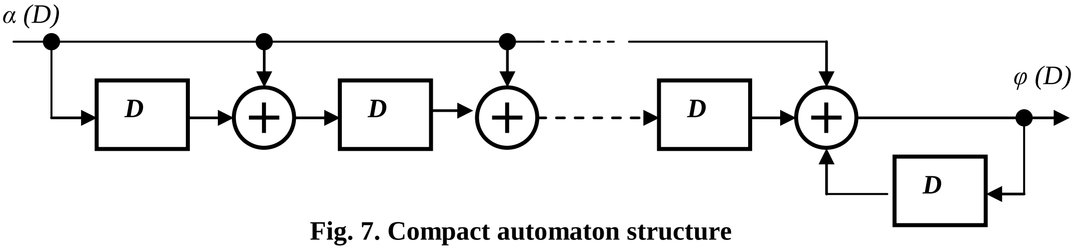

Fig. 7. Compact automaton structure

The number of the finite automaton states is defined by a full set of variables on inputs of delay elements. The quantity of delay elements and, respectively, number of states can be reduced by minimizing the block diagram.

Expression (14) can be transformed to the form:

$$K(D)=\left[p\left(1+D+D^2+...+D^{L-2}+D^{L-1}\right)\frac{1}{1-D}\right]\mathrm{mod}(Lq). \quad (15)$$

In this case, the structure of the minimal automaton is represented on fig. 8.

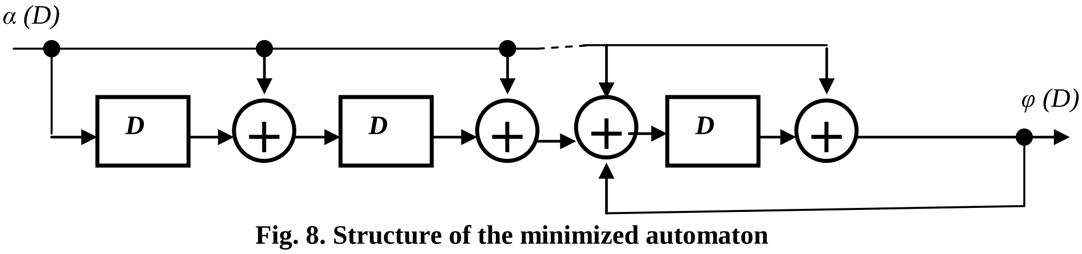

Fig. 8. Structure of the minimized automaton

It follows from fig. 8 structure that total number of automaton states is equal to $S = q^L L$, wherein $S_t = q^{L-1}$ is the quantity of temporary states (caused by the presence of memory elements) and $S_p=qL$ – the quantity of phase states (caused by the phase shift at the end of each clock interval). In solving problems of creating the signal-code constructions using internal CPFM signals and external codes there is a problem of correct connection of multiple outputs of the error-correcting encoder to the CPFM modulator input. Let the error-correcting encoder have v outputs. In this case the CPFM modulator also should have v inputs and symbols arrive on parallel inputs of the modulator as v-length blocks**.** We divide the sequence of clock time intervals $t_k = kT$ in the blocks of duration vT, i.e. imagine the current index as $k = vs + r$. Here v – the block length, s – the current block number, r – the current symbol number within the ($0 \le r \le v–1$) block. Accordingly, the transfer function of the modulator model can be represented as a matrix

$$K(D) = \begin{vmatrix} K_0(D) & K_1(D) & K_2(D) & \dots & K_{\upsilon-1}(D) \\ K_{\upsilon-1}(D) & K_0(D) & K_1(D) & \dots & K_{\upsilon-2}(D) \\ K_{\upsilon-2}(D) & K_{\upsilon-1}(D) & K_0(D) & \dots & K_{\upsilon-3}(D) \\ \dots & \dots & \dots & \dots & \dots \\ K_1(D) & K_2(D) & K_3(D) & \dots & K_0(D) \end{vmatrix} \quad (16)$$

wherein each element of $K_r(D)$ composed of all members of the matrix (16) containing a variable D power ($vs + r$), s = 1,2,3. Elements of the polynomials matrix (16) are shown in table 2. An example of such a model is shown in fig. 9.

Table 2

The elements of matrix (6)

L	ν	*K*0 (D)	K1 (D)	K2 (D)	K3 (D)
1	1	$p/(1–D)$	-	-	-
1	2	p/(1–D2)	pD (/1-D2)	-	-
1	4	p/(1–D4)	pD (/1-D4)	pD2 (/1-D4)	pD3 (/1-D4)
2	2	p (1+D2) (/1-D2)	2pD (/1-D2)	-	-
2	4	p (1+D4) (/1-D4)	2pD (/1-D4)	2pD2 (/1-D4)	2pD3 (/1-D4)

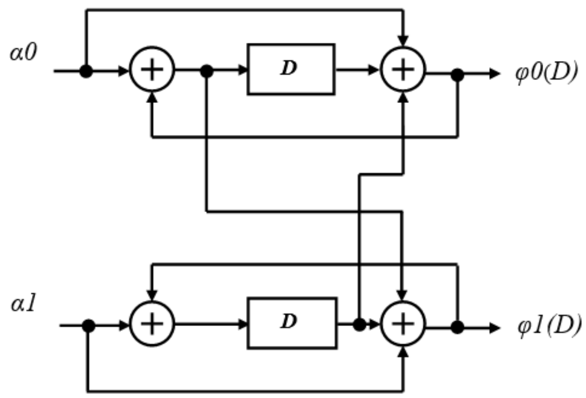

Fig. 9. Modulator model (L = 2, v = 2, p = 1, q = 2, addition modulo 4)

Conclusions

1. The article describes some of the important properties of the frequency-modulated signal with the continuous phase that were not previously investigated in the published literature.

2. The invariance property of CPFM simplifies the exhaustive search of generating polynomials for convolutional codes used in the encoder for better noise immunity in the channels with CPFM signals.

3. Representation of CPFM signals by the differential model facilitates synthesis of a demodulating algorithm for this signal.

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