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EQUILIBRIUM ECONOMETRICS AND POLYMETRICAL ANALYSIS

The basic laws of equilibrium econometrics (Hiks laws and Le Chatelier–Samuelson principle) are analyzed. The principal possibility of using of more expanded methods of polymetric analysis to solve of this problem is discussed. Questions of using of conjugate variables quantities and principles are observed. Features of this concept are observed. The interdependency of basic laws of polymetrical analysis, information theory and econometrics are analyzed too.

Keywords: equilibrium econometrics, Hicks laws, the principle of Le Chatelier–Samuelson, polymetric analysis, evolutionary systems.

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РІВНОВАЖНА ЕКОНОМЕТРИКА ТА ПОЛІМЕТРИЧНИЙ АНАЛІЗ

Проаналізовано основні закони рівноважної економетрики (закони Гікса та принцип ле Шательє–Самуельсона). Показано, що рівноважна економетрика будувалась за аналогією з рівноважною фізико-хімічною термодинамікою. Обговорюється доцільність використання поліметричного аналізу для розв'язання цієї проблеми. Досліджені питання застосування спряжених змінних та принципів. Показано, що поряд з перетвореннями Фур'є та Лапласа доцільно адаптувати принципи Релея та невизначеності для задач рівноважної економетрики. Детально обговорюються особливості цієї концепції. Також проаналізовано проблеми взаємовпливу основних законів поліметричного аналізу, теорії інформації та економетрики.

Ключові слова: рівноважна економетрика, закони Хікса, принцип ле Шательє–Самуельсона, поліметричний аналіз, еволюційні системи.

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РАВНОВЕСНАЯ ЭКОНОМЕТРИКА И ПОЛИМЕТРИЧЕСКИЙ АНАЛИЗ

Проанализировано основные законы равновесной эконометрики (законы Хикса и принцип ле Шателье-Самуельсона). Показано, что равновесная эконометрика создавалась по аналогии с равновесной термодинамикой. Обсуждается целесообразность использования полиметрического анализа для решения этой проблемы. Исследованы вопросы применения сопряженных переменных и принципов. Показано, що наряду с преобразованиями Фурье и Лапласа целесообразно адаптировать принципы Релея и неопределенности для задач равновесной эконометрики. Детально обсуждаются особенности этой концепции. Также проанализированы проблемы взаимовлияния основных законов полиметрического анализа, теории информации и эконометрики.

Ключові слова: равновесная эконометрика, законы Хикса, принцип ле Шателье-Самуельсона, полиметрический анализ, эволюционные системы.

Analysis of recent research and publications

In theoretical and mathematical physics, the use of the polymetric method for analyzing "old" sciences and the creation of "new" is quite successful [1, 2]. The polymetric analysis itself was created by the universal system of synthesis of analysis and formalization of knowledge [1, 2]. It is based on the idea of triple optimization and has allowed to unite at the methodological level into a single system all well-known created and not yet created information systems. This optimization is guided by a measure element, a generalized quadratic form, which, through the (quantitative and qualitative) transformations acting on it, includes a measurement procedure [1, 2]. At the same time, the polymetric method can be used as an expert system of "correctness" and completeness of one or another theory. For this purpose, a hybrid theory of systems was built that allows the classification of existing information systems from the point of view of their operational, including computational, complexity [1, 2]. This is a theory of open systems, although there are only 10 minimal types of formalization systems, the total number of specific systems can be arbitrarily large [1, 2].

Purpose of the study

In this paper, we focus on one of the methods included in the polymetric analysis and is based on the expansion of the Rayleigh principle and its formal quantum-mechanical extension, known as the principle of uncertainty [3], and called the method of conjugate variables. This approach allowed quite simply combining the basic laws of physics and information theory into a single system [1, 4]. This theory is called the theory of information-physical structures [4]. Now we represent this concept more detaily.

Presentation of the main research material

We give Rayleygh principle in a one-dimensional form according by N. Bohr [3]:

$$\Delta k \cdot \Delta x = \Delta \omega \cdot \Delta t = 1, \qquad (1)$$

where Δk, Δx, $\Delta \omega$, Δt ‑ changes of the wave number, coordinate, frequency and time, respectively.

When multiplying this relation by h (Planck constant) and replacing the sign of equality with a sign more-equal ($\geq$), then we will have uncertainty principle

$$\Delta p \cdot \Delta x = \Delta E \cdot \Delta t \geq h\,. \qquad (2)$$

The formula (1) is condition of separation two waves (Rayleygh principle) and mathematical formulation of basic principle in classic theory of coherence [5].

If we change in correlation (2) sign more-equal ($\geq$) on sign equality (=) than we have basic principle of quantum theory of coherence (3) [5]

$$\Delta p \cdot \Delta x = \Delta E \cdot \Delta t = h. \qquad (3)$$

Formula (3) is nothing more than a mathematical expression of Bohr's complementarity and the uncertainty principle.

The main concepts of the theory of information-physical structures are [4]:

1) the principle of fundamental harmonic equilibrium;

2) the equivalence of all canonical parameters: E – energy; p – linear momentum; k – wave number; x – coordinate; ω – frequency; t – time;

3) polymetry, that is, for each physical phenomenon corresponds to its own metric (symmetry, geometry, dimension, etc.).

Other examples of using the conjugated variables are Laplace and Fournier transformations, which are effective computing methods of modern science and techniques [1].

Theoretical basis for the using conjugate variables quantities in science may be de Broglie correlation [6]

$$S_a/h = S_e/k_B = S_g \qquad (4)$$

(equivalence of quantity of ordered and disorder information) [1, 6]. Where S_a – action, S_e – entropy, k_B – Boltzman constant, S_g – dimensionless generalization of action and entropy (generalized measure). Therefore we can go from dimensional quantities (action and entropy) to undimensional quantity ‑ number of proper quanta or after generalization to number of mathematical operations. Thus, theory of informative calculations may be represented as numerical generalization of classical theory of information.

Therefore, it was advisable to use this approach for the "revision" of the form of writing economic laws [1, 7, 8], which was implemented on an example of equilibrium econometrics [9-14]. This theory was called generalized econometrics [1, 7, 8], although it could be called the theory of information-economic structures.

In general, econometrics is called science, which studies quantitative and qualitative economic relationships using mathematical and statistical methods and models of economics [9-14]. The modern definition of the subject of econometrics was formulated in the charter of an econometric society, the main goals of which were the use of statistics and mathematics for the development of economic theory [9-14]. Theoretical econometrics examines the statistical properties of assessments and tests, while applied econometrics deals with the application of econometric methods for the estimation of economic theories. Econometrics provides a toolkit for economic measurements, as well as a methodology for evaluating the parameters of micro- and macroeconomics models. In addition, econometrics is actively used to predict economic processes both in the scale of the economy as a whole, and at the level of individual enterprises [9-14], along with macro and microeconomics [9-14]. In practice, this is a theoretical economy in the narrower sense of the word.

In this paper, we present the main aspects of the application of polymetric analysis for problems of econometrics, especially for a more compact recording of the laws of equilibrium econometrics.

Basic results and discussions

As an example of using the polymetric method, we give a generalized econometric analysis. Compared to physics and information theory, this science seems simpler, but if that were the case then there would not be so many social cataclysms that so often affect the history of mankind [1].

Classical econometrics [9] mainly includes methods of statistical and factor analysis, and the laws of the Walras-Leontiev type (for evolution systems) and von Neumann-Marx (for "revolutionary" systems). However, the current level of development of society requires more advanced theories, as evidenced by the study B. Gavrylishin

[15]. In the modern economy, it is imperative to take into account such factors as ecology, sociology, psychology, etc., that is, a fertile field for the use of a polymetric approach.

First we introduce a new terminology [1, 7-9]:

z_i – total output of the i-th industry;

z_{ij} – total output of the i-th industry, which is consumed by *j*-th industry;

c_i – demand for the *i*-th product;

a_{ij} – constant coefficients of cost of the *i*-th product per unit of output *j*-th industry;

ω – salary;

q_i – profit per unit of output in the second-area industry;

$a_{n+1,i}$ – coefficients of labor costs in the *i*-th industry.

The system of equations of balance and outputs of all products in systems of the Leontief type may be represented as:

$$z_i = \sum_{j=1}^{n} a_{ij} z_j + c_i; (i = 1...n). \tag{5}$$

System of equations of balance of prices, wages and profits is next:

$$p_i = \sum_{j=1}^{n} a_{ji} p_j + a_{n+1,i}\omega + q_i; (i = 1,...n). \tag{6}$$

In systems of the Leontief type there is a twin between outputs and prices in the sense that the matrix of coefficients $\{a_{ij}\}$ of the subsystem for determining prices is obtained by transposing the matrix $\{a_{ij}\}$ subsystem for the determination of issues. Both subsystems have common algebraic properties, so you can use one half of the equations in the future.

Leontief system is a partial case of the general equilibrium system of Walras, consisting of four groups of equations:

I – functions of the market offer for goods;

II – functions of market demand for goods;

III `– query and supply equations for goods and factors;

IV – equations that connect prices to production costs.

In the Walras model, the propositional functions for the factors and the query function for goods are given by the theory of marginal utility in terms of prices. In the Leontief model there are no analogues for these functions, since the vector of final demand ($c_1,...c_n,...$) and the newly created costs $(a_{n+1,1}\omega + q_1,...a_{n+1,n}\omega + q_n)$ are parameters. Issues are determined independently of the prices given by the cost price equations. This independence and the assumption of the continuity of the technical coefficients lead to the simplicity of the Leontief model.

We give the basic laws of equilibrium econometrics according to [9].

The first Hicks law. The excess demand from the zero product to the *j*-th causes an increase of the relation the equilibrium price of the *j* product to the equilibrium price of the zero product, except in the case when the product *j* is free.

The second and third Hicks laws in a weak form. In the result of the shift of priorities from zero product to *j*-th in the indeterminate system $\mathbf{1}'$, the ratio of the price of any product to the price of zero product does not decrease; besides, there is no product for which this ratio would have increased in a larger proportion, as for the *j*-th commodity.

The second and third Hicks laws in a strong form. As a result of shifting the priorities from the zero product to the *j*-th in a highly undisturbed system, the equilibrium price of any product in relation to the price of zero commodity increases in a smaller proportion than the price of the *j*-th commodity.

The principle of Le Chatelier – Samuelson. Let the condition of strong gross substitutability be fulfilled. The increase in the price of any product with the number *j*>*m* due to the shift of priorities, by changing the query of the zero product to the *j*-th (where *j*>*m*), for the case when the prices of all goods with numbers (1, ... *m*) are maintained constant (due to the adaptation of the offer of each these products) is less than the growth of the price of the same commodity with the same shift of priorities for the case when the offer of one of the mentioned n goods, say, a commodity with the number *m*, is not adapted and, accordingly, the price of the *m* commodity may change .

In addition to equilibrium econometric models there are also models of economic growth. These are the models of Smith, von Neumann, Kondratiev, and others. However, in order to illustrate the possibility of applying a polymetric approach to econometrics, we will have enough equilibrium case.

Now let's consider how it is possible in this case to proceed to representations of the polymetric measure. We take the first half of the equations (5) of the Leontief model. We introduce inverse econometric parameters:

$$k_i = \frac{N_1}{z_i}; b_i = \frac{N_2}{c_i}; y_{ij} = \frac{N_3}{a_i z_{ij}}. \tag{7}$$

Generalized econometric parameters then have the form:

$$k_{ij} = k_i z_j; B_{ij} = b_i c_j; Y_{ij} = z_j y_{ij}. \tag{8}$$

In general

$$k_{ij} = \begin{cases} N_1; i = j \\ f_1(k_i, z_j); i \neq j, \end{cases} \tag{9a}$$

$$B_{ij} = \begin{cases} N_2; i = j \\ f_2(b_i, c_j); i \neq j, \end{cases} \tag{9b}$$

$$Y_{ij} = \begin{cases} N_3; i = j, \\ f_3(z_{ij}; y_{mn}); i \neq m; j \neq n. \end{cases} \tag{9c}$$

A generalized econometric equation can be written as follows

$$F(k_{ij}) = F_1(Y_{ij}) + F_2(B_{ij}), \tag{10}$$

where F_i – functional dependence.

In the linear case:

$$k_{ij} = Y_{ij} + B_{ij} \tag{11}$$

or

$$k_i z_j = z_{ij} y_{mn} + b_i c_j. \tag{12}$$

For $i = j = m = n$ we have

$$kz = \tilde{z}y + bc. \tag{13}$$

Minimizing these ratios we will get:

$$\frac{dz}{dy} = \frac{d\tilde{z}}{dk} = A_1; \ \frac{dk}{db} = \frac{dc}{dz} = A_2; \ \frac{d\tilde{z}}{db} = -\frac{dc}{dy} = A_3. \tag{14}$$

After solving of these equations we will be obtained

$$z = z_0 + A_1 y; \qquad c = c_0 + A_2 z; \tag{15a}$$

$$\tilde{z} = \tilde{z}_0 + A_1 k; \qquad \tilde{z} = \tilde{z}_0 + A_3 b; \tag{15b}$$

$$k = k_0 + A_2 b; \qquad c = c_o - A_3 y. \tag{15c}$$

That is, we have linear laws for preserving the amount of costs, demand and inverse quantities. These laws are truie in a specific area. As is easily seen, the relation (15) is also a linear generalization of the Hicks laws and the principle of Le Chatelier – Samuelson.

More complex dependencies are obtained from the next relationship

$$dk \cdot dz = d\tilde{z} \cdot dy + db \cdot dc \tag{16}$$

or in a generalized form –

$$dk_{ij} = dY_{ij} + dB_{ij}. \tag{17}$$

From the equation (12) in a simplified form we can obtain a correlation

$$dk \cdot dz = d\tilde{z} \cdot dy + L_1, \tag{18a}$$

$$dk \cdot dz = db \cdot dc + L_2, \tag{18b}$$

$$d\tilde{z} \cdot dy = db \cdot dc + L_3, \tag{18c}$$

where L_1, L_2, L_3 – constants.

Solutions of the equations (18) are more complicated than the solution of equations (14) – (15). In general, they have the form:

$$k = k_0 + A_1 y; \ \tilde{z} = \tilde{z}_0 + \tilde{A}_1 F(L_1, z) + \tilde{A}_1' F_2(L_1, z, y). \tag{19}$$

That is, in the relations of type (19), unlike the relations (15), we can take into account small changes of any of the econometric parameters. Equations of type (16) yield Volterra-type equations, as well as a more complex type equation. That is, based on equilibrium conditions, dynamic systems can be described: to investigate both economic growth and decline.

By changing the number of corresponding quadratic members in the equation, we can introduce additional factors that affect the economy, including environmental factors, psychology, etc. Many of them can be taken into account with the help of generalized mathematical transformations. For a more complete modeling of such econometric models, you need to use a computer.

Generalized mathematical form of the principle of Le Chatelier – Samuelson according to relationship (4) may be represented in next form

$$\delta S_g \geq 0. \tag{20}$$

Where S_g must be represented through conjugate "economical" quantities.

Sign more (>) is corresponded to non-equilibrium econometrics, sign equal (=) – equilibrium econometrics.

This concept is complemented basic methods of modern econometrics and show principal possibility of application the modified methods of modern mathematical physics for the problems of modern economics and econometrics.

Roughly speaking the principle (20) unites the Quantum Mechanics, thermodynamics and information theory, including econometrics, in one system.

Conclusions

1. The basic concept of Polymetric analysis is discussed.
2. The Rayleygh and uncertainty principles and its modifications are represented.
3. The basic laws of equilibrium econometrics are observed.
4. Problem of using conjugate parameters for the formulation of the basic laws of equilibrium econometrics is analyzed.
5. We show that polymetric method (method of conjugated quantities) may be used for the unification of laws the equilibrium econometrics.
6. The generalized mathematical form of the principle of Le Chatelier – Samuelson is proved and represented.

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