ISSN 2078–5097. Вісн. Львів. ун-ту. Сер. прикл. матем. та інф. 2017. Вип. 25. С. 100–111
Visnyk of the Lviv University. Series Appl. Math. and Informatics. Issue 25. P. 100–111

UDC 532.543

THE (G'/G) – EXPANSION METHOD AND TRAVELING WAVE SOLUTIONS TO SOME INVERSE NONLINEAR DYNAMIC SYSTEMS

I. Mykhailiuk, M. Prytula

Ivan Franko National University of Lviv,
Universytetska Str., 1, Lviv, 79000,
e-mail: iv.mykh@gmail.com, mykola.prytula@gmail.com

The (G'/G) – expansion method [4] is applied to the inverse Korteweg – de Vries and the inverse modified Korteweg – de Vries dynamic systems [2]. For both systems the traveling wave solutions in the form of hyperbolic, rational and trigonometric functions are constructed and analyzed. The obtained results are compared to ones derived by means of the tanh – method.

Key words: the (G'/G) – expansion method, the inverse Korteweg – de Vries dynamic system, the inverse modified Korteweg – de Vries dynamic system, soliton solution.

1. INTRODUCTION

Solutions to nonlinear evolution equations (NEE) play a crucial role in mathematical physics, therefore more and more scientists from all over the world dedicate their studies to investigate such equations. Nonlinear wave phenomena appear in various scientific and engineering fields, such as fluid mechanics, plasma physics, optical fibers, biology, solid-state physics, chemical kinematics, chemical physics and geochemistry.

With the advent of computers many effective numeric methods for finding approximate solutions to partial differential equations (PDEs) appeared. On the other hand, the creation of modern powerful computer algebra systems, such as MATLAB, MATHEMATICA and MAPLE, simplified the analytical investigation of NEEs, assisting mathematicians in their tiny computations. Hence during the past five decades a wide variety of analytical methods for finding exact solutions to NEEs was developed.

Recently, the (G'/G) – expansion method, firstly introduced by Wang et al. [4], has become widely used for many PDEs. It turned out that the method just mentioned provides solutions in a more general form compared to other analytical methods (e.g. the tanh – method). Furthermore, with a certain choice of arbitrary parameters in the (G'/G) – expansion method some well-known solutions to PDEs can be rediscovered.

The rest of the paper is organized as follows. In Section 2, we describe the (G'/G) – expansion method [4] for finding traveling wave solutions to nonlinear evolution equations. In Sections 3 and 4, we apply the method to some inverse nonlinear dynamic systems, namely the inverse Korteweg – de Vries (KdV) and the inverse modified Korteweg – de Vries (mKdV) systems [2], and analyze the obtained solutions. Finally, in Section 5, we summarize our results.

2. DESCRIPTION OF THE (G'/G) – EXPANSION METHOD

Suppose that a nonlinear equation, say in two independent variables x and t, is given by

$$P\left(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, \ldots\right) = 0, \tag{1}$$

where $u = u(x,t)$ is an unknown function, P is a polynomial in $u = u(x,t)$ and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved. In the following we give the main steps of the (G'/G) – expansion method [4].

Step 1. Combining independent variables x and t into one variable

$$\xi = x - Vt, \tag{2}$$

we suppose that $u(x,t) = u(\xi)$. Traveling wave variable (2) permits us to reduce Eq. (1) to an ordinary differential equation (ODE) for $u = u(\xi)$

$$P\left(u, -Vu', u', V^2u'', -Vu'', u'', \ldots\right) = 0. \tag{3}$$

Step 2. Suppose that the solution to ODE (3) can be expressed by a polynomial in (G'/G) as follows:

$$u(\xi) = \sum_{i=0}^{m} \alpha_i \left(\frac{G'}{G}\right)^i, \tag{4}$$

where $G = G(\xi)$ satisfies the second order linear ODE in the form of

$$G'' + \lambda G' + \mu G = 0, \tag{5}$$

$\alpha_i \left(i = \overline{0,m}\right), \lambda, \mu$ are constants to be determined later, $\alpha_m \neq 0.$ The positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in ODE (3).

Step 3. By substituting (4) into Eq. (3) and using the second order LODE (5), collecting all terms with the same order of (G'/G) together, the left-hand side of Eq. (3) is converted into another polynomial in (G'/G). Equating each coefficient of this polynomial to zero yields a set of algebraic equations for $\alpha_i \left(i = \overline{0,m}\right), \lambda, \mu$ and V.

Step 4. Assuming that the constants $\alpha_i \left(i = \overline{0,m}\right), \lambda, \mu$ and V can be obtained by solving the algebraic equations in Step 3, since the general solutions to the second order linear ODE (5) have been well known for us, then substituting $\alpha_i \left(i = \overline{0,m}\right), \lambda, \mu, V$ and the general solutions to Eq. (5) into (4) we obtain travelling wave solutions to the original nonlinear evolution equation (1).

As it was already mentioned, the solution to Eq. (1.5) is well-knows for us and can be easily derived by the Euler method:

$$G(\xi) = \begin{cases} \left(A_1 \sinh\dfrac{\xi\sqrt{\lambda^2 - 4\mu}}{2} + A_2 \cosh\dfrac{\xi\sqrt{\lambda^2 - 4\mu}}{2}\right) e^{-\frac{\lambda\xi}{2}}, & \text{if } \lambda^2 - 4\mu > 0; \\ \left(A_1 + A_2\xi\right) e^{-\frac{\lambda\xi}{2}}, & \text{if } \lambda^2 - 4\mu = 0; \\ \left(A_1 \sin\dfrac{\xi\sqrt{4\mu - \lambda^2}}{2} + A_2 \cos\dfrac{\xi\sqrt{4\mu - \lambda^2}}{2}\right) e^{-\frac{\lambda\xi}{2}}, & \text{if } \lambda^2 - 4\mu < 0; \end{cases} \tag{6}$$

3. INVERSE KORTEWEG – DE VRIES DYNAMIC SYSTEM

Consider the inverse Korteweg – de Vries (KdV) dynamic system [2]

$$\begin{cases} u_t = v \\ v_t = p \\ p_t = u_x + uv, \end{cases} \tag{7}$$

which is obtained from the classical KdV equation with the aid of the inversion mapping $\mathbb{R}^1 \ni x \rightleftarrows t \in \mathbb{R}^1$. Let us solve system (7) by use of the (G'/G) – expansion method.

Step 1. Introducing traveling wave variable $\xi = x - Vt,$ we reduce system (7) to a system of ODE for $u = u(\xi), v = v(\xi), p = p(\xi)$

$$\begin{cases} -Vu' = v \\ -Vv' = p \\ -Vp' = u' + uv. \end{cases} \tag{8}$$

Suppose that the solution to system (8) can be expressed by polynomials in (G'/G) as follows:

$$u(\xi) = \sum_{i=0}^{k} \alpha_i \left(\frac{G'}{G}\right)^i, \quad v(\xi) = \sum_{i=0}^{l} \beta_i \left(\frac{G'}{G}\right)^i, \quad p(\xi) = \sum_{i=0}^{m} \gamma_i \left(\frac{G'}{G}\right)^i. \tag{9}$$

Considering the homogeneous balance between u' and v, v' and p, p' and uv in the first, the second and the third equations of system (8) correspondingly, we obtain a simple system of algebraic equations

$$\begin{cases} k+1 = l \\ l+1 = m \\ m+1 = k+l, \end{cases} \tag{10}$$

from which it can be easily found that $k = 2, l = 3$ and $m = 4.$

Step 2. Considering (9) and (10), we find the solution to system (8) in the following form:

$$\begin{cases} u(\xi) = \alpha_2 \left(\frac{G'}{G}\right)^2 + \alpha_1 \left(\frac{G'}{G}\right) + \alpha_0 \\ v(\xi) = \beta_3 \left(\frac{G'}{G}\right)^3 + \beta_2 \left(\frac{G'}{G}\right)^2 \beta_1 \left(\frac{G'}{G}\right) + \beta_0 \\ p(\xi) = \gamma_4 \left(\frac{G'}{G}\right)^4 + \gamma_3 \left(\frac{G'}{G}\right)^3 + \gamma_2 \left(\frac{G'}{G}\right)^2 \gamma_1 \left(\frac{G'}{G}\right) + \gamma_0, \end{cases} \tag{11}$$

where function $G = G(\xi)$ satisfies the second order linear ODE (5), $\lambda, \mu, V, \alpha_i \left(i = \overline{0,2}\right), \beta_j \left(j = \overline{0,3}\right), \gamma_k \left(k = \overline{0,4}\right)$ are all constants to be determined later, $\alpha_2 \neq 0, \beta_3 \neq 0, \gamma_4 \neq 0.$

Step 3. To substitute (11) into system (8) we need to derive some auxiliary correlations:

$$\begin{aligned}
&v' = -\beta_1\mu - \left(\beta_1\lambda + 2\beta_2\mu\right)F - \left(2\beta_2\lambda + 3\beta_3\mu + \beta_1\right)F^2 - \left(3\beta_3\lambda + 2\beta_2\right)F^3 - 3\beta_3F^4;\\
&u' = -\alpha_1\mu - \left(\alpha_1\lambda + 2\alpha_2\mu\right)F - \left(2\alpha_2\lambda + \alpha_1\right)F^2 - 2\alpha_2F^3;\\
&p' = -\gamma_1\mu - \left(\gamma_1\lambda + 2\gamma_2\mu\right)F - \left(2\gamma_2\lambda + 3\gamma_3\mu + \gamma_1\right)F^2 - \left(3\gamma_3\lambda + 4\gamma_4\mu + 2\gamma_2\right)F^3\\
&\quad - \left(4\gamma_4\lambda + 3\gamma_3\right)F^4 - 4\gamma_4F^5;\\
&uv = -\mu\gamma_1 - \left(\lambda\gamma_1 + 2\mu\gamma_2\right)F - \left(\gamma_1 + 2\lambda\gamma_2 + 3\mu\gamma_3\right)F^2 - \left(2\gamma_2 + 3\lambda\gamma_3 + 4\mu\gamma_4\right)F^3\\
&\quad - (3\gamma_3 + 4\lambda\gamma_4)F^4 - 4\gamma_4F^5;
\end{aligned} \tag{12}$$

In correlations above, $F = \dfrac{G'}{G}$.

By substituting (12) into Eq. (8) and collecting all terms with the same power of $\left(G'/G\right)$ together, the left-hand sides of equations (8) are converted into another polynomial in $\left(G'/G\right)$. Equating each coefficient of these polynomials to zero yields a set of simultaneous algebraic equations for $\lambda, \mu, V, \alpha_i\left(i=\overline{0,2}\right), \beta_j\left(j=\overline{0,3}\right), \gamma_k\left(k=\overline{0,4}\right)$ as follows:

$$\begin{cases}
\alpha_1\mu V - \beta_0 = 0,\\
V\left(\alpha_1\lambda + 2\alpha_2\mu\right) - \beta_1 = 0, \quad V\left(2\alpha_2\lambda + \alpha_1\right) - \beta_2 = 0,\\
2\alpha_2 V - \beta_3 = 0, \quad \beta_1\mu V - \gamma_0 = 0,\\
V\left(\beta_1\lambda + 2\beta_2\mu\right) - \gamma_1 = 0, \quad V\left(2\beta_2\lambda + 3\beta_3\mu + \beta_1\right) - \gamma_2 = 0,\\
V\left(3\beta_3\lambda + 2\beta_2\right) - \gamma_3 = 0, \quad 3\beta_3 V - \gamma_4 = 0,\\
-\alpha_0\beta_0 + \alpha_1\mu + \gamma_1\mu V = 0,\\
-\alpha_1\beta_0 - \alpha_0\beta_1 + \alpha_1\lambda + 2\alpha_2\mu + V\left(\gamma_1\lambda + 2\gamma_2\mu\right) = 0,\\
-\alpha_1\beta_1 - \alpha_2\beta_0 - \alpha_0\beta_2 + 2\alpha_2\lambda + \alpha_1 + V\left(2\gamma_2\lambda + 3\gamma_3\mu + \gamma_1\right) = 0,\\
-\alpha_2\beta_1 - \alpha_1\beta_2 - \alpha_0\beta_3 + 2\alpha_2 + V\left(3\gamma_3\lambda + 4\gamma_4\mu + 2\gamma_2\right) = 0,\\
-\alpha_2\beta_2 - \alpha_1\beta_3 + V\left(4\gamma_4\lambda + 3\gamma_3\right) = 0,\\
4\gamma_4 V - \alpha_2\beta_3 = 0, \quad \alpha_2 \neq 0, \quad \beta_3 \neq 0, \quad \gamma_4 \neq 0.
\end{cases} \tag{13}$$

Step 4. Solving system of algebraic equations (13) with the aid of MATHEMATICA yields

$$\begin{gathered}
\alpha_0 = \frac{\lambda^2V^3 + 8\mu V^3 + 1}{V}, \quad \alpha_1 = 12\lambda V^2, \quad \alpha_2 = 12V^2,\\
\beta_0 = 12\lambda\mu V^3, \quad \beta_1 = 12\left(\lambda^2V^3 + 2\mu V^3\right), \quad \beta_2 = 36\lambda V^3, \quad \beta_3 = 24V^3,\\
\gamma_0 = 12\mu V^4\left(\lambda^2 + 2\mu\right), \quad \gamma_1 = 12\lambda V^4\left(\lambda^2 + 8\mu\right),\\
\gamma_2 = 12V^4\left(7\lambda^2 + 8\mu\right), \quad \gamma_3 = 144\lambda V^4, \gamma_4 = 72V^4,
\end{gathered} \tag{14}$$

where λ, μ, V are arbitrary constants.

Finally, substituting (14) with the general solution to linear ODE (5) into representation (11) we obtain three types of traveling wave solutions to the inverse KdV dynamic system as follows.

When $\lambda^2 - 4\mu > 0$, we get the family of hyperbolic functions solutions

$$\begin{cases} u(\xi) = \dfrac{3\left(A_1^2 - A_2^2\right)V^2\sigma}{\left(A_1 \sinh \frac{1}{2}\xi\sqrt{\sigma} + A_2 \cosh \frac{1}{2}\xi\sqrt{\sigma}\right)^2} + V^2\sigma + \dfrac{1}{V} \\ v(\xi) = \dfrac{3\left(A_1^2 - A_2^2\right)V^3\sigma\sqrt{\sigma}\left(A_2 \sinh \frac{1}{2}\xi\sqrt{\sigma} + A_1 \cosh \frac{1}{2}\xi\sqrt{\sigma}\right)}{\left(A_1 \sinh \frac{1}{2}\xi\sqrt{\sigma} + A_2 \cosh \frac{1}{2}\xi\sqrt{\sigma}\right)^3} \\ p(\xi) = \dfrac{3\left(A_1^2 - A_2^2\right)V^4\sigma^2\left(2A_1A_2 \sinh \xi\sqrt{\sigma} + \left(A_1^2 + A_2^2\right)\cosh \xi\sqrt{\sigma} + 2\left(A_1^2 - A_2^2\right)\right)}{2\left(A_1 \sinh \frac{1}{2}\xi\sqrt{\sigma} + A_2 \cosh \frac{1}{2}\xi\sqrt{\sigma}\right)^4} \end{cases}, \qquad (15)$$

where $\xi = x - Vt,\ \sigma = \lambda^2 - 4\mu,\ A_1, A_2, V$ are arbitrary constants. In particular, setting $A_1 = 0, V = -\dfrac{k_2}{k_1}, \sigma = 4k_1^2$ we obtain soliton solution, found by means of the tanh – method in [1].

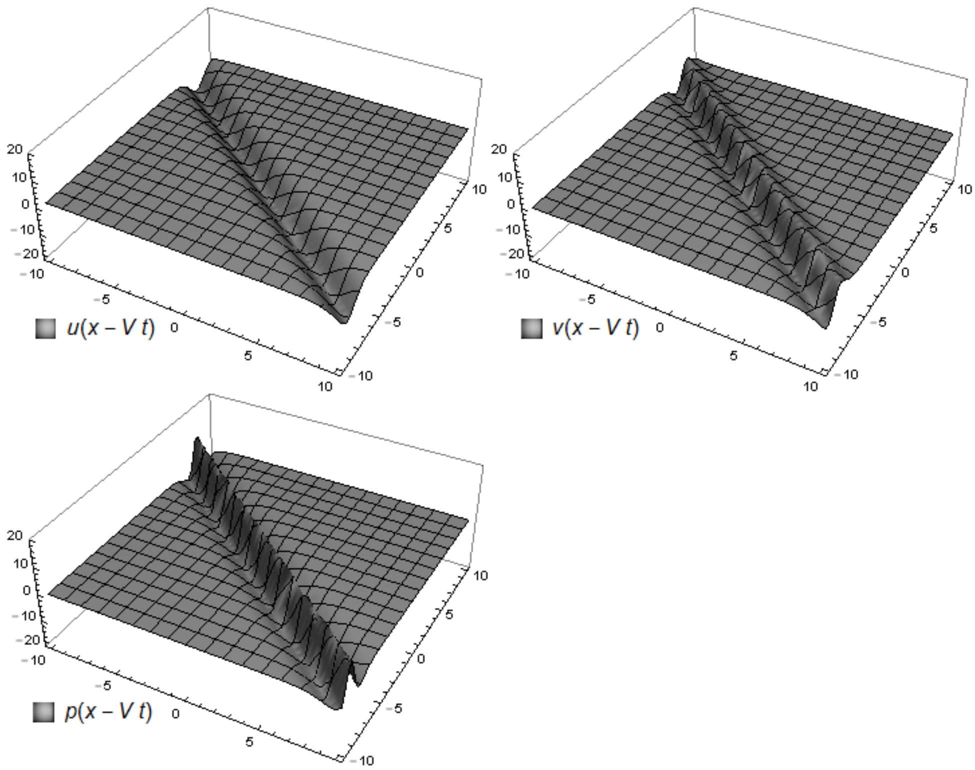

Fig. 1. Hyperbolic functions solution (15) when $A_1 = 1,\ A_2 = 2,\ \lambda = 2.5,\ \mu = 1.1, V = -1.2$

When $\lambda^2 - 4\mu = 0$, we get the family of rational functions solutions

$$\begin{cases} u(\xi) = \dfrac{A_2^2\xi^2 + 2A_2^2V^3\left(6-\xi^2\sigma\right) + 2A_2A_1\xi\left(1-2V^3\sigma\right) + A_1^2\left(1-2V^3\sigma\right)}{V\left(A_2\xi + A_1\right)^2} \\ v(\xi) = -\dfrac{6A_2V^3\left(A_2^2\left(\xi^2\sigma - 4\right) + 2A_2A_1\xi\sigma + A_1^2\sigma\right)}{\left(A_2\xi + A_1\right)^3} \\ p(\xi) = 12V^4\left(\dfrac{3\psi^4}{8} - \dfrac{3\lambda\psi^3}{2} + \dfrac{\left(7\lambda^2 + 8\mu\right)\psi^2}{4} - \dfrac{\left(\lambda^3 + 8\lambda\mu\right)\psi}{2} + \left(\lambda^2\mu + 2\mu^2\right)\right), \end{cases} \tag{16}$$

where $\xi = x - Vt$, $\psi = \dfrac{A_2\left(\lambda\xi - 2\right) + A_1\lambda}{A_2\xi + A_1}$, $\sigma = \lambda^2 - 4\mu$, A_1, A_2, V are arbitrary constants.

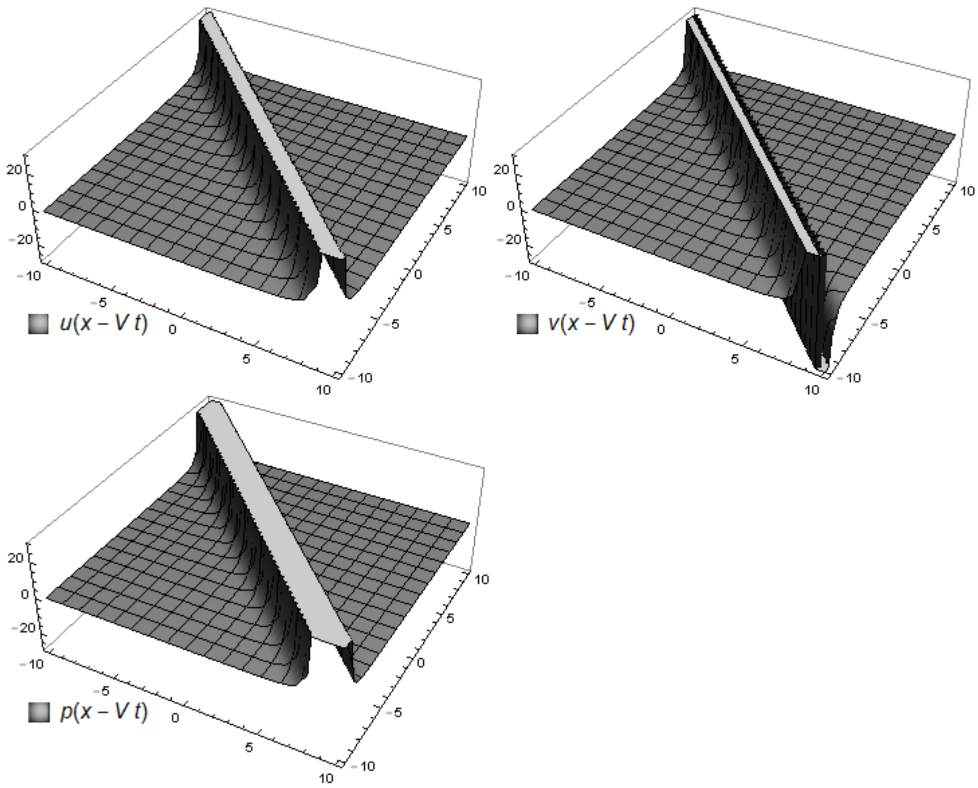

Fig. 2. Rational functions solution (16) when $A_1 = 1$, $A_2 = 2$, $\lambda = 2$, $\mu = 1$, $V = -1$

When $\lambda^2 - 4\mu < 0$, we get the family of trigonometric functions solutions

$$\begin{cases} u(\xi)=\dfrac{3\left(A_1^2+A_2^2\right)V^2\sigma}{V\left(A_1\sin \frac{1}{2}\xi\sqrt{\sigma}+A_2\cos \frac{1}{2}\xi\sqrt{\sigma}\right)^2}+V^2\sigma-\dfrac{1}{V} \\ v(\xi)=\dfrac{3\left(A_1^2+A_2^2\right)V^3\sigma^{3/2}\left(A_1\cos \frac{1}{2}\xi\sqrt{\sigma}-A_2\sin \frac{1}{2}\xi\sqrt{\sigma}\right)}{\left(A_1\sin \frac{1}{2}\xi\sqrt{\sigma}+A_2\cos \frac{1}{2}\xi\sqrt{\sigma}\right)^3} \\ p(\xi)=\dfrac{3\left(A_1^2+A_2^2\right)V^4\sigma^2\left(-2A_1A_2\sin\xi\sqrt{\sigma}+\left(A_1^2-A_2^2\right)\cos\xi\sqrt{\sigma}+2\left(A_1^2+A_2^2\right)\right)}{2\left(A_1\sin \frac{1}{2}\xi\sqrt{\sigma}+A_2\cos \frac{1}{2}\xi\sqrt{\sigma}\right)^4}, \end{cases} \tag{17}$$

where $\xi=x-Vt,\ \sigma=4\mu-\lambda^2,\quad A_1,A_2,V$ are arbitrary constants.

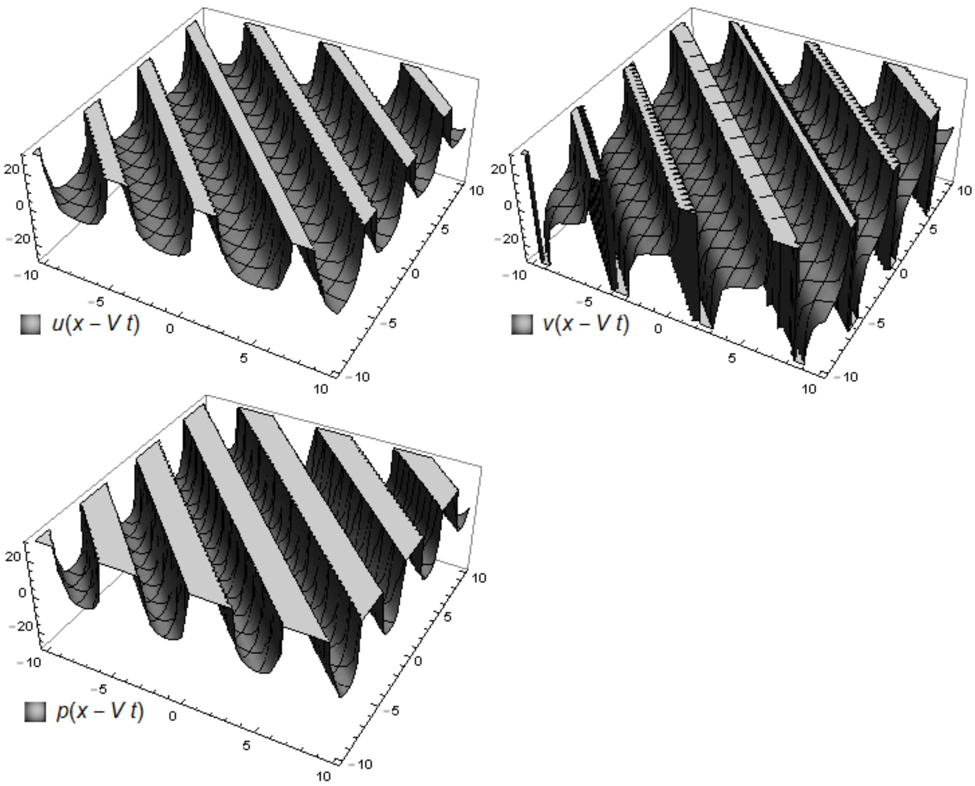

Fig. 3. Trigonometric functions solution (17) when $A_1=1,\ A_2=2,\ \lambda=1.7,\ \mu=1,\ V=-1$

4. INVERSE MODIFIED KORTEWEG – DE VRIES DYNAMIC SYSTEM

Consider the inverse modified Korteweg – de Vries (mKdV) dynamic system [2]

$$\begin{cases} u_t = v \\ v_t = p \\ p_t = u_x + u^2 v, \end{cases} \quad (18)$$

which is obtained from the classical mKdV equation with the aid of the inversion mapping $\mathbb{R}^1 \ni x \rightleftarrows t \in \mathbb{R}^1$. Let us solve system (18) by use of the (G'/G) – expansion method.

Step 1. Introducing traveling wave variable $\xi = x - Vt$, we reduce system (18) to a system of ODE for $u = u(\xi), v = v(\xi), p = p(\xi)$

$$\begin{cases} -Vu' = v \\ -Vv' = p \\ -Vp' = u' + u^2 v. \end{cases} \quad (19)$$

Suppose that the solution to system (19) can be expressed by polynomials in (G'/G) as follows:

$$u(\xi) = \sum_{i=0}^{k} \alpha_i \left(\frac{G'}{G}\right)^i, \quad v(\xi) = \sum_{i=0}^{l} \beta_i \left(\frac{G'}{G}\right)^i, \quad p(\xi) = \sum_{i=0}^{m} \gamma_i \left(\frac{G'}{G}\right)^i. \quad (20)$$

Considering the homogeneous balance between u' and v, v' and p, p' and $u^2 v$ in the first, the second and the third equations of system (19) correspondingly, we obtain a simple system of algebraic equations

$$\begin{cases} k + 1 = l \\ l + 1 = m \\ m + 1 = 2k + l, \end{cases} \quad (21)$$

from which it can be easily found that $k = 1, l = 2$ and $m = 3.$

Step 2. Considering (20) and (21), we find the solution to system (19) in the following form:

$$\begin{cases} u(\xi) = \alpha_1 \left(\frac{G'}{G}\right) + \alpha_0 \\ v(\xi) = \beta_2 \left(\frac{G'}{G}\right)^2 \beta_1 \left(\frac{G'}{G}\right) + \beta_0 \\ p(\xi) = \gamma_3 \left(\frac{G'}{G}\right)^3 + \gamma_2 \left(\frac{G'}{G}\right)^2 \gamma_1 \left(\frac{G'}{G}\right) + \gamma_0, \end{cases} \quad (22)$$

where function $G = G(\xi)$ satisfies the second order linear ODE (5), $\lambda, \mu, V, \alpha_i \left(i = \overline{0,1}\right), \beta_j \left(j = \overline{0,2}\right), \gamma_k \left(k = \overline{0,3}\right)$ are all constants to be determined later, $\alpha_1 \neq 0, \beta_2 \neq 0, \gamma_3 \neq 0.$

Step 3. To substitute (22) into system (19) we need to derive some auxiliary correlations:

$$v' = -\alpha_1 \mu - \alpha_1 \lambda F - \alpha_1 F^2;$$
$$u' = -\beta_1 \mu - (\beta_1 \lambda + 2\beta_2 \mu) F - (2\beta_2 \lambda + \beta_1) F^2 - 2\beta_2 F^3;$$

$$p' = -\gamma_1\mu - \left(\gamma_1\lambda + 2\gamma_2\mu\right)F - \left(2\gamma_2\lambda + 3\gamma_3\mu + \gamma_1\right)F^2 - \left(3\gamma_3\lambda + 2\gamma_2\right)F^3 - 3\gamma_3 F^4;$$
$$u^2 v = \alpha_0^2\beta_0 + \left(\alpha_0^2\beta_1 + 2\alpha_1\alpha_0\beta_0\right)F + \left(\alpha_0^2\beta_2 + 2\alpha_1\alpha_0\beta_1 + \alpha_1^2\beta_0\right)F^2 \qquad (23)$$
$$+\left(\alpha_1^2\beta_1 + 2\alpha_0\alpha_1\beta_2\right)F^3 + \alpha_1^2\beta_2 F^4.$$

In the correlations above, $F = \dfrac{G'}{G}$.

By substituting (23) into Eq. (19) and collecting all terms with the same power of $\left(G'/G\right)$ together, the left-hand sides of equations (19) are converted into another polynomial in $\left(G'/G\right)$. Equating each coefficient of these polynomials to zero yields a set of simultaneous algebraic equations for $\lambda, \mu, V, \alpha_i\left(i=\overline{0,1}\right), \beta_j\left(j=\overline{0,2}\right), \gamma_k\left(k=\overline{0,3}\right)$ as follows:

$$\begin{cases} \alpha_1\mu V - \beta_0 = 0, \quad \alpha_1\lambda V - \beta_1 = 0, \quad \alpha_1 V - \beta_2 = 0, \\ \beta_1\mu V - \gamma_0 = 0, \quad V\left(\beta_1\lambda + 2\beta_2\mu\right) - \gamma_1 = 0, \quad V\left(2\beta_2\lambda + \beta_1\right) - \gamma_2 = 0, \\ 2\beta_2 V - \gamma_3 = 0, \quad \alpha_0^2\left(-\beta_0\right) + \alpha_1\mu + \gamma_1\mu V = 0, \\ \alpha_0^2\left(-\beta_1\right) - 2\alpha_1\alpha_0\beta_0 + \alpha_1\lambda + V\left(\gamma_1\lambda + 2\gamma_2\mu\right) = 0, \\ \alpha_0^2\left(-\beta_2\right) - 2\alpha_1\alpha_0\beta_1 - \alpha_1^2\beta_0 + \alpha_1 + V\left(2\gamma_2\lambda + 3\gamma_3\mu + \gamma_1\right) = 0, \\ -2\alpha_0\alpha_1\beta_2 - \alpha_1^2\beta_1 + V\left(3\gamma_3\lambda + 2\gamma_2\right) = 0, \\ 3\gamma_3 V - \alpha_1^2\beta_2 = 0, \quad \alpha_1 \neq 0, \quad \beta_2 \neq 0, \quad \gamma_3 \neq 0. \end{cases} \qquad (24)$$

Step 4. Solving system of algebraic equations (24) with the aid of MATHEMATICA yields two sets of parameters as follows. Set 1.

$$\lambda = \mp\sqrt{\frac{2}{3}}\frac{\alpha_0}{V}, \quad \mu = \frac{\alpha_0^2 V - 3}{6V^3}, \quad \alpha_1 = \mp\sqrt{6}V,$$
$$\beta_0 = \pm\frac{3\sqrt{6} - \sqrt{6}\alpha_0^2 V}{6V}, \quad \beta_1 = 2\alpha_0 V, \quad \beta_2 = \mp\sqrt{6}V^2, \qquad (25)$$
$$\gamma_0 = \frac{\alpha_0\left(\alpha_0^2 V - 3\right)}{3V}, \quad \gamma_1 = \pm\sqrt{6}\left(1 - \alpha_0^2 V\right), \quad \gamma_2 = 6\alpha_0 V^2, \quad \gamma_3 = \mp 2\sqrt{6}V^3,$$

where α_0, V are arbitrary constants. Set 2.

$$\lambda = 0, \quad \mu = -\frac{1}{2V^3}, \quad \alpha_0 = 0, \quad \alpha_1 = \mp\sqrt{6}V,$$
$$\beta_0 = \pm\sqrt{\frac{3}{2}}\frac{1}{V}, \quad \beta_1 = 0, \quad \beta_2 = \mp\sqrt{6}V^2, \qquad (26)$$
$$\gamma_0 = 0, \quad \gamma_1 = \pm\sqrt{6}, \quad \gamma_2 = 0, \quad \gamma_3 = \mp 2\sqrt{6}V^3,$$

where V is arbitrary constant.

Finally, substituting (25) and (26) with the general solution to linear ODE (5) into representation (22) we obtain three types of traveling wave solutions to the inverse mKdV dynamic system. It is worth mentioning that both parameters sets (25) and (26) yield the same solutions. Now let us discuss them in detail.

When $V > 0$, we get the family of hyperbolic functions solutions

$$\begin{cases} u(\xi) = \dfrac{\mp\left(A_1\sqrt{3/V}\cosh\left(\xi/\sqrt{2V^3}\right) + A_2\sqrt{3/V}\sinh\left(\xi/\sqrt{2V^3}\right)\right)}{A_1\sinh\left(\xi/\sqrt{2V^3}\right) + A_2\cosh\left(\xi/\sqrt{2V^3}\right)} \\ v(\xi) = \dfrac{\mp\sqrt{3/2}\left(A_1^2 - A_2^2\right)}{V\left(A_1\sinh\left(\xi/\sqrt{2V^3}\right) + A_2\cosh\left(\xi/\sqrt{2V^3}\right)\right)^2} \\ p(\xi) = \mp\dfrac{\sqrt{3}\left(A_1^2 - A_2^2\right)\sqrt{1/V^3}\left(A_1\cosh\left(\xi/\sqrt{2V^3}\right) + A_2\sinh\left(\xi/\sqrt{2V^3}\right)\right)}{\left(A_2\cosh\left(\xi/\sqrt{2V^3}\right) + A_1\sinh\left(\xi/\sqrt{2V^3}\right)\right)^3} \end{cases}, \quad (27)$$

where $\xi = x - Vt$, V is arbitrary constant. In particular, setting $A_1 = 0, V = 0.5$ in (27) we obtain soliton solution, found by means of the tanh – method in [3].

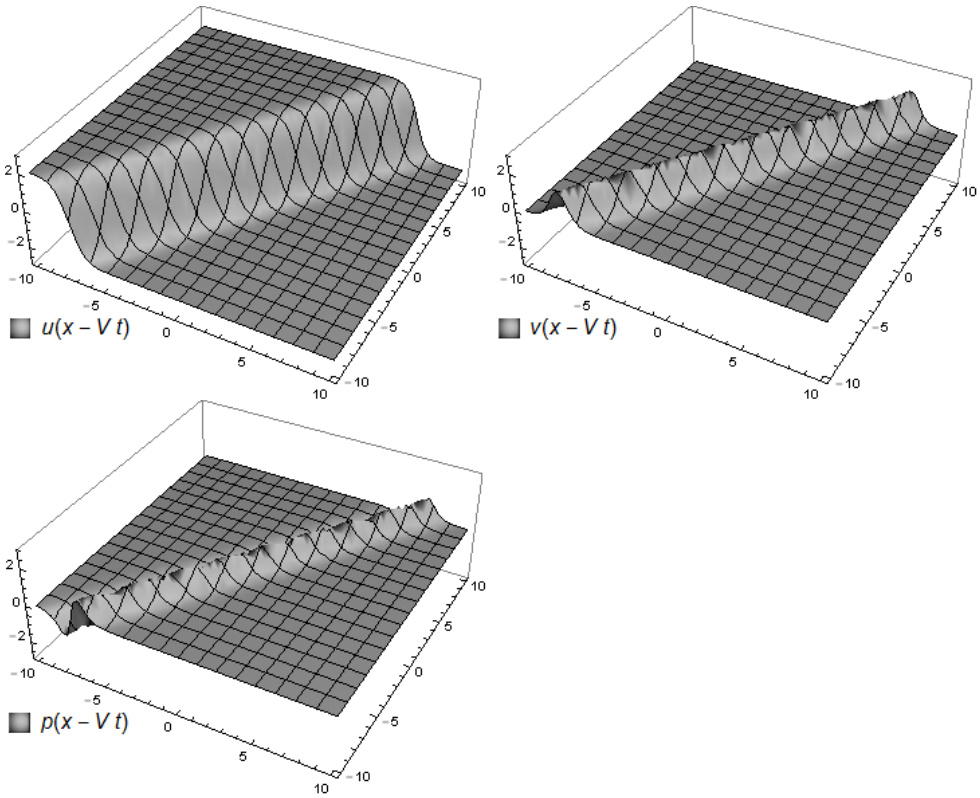

Fig. 4. Hyperbolic functions solution (27) when $A_1 = 1, A_2 = 2, V = 0.7$

When $V < 0$, we get the family of trigonometric functions solutions

$$\begin{cases} u(\xi)=\dfrac{\pm A_2\sqrt{-3/V^3}V\sin\left(\xi/\sqrt{-2V^3}\right)\mp A_1\sqrt{-3/V^3}V\cos\left(\xi/\sqrt{-2V^3}\right)}{A_1\sin\left(\xi/\sqrt{-2V^3}\right)+A_2\cos\left(\xi/\sqrt{-2V^3}\right)} \\ v(\xi)=\dfrac{\pm\sqrt{3/2}\left(A_1^2+A_2^2\right)}{V\left(A_1\sin\left(\xi/\sqrt{-2V^3}\right)+A_2\cos\left(\xi/\sqrt{-2V^3}\right)\right)^2} \\ p(\xi)=\pm\dfrac{\left(A_1^2+A_2^2\right)\sqrt{-3/V^3}\left(A_1\cos\left(\xi/\sqrt{-2V^3}\right)-A_2\sin\left(\xi/\sqrt{-2V^3}\right)\right)}{\left(A_1\sin\left(\xi/\sqrt{-2V^3}\right)+A_2\cos\left(\xi/\sqrt{-2V^3}\right)\right)^3}, \end{cases} \tag{28}$$

where $\xi = x - Vt$, V is arbitrary constant.

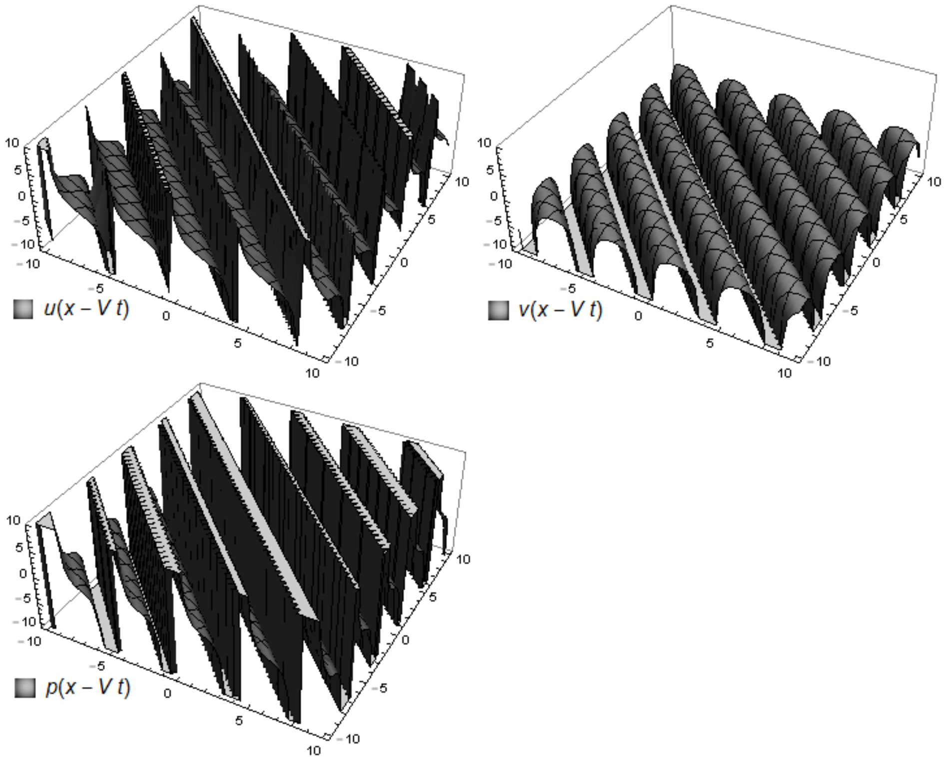

Fig. 5. Trigonometric functions solution (28) when $A_1 = 1, A_2 = 2, V = -1$

5. CONCLUSION

The (G′/G) – expansion method was successfully used to derive exact traveling wave solutions to two inverse nonlinear dynamic systems, namely the inverse KdV and the inverse mKdV systems.

The method was implemented in computer algebra system MATHEMATICA, with the aid of which we obtained the solutions as follows. For the inverse KdV system we constructed the solutions in the form of hyperbolic, rational and trigonometric functions, and for the inverse mKdV system we derived the hyperbolic and trigonometric functions solutions. The correctness of the obtained results was assured by putting them back into the original systems with the aid of MATHEMATICA. All the obtained solutions were graphically analyzed. Moreover, it is shown that with a certain choice of arbitrary parameters the soliton solutions for both dynamic systems can be rediscovered.

The main advantage of the method is that it provides solutions with relatively many arbitrary parameters, and thus these solutions are often more general compared to other analytical methods.

Finally, the method is affirmed to be suitable for computer implementation.

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Стаття: надійшла до редколегії 16.11.2016
доопрацьована 15.03.2017
прийнята до друку 14.06.2017

МЕТОД (G'/G) – РОЗВИНЕННЯ ТА РОЗВ’ЯЗКИ У ВИГЛЯДІ БІЖУЧИХ ХВИЛЬ ДЛЯ ДЕЯКИХ ІНВЕРСНИХ НЕЛІНІЙНИХ ДИНАМІЧНИХ СИСТЕМ

І. Михайлюк, М. Притула

Львівський національний університет імені Івана Франка,
вул. Університетська, 1, Львів, 79000, e-mail: iv.mykh@gmail.co

Метод (G'/G) – розвинення [4] застосовано до інверсної динамічної системи Кортевега – де Фріза та інверсної модифікованої динамічної системи Кортевега – де Фріза [2]. Для обох систем побудовано та проаналізовано розв’язки у вигляді біжучих хвиль, що виражені через гіперболічні, раціональні та тригонометричні функції. Отримані результати порівняно із розв’язками, знайденими методом tanh – функцій.

Ключові слова: метод (G'/G) – розвинення, інверсна нелінійна динамічна система Кортевега – де Фріза (КдФ), інверсна модифікована нелінійна динамічна система Кортевега – де Фріза (мКдФ), солітонний розв’язок.