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USAGE OF VECTOR PARAMETRIC OPTIMIZATION FOR ROBUST STABILIZATION OF GROUND VEHICLES INFORMATION-MEASURING DEVICES

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Abstract. *The statement of the vector robust parametric optimization problem taking into consideration two groups of constraints such as the stability conditions and the requirements to performance, as well as the uncertainties of the mathematical models of the controlled plant and external disturbances is represented. It was applied to the parametric synthesis of the robust system for stabilization of the information-measuring devices at the moving base is implemented. The interactive heuristic two-step procedure for this problem solution is proposed. Efficiency of the suggested procedure is proved by the simulation results.*

Keywords: information-measuring devices; robust systems; vector optimization.

1. Introduction

Competitive capacity of the systems for stabilization of the information-measuring devices operated at the ground vehicles is defined by the possibility to operate in the difficult conditions, which are associated with various disturbances caused by the irregularities of the road or terrain relief.

Moreover, the platforms used at the ground vehicles are characterized by some parameters which may change in the wide range (to 50 %). The platform inertia moment and the rigidity of the elastic linkage between the moving base of the platform with paying load mounted on it and the actuator belong to such parameters. Influence of the difficult operation conditions may lead not only to failure to execute the specification requirements but to the loss of the stability. Application of the robust control allows successful withstanding of the studied systems to these adverse operational factors. The important advantage of the robust system is its ability to satisfy the specification requirements during the system parameters change in the certain range and the external disturbance influence without using of the adaptation techniques. Such approach ensures ease of control and increases the competitive capacity correspondingly.

It should be noted, that the various and conflicting requirements are immanent to the systems of the studied type. The most important conflict is the necessity to ensure the accuracy and the robustness at the same time. To solve this conflict it is necessary to find the compromise solution. The requirements to the system accuracy during influence of the deterministic and stochastic disturbances are mutually conflicting too.

Thus, the modern approach to the studied system parametric synthesis is the vector robust parametric optimization, because it allows successful modification of existing systems via changing their adjustable parameters.

2. Analysis of last researches

Approaches to determination of the optimization criteria for the control systems of the wide class are represented in [Tunik et al. 2001]. The criteria represented in this paper are able to ensure such conflicting requirements as the control system accuracy and robustness at the same time. They are based on the H_2, and H_∞-norms of the closed loop control system sensitivity and complementary sensitivity matrix functions respectively. The algorithm and basic stages of the robust parametric mixed H_2/H_∞-optimization of the system for stabilization of the information-measuring devices assigned for operation at the ground vehicles are given in the paper [Sushchenko 2008]. At the above stated papers the combined optimization criterion consisting of the local scalar criteria is considered. The optimization problem taking into account some optimization criteria represents the vector optimization problem [Egupov 2002]. In this paper it is also mentioned, that application of the vector optimization is convenient for systems which operate in different operation modes, for example, nominal and parametrically disturbed.

The **goal** of the given paper is the development of the procedure for the vector mixed H_2/H_∞-optimization of the systems for stabilization of the information-measuring devices operated at the ground vehicles.

The problems of the optimization criterion vectorization and way of the optimization problem solution by the chosen criterion are considered in the paper.

3. The nominal and parametrically disturbed models of the stabilization system

To design the robust systems for stabilization of the information-measuring devices operated at the ground vehicles it is necessary to determine the change range of the system parameters having the most significant influence on the system stability and performance. For the studied stabilization system changes of these parameters could be estimated as follows:

1) change of the stabilized plant inertia moment during system functioning in the range $\pm$ 50%;

2) change of the rigidity of the elastic linkage between the moving base of the platform with paying load (the stabilized plant) mounted on it and the actuator in the range $\pm$ 50%.

The system of the studied type is characterized by such features as the high requirements to the accuracy and the necessity to keep these requirements for sufficient changes of the stabilized plant. For the systems of stabilization of the information-measuring devices for the ground vehicles the variations of the above stated parameters in the sufficiently wide range ($\pm$ 50%) underlines actuality of the robust approaches to such systems design. This fact leads to the necessity of development the parametrically disturbed models of the systems for stabilization of the information-measuring devices.

To carry out the robust parametric optimization it is necessary to have the set of models in the state space, given by quadruple of matrices $\mathbf{A},\mathbf{B},\mathbf{C},\mathbf{D}$, which describe the stabilized plant and actuator, united by the elastic linkage as the single device, within the boundary values of the changed parameters. These matrices generate the set of the closed loop system matrices which must be used for the stabilization system parametric synthesis.

In accordance with the model of the stabilization system of the information-measuring devices for the ground vehicle represented in [Sushchenko 2008], the state space model of the stabilization system horizontal channel with the nominal values of parameters such as the plant inertia moment (J_p=882 Nms^2) and the rigidity of elastic linkage between the actuator and the platform moving base (c_p=25·10^4 Nm/rad) for the state vector

$$\mathbf{x}^{\mathrm{T}} = \begin{bmatrix} \dot{\varphi}_{sm} & \dot{\varphi}_p & \varphi_{sm} & \varphi_p & U \end{bmatrix},$$

where the state variables x_i, $i=\overline{1,5}$ characterize the angular rates, and angles of turn of the servomotor and stabilized plant, and the control voltage respectively, could be determined by the following state and control matrices:

$$\mathbf{A}_0 = \begin{bmatrix} -1{,}62 & 0 & -3225{,}37 & -1{,}64\cdot10^6 & 1602{,}56 \\ 0 & -0{,}232 & 0{,}55 & -281{,}85 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -41{,}67 & 0 & 0 & 0 & -666{,}67 \end{bmatrix}; \tag{1}$$

$$\mathbf{B}_0^{\mathrm{T}} = \begin{bmatrix} 0 & -0{,}28 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 666{,}67 \end{bmatrix}, \tag{2}$$

The matrices of the nominal model are marked by the index 0.

The appropriate matrices for parametrically disturbed model of the horizontal channel look like

$$\mathbf{A}_{p1} = \begin{bmatrix} -1{,}62 & 0 & -3225{,}37 & -1{,}64\cdot10^6 & 1602{,}56 \\ 0 & -0{,}155 & 0{,}367 & -187{,}9 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -41{,}67 & 0 & 0 & 0 & -666{,}67 \end{bmatrix}; \tag{3}$$

$$\mathbf{B}_{p1}^{\mathrm{T}} = \begin{bmatrix} 0 & -0{,}187 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 666{,}67 \end{bmatrix}; \tag{4}$$

$$\mathbf{A}_{p2} = \begin{bmatrix} -1{,}54 & 0 & -4607{,}76 & -2{,}34\cdot10^6 & 2289{,}42 \\ 0 & -0{,}232 & 0{,}55 & -281{,}85 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -41{,}67 & 0 & 0 & 0 & -666{,}67 \end{bmatrix}; \tag{5}$$

$$\mathbf{B}_{p2}^{\mathrm{T}} = \begin{bmatrix} 0 & -0{,}28 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 666{,}67 \end{bmatrix}. \tag{6}$$

In contrast to the nominal model determined by matrices (1), (2), the model described by the expressions (3), (4) is characterized by the increase on 50% of the stabilized plant inertia moment (J_p=1330,5 Nms^2) and the rigidity of the elastic linkage between the stabilized plant and actuator (c_p=37,5·10^4 Nm/rad). The model described by the expressions (5), (6) is characterized by the decreased values of the above stated parameters (J_p=443,5 Nms^2, c_p=12,5·10^4 Nm/rad correspondingly).

In expressions (2), (4), (6) the first row corresponds to disturbance moment and the second – to the control signal.

The matrix of real measurements looks like

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

A direct transfer matrix **D** is the null matrix.

In a similar way it is possible to develop the nominal and parametrically disturbed models of the vertical channel of the system for stabilization of the information-measuring devices.

The solutions obtained with taking into account of the maximum and minimum permissible parametric structured disturbances are true for any other parameter values lying in the range of the given acceptable domain. This statement is true, if the studied system is detected and stabilized [Skogestad, Postlethwaite 1997]. This fact takes place in the most of practical situations.

Usage of parametrically disturbed models for the optimization procedure allows including some parametric uncertainties in the mathematical models. These uncertainties are caused by real system parameters changes in some given ranges and the impossibility to take into consideration all peculiarities of the real systems in the mathematical form. Usually, such uncertainties are divided into two groups such as parametric structured and parametric unstructured (non-simulated dynamics of the system) [Chapellat et al. 1990].

Taking into consideration the parametric structured disturbances in the parametric optimization procedure is easier than parametric unstructured disturbances.

Systems of the studied type must satisfy the specification requirements both in conditions of the parametric disturbances and under action of the external stochastic disturbances. This requires taking into consideration during optimization both the deterministic and stochastic cases. A system of the studied type is subjected to the influence of the stochastic disturbances caused by the irregularities of the road and terrain relief. The features of these stochastic disturbances mathematical description were considered in the paper [Sushchenko 2008].

In accordance with results obtained in this paper, the disturbances due to the irregularities of the road and terrain relief may be represented as the time-invariant random processes with some spectral densities. The expressions for these spectral densities description depend on the type of the irregularities of the terrain by which the ground vehicle is moving on [Dynamic…1976].

To simulate the influence of such disturbances on the studied system it is necessary to obtain the transfer function of the forming filter, which converts the white noise at its input into a random process with the given spectral density.

For determination of such filter the Wiener factorization is used. In this case the forming filter for the road relief irregularities simulation will be obtained by the formula [Dynamic…1976]

$$K_q(\omega) = K_h(j\omega) H_q(j\omega) H_\kappa(j\omega) H(j\omega)\,, \tag{7}$$

where $K_h(j\omega)$ – the spectral density of the macro-profile;

$H_q(j\omega)$ – the transfer function, which corresponds to conversion into the micro-profile;

$H_\kappa(j\omega)$ – the transfer function of averaging by the contact area;

$H(j\omega)$ – the transfer function of the suspension.

Based on the expression (7) the forming filter for the road with the medium irregularities becomes [Dynamic…1976; Sushchenko 2008]

$$K_q(j\omega) = \sqrt{\frac{D_2}{v}}\, j\omega \frac{(3{,}1v + j\omega)}{(\omega_2 + j\omega)} \times$$
$$\times \frac{(j\omega)^2}{(j\omega)^2 + \sqrt{2}\omega_{\mathrm{m}} j\omega + \omega_{\mathrm{m}}^2} \times \frac{\omega_{\mathrm{c}}^2}{(j\omega)^2 + \sqrt{2}\omega_{\mathrm{c}} j\omega + \omega_{\mathrm{c}}^2} \times$$
$$\times \frac{\omega_0^2}{(j\omega)^2 + \sqrt{2}\omega_0 j\omega + \omega_2^2}\,, \tag{8}$$

where D_2 – a coefficient depending on a type of the road [Dynamic…1976];

ω_2 – a coefficient depending on a type of both the road and the terrain [Dynamic…1976];

v – the vehicle speed;

$\omega_m = 2\ \mathrm{rad/s}$;

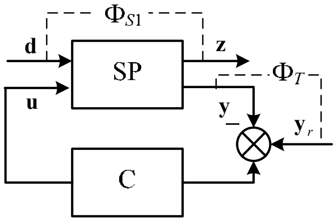
$\omega_c = v/a$;

a – one half of the contact area length;

ω_0 – the eigen-frequency of the angular oscillations.

The expression for the forming filter depends on the ground vehicle speed. From this it follows that the stochastic model makes an additional uncertainty due to disturbance dependence on speed changing during the vehicle motion.

The structural schemes of the deterministic and stochastic stabilization systems are represented in Fig. 1.



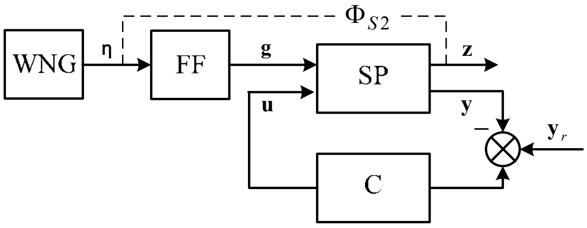


Fig. 1. The deterministic (*a*) and stochastic (*b*) stabilization systems:
SP – the stabilized plant;
C – the controller;
FF – the forming filter;
WNG – the white noise generator;
d – the input (command) signal;
z – the observation signal;
u – the control signal;
y – the output signal;
η – the white noise;
g – the disturbance;
Φ_{S1} – the sensitivity function for the deterministic case;
Φ_{S2} – the sensitivity function for the stochastic case;
Φ_T – the complementary sensitivity function

The controller of the researched system may be represented in the state space by following four matrices

$$\mathbf{A}=\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & -1/T_2 & 0 & 0\\ 0 & 0 & -1/T_3 & 0\\ 0 & 0 & 0 & -1/T_5\end{bmatrix};$$

$$\mathbf{B}^T=\begin{bmatrix}k_g k_1 R_1 & k_g k_2 R_2 b_1 & 0 & 0\\ 0 & 0 & k_3 R_3 b_2 & k_3 R_3 b_3\end{bmatrix};$$

$$\mathbf{C}=[1 \quad 1 \quad 1 \quad 1];$$

$$\mathbf{D}=[k_g k_2 R_2 T_1 / T_2 \quad 0], \qquad (9)$$

where

$$b_1=\frac{T_2-T_1}{T_2^2};$$

$$b_2=\frac{T_4-T_3}{T_5-T_3};$$

$$b_3=\frac{T_5-T_4}{T_5-T_3}.$$

In the represented controller the control is implemented by the plant angular rate measured by the gyro (the main feedback) and by the current of the servomotor armature circuit (the local feedback). In the represented matrices (9) k_g is the transfer constant of the angular rate sensor; k_1, k_2, k_3 are the gain coefficients; R_1, R_2, R_3 are the adjustable coefficients, T_1, T_2, T_3, T_4, T_5 are the time constants. The goal of the optimization procedure is to find the optimal values of the adjustable coefficients R_1, R_2, R_3 and the time constants T_2, T_3, T_5.

4. Statement of the robust parametric optimization as the vector optimization problem

For the system parametric optimization it is necessary to choose the optimization criterion taking into consideration the various aspects of the system functioning. Both accuracy and stability in the presence of the internal and external disturbances have great importance for the systems of the studied type.

For calculation of the local performance indices of the robust stabilization systems the H_2-norms are used. It should be noted, that the H_2-norms represent the square roots from the integral quadratic performance criteria. In the general form these criteria look like:

– for the deterministic dynamic systems and signals [Kwakernaak 1993]:

$$J_d=\int_0^\infty(\mathbf{x}^{\mathrm{T}}\mathbf{Q}\mathbf{x}+\mathbf{u}^{\mathrm{T}}\mathbf{R}\mathbf{u})dt, \qquad (10)$$

where **Q, R** – the weighting matrices, which take into consideration the weights of the state variables and the external actions;

– for the stochastic dynamic systems and signals [Kwakernaak 1993]:

$$J_s=\int_0^\infty(\mathrm{M}[\mathbf{x}^{\mathrm{T}}\mathbf{Q}\mathbf{x}+\mathbf{u}^{\mathrm{T}}\mathbf{R}\mathbf{u}])dt, \qquad (11)$$

where M – the symbol of the mathematical expectation.

For calculation of the stabilization systems robustness criteria the H_∞-norms of the closed loop systems complementary sensitivity matrix function $\Phi_T(j\omega)$ [Skogestad, Postlethwaite 1997] is used

$$H_\infty = \sup_{\omega \in [-\infty, \infty]} \overline{\sigma}(j\omega), \tag{12}$$

where $\overline{\sigma}$ – the maximum singular value of the closed loop system matrix transfer function $\Phi_T(j\omega)$ at the circular frequency ω.

The function $\overline{\sigma}(j\omega)$ in (12) is called the singular characteristic of the multi-dimensional system

$$\overline{\sigma}(j\omega) = \max_{i \in 1,\dots,n} \sqrt{eig_i \Phi^*(j\omega)\Phi(j\omega)}_{\forall \omega \in [-\infty, \infty]}, \tag{13}$$

where *eig* means operation of the eigenvalues determination. The H_∞-norm characterizes the upper boundary of the system singular frequency characteristic maximum value.

Combination of the H_2 and H_∞-optimization allows to form a problem of the synthesis of the stabilization system with the optimal performance level based on the quadratic criteria for deterministic and stochastic cases under condition of preserving this performance level during action of parametric disturbances [Egupov 2002; Skogestad, Postlethwaite 1997; Tunik et al. 2001]. That is why the combined optimization criterion for the studied system must include the performance indices of the nominal and parametrically disturbed systems for the deterministic and stochastic cases (9, 10). The optimization criterion must also include the robustness indices of the nominal and disturbed systems determined by (12), (13).

Then the combined criterion becomes [Tunik et al. 2001]:

$$\begin{aligned} J_{H_2/H_\infty} &= \lambda_2^{nom\,d} \| \Phi_{S1}(\mathbf{K}, \mathbf{x}, \mathbf{u}, j\omega)^{nom\,d} \|_2 + \\ &+ \lambda_2^{nom\,s} \| \Phi_{S2}(\mathbf{K}, \mathbf{x}, \mathbf{u}, j\omega)^{nom\,s} \|_2 + \\ &+ \lambda_\infty^{nom} \| \Phi_T(\mathbf{K}, \mathbf{x}, \mathbf{u}, j\omega)^{nom} \|_\infty + \\ &+ \sum_{i=1}^{n} \lambda_{2_i}^{par\,d} \| \Phi_{S1}(\mathbf{K}, \mathbf{x}, \mathbf{u}, j\omega)_i^{par\,d} \|_2 + \\ &+ \sum_{i=1}^{n} \lambda_{2_i}^{par\,s} \| \Phi_{S2}(\mathbf{K}, \mathbf{x}, \mathbf{u}, j\omega)_i^{par\,s} \|_2 + \\ &+ \sum_{i=1}^{n} \lambda_{\infty_i}^{par} \| \Phi_T(\mathbf{K}, \mathbf{x}, \mathbf{u}, j\omega)_i^{par} \|_\infty + PF, \end{aligned} \tag{14}$$

where $\lambda_2^{nom\,d}, \lambda_2^{nom\,s}, \lambda_\infty^{nom}, \lambda_{2_i}^{par\,d}, \lambda_{2_i}^{par\,s}, \lambda_{\infty_i}^{par}$ – the weighting coefficients of the appropriate norms;

$\| \Phi_{s1}^{nom\,d} \|_2, \| \Phi_{s2}^{nom\,s} \|_2, \| \Phi_{s1}^{par\,d} \|_{2_i}, \| \Phi_{s2}^{par\,s} \|_{2_i}$ – the H_2-norms of the matrix sensitivity functions for the nominal and disturbed by the parametric structured disturbances closed loop systems including the deterministic and stochastic cases;

$\mathbf{K}$ – the vector of the controller parameters to be optimized;

$\| \Phi_T^{nom} \|_\infty, \| \Phi_T^{par} \|_{\infty_i}$ – the H_∞-norms of the matrix complementary sensitivity functions for the nominal and disturbed by the parametric structured disturbances closed loop systems;

n – the amount of models of the system disturbed by the parametric structured disturbances;

PF – the penalty function, that guarantees system stability during the optimization process.

The H_∞-norms taken into consideration in the combined optimization criterion (14) ensure the certain insensitivity of the synthesized system to the parameter changes in the range of the accepted domain. It is known [Skogestad, Postlethwaite 1997; Tunik et al. 2001], that the requirements to the control accuracy (performance) and robustness are mutually conflicting. Therefore the H_2/H_∞ optimization of the stabilization system lies in the search of the compromise between the accuracy and robustness of the system.

Usage of the combined criterion for parametric optimization execution allows finding solution able to ensure a compromise between the requirements to the system accuracy and robustness. Such approach to synthesis problem solving is called multi-purpose [Tunik et al. 2001].

The goal of the robust parametric optimization is the minimization of the criterion (14) for the various combinations of the system parameter values. It is important to provide the presence of the system state variables in the range given by the specification of requirements and to obtain aforementioned compromise between robustness and performance.

Consider features of the robust H_2/H_∞-optimization with criterion (14) more in detail. In compliance with the definition given in the paper [Poliak, Shcherbakov 2005] this problem belongs to the so-called “difficult” problems of the control theory. Such problems are characterized by such properties as the non-convexity and NP-complexity (*NP* – non-deterministic polynomial hard). The last property means that the solution of the problem characterized by the input data amount n may not be obtained by means of the Turing machine during the time of the order $O(^k_n)$, where k is an arbitrary constant, which does not depend on the input data. These factors make impossible receiving the exact solution of the optimization problem. It should be noted, that in some cases such solution is absent. Nevertheless, the rejection of the exact solution search leads to the necessity to find technically reasonable solution, which will satisfy the input data of pre-designing and will be efficient from the point of view of a designer.

The criterion (14) represents a conversion of the set of scalar (local) criteria in the single global criterion. In this problem the separate scalar criteria are conflicting. It is known [Egupov 2002; Skogestad, Postlethwaite 1997], that minimization of the accuracy criterion (the H_2-norm of the nominal system sensitivity function $\mathbf{S}$) leads to maximization of the robustness criterion (the H_∞-norm of the nominal system complementary sensitivity function $\mathbf{T}$) [Kwakernaak 1993].

Moreover, the performance (accuracy) criteria based on the H_2-norms of the sensitivity function $\mathbf{S}$ for the deterministic and stochastic cases are conflicting too. Really, for the deterministic case the H_2-norm minimization of the closed loop system sensitivity function for improvement of its accuracy means increasing of its bandwidth. At the same time for the stochastic case it means increasing of the variance of the closed loop system error during the white noise action at its input, thus degrading its accuracy.

Taking into consideration above stated factors the criterion (14) represents the scalar conversion of the vector criterion [Balandin, Kogan 2007; Egupov 2002] by means of the weighting coefficients λ_i, which in the multiobjective optimization theory are called the coefficients of the local criteria importance. Therefore it is convenient to consider the H_2/H_∞-criterion more in detail from the multi-objective optimization viewpoint.

It is possible to represent the local criteria (14) in the vector form:

$$J^d_{H_2} = \begin{bmatrix} \|\Phi_{S1}(\mathbf{K},\mathbf{x},\mathbf{u},j\omega)^{nom\,d}\|_2 \\ \|\Phi_{S1}(\mathbf{K},\mathbf{x},\mathbf{u},j\omega)^{par\,d}_1\|_2 \\ \cdots \\ \|\Phi_{S1}(\mathbf{K},\mathbf{x},\mathbf{u},j\omega)^{par\,d}_n\|_2 \end{bmatrix}; \qquad (15)$$

$$J^s_{H_2} = \begin{bmatrix} \|\Phi_{S2}(\mathbf{K},\mathbf{x},\mathbf{u},j\omega)^{nom\,s}\|_2 \\ \|\Phi_{S2}(\mathbf{K},\mathbf{x},\mathbf{u},j\omega)^{par\,s}_1\|_2 \\ \cdots \\ \|\Phi_{S2}(\mathbf{K},\mathbf{x},\mathbf{u},j\omega)^{par\,s}_n\|_2 \end{bmatrix}; \qquad (16)$$

$$J_\infty = \begin{bmatrix} \|\Phi_{T}(\mathbf{K},\mathbf{x},\mathbf{u},j\omega)^{nom}\|_\infty \\ \|\Phi_{T}(\mathbf{K},\mathbf{x},\mathbf{u},j\omega)^{par}_1\|_\infty \\ \cdots \\ \|\Phi_{T}(\mathbf{K},\mathbf{x},\mathbf{u},j\omega)^{par}_n\|_\infty \end{bmatrix}. \qquad (17)$$

The vectors of the weighting coefficients for the local criteria (15) – (17) look like

$$\lambda_d = [\lambda_2^{nom\,d} \quad \lambda_{21}^{par\,d} \quad \ldots \quad \lambda_{2n}^{par\,d}]; \qquad (18)$$

$$\lambda_s = [\lambda_2^{nom\,s} \quad \lambda_{21}^{par\,s} \quad \ldots \quad \lambda_{2n}^{par\,s}]; \qquad (19)$$

$$\lambda_\infty = [\lambda_\infty^{nom\,s} \quad \lambda_{1\infty}^{par\,s} \quad \ldots \quad \lambda_{n\infty}^{par\,s}]. \qquad (20)$$

Using the expressions (15) – (20), it is possible to rewrite the criterion (14) in the form

$$J_{H_2/H_\infty} = \lambda_d J^d_{H_2} + \lambda_s J^s_{H_2} + \lambda_\infty J_\infty + PF. \qquad (21)$$

The first three local criteria in the global criteria (21) are conflicting as it was mentioned above.

Denote K_p the Pareto-optimum solution of the design problem for the controller with the vector of the adjustable parameters $\mathbf{K}$. Here it is expedient to remember the property of the solved problem NP-complexity. The rigorous solution of this problem may not exist in accordance with this property. In this case it is reasonable to find the problem solution acceptable from the point of view of the engineering requirements to the design controller. This solution must be compromise between accuracy and robustness on the one hand and between accuracy in deterministic and stochastic cases on the other hand on the basis of the system designer's preferences. Such solution may be considered as some engineering analogue of the Pareto-optimal solution. Search of this solution is implemented in the space of coefficients defined by the vectors of weighting coefficients λ_d, λ_s and λ_∞ in criterion (21) and elements of the weighting matrices $\mathbf{Q}$ and $\mathbf{R}$ of the H_2-norms, calculated for the local criteria (10) and (11). Denote $\Lambda = [\lambda_d \quad \lambda_s \quad \lambda_\infty]$.

Then the problem for design of the H_2 / H_∞-controller for the platform stabilized in the inertial space taking into consideration the expression (21) may be formulated in the following way:

$$\mathbf{K}_p = \operatorname{argmin}_p J_{H_2/H_\infty}(\mathbf{K}, \mathbf{Q}, \mathbf{R}, \Lambda, \mathbf{x}, \mathbf{u}, j\omega); \tag{22}$$

$$\mathbf{K} \in D,\ D: \mathrm{Re}[\mathrm{eig}_i(\mathbf{I} + \mathbf{L}(s))] < 0,\ i \in 1,...,i_0; \tag{23}$$

$$x_i < x_{i0},\ i = 1,...,n_0;\ u_j < u_{j0},\ j = 1,...,m_0; \tag{24}$$

$$\mathbf{L}(s) = \mathbf{P}(s)\mathbf{W}(s),$$

where D – the stability region in the space of parameters;

$\mathbf{P}(s)$ – the controller;

$\mathbf{W}(s)$ – the transfer function of the plant taking into consideration the actuator and the stabilized plant;

i_0 – the order of the set of the differential equations in the Cauchy form;

$n_0 \times 1$ – the dimension of the vector of states;

$m_0 \times 1$ – the dimension of the control vector.

It should be noted, that in the optimization problem (22) the constraints (23) are defined by the conditions of the closed loop system stability and the constraints (24) are defined by the specification requirements to the designed system. The symbol $\min_p$ denotes a “nonstrict” minimum, which is determined by the concrete requirements to the designed system.

The process of the solution search represents a procedure of the multiple minimization of the criterion (14) by one of the known methods. The Nelder-Mead method or the genetic algorithm may be used as such methods. The advantages of the genetic algorithm lie in the possibility to find the global minimum in every concrete case.

Now two trends of the optimization by the vector criteria are known [Egupov 2002]. The first trend uses the interactive procedures, the second lies in development of the specialized criteria for the system of the certain class. For the studied problem connected with the compromise solution search and use of the specific criteria for the robust systems it is expedient to use the optimization based on combination of both trends.

The interactive heuristic procedure of the vector optimization for the system of the studied type is divided into two steps.

At the first step the vector optimization of the system acceptable from the point of view of the specification requirements is realized by means of the appropriate weighting matrices **Q** and **R** search. This process is accompanied by check of the constraints (24). If these constraints are not satisfied, the elements of the weighting matrices q_i (for $x_i > x_{i0}$) and r_j (for $u_j > u_{j0}$) are changed. The procedure is repeated while the non-equalities (24) will be true. Satisfaction of the expressions (24) for all x_i and u_j means determination of the solution acceptable from the point of view of the specification requirements given to the system.

At the second step for the weighting matrices **Q** and **R** obtained at the first step the vector optimization with variation of the criteria weighting coefficients defined by the vector Λ is implemented. This provides search of solution ensuring compromise between the system robustness and accuracy.

A compromise between the local criteria by means of the weighting coefficients Λ is defined by the designer’s preferences, which are directly connected with the operation conditions of the system for stabilization of the information-measuring devices and its design features.

For example, in most cases, the preference is shown to the criterion $J_{H_2}^s$ if the operation is carried out in conditions of the intensive random disturbances. In other cases, it is desirable to decrease the criterion J_∞ for the sufficient parametric disturbances. To achieve the desired compromise, it is sufficient to carry out some cycles of the minimization procedure execution in conditions of variation of the criteria weighting coefficients (components of the vector Λ).

It should be noted, that the given procedure has the heuristic nature. It is impossible to prove its convergence but it is checked for the sufficiently quantity of the practical situations that have been proved its efficiency. As it follows from the constraints (23), during the parametric optimization procedure and the vector $\mathbf{K}^*$ optimal values search it is necessary to ensure the closed loop system stability during variation of the plant characteristics and the controller parameters. To achieve this purpose the penalty function is added to the optimization criterion (14). This function provides location of the closed loop system poles in the left half-plane of the complex variable [Balandin, Kogan 2007].

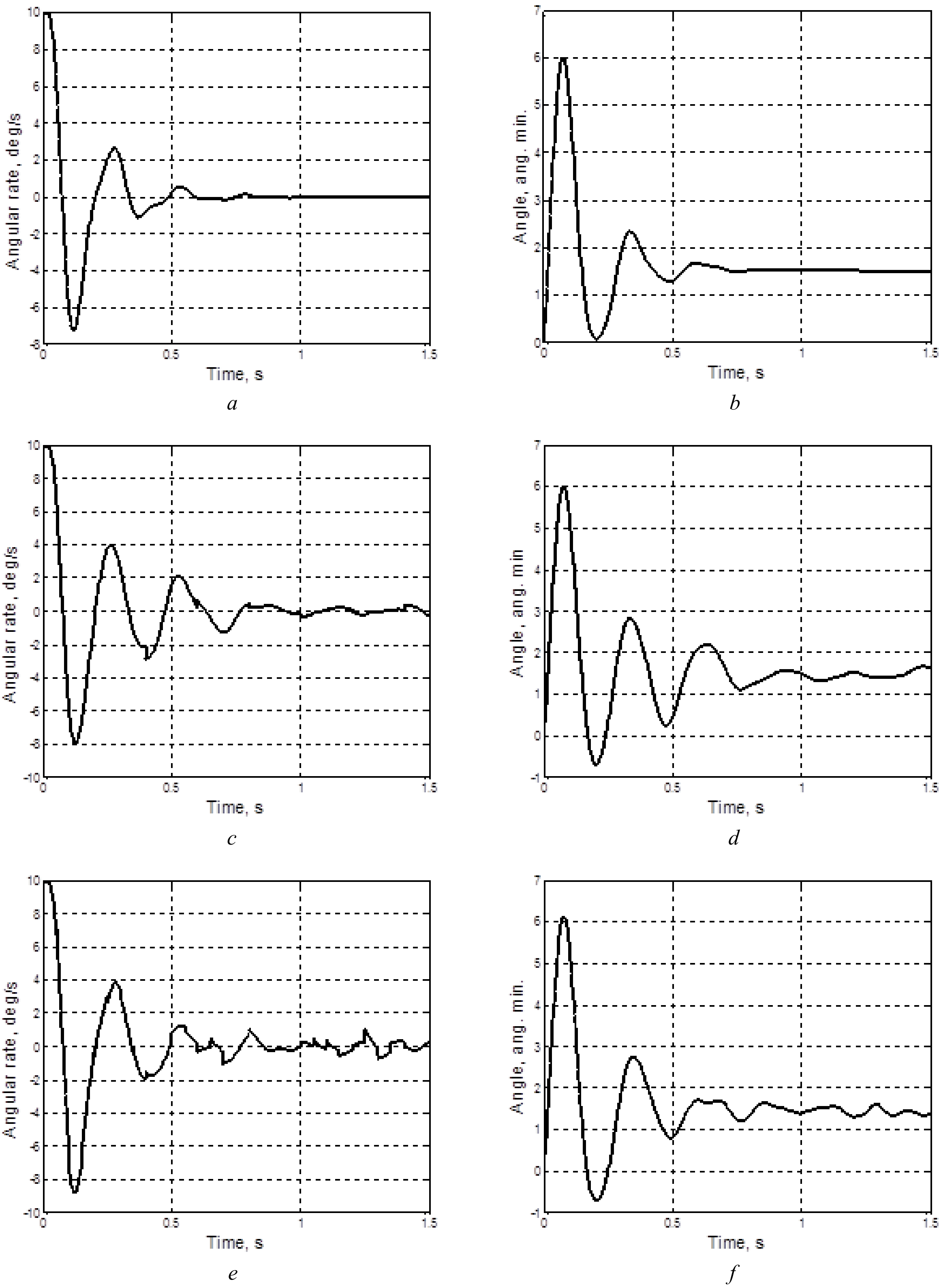


Fig. 2. Results of simulation of the synthesized stabilization system (the horizontal channel) for the deterministic case (*a*, *b*) and for the stochastic case taking into consideration such disturbance as the roads with the tussocks (*c*, *d*) and the medium irregularities (*e*, *f*):
a, *c*, *e* – the stabilization angular rate;
b, d, f – the stabilization angular position error

To determine the penalty function it is necessary to check out the presence of the closed loop system poles in some area in the left half-plane of the complex variable. This area must correspond to the criteria of the system stability.

For a system of the studied type it is expedient to use the transient process quality indices as boundary conditions, which must be satisfied in any case, by means of their introduction in the penalty function. This defines the certain requirements to the distribution of the closed loop system transfer function poles.

The parameters bounded this area represent the least distance to the imaginary axis and the greatest distance to the imaginary axis. The expression for the penalty function may include some additional conditions, for example, the constraint by the value of the coefficient in the gain circuit, which enters in the vector of the adjustable parameters. This coefficient ensures the acceptable value of the angular rigidity by the moment.

5. Results of the vector robust optimization

Results of the vector robust optimization of the system with the controller (9) are represented in the Tables 1, 2.

Table 1. Values of the stabilization system optimization parameters

R_1	R_2	R_3	T_2	T_3	T_5
0,18	0,007	0,32	0.005	0.015,3	0,0015

Table 2. Performances of the optimized system

H_2 norm	H_∞ norm	ΔA, dB	$\Delta\varphi$, degree	C_m, Nm/ang. min
0,3182	0,1261	59,4	91,1	84,7

In Tables 1, 2 the optimal values of the adjustable parameters and the time constants are given. The Table 2 includes such performances of the optimized system as norms, margins by the amplitude and the phase and the angular rigidity C_m [Sushchenko 2008]. The simulation results for the stabilization system horizontal channel for deterministic and stochastic cases taking into consideration the disturbances caused by the medium road relief irregularities, which are given by the expression (8), are represented in Fig. 2.

The results of simulation for the road with tussocks are also represented in Fig. 2. At that description of the disturbance for the roads with tussocks given in [Dynamic…1976] was used.

As it follows from the obtained results the transient processes satisfy the requirements by the accuracy and speed. Values of the norms $H_2 = 0{,}318$ and $H_\infty = 0{,}126$ are also acceptable.

Confidence of the H_2 / H_∞-optimization result is based on the principle of the guaranteed result [Balandin, Kogan 2007], which does not depend on the spectral properties of the external disturbance and depends on its H_∞-norms only:

$$\frac{\|\Phi(s)\mathbf{w}(s)\|_\infty}{\|\mathbf{w}(s)\|_\infty} < \gamma,$$

where $\Phi(s)$ – the closed loop system matrix transfer function;

$\mathbf{w}(s)$ – the Laplace transformation of the disturbance signal;

γ – a small number.

Such approach is widely used in the practice of the robust systems design.

6. Conclusions

The new method of problem solving for the multiobjective optimization of the robust systems for stabilization of the information-measuring-devices at the moving base is developed. The method is based on the idea of the vector parametric H_2 / H_∞-optimization. The goal of the method is achievement of the compromise between the robustness and accuracy of the system operated in difficult conditions of the internal parametric and external coordinated disturbances action. The developed procedure takes into consideration constraints by the phase coordinates, which are caused by the specification requirements, and the necessity to satisfy the system stability conditions. Usage of the procedure allows to implement modernization of the systems for stabilization of the information-measuring devices operated at the ground vehicles.

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А.А. Тунік[1], О.А. Сущенко[2]. Векторна параметрична оптимізація робастних систем стабілізації інформаційно-вимірювальних пристроїв наземних рухомих об'єктів
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Розглянуто особливості постановки задачі векторної параметричної оптимізації робастних систем стабілізації з урахуванням двох груп обмежень: умов стійкості системи та виконання надаваних до неї технічних вимог. Здійснено векторизацію критерію оптимізації робастної системи стабілізації інформаційно-вимірювальних пристроїв, експлуатованих на рухомих об'єктах. Запропоновано інтерактивну евристичну процедуру розв'язання цієї задачі, що складається з двох етапів. Ефективність запропонованої процедури підтверджено результатами моделювання.

Ключові слова: векторна оптимізація; інформаційно-вимірювальні пристрої; робастні системи стабілізації.

А.А. Туник[1], О.А. Сущенко[2]. Векторная параметрическая оптимизация робастных систем стабилизации информационно-измерительных устройств наземных подвижных объектов
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Рассмотрены особенности постановки задачи векторной параметрической оптимизации робастных систем стабилизации с учетом двух групп ограничений: условий устойчивости системы и предъявляемых к ней требований. Осуществлена векторизация критерия оптимизации робастной системы стабилизации информационно-измерительных устройств, эксплуатируемых на подвижных объектах. Предложена интерактивная эвристическая процедура решения этой задачи, состоящая из двух этапов. Эффективность предложенной процедуры подтверждена результатами моделирования.

Ключевые слова: векторная оптимизация; информационно-измерительные устройства; робастные системы стабилизации.

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