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MODELING OF AN AIRPLANE WING MOMENTS INDUCED BY ATMOSPHERIC TURBULENCE

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Abstract. *We have used Diederich's theory of wingspan average correlation functions to obtain analytical expressions for the local spectral density of aircraft wing moments induced by horizontal and vertical wind gusts. We have assumed that the correlation functions of atmospheric turbulence belong to the Bullen family which includes both partial cases of known Dryden's model as well as von Karman's model.*

Keywords: power spectral density; turbulence models; von Karman model.

1. Introduction

For modeling the effect of atmospheric turbulence on the movement of large aircraft it is necessary to take into account the influence of the size of an aircraft on the forces and moments of forces induced by turbulent gusts.

Different approaches have been developed by many authors [1, 2, 3, 5, 6, 7, 8].

However, the Diederich approach [1] based on the averaged wing span correlation functions and its power spectral densities can be considered as the most physically obvious.

This approach assumes that the atmospheric turbulence is the stationary random Gaussian process.

We will suppose also that the air turbulence under consideration is homogeneous and isotropic.

"Homogeneous turbulence" is understood to mean turbulent motion, the probability characteristics of which are identical for the entire wind velocity field under consideration.

"Isotropic turbulence" is understood to mean turbulent motion, the probability characteristics of which do not depend on the direction along which the correlation between the velocities at two points in the field is being considered.

It follows from the general theory of homogeneous and isotropic turbulence that all that should be known to describe the velocity field in this case is the two correlation functions: longitudinal function $f(r)$ and lateral function $g(r)$ [5], where

$$g(r) = f(r) + \frac{r}{2}\frac{df(r)}{dr}.$$

If we neglect the size of an aircraft and accept Taylor's hypothesis of a frozen field of atmospheric turbulence then the correlation functions will depend only on the argument $U\tau$, where U is the flight path velocity, τ is the time.

As it has been shown in [1] it is possible to take into account the wing span effect by means of modification of an argument of correlation functions and accept the argument as

$$r_{eff} = \sqrt{U^2 \tau^2 + (y_2 - y_1)^2},$$

where y_1 and y_2 – coordinates of two points along the span.

Then it is possible to obtain an analytical expressions for the correlation functions of the forces and moments acting on the wing which depend on y_1, y_2 and τ.

Integrating these expressions over the span and performing then the Fourier transform in time, it is possible to evaluate the correspondent power density spectra.

This spectra is the base for further analysis of the effects of the turbulent gusts on the aircraft motion.

However such a program can be realized only for the Dryden turbulence model, where

$$f(r) = f_D(r) = \sigma_w^2 e^{-\frac{r}{L}}, \tag{1}$$

where σ_w^2 – a variance (mean value of a square of an arbitrary velocity component of a turbulent wind);

L – a turbulence integral scale:

$$L = \frac{1}{\sigma_w^2} \int_0^{\infty} f(x) dx.$$

The experiments suggest that the more appropriate model of air turbulence is the von Karman model,

$$f_K(r) = \sigma_w^2 \frac{2^{2/3}}{\Gamma(1/3)} \left(\frac{r}{a_K}\right)^{1/3} K_{1/3}\left(\frac{r}{a_K}\right), \quad (2)$$

$$a_K = \frac{\Gamma(1/3)}{\sqrt{\pi}\Gamma(5/6)} L \approx 1{,}339L,$$

where $K_{1/3}(x)$ – modified Bessel function of the second kind of order 1/3;

$\Gamma(x)$ – gamma function.

However the power spectral densities of forces and moments for the von Karman model in [2, 3] have been found only numerically.

This leads to difficulties when finding a Fourier transformation of the correlation functions.

The **aim** of this paper is to obtain the analytical expressions for the local power density spectra of an aircraft wing moments induced by horizontal and vertical wind gusts for the Bullen family of turbulence models.

2. The local power spectra

We will assume that the longitudinal correlation function has the form [9]

$$f(r) = \sigma_w^2 \frac{2^{1-s}}{\Gamma(s)} \left(\frac{r}{a}\right)^s K_s\left(\frac{r}{a}\right), \quad (3)$$

$$a = \pi^{-\frac{1}{2}} \Gamma(s) \Gamma\left(\frac{1}{2} + s\right)^{-1} L.$$

If $s = 1/2$, the Dryden formula (1) follows from (3), and when $s = 1/3$ the von Karman formula (2).

As shown in [1, 2, 3] turbulent gusts generate the random moments of forces that may be separated into two types.

One of them is due to the vertical gusts with correlation function of the form

$$\Psi_{C_l}(\tau) = \frac{C_{l_p}^2}{8U^2} \int_0^2 \gamma(\eta) g_V(\rho(\tau,\eta)) d\eta, \quad (4)$$

$$\gamma(\eta) = 6(4 - 6\eta + \eta^3); \; g_V \equiv g = f + \frac{\rho}{2} \frac{df}{d\rho},$$

$$\rho = \sqrt{\frac{U^2\tau^2}{a^2} + \left(\frac{b\eta}{2a}\right)^2},$$

where C_{l_p} – coefficient of damping in roll coefficient;

$\gamma(\eta)$ – the weight function;

b – wingspan.

Another one is caused by horizontal gusts and the correlation function of the moment has the form

$$\Psi_{C_l}(\tau) = \frac{\alpha_0^2 C_{l_p}^2}{2U^2} \int_0^2 \gamma(\eta) g_H(\rho(\tau,\eta)) d\eta, \quad (5)$$

$$g_H(\rho) = \frac{U^2\tau^2}{U^2\tau^2 + (b\eta/2)^2} f(\rho) + \frac{(b\eta/2)^2}{U^2\tau^2 + (b\eta/2)^2} g_V(\rho),$$

where α_0 – angle of attack.

It turns out that despite of complicated dependence of the local correlation functions $g_V(\rho)$ and $g_H(\rho)$ on time τ, it is possible to find out analytical formulas for the power spectral densities which are written in the form

$$G_V(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} e^{i\omega\tau} g_V(\rho) d\tau,$$

$$G_H(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} e^{i\omega\tau} g_H(\rho) d\tau.$$

To find the functions $G_V(\omega)$ and $G_H(\omega)$ it is convenient to represent $g_V(\rho)$ and $g_H(\rho)$ in the form

$$g_V = \sigma_w^2 \frac{2^{-s}}{\Gamma(s)} \left\{2(1+s)\rho^s K_s(\rho) - \rho^{s+1} K_{s+1}(\rho)\right\}$$

$$g_H = \sigma_w^2 \frac{2^{-s}}{\Gamma(s)} \left(2\rho^s K_s(\rho) - (b\eta/2a)^2 \frac{K_{1-s}(\rho)}{\rho^{1-s}}\right).$$

Using the integral transform [4]

$$\int_0^{\infty} \cos(cx) \left(x^2 + p^2\right)^{\frac{s}{2}} K_s\left(\sqrt{x^2+p^2}\right) dx =$$
$$= \sqrt{\frac{\pi}{2}} p^{s+\frac{1}{2}} \left(\sqrt{1+c^2}\right)^{-s-\frac{1}{2}} K_{s+\frac{1}{2}}\left(p\sqrt{1+c^2}\right)$$

we obtain the following expression for the power spectral density

$$G_V(\nu,\eta) = \frac{A_s \beta^{s+\frac{1}{2}}}{\left(1+\nu^2\right)^{s+\frac{3}{2}}} \left(-\beta K_{s+\frac{3}{2}}(\beta) + 2(1+s)(1+\nu^2) K_{s+\frac{1}{2}}(\beta)\right),$$

where

$$A_s = \frac{\sigma_w^2 L}{\pi U} \frac{2^{\frac{1}{2}-s}}{\Gamma\left(s+\frac{1}{2}\right)},$$

$$\nu = \frac{\omega a}{U},$$

$$\beta = \frac{b\eta}{2a}\sqrt{1+\nu^2}.$$

Using the recurrence relation for modified Bessel's functions,

$$\beta K_{q+1}(\beta) = \beta K_q(\beta) + 2qK_{q-1}(\beta),$$

we find an expression for the local power spectral density of the moment, which is caused by vertical gusts

$$G_V = \frac{A_s \beta^{s+\frac{1}{2}}}{\left(1+\nu^2\right)^{s+\frac{3}{2}}} \Bigg(\left(2(1+s)\nu^2+1\right) K_{s+\frac{1}{2}}(\beta) - \\ -\beta K_{s-\frac{1}{2}}(\beta) \Bigg). \quad (6)$$

Similarly we find an expression for the power spectral density of the moment due to horizontal gusts:

$$G_H = \frac{A_s \beta^{s+\frac{1}{2}}}{\left(1+\nu^2\right)^{s+\frac{1}{2}}} \left(2K_{s+\frac{1}{2}}(\beta) - \beta K_{s-\frac{1}{2}}(\beta) \right). \quad (7)$$

For the Dryden model, $s = 1/2$, from (6), (7) we find

$$G_V = \frac{\sigma_w^2 L}{\pi U} \frac{\beta}{\left(1+\nu^2\right)^2} \left(\left(3\nu^2+1\right) K_1(\beta) - \beta K_0(\beta) \right), \quad (8)$$

$$G_H = \frac{\sigma_w^2 L}{\pi U} \frac{\beta}{1+\nu^2} \left(2K_1(\beta) - \beta K_0(\beta) \right). \quad (9)$$

These expressions are identical with the formulas given in [3].

Substituting s = 1/3 into (6) and (7) we find an expression for the von Karman turbulence model

$$G_V = \frac{\sigma_w^2 L}{3\pi U} \frac{C_K \beta^{\frac{5}{6}}}{\left(1+\nu^2\right)^{\frac{11}{6}}} \Bigg(\left(8\nu^2+3\right) K_{\frac{5}{6}}(\beta) - \\ -3\beta K_{\frac{1}{6}}(\beta) \Bigg), \quad (10)$$

$$G_H = \frac{\sigma_w^2 L}{\pi U} \frac{C_K \beta^{\frac{5}{6}}}{\left(1+\nu^2\right)^{\frac{5}{6}}} \left(2K_{\frac{5}{6}}(\beta) - \beta K_{\frac{1}{6}}(\beta) \right), \quad (11)$$

where

$$C_K = 2^{\frac{1}{6}} \Gamma\left(\frac{5}{6}\right)^{-1} \approx 1.26702.$$

3. Discussion

Let us discuss briefly the physical meaning of the formulae obtained. It can be shown that the function $G_V(\nu,\beta)$ is positive and the function $G_H(\nu,\beta)$ has only a single root, which is a solution of the equation

$$2K_{s+\frac{1}{2}}(\beta_*) - \beta_* K_{s-\frac{1}{2}}(\beta_*) = 0.$$

For Dryden's model $\beta_* \approx 1.332$ and for von Karman's model $\beta_* \approx 1.227$.

Formulae (6) and (7) allow us to analyze the properties of local power spectral density at high frequencies $\nu >> 1$.

In the limit of zero span magnitude, $b \to 0$,

$$\lim_{x\to 0} x^q K_q(x) = 2^{1-q}\Gamma(q), \quad \lim_{x\to 0} x^{q+1} K_q(x) = 0,$$

formulae (6) and (7) reduce to well known expressions for the lateral and longitudinal spectral densities for the airplane point model:

$$G_{V0}(\nu) = \frac{\sigma_w^2 L}{\pi U} \frac{\left(2(1+s)\nu^2+1\right)}{\left(1+\nu^2\right)^{s+\frac{3}{2}}},$$

$$G_{H0}(\nu) = \frac{\sigma_w^2 L}{\pi U} \frac{2}{\left(1+\nu^2\right)^{s+\frac{1}{2}}}.$$

At higher frequencies the functions $G_{V0}(\nu)$ and $G_{H0}(\nu)$ decrease by the power law

$$G_{V0}(\nu) \propto \frac{1}{\nu^{2s+1}},$$

$$G_{H0}(\nu) \propto \frac{1}{\nu^{2s+1}}.$$

It is clear from the formulae (6) and (7) that the effect of the finite span of the aircraft leads to exponential decay of the local power spectral density at high frequencies. Indeed, since in the region of the high frequencies $\nu >> 1$

$$\beta \approx \frac{b\eta}{2a}\nu,$$

it is possible to use the well known asymptotic expression for the Bessel functions in the formulae for the local spectral density

$$K_q(x) \approx \sqrt{\frac{\pi}{2x}} e^{-x}, \quad x >> 1.$$

Then we obtain the following asymptotic behavior of the functions $G_{V0}(\nu)$ and $G_{H0}(\nu)$ at $\nu >> 1$:

$$G_V(\nu,\beta) \to \frac{\sigma_w^2 L}{\sqrt{\pi} U} \frac{2^{1-s}(1+s)}{\Gamma\left(s+\frac{1}{2}\right)} \left(\frac{b\eta}{2a}\right)^s \frac{e^{-\frac{b\eta}{2a}\nu}}{\nu^{s+1}},$$

$$G_H(\nu,\beta) \to -\frac{\sigma_w^2 L}{\sqrt{\pi} U} \frac{2^{-s}}{\Gamma\left(s+\frac{1}{2}\right)} \left(\frac{b\eta}{2a}\right)^{s+1} \nu e^{-\frac{b\eta}{2a}}.$$

4. Conclusions

The expressions obtained for the local spectral densities $G_V(\nu,\eta)$ and $G_H(\nu,\eta)$ (formulae (8)–(11)) may be used for calculation of the spectral densities of the moments induced by the wind gusts according to the procedure described in [3].

We are going to carry out a detailed comparison of the results for the spectral densities for Dryden and von Karman turbulence models in our next paper.

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А.О. Антонова[1], М.С. Кулик[2], І.О. Ластівка[3]. Моделювання моментів, обумовлених дією атмосферної турбулентності на крило літака

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На основі методу усереднених за розмахом крила кореляційних функцій Дідеріха отримано аналітичні вирази для локальних спектральних щільностей моментів крила літака, обумовлених горизонтальними і вертикальними поривами вітру. Припущено, що кореляційні функції атмосферної турбулентності належать до сім'ї Булена, яка включає як частинні випадки відомі моделі турбулентності Драйдена і фон Кармана.

Ключові слова: моделі турбулентності; спектральна щільність; фон Кармана модель.

А.О. Антонова[1], Н.С. Кулик[2], И.А. Ластивка[3]. Моделирование моментов, обусловленных воздействием атмосферной турбулентности на крыло самолета

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На основе метода усредненных по размаху крыла корреляционных функций Дидериха получены аналитические выражения для локальных спектральных плотностей моментов крыла самолета, обусловленных горизонтальными и вертикальными порывами ветра. Допущено, что корреляционные функции атмосферной турбулентности относятся к семейству Буллена, которое включает как частные случаи известные модели турбулентности Драйдена и фон Кармана.

Ключевые слова: модели турбулентности; спектральная плотность; фон Кармана модель.

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