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MATHEMATICAL MODEL OF TRIAXIAL MULTIMODE ATTITUDE AND HEADING REFERENCE SYSTEM

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Abstract

Purpose: *The paper deals with the mathematical description of the gimballed attitude and heading reference systems, which can be applied in design of strategic precision navigation systems. The main goal is to created mathematical description taking into consideration the necessity to use different navigations operating modes of this class of navigation systems. To provide the high accuracy the indirect control is used when the position of the gimballed platform is controlled by signals of gyroscopic devices, which are corrected using accelerometer's signals.* ***Methods:*** *To solve the given problem the methods of the classical theoretical mechanics, gyro theory, and inertial navigation are used.* ***Results:*** *The full mathematical model of the gimballed attitude and heading reference system is derived including descriptions of different operating modes. The mathematical models of the system Expressions for control and correction moments in the different modes are represented. The simulation results are given.* ***Conclusions:*** *The represented results prove efficiency of the proposed models. Developed mathematical models can be useful for design of navigation systems of the wide class of moving vehicles.*

Keywords: accelerometers; attitude and heading reference systems; control and correction moments; dynamically tuned gyros; inertial navigation systems.

1. Introduction

The paper deals with navigation problem, which can be solved by means of gimbaled platform with navigation measuring instruments.

Motion of such a platform is controlled by the servo-system, which keeps orientation relative to the basic reference frame based on gyro devices signals. In this case the controlled platform is used as basic navigation reference frame. And there is no necessity to calculate platform orientation by program means in real time.

On the one hand this decreases requirements to the airborne computer. On the other hand requirements to accuracy of control platform sufficiently increase. Therefore gimbaled attitude and heading reference systems (AHRS) are used in high-precision applications, namely, strategic spacecraft and marine vehicles [1].

The error of platform orientation control is ignored in such systems. This leads to increase of the navigation error. Therefore to achieve the high navigation accuracy it is necessary to control by platform orientation with a very small error. To provide such requirements it is possible using high-precision measuring instruments (dynamically tuned gyros), and correction techniques (integral correction, indirect control with correction by accelerometers), and multi-mode operation. The list of modes of the researched system is represented in Table.

Table

Features of AHRS modes

Mode	Sensors	Concurrent execution
Calibration	Dynamically tuned gyros	Previous levelling
Previous levelling	Accelerometers	Calibration
Precision levelling	Dynamically tuned gyros, accelerometer	
Setting to the meridian	Dynamically tuned gyros, accelerometer	Precision levelling
Gyroscopic compass	Dynamically tuned gyros, accelerometer	Precision levelling

Specific feature of the high-precision gimbaled navigation system is the necessity to carry out the initial alignment of the platform relative to the basic navigation reference frame. This process includes levelling (setting to the horizon) and setting to the meridian (gyro compassing) [1]. Levelling provides setting of the platform with the navigation measuring instruments to the horizon plane. This process error is defined by the zero signals of accelerometers mounted on the platform. Navigation of marine vehicles requires both previous and high-precision levelling. The previous levelling is implemented by means of accelerometers signals. The high-precision levelling is provided by means of accelerometers and gyroscopic devices. Gyro compassing is setting to the meridian. This process is implemented by means of platform rotation relative to the platform vertical axis. In its turn, orientation of this axis is defined by the levelling process. Therefore gyro compassing is less accurate in comparison with the levelling.

2. Analysis of the latest research and publications

Features of creation of mathematical models of gimbaled AHRS assigned for determination of vehicles are analyzed in many papers, for example in [2]. The mathematical model of the small-size corrected gyro compass based on the dynamically tuned gyro is represented in [3]. Mathematical models of the gimbaled AHRS including modes of setting to horizon and heading measurement are given in [4], which researches AHRS with the biaxial platform. The given paper differs by detailed analysis of controls actions and full representation of all AHRS modes with triaxial platform. Research of the separate gimbaled AHRS modes was carried out in [5–7]. This paper represents generalized representation of gimbaled AHRS. Its feature is description of control and correction moments in all AHRS modes.

3. Research tasks

The main goal of this research is to represent the full mathematical model of the gimbaled AHRS including modes of calibration, previous and high-precision levelling, gyroscopic compass and setting to the meridian. Differences of modes are defined by type of sensors, basic functions and accuracy requirements.

4. Calibration

Calibration of the dynamically tuned gyro, which carries out functions of the vertical gyroscope is carried out against a background of the previous levelling. To measure the gyroscopic vertical drift the platform is turned at the horizon plane on angles 0^o … 360^o with some given interval (10^o). This provides measuring earth angular rates ω_x, ω_y projections onto the measuring axes of the gyroscopic vertical. Calculation of drifts $\Delta\dot{\alpha}_{\text{др}}$, $\Delta\dot{\beta}_{\text{др}}$ is carried out by the formulas

$$\Delta\dot{\alpha} = \frac{\sum_{i=1}^{18}(\omega_{y_i} + \omega_{y_{i+18}})}{18}; \quad \Delta\dot{\beta} = \frac{\sum_{i=1}^{18}(\omega_{y_i} + \omega_{y_{i+18}})}{18}. \qquad (1)$$

Further calibration of the dynamically tuned gyro, which carries out functions of the directional gyroscope, is implemented. For calibration of the gyroscopic device the platform is rotated with the given angular rate and comparison of calculated and measured rotation angles is implemented at the fixed instant of time ($t_1 = \Delta t$=90 s, $t_1 = 2\Delta t$=180 s, $t_1 = 3\Delta t$=270 s). The drift of the directional gyroscope is

$$\Delta\dot{\psi} = \frac{a_1 + a_2 + a_3}{3\Delta t}, \qquad (2)$$

where $a_1 = \psi_1 - \omega_\text{к}\Delta t$, $a_2 = \psi_2 - \omega_\text{к} 2\Delta t$, $a_3 = \psi_3 - \omega_\text{к} 3\Delta t$.

The expressions (1), (2) provide calibration of the researched AHRS.

5. Mathematical model of multimode AHRS: previous levelling mode

Basic function of this mode is levelling of the gimbaled platform to the horizon plane. The platform in this mode is controlled by information of accelerometers and angle-data transmitters mounted at the gimbaled platform. Respectively, mathematical mode of AHRS in levelling mode must include the Euler dynamic and kinematic equations, which describe the mathematical model of the platform, and the accelerometers models [8].

Using transformations given in [6] the AHRS mathematical model in the levelling mode can be described by the platform model:

$$\dot{\omega}_{xp} = [-(J_z - J_y)\omega_{yp}\omega_{zp} - f_x\omega_{xp} - M_0 \text{sign}\omega_0 - $$
$$ - M_{xcon}]/J_x;$$
$$\dot{\omega}_{yp} = [-(J_x - J_z)\omega_{xp}\omega_{zp} - f_y\omega_y - M_0 \text{sign}\omega_0 - k_2\delta_1$$
$$ - k_4(-\delta_2 + k_3\alpha)/T]/J_y;$$
$$\dot{\omega}_{zp} = [-(J_y - J_x)\omega_{xp}\omega_{yp} - f_y\omega_{zp} + M_{zcon}]/J_z;$$
$$\dot{\alpha} = (\omega_{xp}\sin\gamma + \omega_{yp}\cos\gamma)/\cos\beta;$$
$$\dot{\beta} = \omega_{xp}\cos\gamma - \omega_{yp}\sin\gamma;$$

$\dot{\gamma} = \omega_{zp} + \mathrm{tg}\beta(\omega_{xp}\sin\gamma + \omega_{yp}\cos\gamma)$; (3)

and the accelerometer models

$\dot{\delta}_{xp} = (-\delta_{xp} + k_A\beta)/T_A$; $\dot{\delta}_{yp} = (-\delta_{yp} + k_A\alpha)/T_A$, (4)

where J_x, J_y, J_z are moments of platform inertia; f_x, f_y, f_z are moments of viscous friction; M_0 is the moment of friction of stabilizing motors; ω_0 is the external angular rate, δ_{xp}, δ_{yp} are accelerometer signals, T_A , k_A are accelerometer time and transfer constants. It is known that the mathematical model depends on type of the accelerometer. The researched model uses the pendulum accelerometers. Expressions for control of stabilizing motors can be represented in the following form:

$M_{xcon} = k_{x1}\delta'_{xp} + k_{x2}\dot{\beta}$; $M_{ycon} = k_{y1}\delta'_{yp} + k_{y2}\dot{\alpha}$, (5)

where δ_{xp}, δ_{yp} are accelerometer signals; a_{xp}, a_{yp} are given accelerometer drifts; a_{xt}, a_{yt} are accelerometer temperature drifts; $k_{x1}, k_{x2}, k_{y1}, k_{y2}, k_{z1}$ are transfer constants.

If axes of measuring and navigation reference frames coincide, the control moments can be described by the following expressions

$M_{xcon} = k_{x1}(\delta'_{xp}\cos\gamma - \delta'_{yp}\sin\gamma) + k_{x2}\dot{\beta}$;

$Mon_{ycon} = k_{y1}(\delta'_{xp}\sin\gamma + \delta'_{yp}\cos\gamma) + k_{y2}\dot{\alpha}$;

$\delta'_{xp} = \delta_{xp} - |a_{xp}|\,\mathrm{sign}a_{xp} - |a_{xt}|\,\mathrm{sign}a_{xt}$;

$\delta'_{yp} = \delta_{yp} - |a_{yp}|\,\mathrm{sign}a_{yp} - |a_{yt}|\,\mathrm{sign}a_{yt}$;

$M_{zcon} = k_{z1}\gamma$. (6)

Angles α, β, γ are measured by angle-data transmitters mounted at gimbals axes. Information about platform accelerations enters from accelerometers. It should be noted the pendulum accelerometer output signals represent angles of platform deviations from the horizon plane. Control relative to the axis Oz is carried out to prevent the azimuth drift. The range of change of coefficients k_{x1}, k_{y1} depends on ratio between maximal level of friction moment and permissible error of platform slope to horizon plane. The range of change of coefficients k_{x2}, k_{y2} depends on ratio of k_{y2}/k_{x2} and a linear zone of the angular rate measuring instrument. Expressions (5), (6) represent controls in previous levelling mode.

6. Mathematical model of multimode AHRS: precision levelling mode

Basic function of this mode is precision levelling of the gimbaled platform to the horizon plane. The platform in this mode is controlled by information of attitude gyroscope. And this gyroscopic device is corrected by means of accelerometer signals. Such a control is called indirect. The precision stabilization in this mode is provided by sharing of controls and corrections. For example, the integral correction makes the platform resistant to the external disturbances. The mutual location of platform and gyro device reference frames is represented in Fig. 1.

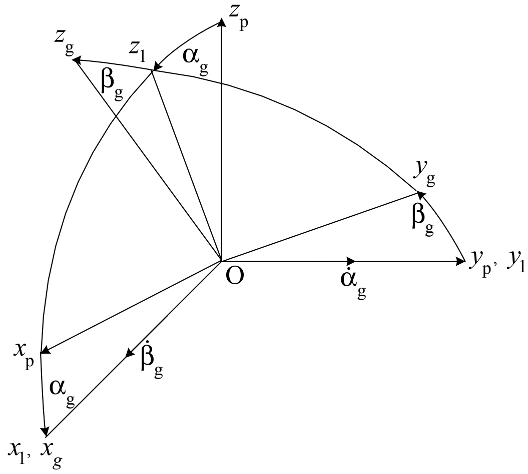


Fig. 1. Mutual location of reference frames: $Ox_py_pz_p$ is the platform reference frame; $Ox_gy_gz_g$ is the reference frame defined by the Resal axes of the attitude gyroscope

The mathematical model of the gimbaled AHRS in the precision levelling mode includes dynamic and kinematics equation, which describe the platform, accelerometer models, and the model of the dynamically tuned gyroscope, which carries out functions of the attitude gyroscope. In this case the models (1) and (2) must be supplemented with model of the dynamically tuned gyroscope [2]

$$\ddot{\alpha}_g = (-d\dot{\alpha}_g + H\dot{\beta}_g + \frac{H}{T_g}\beta_g - c\alpha_g + H_1\omega_{xp} - \sum_{i=1}^{4} M_{xi})/J_g;$$

$$\ddot{\beta}_g = (-d\dot{\beta}_g - H\dot{\alpha}_g - \frac{H}{T_g}\alpha_g - c\beta_g - H_1\omega_{yp} - \sum_{i=1}^{4} M_{yi})/J_g; (7)$$

here J_g is a sum of equatorial moments of a rotor and gyroscope gimbals; d is damping coefficient; H is the gyroscope kinetic moment; T_g is time constant of attitude gyroscope; c is the residual rigidity of the gimbals; $H_1 = H(1-10^{-3})$; $\omega_{xp}, \omega_{yp}, \omega_{zp}$ are projections of the platform angular rate.

The equations, which describe dynamics and kinematics of the platform for different modes differ by control moments. These moments for precision levelling mode can be described by the following expressions

$$M_{xcon} = k_{x1}\beta_g + k_{x2}\dot{\beta}_g \text{ ; } M_{ycon} = k_{y1}\alpha_g + k_{y2}\dot{\alpha} \text{ ;}$$
$$M_{zcon} = k_{z1}\dot{\gamma} \text{ .} \quad (8)$$

To provide precision stabilization it is necessary to use correction of the attitude gyroscope supplement equations (8) by the appropriate relations. There are different approaches to such a correction implementation. For the system to be researched it is convenient to use corrections taking into consideration the diurnal rotation of the Earth, systematic gyroscope drift, external information, and also integral correction [9].

To form moments of integral correction it is necessary to derive expressions for accelerometer indications. These accelerometers are mounted on the platform along the three orthogonal axes, which define the platform-axis reference frame [9]

$$a_{xp} = w_{xp} - (w_{zp} + g)\alpha \text{ ;}$$
$$a_{yp} = w_{yp} - (w_{yp} + g)\beta \text{ ;}$$
$$a_{zp} = w_{xp}\alpha + w_{yp}\beta + (w_{yp} - g) \text{ ,} \quad (9)$$

here $a_{xp} = \delta_x g,\ a_{yp} = \delta_y g,\ a_{zp} = \delta_z g$ are accelerometer indications, w_{xp}, w_{yp}, w_{zp} are the moving vehicle accelerations; g is the acceleration of gravity, α, β are errors of determination of the local vertical line.

The moving vehicle accelerations can be defined by the expressions

$$w_{xp} = \dot{v}_{xp} + \Delta w_{xp},\ w_{yp} = \dot{v}_{yp} + \Delta w_{yp},\ w_{zp} = \dot{v}_{zp} + \Delta w_{zp} \quad (10)$$

here v_{xp}, v_{yp}, v_{zp} are relative linear speeds of the moving vehicle; $\Delta w_{xp}, \Delta w_{yp}, \Delta w_{zp}$ are corrections taking into consideration influence of relative and Coriolis accelerations, the vertical acceleration, nonsphericity of the Earth. Then the moments of the integral correction can be represented in the following form

$$M_{xcor1} = \mu \int_0^{\Delta t} (\dot{v}_{xp} - g\alpha)dt = \mu \int_0^{\Delta t} (a_{xp} - \Delta w_{xp} - w_{zp}\alpha_g)dt \text{ ,}$$
$$M_{ycop1} = \mu \int_0^{\Delta t} (\dot{v}_{yp} - g\beta)dt = \mu \int_0^{\Delta t} (a_{yp} - \Delta w_{yp} - w_{zp}\beta_g)dt \text{ ,} \quad (11)$$

here $\mu = H_1 / R_2$ is the coefficient of the integral correction, $\Delta w_{xp}, \Delta w_{yp}, \Delta w_{zp}$ can be determined based on equations (9), (10).

Correction moments taking into consideration the Earth rotation look like [9]:

$$M_{xcop2} = H\omega_e \cos\varphi \sin k \text{ ;}$$
$$M_{ycop2} = H\omega_e \cos\varphi \cos k \text{ ,} \quad (12)$$

here φ is the geographical latitude of the vehicle location.

To achieve the high accuracy of the transient processes in the precision levelling mode it is necessary to compensate systematic components of the gyroscopic device

$$M_{xcop3} = k_{\Delta x}\dot{\alpha}_{g\Delta} \text{ ; } M_{ycop3} = k_{\Delta y}\dot{\beta}_{g\Delta} \text{ ,} \quad (13)$$

here $k_{\Delta x}, k_{\Delta y}$ are transfer constants, $\dot{\alpha}_{\Delta}, \dot{\beta}_{\Delta}$ are drift components.

The correction by the external information must be used too

$$M_{xcop4} = k_1 \int_0^t (v_{exty} \cos\gamma - v_y \sin\gamma) / R_2 dt \text{ ;}$$
$$M_{ycop4} = k_2 \int_0^t (v_{extx} \sin\gamma - v_x \cos\gamma) / R_1 dt \text{ ,} \quad (14)$$

here k_1, k_2 are transfer constants, v_{extx}, v_{exty} are components of the vehicle linear speed, determined by means of the external correction facilities, v_x, v_y are components of the vehicle linear speed.

Expressions (11)–(14) describe features of correction in the precision levelling mode.

The mutual location of the platform-axis, geographical, and trajectory reference frames is shown in Fig. 2. The longitudinal axis of the researched object is directed along the axis OY. Figure 2 uses the following notations. The reference frame OXYZ is the trajectory reference frame, for which the axis OY s directed along the vehicle longitudinal axis; and the axis OZ – along the vertical axis. The reference frame $OX_gY_gZ_g$ is the geographical reference frame, for which the axis OY_g is directed to the North, and the axis OX_g – to the East respectively. The reference frame $OX_pY_pZ_p$ is the platform-axis reference frame deviated relative to the trajectory reference frame on the angle ψ_0. Here k_p is the calculated heading.

Precision levelling to the horizon plane must be carried out during modes of setting to the meridian and determination of the heading. In the first case the platform-axis reference frame is deviated relative to the geographical reference frame on the calculated heading. In the second case this reference frame coincides with the geographical reference frame. Respectively, $k = -(\psi_0 + \dot{\psi}t) + k_p$ for the first case, and $k = -(\psi_0 \pm \dot{\psi} t)$ for the second case, where $\dot{\psi}$ is the rate of rotation in the horizon plane, $\Delta\psi = \dot{\psi} t$.

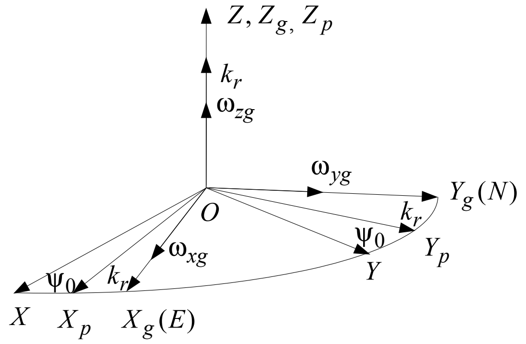


Fig. 2. Mutual location of the platform-axis, geographical and trajectory references frames

Control moments applied to the stabilizing motors become

$$M_x = k_1(\theta_{VG}\cos\psi - \gamma_{VG}\sin\psi) + k_3\dot{\theta}_{vg};$$

$$M_{ycon} = k_{y1}(\gamma_g\cos\psi + \theta_g\sin\psi) + k_{y2}\dot{\gamma}_g; \quad (15)$$

here $k_{x1}, k_{x2}, k_{y1}, k_{y2}$ are transfer constants; $\gamma_g = k_{\text{ADT}}\alpha_g, \theta_g = k_{\text{ADT}}\beta_g$ are signals formed by angle-data transmitters of gyro, which carries out functions of the attitude gyroscope; $\psi = \psi_0 \pm \dot{\psi} t$.

Moments of the integral correction can be determined in the general form, which corresponds to both setting to the meridian and heading determination. Obviously, that in the latter case it is necessary to believe $k_p = 0$. Indications of accelerometers mounted on the platform can be defined by the expressions [9]

$$a_{xv} = w_{xp} - (w_{zp} + g)\gamma_{\text{VG}};\ a_{y\text{в}} = w_{y\text{п}} - (w_{z\text{п}} + g)\theta_{\text{гв}};$$

$$a_{zv} = w_{xp}\gamma_g + w_{yp}\theta_g + w_{zp} - g, \quad (16)$$

here a_{xv}, a_{yv}, a_{zv} are accelerometers indications, w_{xp}, w_{yp}, w_{zp} are accelerations of the vehicle, the g is acceleration of gravity. As

$$w_{xp} = \dot{v}_{xp} + \Delta w_{xp},\ w_{yp} = \dot{v}_{yp} + \Delta w_{yp},\ w_{zp} = \dot{v}_{zp} + \Delta w_{zp}, \quad (17)$$

where v_{xp}, v_{yp}, v_{zp} are relative speeds of the vehicle, $\Delta w_{xp}, \Delta w_{yp}, \Delta w_{zp}$ are corrections, the moments for the integral correction implementation become

$$M_{x1} = k_i\int_0^{\Delta t}(a_{xv} - \Delta w_{xp} - w_{zp}\gamma_{VG})dt,$$

$$M_{ycop1} = K_i\int_0^{\Delta t}(a_{yv} - \Delta w_{yp} - w_{zp}\theta_g)dt, \quad (18)$$

where k_i is a coefficient of the integral correction.

Based on (16), (17) and taking into consideration some concepts of [9] projections of the full accelerations on the platform axes become

$$\begin{aligned}
w_{xp} = {} & \dot{v}_{xp} - \frac{(v_{yp}\cos k_r - v_{xp}\sin k_r)\omega_e\cos k_r}{R_2} \\
& \times\left(\frac{e^2\sin\varphi\cos^2\varphi}{1-e^2} + \frac{R_1}{R_2}\sin\varphi\right) - \\
& -\frac{v_{yp}\cos k_r - v_{xp}\sin k_r}{R_2}\sin k_P v_{zp} + \\
& +\frac{v_{xp}\cos k_r + v_{yp}\sin k_r}{R_1}\cos k_r v_{zp} + \\
& 2\omega_e\cos\varphi\cos k_r v_{zp} - \\
& -\frac{v_{xp}\cos k_r + v_{yp}\sin k_r}{R_1}\text{tg}\varphi v_{yp} - 2\omega_e\sin\varphi v_{yp} + \dot{k}_P v_{yp}; \\
w_{yp} = {} & \dot{v}_{yp} + \frac{v_{xp}\cos k_r + v_{yp}\sin k_r}{R_1}\text{tg}\varphi v_{xp} + \\
& +2\omega_e\sin\varphi v_{xp} + R_1\omega_e^2\sin\varphi\cos\varphi\cos k_r - \\
& \dot{k}_r v_{xp} - \dot{k}_P R_1\omega_e\cos\varphi\cos k_r + \\
& +\frac{v_{yp}\cos k_r - v_{yp}\sin k_r}{R_2}\cos k_P v_{zp} + \\
& \frac{v_{xp}\cos k_r + v_{yp}\sin k_r}{R_1}\sin k_P v_{zp} + \\
& +2\omega_e\cos\varphi\sin k_r v_{zp}; \\
w_{zp} = {} & \dot{v}_H - \frac{v_{yp}\cos k_r - v_{xp}\sin k_r}{R_2}\cos k_r v_{yp} - \\
& -\frac{v_{xp}\cos k_r + v_{yp}\sin k_r}{R_1}\sin k_r v_{yp} - \\
& 2\omega_e\cos\varphi\sin k_r v_{yp} + \\
& +\frac{v_{yp}\cos k_r - v_{xp}\sin k_r}{R_2}\sin k_r v_{xp} - \\
& \frac{v_{xp}\cos k_r + v_{yp}\sin k_r}{R_1}\cos k_r v_{xp} - \\
& -2\omega_e\cos\varphi\cos k_r v_{xp} - R_1\omega_e^2\cos^2\varphi\cos^2 k_r,
\end{aligned} \quad (19)$$

here e is eccentricity of the Earth spheroid.

Correction moments taking into consideration rotation of the Earth are

$$M_{xcop2} = H\omega_e\cos\varphi\cos k;$$

$$M_{ycop2} = -H\omega_e\cos\varphi\sin k. \quad (20)$$

The appropriate moments, which compensate systematic drift in the mode of precision levelling to the horizon plane, look like

$$M_{xcop3} = k_{\Delta x}\dot{\gamma}_{\Delta};\; M_{ycop3} = k_{\Delta y}\dot{\theta}_{\Delta}, \qquad (21)$$

here $k_{\Delta x}, k_{\Delta y}$ are transfer constants, $\dot{\gamma}_{\Delta}, \dot{\theta}_{\Delta}$ are drift components

$$M_{xcop4} = k_{13}\int_{0}^{\Delta t}(v_{ext}\cos\psi - v_{yp})/R_2\,dt;$$

$$M_{ycop4} = k_{23}\int_{0}^{\Delta t}(v_{ext}\sin\psi - v_{xp})/R_1\,dt, \qquad (22)$$

here k_{13}, k_{23} are transfer constants, v_{ext} is the vehicle linear speed, determined by the external correction facilities, v_{xp}, v_{yp} are components of the vehicle linear speed based on accelerometer indications and taking into consideration deviation of their measuring axes relative to the trajectory reference frame.

The expressions (18)–(22) describe control in the mode of the precision levelling.

7. Mathematical model of multimode attitude and heading reference system: mode of gyroscopic compass

Correction of the dynamically tuned gyro, which carries out functions of the directional gyro, is implemented by means of accelerometers and angle-data transmitters mounted at the gimbals axes. The mode of heading determination is combined with the mode of levelling to the horizon plane. Such an approach provides location of the directional gyro in the horizon plane and respectively the high accuracy of heading determination.

To describe the above stated mode it is necessary to introduce the following reference frames:

1)geographical reference frame $O\xi\eta\zeta$, in which the axis $O\eta$ is directed to the North, the axis $O\xi$ is directed ton the East, and the axis $O\zeta$ represents the local vertical line;

2)body-axis reference frame $O\xi_1\eta_1\zeta_1$, which is turned in the geographical reference frame $O\xi\eta$ relative to the axis $O\eta$ on the angle of heading k_0;

3)platform-axis reference frame $O\xi_2\eta_2\zeta_2$, which is turned in the geographical reference frame relative to the axis $O\eta$ on the angle of the initial azimuth A_0;

4)platform-axis reference frame $Ox_py_pz_p$ deviated relative to the reference frame $O\xi_2\eta_2\zeta_2$ on small angles α, β, γ.

Mutual location of the reference frames $O\xi\eta\zeta$, $O\xi_1\eta_1\zeta_1$, $O\xi_2\eta_2\zeta_2$ is given in Fig. 3.

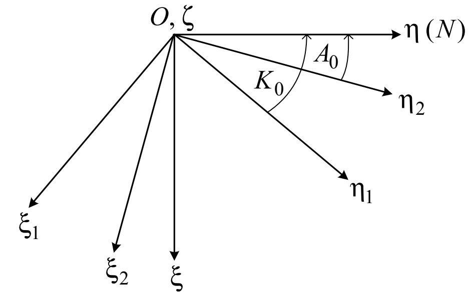


Fig. 3. Mutual location of reference frames $O\xi\eta\zeta$, $O\xi_1\eta_1\zeta_1$, $O\xi_2\eta_2\zeta_2$

The angular location of the platform in the inertial space is shown in Fig. 4.

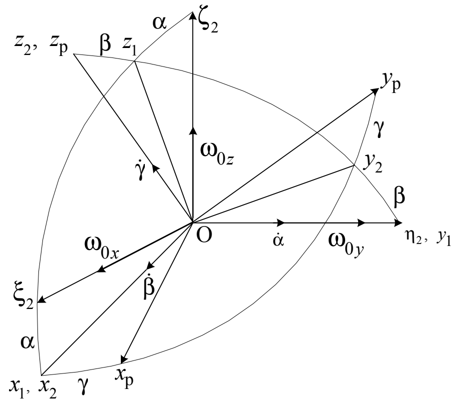


Fig. 4. The angular position of the platform in the inertial space

The angles α, β define accuracy of location of the platform in the horizon plane, and the angle γ defines deviation of the platform from the meridian plane. It follows from Fig. 3 that the platform azimuth is $A = A_0 - \gamma$. A matrix of the directional cosines between the reference frames $Ox_py_pz_p$, $O\xi_2\eta_2\zeta_2$ based on Fig. 4 becomes

$$C = \begin{bmatrix} \begin{matrix}\cos\alpha\cos\gamma + \\ +\sin\alpha\sin\beta\sin\gamma\end{matrix} & \cos\beta\sin\gamma & \begin{matrix}-\sin\alpha\cos\gamma + \\ +\cos\alpha\sin\beta\sin\gamma\end{matrix} \\ \begin{matrix}-\cos\alpha\sin\gamma + \\ +\sin\alpha\sin\beta\cos\gamma\end{matrix} & \cos\beta\cos\gamma & \begin{matrix}\sin\alpha\sin\gamma + \\ +\cos\alpha\sin\beta\cos\gamma\end{matrix} \\ \sin\alpha\cos\beta & -\sin\beta & \cos\alpha\cos\beta \end{bmatrix}. \qquad (23)$$

Taking into consideration smallness of angles α, β, γ, this matrix can be transformed to the following form

$$C=\begin{bmatrix}1 & \gamma & -\alpha\\ -\gamma & 1 & \beta\\ \alpha & -\beta & 1\end{bmatrix}. \qquad (24)$$

Taking into consideration reference frames represented in Figures 3, 4, matrices of directional cosines and known expressions for determination of angular rates of the geographical reference frame, it is possible to represent the mathematical model of the AHRS in the mode of the gyroscopic compass. In addition to the differential equations, which represent the mathematical model of the vertical gyroscope (7) and the mathematical models of the accelerometers (4), in the above mentioned mode it is necessary to use the following relations by the mathematical model of the directional gyro (heading determination) [2]; equations, which describe dynamics and kinematics of the platform; and also expressions for determination of the initial angular rates

$$\ddot{\alpha}_h=(-d\dot{\alpha}_h-H\dot{\beta}_h-\frac{H}{T_h}\beta_h-c\alpha_h-H_1\omega_{xp}+M_z^y+M_z^k)/J_h;$$

$$\ddot{\beta}_h=(-d\dot{\beta}_h+H\dot{\alpha}_h+\frac{H}{T_h}\alpha_h-(c+k_8)\beta_h+H_1\omega_{zp}+M_x^y+M_x^k)/J_h,$$

$$\dot{\omega}_{xp}=[-(J_z-J_y)\omega_{yp}\omega_{zp}-f_x\omega_{xp}-M_c\mathrm{sign}\omega_0-M_{xcon}]/J_x;$$

$$\dot{\omega}_{yp}=[-(J_x-J_z)\omega_{xp}\omega_{zp}-f_y\omega_y-M_c\mathrm{sign}\omega_0-M_{ycon}]/J_y;$$

$$\dot{\omega}_{zp}=[-(J_y-J_x)\omega_{xp}\omega_{yp}-f_y\omega_{zp}-M_c\mathrm{sign}\omega_0-M_{zcon}]/J_z;$$

$$\dot{\alpha}=\omega_{xp}\sin\gamma+\omega_{yp}\cos\gamma-\omega_{y0}-\omega_{z0}\beta;$$

$$\dot{\beta}=\omega_{xp}\cos\gamma-\omega_{yp}\sin\gamma-\omega_{x0}+\omega_{z0}\alpha;$$

$$\dot{\gamma}=\omega_{zp}-\omega_{x0}\alpha+\omega_{y0}\beta-\omega_{z0};$$

$$\omega_{x0}=-\Omega\cos\varphi\sin A_0-\frac{V_p\cos k_0}{R}\cos A_0-\frac{V_p\sin k_0}{R}\sin A_0$$

$$\omega_{y0}=\Omega\cos\varphi\cos A_0+\frac{V_p\sin k_0}{R}\cos A_0-\frac{V_p\cos k_0}{R}\sin A_0$$

$$\omega_{z0}=\Omega\sin\varphi+\frac{V_p\sin k_0}{R}\mathrm{tg}\varphi, \qquad (25)$$

here M_z^y, M_x^y are control moments;

In the described mode the moments of stabilizing engines control can be determined based on signals of the gyroscopic vertical and angle data transmitter

$$M_{xcon}=k_{x1}\beta_g+k_{x2}\dot{\beta};\ M_{ycon}=k_{y1}\alpha_g+k_{y2}\dot{\alpha};$$

$$M_{zcon}=k_{z1}\alpha_h. \qquad (26)$$

Control moments (18)–(22) provide correction of the vertical gyroscope.

In the mode of the gyroscopic compass the control is implemented by the accelerometer signal, which measures the angle of platform deviation β_p. And the signal, which enters to the torque motor mounted at the vertical axis, is sufficiently weakened. The additional control signal β_h enters from the angle-data transmitter. Then control moments will be defined by the expressions

$$M_{hxcop}=k_{hx1}\beta_к-k_{x2}\varepsilon\delta_{xp};\ M_{hycop}=k_y\beta_h-k\delta_{xp}, \qquad (27)$$

here ε is some small value.

8. Mathematical model of multimode AHRS: mode of setting to the meridian

Mode of setting to the meridian is one of the most important modes of the researched AHRS. There are different ways to set AHRS to a meridian. The most widespread are gyro compassing and turn at some in advance calculate angle [10]. The latter way is the most convenient for the marine AHRS but its implementation is connected with some problems. In the first place, to calculate the turn angle it is necessary to determine vehicle's speeds. Such information can be measured by the dynamically tuned gyro as it can operate in the mode of the angular rate measurement. The assessment of the angle of the setting to the meridian by means of measurement of the earth angular rate projections

$$\delta_{npoc}=\mathrm{artcg}(\omega_{x\,oc}/\omega_{y\,oc}). \qquad (28)$$

The expression (28) can be divided by some relations

$$\delta_{np}=\delta_{npoc}-180^{\circ}\ \text{для}\ \omega_{xoc}\le 0;\ \omega_{yoc}\le 0;$$

$$\delta_{np}=-\delta_{npoc}\ \text{для}\ \omega_{xoc}\le 0;\ \omega_{yoc}>0;$$

$$\delta_{np}=180^{\circ}-\delta_{npoc}\ \text{для}\ \omega_{xoc}\le 0;\ \omega_{yoc}\le 0;$$

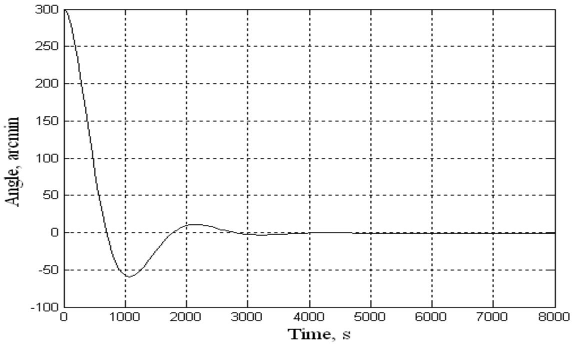
$$\delta_{np}=\delta_{npoc}\ \text{для}\ \omega_{xoc}>0;\ \omega_{yoc}>0.$$

But such its implementation requires additional electronics, complication of data processing, and computer resources.

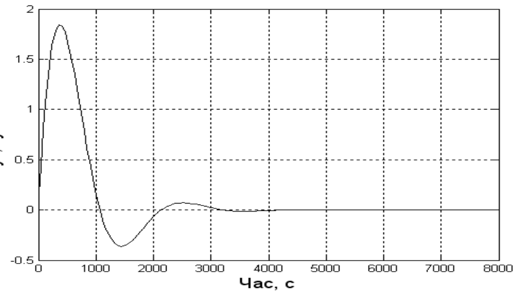
As regards gyro compassing the proposals given in [10] stays relevant today. But vehicle's deviations are believed to be small in this paper. Mathematical model and control features basically correspond to mode of the gyro compass (23)–(27). The accelerated setting to the meridian is provided by increasing the coefficient k_{x2} in the expression (26).

9. Simulation results

Transient processes by the azimuth and pitch are given in Fig. 5.



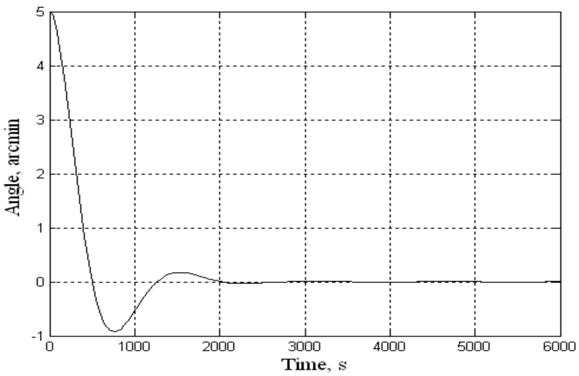
a



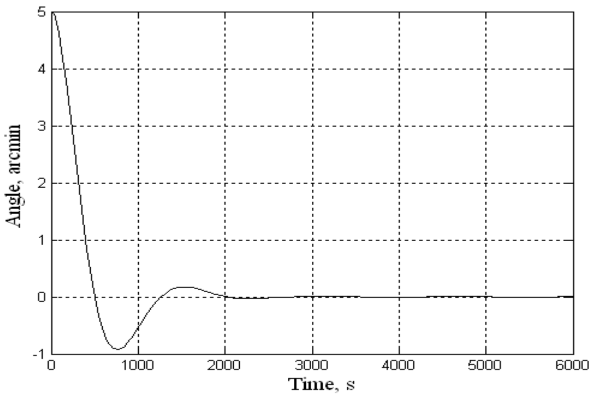
b

Fig. 5. Transient processes in the mode of the gyroscopic compass: *a* – azimuth channel; *b* – pitch channel

Results of accelerated setting to the meridian are given in Fig. 6.



a



b

Fig. 7. Accelerated setting to the meridian

10. Conclusions

The full mathematical model of the triaxial multi-mode AHRS is derived. The control expressions for every mode are given. The obtained results represent generalization of the mathematical description of the gimbaled AHRS.

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Математична модель багаторежимної системи визначення повної просторової орієнтації рухомого об'єкта

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Мета: У статті представлено математичний опис системи визначення просторової орієнтації рухомого об'єкта, яка може бути використана у високоточних системах стратегічного призначення. **Методи дослідження:** Для розв'язання даної проблеми було використано класичну теорію механіки, теорію гіроскопів та теорію інерціальної навігації. **Результати:** Отримано повну математичну модель платформної системи визначення просторової орієнтації, включаючи опис різних режимів функціонування. Представлено результати моделювання. **Висновки:** Представлені результати підтверджують ефективність запропонованих моделей. Розроблені математичні моделі можуть бути корисними для навігаційних систем рухомих об'єктів широкого класу.

Ключові слова: акселерометри; динамічно настроювані гіроскопи; інерціальні навігаційні системи; моменти управління та корекції; системи визначення просторової орієнтації.

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Математическая модель многорежимной системы определения полной пространственной ориентации объекта

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Цель: В статье представлено математическое описание системы определения пространственной ориентации подвижного объекта, которое может быть использовано в высокоточных системах стратегического назначения. **Методы исследования:** Для решения данной проблемы были использованы классическая теория механики, теория гироcколов и теория инерциальной навигации. **Результаты:** Получена полная математическая модель платформной системы определения пространственной ориентации, включая описание разных режимов функционирования. Представлены результаты моделирования. **Выводы:** Представленные результаты подтверждают эффективность предложенных моделей. Разработанные математические модели могут быть полезными для навигационных систем подвижных объектов широкого класса.
Ключевые слова: акселерометры; динамически настраиваемые гироскопы; инерциальные навигационные системы; моменты управления и коррекции; системы определения пространственной ориентации.

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