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РЕЛАКСАЦІЙНІ АВТОКОЛИВАННЯ КРУТІННЯ ГЛИБОКИХ БУРИЛЬНИХ КОЛОН

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RELAXATIONAL TORSIONAL AUTOVIBRATIONS OF DEEP DRILL STRINGS

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РЕЛАКСАЦИОННЫЕ АВТОКОЛЕБАНИЯ КРУЧЕНИЯ ГЛУБОКИХ БУРИЛЬНЫХ КОЛОНН

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Statement of the problem.

Autovibration of dynamical systems is one of the most widespread self-organization phenomena in nature. It can play both the positive and negative roles in many devices, beginning, for example, from bow and wind musical instruments and to complex objects of modern industry and electronic technical equipment. The simplest and clearest model, illustrating the process of mechanical autovibration generating, is the 1 DOF oscillator including a conveyor belt with a load on it, restrained by elastic weightless spring [1]. Between the belt and load, the conditions of nonlinear frictional interaction are realized, which at certain constant values of the belt velocity v cause self-excitation of periodic reciprocating motions of the weight. Owing to its simplicity, this system was exhaustively investigated for different laws of nonlinear friction and elastic extension-compression of the spring. With its use, some general regularities of the autovibrational process self-excitation and proceeding were established.

But the considered model undergoes qualitative alterations if the spring is long. Then, its mass may be comparable or even larger the body mass, it ceases to be a simple elastic element, and becomes an elastic waveguide transmitting longitudinal extension-compression waves. Such device should be simulated by distributed systems with vibrations possessing modes arranged in an ordered (wave) fashion. In practice, such phenomena may appear, for example, in towing a transport facility on a water or solid surface.

Analysis of Publications. Similar processes also occur in the devices of deep drilling [2,3]. At the drill string extraction from the bore-hole cavity, the drill bit grates with its surface and the string begins to play the role of a waveguide. However, apparently the most distinctive autovibrational wave processes are generated in drilling the deep vertical bore-holes [4-9]. When, as a result of non-linear frictional interaction between the rotating bit and the near bottom surface of the well, the bit begins to commit torsional vibrations and torsional waves begin to propagate along the drill string. Analysis of these vibrations was performed on the basis of the non-linear model of torsional wave pendulum in [10]. It was shown in this reference that the self-oscillations were realized inside some diapason of change of the system rotation velocity and transitions from stationary rotations to periodic rotational motions were accomplished in the forms of the Hopf bifurcations [1].

The basic attention is paid to the systems with small inertance where the vibrating body mass is much less the waveguide inertance. In this connection, the constructed differential equation has the small parameter before the senior (second) derivative. The equations of this type are called singularly perturbed [11,12]. Their solutions have the shapes of saw-tooth functions and the vibrations described by them are called relaxational [13]. The second feature of the constructed solutions consists in the fact that the velocity function of the vibrating body has the quantized character in time with the quantum duration equaled the duration of the waveguide double length running by the wave [14].

Constitutive equation.

For the purpose of theoretical simulating the phenomenon of self-excitation of a waveguide vibration, the wave model of a dragging device with elastic cable of length L is used. It should be remarked that this model can be easily extended to other mechanic or electronic waveguide systems. In the considered case the right-hand end of the cable is considered to move with constant velocity v along the immovable OX axis. The longitudinal vibrations of the dragged body are excited through its frictional interaction with the horizontal surface.

To describe the body motion, introduce also the O_1x axis moving with speed v. Then, the distance travelled by the body along the Ox axis is $vt + u(0,t)$, where vt is the distance covered by the O_1 point; t is the time; $u = u(x,t)$ is the elastic displacement of the cable element along the O_1x axis.

By treating the elastic cable as an elastic waveguide, its axial vibrations can be described by the wave equation

$$\rho A \frac{\partial^2 u}{\partial t^2} - EA \frac{\partial^2 u}{\partial x^2} = 0 \tag{1}$$

where A is the cross-section area of the cable, ρ is its material density, and E is its elasticity modulus.

Eq. (1) can be brought to the standard form:

$$\frac{\partial^2 u}{\partial t^2} - \alpha^2 \frac{\partial^2 u}{\partial x^2} = 0 \tag{2}$$

Here $\alpha = \sqrt{E/\rho}$ is the velocity of the longitudinal wave.

The solution to Eqs. (1), (2) is

$$u(x,t) = f(x - \alpha t) + g(x + \alpha t) \tag{3}$$

expressed through the phase variables $x - \alpha t$ and $x + \alpha t$. In Eq. (3), $f(x - \alpha t)$ is the longitudinal elastic wave emanated towards right end $x = L$ from the body, $g(x + \alpha t)$ is the wave propagating to the body from the right end $x = L$ of the cable.

Since the end $x = L$ is moving with constant velocity v, it can be considered as clamped one for the elastic displacement. Then,

$$u(L,t) = 0 \qquad \text{or} \qquad f(L - \alpha t) + g(L + \alpha t) = 0 \tag{4}$$

To deduce the boundary condition at the left end $x = 0$, consider the dynamic equilibrium of the forces applied to it. So, one has

$$F^{in} + F^{fr} + F^{el} = 0\,, \tag{5}$$

where $F^{in} = -m\ddot{u}$ is the inertia force acting on the body, $F^{fr} = F^{fr}(v + \dot{u})$ is the friction force formed between the body and horizontal surface, it will be defined later. Here, the dot over a symbol denotes derivative with respect to time.

The elastic force F^{el} is calculated with the help of equality

$$F^{el} = EA\, \partial u / \partial x\big|_{x=0} \tag{6}$$

where the strain $\partial u / \partial x$ value is calculated as follows

$$\left.\frac{\partial u}{\partial x}\right|_{x=0} = \frac{\partial}{\partial x}\left[f(x - \alpha\, t) + g(x + \alpha\, t)\right]_{x=0} \tag{7}$$

Since the independent variables u and t are connected by the phase variables $x - \alpha t$ and $x + \alpha t$, the partial derivative $\partial u / \partial x$ can be expressed via the $\partial u / \partial t$ derivative. Really, it issues from Eqs. (3) and (4)

$$g(L + \alpha\, t) = -f(L - \alpha\, t). \tag{8}$$

However,

$$g(x+\alpha t)=g\left[L+\alpha\left(t-\frac{L-x}{\alpha}\right)\right]. \tag{9}$$

From (8) and (9), it follows $g\left[L+\alpha\left(t-\frac{L-x}{\alpha}\right)\right]=-f\left[L-\alpha\left(t-\frac{L-x}{\alpha}\right)\right]$.

So

$$g(x+\alpha t)=-f(2L-x-\alpha t). \tag{10}$$

Introduce the notations $x-\alpha t=p$, $2L-x-\alpha t=q$. Then, instead of (3), the presentation is gained

$$u(x,t)=f(x-\alpha t)-f(2L-x-\alpha t)=f(p)-f(q). \tag{11}$$

With its use, the derivatives can be calculated

$$\begin{aligned}\frac{\partial u}{\partial x}&=\frac{\partial f[p(x,t)]}{\partial x}-\frac{\partial f[q(x,t)]}{\partial x}=\frac{\partial f(p)}{\partial p}\frac{\partial p}{\partial x}-\frac{\partial f(q)}{\partial q}\frac{\partial q}{\partial x}=\frac{\partial f(p)}{\partial p}+\frac{\partial f(q)}{\partial q}\\ \frac{\partial u}{\partial t}&=\frac{\partial f[p(x,t)]}{\partial t}-\frac{\partial f[q(x,t)]}{\partial t}=\frac{\partial f(p)}{\partial p}\frac{\partial p}{\partial t}-\frac{\partial f(q)}{\partial q}\frac{\partial q}{\partial t}=-\alpha\frac{\partial f(p)}{\partial p}+\alpha\frac{\partial f(q)}{\partial q}\end{aligned} \tag{12}$$

Through correlation these equalities, one can represent

$$\frac{\partial u}{\partial x}=-\frac{1}{\alpha}\frac{\partial f(p)}{\partial t}+\frac{1}{\alpha}\frac{\partial f(q)}{\partial t} \tag{13}$$

and express the F^{el} force in terms of the derivatives with respect to t:

$$F^{el}=\frac{EA}{\alpha}\left[-\frac{\partial}{\partial t}f(-\alpha t)+\frac{\partial}{\partial t}f(-\alpha t+2L)\right].$$

After performing these substitutions and transformations, Eq. (5) will look like to the following non-linear relationship

$$m\left[\ddot{f}(-\alpha t)-\ddot{f}(-\alpha t+2L)\right]+\frac{EA}{\alpha}\left[\dot{f}(-\alpha t)-\dot{f}(-\alpha t+2L)\right]-F^{fr}(v+\dot{u})=0. \tag{14}$$

Here, $f=f(0,t)$, $u=u(0,t)$. Types of the F^{fr} function will be discussed later.

Solutions of the constructed equation display a series of features stemming from the type of the $F^{fr}(v+\dot{u})$ non-linear function. In the first place, it has a stationary solution for every value of v. Indeed, if $\dot{u}=0$ and $\ddot{u}=0$, then Eq. (2) is reduced to the form

$$\frac{d^2u}{dx^2}=0$$

with stationary solution $du/dx=c_1$, $u=c_2+c_1x$.

The constants c_1 and c_2 are found from boundary conditions (4) and (5)

$$u(L) = 0, \qquad EA\frac{du}{dx} = -F^{fr}(v)$$

or

$$c_2 + c_1 L = 0, \quad c_1 = -F^{fr}(v)/EA.$$

Thus,

$$u(x) = (L - x) \cdot F^{fr}(v)/EA \tag{15}$$

Secondly, a diapason $v_b \le v \le v_l$ exists, where stable periodic solutions occur in addition to stationary ones (15), which become unstable. Outside this diapason, stationary solutions (15) in the form of balanced motion $v = const$, $u(0) = L \cdot F^{fr}(v)/EA$ are stable. The states $v = v_b$, $v = v_l$, where the stationary motion is changed by autovibration and vice versa, are called the bifurcations of limit cycle birth and limit cycle loss or the Hopf (Poincare-Andronov-Hopf) bifurcations [1].

In parallel with these two traits, the third one exists for small inertance (mass m) of the body in comparison with large inertance (or acoustic stiffness $\rho A \cdot \alpha = A\sqrt{E\rho}$) of the waveguide and values of $F^{fr}(v)$. So, as indicated in [13], the problem of integration the equation with small coefficient before the senior derivative is singularly perturbed, the autovibrations are of relaxation type, and have nearly discontinuous velocities.

Beginning from the classic works by A. Poincare and A.M. Liapunov, the so called regular type of equations

$$\ddot{x} = F(t, x, \dot{x}, \varepsilon) \qquad (0 \le t \le 1) \tag{16}$$

was analyzed in details. Here, it is assumed that right-hand term regularly depends on the parameter ε in the vicinity of $\varepsilon = 0$ and the solutions are studied inside the segment $0 \le t \le 1$. However the equation solutions become less regular and more diversified when the small parameter $0 < \varepsilon << 1$ occurs before the second derivative

$$\varepsilon\ddot{x} = F(t, x, \dot{x}) \qquad (0 \le t \le 1). \tag{17}$$

In this case, the influence of the left-hand member on the solution becomes significant only for large values of $\ddot{x}$, related to the states of fast change in the system motion. So, the distinguishing property of these type equations is that they have periodic solutions in the shape of nearly broken straight lines or saw tooth curves.

Ultimately, one more feature of Eq. (14) is that it includes the delay argument $-\alpha(t - 2L/\alpha)$. By virtue of this, the system remembers the perturbations imposed previously on it with the $2L/\alpha$ delay and is self-adjusted to quantized vibrations with time quantum $\Delta\tau = 2L/\alpha$ [14]. So, it is of interest to follow the evolution of the autovibration modes with the change in parameter v.

Yet, the principal cause of the irregular dynamics agitation is non-linear frictional interaction between the movable body and foundation surface which is responsible for the antidumping and excitation of relaxation oscillations. Thus, mechanism of this kind interaction should be considered especially.

Discussion of results.

In mechanics, the waveguiding autovibrational systems are rarely met. The sole example of the singularly perturbed problem in this field, which plays a large role in practical applications, is the problem about the torsional auto-oscillations of long drill strings at their rotation with angular velocity ω. Such vibrations are generated as a consequence of nonlinear frictional interaction of their bits with the bore-hole bottom surfaces at the rock cutting. Because of the fact that the bit (vibrating body) mass is much less than the drill string (waveguide) mass, the system has small inertance, the coefficient before the inertia member (the second derivative) of the corresponding vibration equation is very small and its solution has the shape of a broken line. In connection with the discontinuous character of the relaxation vibrations, they are dangerous for the strength of the bit and drill string. Yet, the structure of the equation, describing these phenomena, is complicated, there are no universal methods for their investigation, and because of this, they are poorly understood.

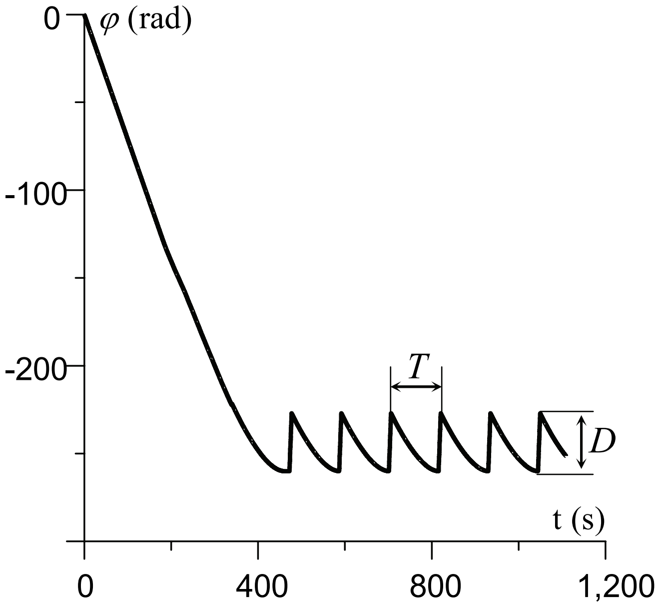

Figure 1 − The mode of the bit auto-oscillation ($L = 8000\ m$, $\omega_b = 0.71\ \ rad/s$)

Firstly, consider the homogeneous drill string 8000 m in length. The characteristic parameters used for its analysis are selected as follows: $G = 8.077 \cdot 10^{10}\ Pa$, $\rho = 7.8 \cdot 10^3\ kg/m^3$. External and internal radii of the tube cross-section are $r_1 = 0.0841\, m$ and $r_2 = 0.0741\, m$, then $I_z = 3.12 \cdot 10^{-5}\ m^4$.

One of the main features, influencing on the process of the bit torsion vibration, is the law of the friction moment M^{fr} dependence on the total velocity $\omega + \dot{\varphi}$ of its rotation. The shape of function $M^{fr}(\omega + \dot{\varphi})$ is determined by many factors, what is more, the values of their parameters vary during the drilling process.

In this reason, it is conceivable that no universal functions of this kind can be chosen for analysis of the system dynamics. The most commonly encounted relationships between M^{fr} and $\omega + \dot{\varphi}$ are represented by the Coulomb friction law shown in. It is used in our investigation for analysis of general regularities of autovibration proceeding. In its diagram, the vertical segment determines the static friction moment M_{st}, it is realized in the absence of sliding between bodies. After achieving some limit value $M_{\lim}$, the static friction moment M_{st} is replaced by the dynamic friction moment M_{dyn}, which is accompanied by sliding between rubbing surfaces. then, the friction moment M^{fr} can be represented with the aid of the following approximate function [10]

$$M^{fr} = M_{\lim} - \left(m \frac{a_1 k(\omega+\dot{\varphi}) + a_3 k^3 (\omega+\dot{\varphi})^3 + a_5 k^5 (\omega+\dot{\varphi})^5 + a_7 k^7 (\omega+\dot{\varphi})^7 + a_9 k^9 (\omega+\dot{\varphi})^9}{1 + a_2 k^2 (\omega+\dot{\varphi})^2} \right)^{1/2}$$

where the coefficients $a_i\ (i = 1,2,...,9)$ are found by the trial-and-error method. For the considered cases they have the following values: $a_1 = 2400\ N \cdot m \cdot s,\ a_2 = 225\ s^2,\ a_3 = 15000\ N \cdot m \cdot s^3,\ a_5 = 1\ N \cdot m \cdot s^5$, $a_7 = 4\ N \cdot m \cdot s^7$, $a_9 = -130\ N \cdot m \cdot s^9$, $k = 0.025$, $m = 1000$, $M_{\lim} = -41250\ N \cdot m$, $M^{fr}_{\min} = -8.25 \cdot 10^4\ N \cdot m$.

The analysis of the bit dynamics was performed by integrating the appropriate equation by the Runge-Kutta method with the initial conditions $\varphi(0) = 0$, $\dot{\varphi}(0) = 0$ for different values of ω. The integration step was selected to be $\Delta t = 7.769 \cdot 10^{-6}\ s$.

The calculation results permit us to formulate some regularities. On the one hand, in the process of functioning, the drill string can be either in the states of stationary rotation or of torsional self-induced elastic oscillation, depending on the chosen regime of drilling. As this takes place, the value ω_b of the angular velocity ω corresponding to the bifurcation state of the limit cycle birth equals the value $\omega = 0.71\, rad/s$, which conforms to the minimum point of the $M^{fr}(\omega + \dot{\varphi})$ diagram. The regimes of motion with $\omega < \omega_b$ are characterized by the stationary rotation without any oscillation when the system changes from its initial state $\varphi(0) = 0$, $\dot{\varphi}(0) = 0$ to some quasi-static equilibrious state $\varphi(t) = \varphi_{st}$, $\dot{\varphi}(t) = 0$ and self-induced vibrations do not take place. But during the system transition from outside to inside this diapason through the value $\omega = \omega_b$, the Hopf bifurcation occurs and limit cycles appear together with the unstable stationary solutions $\varphi(t) = const$, $\dot{\varphi}(t) = 0$.

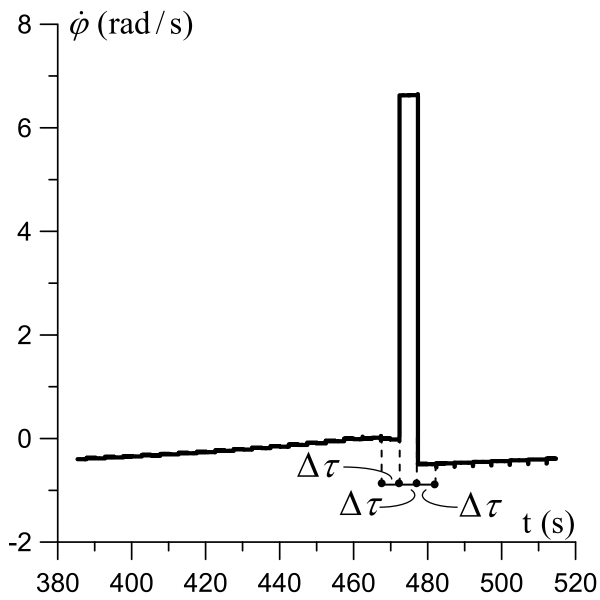

Figure 2 − The diagram of the angular velocity change ($L = 8000\, m$, $\omega_b = 0.71\ rad/s$)

The mode of the bit angular vibration in the result of bifurcation of the limit cycle birth ($\omega_b = 0.71\, rad/s$) is shown in Fig. 1. It is realized with comparatively large period $T \approx 115\, s$ and swing $D \approx 35\, rad$. But the more interesting feature of this process is that the auto-oscillations are of the relaxational (nearly discontinuous) type and include time segments of fast and slow motions inside every period.

The diagram of angular velocity $\dot{\varphi}(t)$ in the time diapason $380 \le t \le 520\, s$ is presented in Fig. 2. It illustrates a principally new, subtler peculiarity, which is unique only to waveguiding systems [14]. This feature consists in the fact that the self-excited oscillations proceed in the manner of quantized time and the time quantum duration $\Delta\tau$ is equal to the time segment of the wave passing the path from the bit to the top end of the DS and backward, i.e. $\Delta\tau = 2L/\beta$.

Together with the phenomena of self-excitation of stationary auto-oscillations, the problem about transient processes under conditions of the moving body speeding up or braking represents certain interest. It can be imagined that if a DS begins to rotate with small constant angular acceleration ε, then it will gradually pass through the bifurcation velocity ω_b, enter into the diapason of auto-oscillations and afterwards again go out through the point ω_l of limit cycle loss to the domain of pure rotation without oscillations. But the situation changes when acceleration ε is not small. Then, owing to existence of the system inertance, all the observed effects can take place, though with some delay for homogeneous DS 1000 m in length and angular acceleration $\varepsilon = 0.05\, rad/s^2$). The found effect becomes more visible with further ε enlargement and for the values $\varepsilon \ge 0.5\, rad/s^2$ the auto-oscillation phenomena do not occur at all.

Conclusions.

The analysis of the limit cycle birth bifurcations in the models of homogeneous and sectional waveguiding systems is presented in this paper. The constitutive differential equations with delay argument are constructed, which are shown to be singularly perturbed. Based on analysis of an applied example associated with self-excitation of deep drill string torsion oscillation, one can draw the following conclusions:

1. The auto-oscillations of homogeneous and sectional DSs prevail at low values of the their angular velocity ω, the boundaries of the ω segments of their self-excitation do not depend on the number of the DS sections and are determined by the outline of the friction moment function.

2. The autovibrations are of the relaxation type and contain fast and slow motions.

3. The self-excited oscillations proceed in the manner of quantized time. The time quantum durations equal the time of the torsional wave propagating through the doubled length of the DS.

4. The velocity - time quanta in sectional drill strings have additional fragmentations caused by the multiple diffractions of the torsional waves at the points of the section joints.

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РЕФЕРАТ

Глушакова О.В. Релаксаційні автоколивання крутіння глибоких бурильних колон. / О.В. Глушакова // Вісник Національного транспортного університету. Серія «Технічні науки». Науково-технічний збірник. – К. : НТУ, 2015. – Вип. 1 (31).

Розглянута проблема про самозбудження крутильних коливань бурильних колон в результаті фрикційної взаємодії долота з руйнівною породою.

Об'єкт дослідження − ефекти біфуркаційного переходу від стаціонарного обертання бурильної колони до автоколивань крутіння і назад.

Мета роботи − дослідити біфуркаційні стани бурильних колон, що обертаються, і побудувати моди автоколивань.

Метод дослідження − метод чисельного інтегрування рівнянь крутіння в перед- і пост-критичних станах.

КЛЮЧОВІ СЛОВА: ХВИЛЕВОДНІ СИСТЕМИ, СИНГУЛЯРНО ЗБУРЕНІ ЗАДАЧІ, САМОЗБУДЖЕННЯ КОЛИВАНЬ, БІФУРКАЦІЯ ХОПФА, РЕЛАКСАЦІЙНІ КОЛИВАННЯ.

ABSTRACT

Glushakova O.V. Relaxational torsional autovibrations of deep drill strings. Visnyk National Transport University. Series «Technical sciences». Scientific and Technical Collection. – Kyiv: National Transport University, 2015. – Issue 1 (31).

In this paper the analysis of the limit cycle birth bifurcations in the models of homogeneous and sectional waveguiding systems is presented. The constitutive differential equations with delay argument are constructed, which are shown to be singularly perturbed. Based on analysis of an applied example associated with self-excitation of deep drill string torsion oscillation, one can draw the following conclusions:

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The autovibrations are of the relaxation type and contain fast and slow motions.

KEYWORDS: WAVEGUIDING SYSTEMS, SINGULARLY PERTURBED PROBLEM, SELF-INDUCED VIBRATIONS, HOPF'S BIFURCATION, RELAXATION VIBRATIONS.

РЕФЕРАТ

Глушакова О.В. Релаксационные автоколебания кручения глубоких бурильных колонн. / О.В. Глушакова // Вестник Национального транспортного университета. Серия «Технические науки». Научно-технический сборник. – К. : НТУ, 2015. – Вып. 1 (31).

Рассмотрена проблемма о самовозбуждении крутильных колебаний бурильных колонн в результате фрикционного взаимодействия долота с разрушаемой породой.

Объект исследования − эффекты бифуркационного перехода от стационарного вращения бурильной колонны к автоколебаниям кручения и обратно.

Цель работы − исследовать бифуркационные состояния вращающихся бурильных колонн и построить моды автоколебаний.

Метод исследования − метод численного интегрирования уравнений кручения в пред- и пост-критических состояниях.

КЛЮЧЕВЫЕ СЛОВА: ВОЛНОВОДНЫЕ СИСТЕМЫ, СИНГУЛЯРНО ВОЗМУЩЕННЫЕ ЗАДАЧИ, САМОВОЗБУЖДЕНИЕ КОЛЕБАНИЙ, БИФУРКАЦИЯ ХОПФА, РЕЛАКСАЦИОННЫЕ КОЛЕБАНИЯ.

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