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CALCULATION OF NON-HOMOGENEOUS ANISOTROPIC RECTANGULAR PLATES WITH ARBITRARY FIXATION ON THE CONTUR

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Abstract. For practical applications in engineering are important tables for determining deflections and internal forces of structures. Such tables for the isotropic case under various conditions of plate support on the contour are given in many works. As for the anisotropic plates, there are no such tables, with the exception of one Huber table compiled for a freely supported rectangular orthotropic plate, depending on the relationship between the stiffness values $s = \sqrt{D_{11}D_{22}} / D_{12} = 1$. Here is a method of calculating the non-homogeneous anisotropic rectangular plates with arbitrary fixation on the contour is set forth, which is reduced to a boundary value problem. To solve a system of equations in terms of displacements using finite difference method (FDM) in combination with different variations of analytical solutions. It is advisable to construct a numerical solution of the problem so that in difficult cases the support fixing and uploading solution sought, not directly, but in the form of amendments to the known solution for simple cases of reference to consolidate and uploading at finding the solutions which the analytical methods or the FDM with sparse mesh may be used. Given as examples are the results of calculation for a series of square orthotropic plates with a fixed boundary under the action of uniformly distributed load.

Keywords: regional task, non-homogeneous anisotropic rectangular plates, supporting fixing, contour.

РАСЧЕТ НЕОДНОРОДНЫХ АНИЗОТРОПНЫХ ПРЯМОУГОЛЬНЫХ ПЛАСТИН С ПРОИЗВОЛЬНЫМ ЗАКРЕПЛЕНИЕМ НА КОНТУРЕ

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Аннотация. Излагается методика расчета неоднородных анизотропных прямоугольных пластин с произвольным закреплением на контуре, которая сводится к краевой задаче. Для решения системы уравнений в перемещениях используется метод конечных разностей (МКР) в комбинации с различными вариантами аналитических решений. Целесообразно строить численное решение краевой задачи так, чтобы в сложных случаях опорного закрепления и загружения решение искалось не непосредственно, а в виде поправок к известному решению для простых случаев опорного закрепления и загружения, при разыскании решений которых могут быть использованы аналитические способы или МКР с редкой сеткой. В качестве примеров приведены результаты расчета для серии квадратных ортотропных пластин с жестко закрепленным краем под действием равномерно распределенной нагрузки.

Ключевые слова: краевая задача, неоднородные анизотропные прямоугольные пластины, опорное закрепление, контур.

РОЗРАХУНОК НЕОДНОРІДНИХ АНІЗОТРОПНИХ ПРЯМОКУТНИХ ПЛАСТИН З ДОВІЛЬНИМ ЗАКРІПЛЕННЯМ НА КОНТУРІ

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Анотація. Викладається методика розрахунку неоднорідних анізотропних прямокутних пластин з довільним закріпленням на контурі, яка зводиться до крайової задачі. Для вирішення системи рівнянь в переміщеннях використовується метод кінцевих різниць (МКР) в комбінації з різними варіантами аналітичних рішень. Доцільно будувати чисельне розв'язання крайової задачі так, щоб у складних випадках опорного закріплення і навантаження рішення шукалося не безпосередньо, а у вигляді поправок до відомого рішення для простих випадків опорного закріплення і навантаження, при розшуку рішень яких можуть бути використані аналітичні методи або МКР з рідкою сіткою. В якості прикладів наведені результати розрахунку для серії квадратних ортотропних пластин з жорстко закріпленим краєм під дією рівномірно розподіленого навантаження.

Ключові слова: крайова задача, неоднорідні анізотропні прямокутні пластини, опорне закріплення, контур.

Introduction. Despite of a great variety of non-homogeneous anisotropic plates calculation methods, one meet significant difficulties while their practical realization. They are defined not only by the complexity of differential equations integration with the variable coefficients, but also by the complexity of concomitant boundary conditions satisfaction. Very seldom, while solving practical tasks, such a calculation may be done in an analytical way. It's purposefully, and sometimes only possible to use modern numeral methods. The simplest from them is the method of final differences (MFD). Thus, while using this method, it turns out that the solution may be found exactly enough in one cases of bearing fixing, for instance, – hinged–flexible, – on a simple and rare enough net, in other ones, for instance – rigid jamming, – only on a complicated or too dense net, that makes a corresponding calculation too labor-consuming, even while using modern calculation techniques.

For the shortage weakening it's purposefully to build a numerous solution so that in difficult cases of bearing fixing and loading the solution would be found not directly, but in the view of amendments to the known solution for simple cases of bearing fixing and loading, while searching of which analytical means or MFD with a rare net can be used. Such a calculation method, though somewhat complicated finding of the searched solution for difficult cases of fixing, because it will have to be done in two stages, nevertheless, finally, turns out to be more effective, as it lets find composite parts searched solution with the use of analytical correlation or comparatively simple in the structure and the biggest in number of finally differential equations systems.

The analysis of the last publications. For the opportunity of practical use in engineering important meaning have tables for bending's definition and constructions' internal forces. In many works [1, 5] such tables are given for an isotropic case in different conditions of leaning on the contour. What concerns anisotropic plates, there are no such tables, except for one Guber's table, composed for a freely-leant rectangular orthotropic plate depending upon the correlation between rigidity values $s = \sqrt{D_{11}D_{22}} / D_{12} = 1$. Up till now it remains the only one and is a compulsory component in all the guidance in plates' calculation. The given article is addressed to the engineers and scientific workers, dealing with the projecting of thin-walled constructions. Their important setting is the continuation of particular tasks study, for different sides' correlation *b*/*a* between plates' sizes and the ways of fixing on edges and some of them were looked through in the work [3].

The purpose of this research was to expound calculations method, realizing the formulated idea and to illustrate the results of its use on the examples.

The objects and the methods of investigation. The task of a non-homogeneous anisotropic rectangular plate calculation with free fixing on the contour may be taken to the finding of an edge task of a general view solution:

$$L(w)\big|_{\Omega} = f, \qquad R_i(w)\big|_{S_i^{\Gamma}} = \rho_i, \qquad \Gamma_i(w)\big|_{S_i^{\Gamma}} = \varphi_i \tag{1}$$

where:

$$L(w) = \vec{B}^T D \vec{B} w, \qquad \vec{B} = \begin{bmatrix} \dfrac{\partial^2}{\partial x^2} \\ \dfrac{\partial^2}{\partial y^2} \\ 2\dfrac{\partial^2}{\partial x \partial y} \end{bmatrix} \qquad R_1(w) = \begin{cases} \mp \dfrac{\partial w}{\partial x} & \\ & \text{when} \\ \mp \dfrac{\partial w}{\partial y} & \end{cases} \begin{matrix} x = 0 \ и \ a \\ \\ y = 0 \ и \ b \end{matrix}; \tag{2}$$

$$R_2(w) = w \Big\{ \ \text{when} \ \begin{matrix} x = 0 \ \ и \ \ a \\ y = 0 \ \ и \ \ b \end{matrix}; \quad \Gamma_{23} = \begin{cases} Q_x + \dfrac{\partial M_{xy}}{\partial y} & \\ & \text{when} \\ Q_y + \dfrac{\partial M_{xy}}{\partial x} & \end{cases} \begin{matrix} x = 0 \ и \ a \\ \\ y = 0 \ и \ b \end{matrix};$$

$$\Gamma_1(w) = \begin{cases} M_x \\ M_y \end{cases} \text{when} \ \begin{matrix} x = 0 \ \ и \ \ a \\ y = 0 \ \ и \ \ b. \end{matrix}$$

Ω $=\{0 \le x \le a,\ 0 \le y \le b\}$ – a rectangular area, which a plate takes; S = $\{x = 0$ и a, $y = 0$ и $b\}$ – rectilinear parts of contour, limiting this area; f – the intensity of a crossed load, applied to a plate; ρ_i и φ_i – given on the sections $S_i^R < S$ и $S_i^{\Gamma} = S - S_i^R$ of a contour the meanings of a turn's edge, a bend, the moments and the given crossed forces ; D – rigidity's square matrix of order 3 with the elements, depending on coordinates x and y.

The variables S_i^R and S_i^{Γ}, from formulas (1) and (2) one may get edge differential tasks, describing the state of a looked-through plate in all possible ways of fixing and loading its edges.

Probably, the solution of a task is being solved (1), thus the solution of a corresponding "rigid" task is known:

$$L(v)\big|_{\Omega} = f, \qquad R_i(v)\big|_{S} = \rho_i. \tag{3}$$

In this work (1) it is shown that between the solutions (1) and (3) the dependence takes place:

$$u(x,y) = v(x,y) + \sum_{j=1}^{\infty} X_j \chi_j(x,y), \tag{4}$$

where:

$$X_j = \iint_{\Omega} \chi_j f d\omega - \sum_{i=1}^{2} \int_{S_i^R} T_i(\chi_j)\rho_i ds - \sum_{i=1}^{2} \int_{S_i^{\Gamma}} \Gamma_i(\chi_j)\rho_i ds + \sum_{i=1}^{2} \int_{S_i^{\Gamma}} R_i(\chi_j)\varphi_i ds, \tag{5}$$

and $\chi_j(x,y)$, (j=1,2,…) – a system of the coordinate functions, built in a special way. For their composing it's necessary to find a full system of an auxiliary homogeneous edge task solution:

$$L(\psi_k)\big|_{\Omega} = 0, \qquad R_i(\psi_k)\big|_{S_i^R} = 0 \tag{6}$$

and prorogation it in bilinear expression:

$$F(p,q) = \iint_{\Omega} (\vec{B}p)^T D\vec{B}q d\omega . \tag{7}$$

The last may be done according to the formula:

$$\chi_j(x,y) = \sum_{k=j}^{1} b_{jk}\psi_k(x,y) \qquad j = 1, 2, \ldots , \tag{8}$$

where:

$$b_{jk} = c_j \text{ when } k = j; \qquad b_{jk} = -c_j \sum_{i=j-1}^{k} \alpha_{jl} b_{lk} \quad \text{when } k = j-1, \ldots , 2, 1, \tag{9}$$

further:

$$\alpha_{jl} = c_l \left\{ \beta_{jl} - \sum_{m=l-1}^{1} \alpha_{jm}\alpha_{lm} \right\}; \qquad c_j = \left(\beta_{jj} - \sum_{l=j-1}^{1} \alpha_{jl}^2 \right)^{-1/2}, \tag{10}$$

and, finally, $\beta_{jk} = F(\psi_j, \psi_l)$.

An important feature of the described way of the solution is in the following: if the system of coordinate elements is built as χ_j, then with its help one can get the function of bends *and at any loads f*, φ_i and displacements ρ_i. For this it is only necessary to calculate the corresponding coefficients X_j in formula (5).

In the case, when a plate is homogeneous, the matrix rigidity elements *D* doesn't depend on the coordinates *x, y*; here the task solutions (6) may be built in an analytical way [4]. The final differences method for plates' calculation was developed and applied in the works successfully [1], [2]. The case of a non-homogeneous plate, being looked through now, is much more difficult. Thus, the equation's coefficients (6) the variable, so its solutions can be, as a rule, found only in the result of any numeral method applying.

Evidently, formula (4) may be used not only for function's finding *u* by the known formula *v*, but also for function *v* definition, if we know any function *u*. The last may be sometimes built in an analytical form for non-homogeneous plate, all four edges of which are fixed hinged to the non-deformed support.

In this work [3] the results of rectangular orthotropic plates with the correspondence between rigidity values calculation are given $s = \sqrt{D_{11}D_{22}} / D_{12} = 1$, $e_1 = \sqrt{D_{12}/D_{11}} = 0{,}91$, $e_2 = \sqrt{D_{12}/D_{22}} = 1{,}1$ for v_1 = 0,3, v_2 = 0,2 and in different sides' correspondences *b*/*a* under the action of equally distributed load q when rigid fixing.

The results of the research. Here are given the results of rectangular orthotropic plates' calculation with the correspondence between rigidities values $s = 1/(e_1 e_2) = \sqrt{D_{11}D_{22}} / D_{12} < 1$, $e_1 = \sqrt{D_{12}/D_{11}} = 0{,}93$, $e_2 = \sqrt{D_{12}/D_{22}} = 1{,}26$ and in different sides' correspondences *b*/*a* under the action of equally distributed load q when rigid fixing.

For the convenient appliance of calculations results numeral meanings of bends and bending moments in the center of supporting edges of orthotropic rectangular plates for v_1 = 0,3, v_2 = 0,2 are given in table 1. In table 1 the efforts from equally distributed load actions are mentioned.

Table 1 – The forces from the action of an equally distributed load

a / b	The bend in the center	The moments in the center of supporting sides		The moments in the center	
	$w = \alpha \frac{qb^4}{D_{22}}$	$M_x = \beta_1 qb^2$ $x = \pm \frac{a}{2}, y = 0$	$M_y = \beta_2 qb^2$ $y = \pm \frac{b}{2}, x = 0$	$M_x = \gamma_1 qb^2$ $x = 0,\ y = 0$	$M_y = \gamma_2 qb^2$ $y = 0,\ x = 0$
	α	β_1	β_2	γ_1	γ_2
1	0,00056	-0,04394	-0,03860	0,06231	0,01149
1,1	0,00070	-0,04738	-0,04306	0,06309	0,01375
1,2	0,00081	-0,05035	-0,04710	0,06360	0,01593
1,3	0,00094	-0,05281	-0,05072	0,06392	0,01802
1,4	0,00106	-0,05511	-0,05395	0,06412	0,01999
1,5	0,00117	-0,05699	-0,05683	0,06425	0,02181
1,6	0,00121	-0,05861	-0,05938	0,06435	0,02351
1,7	0,00128	-0,05998	-0,06166	0,06444	0,02508
1,8	0,00139	-0,06115	-0,06369	0,06454	0,02652
1,9	0,00157	-0,06213	-0,06551	0,06463	0,02784
2,0	0,00165	-0,06296	-0,06714	0,06474	0,02907

The conclusion. From the data, given in this table, it follows that at the action of an equally distributed load and almost all *a*/*b*, i.e. plates sides correspondences, with the increase of *a*/*b* the bend in the center, the moment M_y in the center and the moments M_x, M_y on the edges grow; the moment M_x in the center at an equally distributed load and in the center of a supporting side in case of rigid fixing of a plate on the whole contour decreases. In case of an equally distributing load action q the moment M_x on the edges increases equally on a small value approximately in 1.4 – 1.5 times.

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