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EVALUATION METHOD OF FRICTION OF GRAVITATIONAL TYPE TOWING TANK TOWING SYSTEM

The method of determination of the coefficients of friction forces of the gravity type towing tank towing system, based on the study of kinematic characteristics of the equivalent simulating system. The technique is based on a comparison of the values of velocity and acceleration of the towing load, obtained experimentally and calculated from the analytical solution of the problem of the motion of the towing system.

Keywords: *gravitational type towing tank, towing system, coefficients of forces and moments of friction.*

Запропоновано методику визначення коефіцієнтів сил тертя буксирувальної системи дослідного басейну гравітаційного типу, заснована на дослідженні кінематичних характеристик еквівалентної моделюючої системи. Методика побудована на порівнянні значень швидкості і прискорення буксирувального вантажу, отриманих експериментально і отриманих з аналітичного рішення задачі про рух буксирувальної системи.

Ключові слова: *дослідний басейн гравітаційного типу, буксирувальна система, коефіцієнти сил і моментів тертя.*

Предложена методика определения коэффициентов сил трения буксировочной системы опытового бассейна гравитационного типа, основанная на исследовании кинематических характеристик эквивалентной моделирующей системы. Методика построена на сравнении значений скорости и ускорения буксирующего груза, полученных экспериментально и вычисленных из аналитического решения задачи о движении буксировочной системы.

Ключевые слова: *опытовый бассейн гравитационного типа, буксировочная система, коэффициенты сил и моментов трения.*

Introduction. The Odessa National Maritime University (further ONMU) towing tank was founded in 1930, it came into operation in 1932. Structurally, it belongs to the gravitational type towing tanks of Wellenkamp system [1].

Unlike the basins of Froude type [2] the main feature of this type of towing tanks is the presence of a rope system, via which the vessels model is being towed. In this case the model is attached to the looped rope and moves under the action of the towing load, descending into a special shaft.

The scheme of the towing system of ONMU towing tank is shown on a Figure 1.

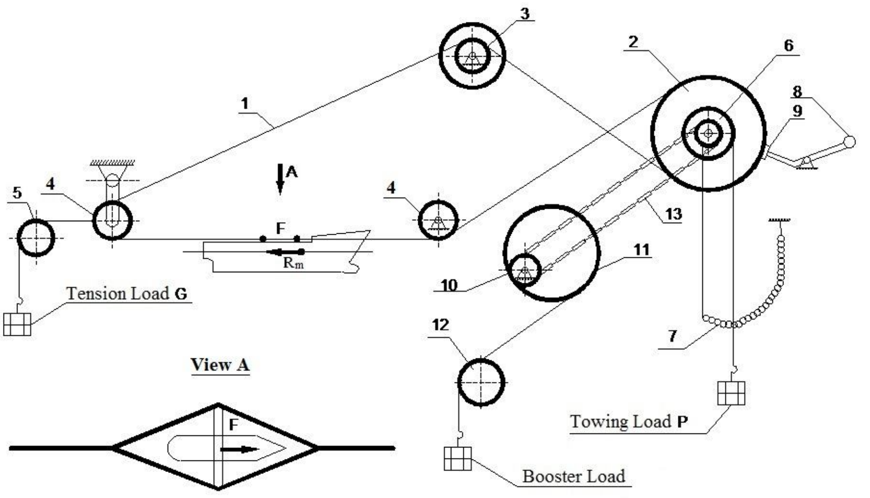

Fig. 1. The scheme of the ONMU towing tank towing system:
1 – main towing rope; 2 – driving pulley; 3 – measuring pulley;
4 – discharging pulley; 5 – idler; 6 – towing load pulley;
7 – compensating chain load; 8 – starting lever; 9 – friction brake;
10 – accelerating device; 11 – accelerating load pulley;
12 – discharging pulley;13 – chain belt

The presence of the rope wiring causes appearance of inevitable forces of friction in pulleys. Thus, the control of friction of the towing system in the process of realization of towing tests is an important technological process influencing on the results of experimental researches.

Usually control of friction in the ONMU towing tank is conducted by running the system in free oscillation mode, i.e. without a model under the influence of small-sized towing load (0.1-0.2 kg). Force of friction was determined in dimensional form the in assumption of its permanence in a narrow range of velocity of the system (about 1.0 m/s). The disadvantage of such method is a necessity for the hand selection of ranges of speeds at the periods of acceleration and braking of the system, in the range of which will be valid the resulting values of the friction forces. Also, the disadvantage is the complexity of the accounting inertial effects on the area of the system braking, which is caused by motion of load point of a suspension on the phase of changing the direction of its motion.

The measuring system of ONMU towing tank was modernized in 2011-2012. During the modernization, a software «TowDrag» was developed. This program visualizes the process of recording the speed of the model, automates the process of test results data processing. [3]. «TowDrag» has a special mode to control the system friction.

The velocity changing of the main rope of the towing system in the free oscillation mode is shown in Figure 2.

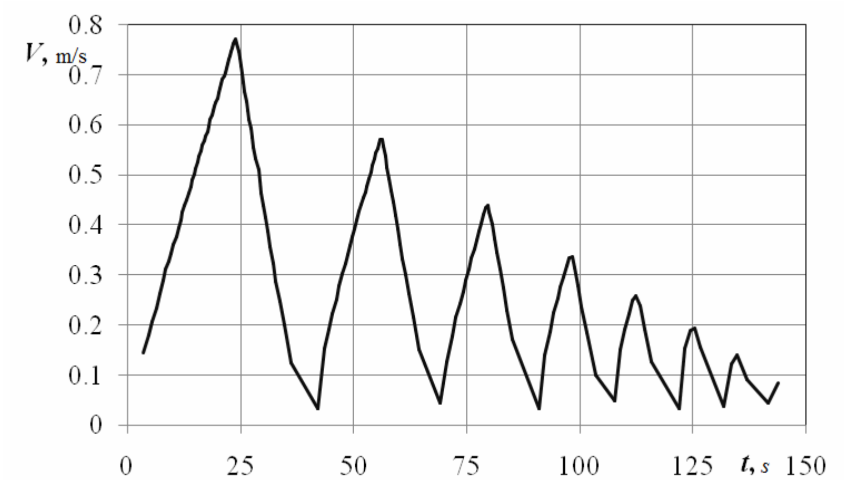

Fig. 2. A typical view of the process of changing the velocity of towing system in the free oscillation mode

The aim of the present investigation is development of methodology to determine the coefficients of friction of the towing system of towing tank that allows to determine force and moment of friction of the system depending on the rate of movement of towing load.

Methods of determining the coefficients of friction of the towing system is as follows.

Let $D_i[\varphi(\mu_0,\mu_1,\mu_2,t),\alpha(\mu_0,\mu_1,\mu_2,t)]=0,\ i=1{,}2$ − a system of equations of motion of the tow $\varphi(\mu_0,\mu_1,\mu_2,t),\alpha(\mu_0,\mu_1,\mu_2,t)$ − the generalized coordinate system, depending on the coefficients of friction μ_0,μ_1,μ_2. Let $l=f(\varphi(\mu_0,\mu_1,\mu_2,t),\alpha(\mu_0,\mu_1,\mu_2,t))=l(\mu_0,\mu_1,\mu_2,t)$ − the law of the vertical movement of the towing load of a system also depends on the coefficient of friction due to the geometric and kinematic relationships with generalized coordinates. Also, experimental studies allow us to determine the values of velocity $\dot{l}_э(t)$ and acceleration $\ddot{l}_э(t)$ of towing load at a time interval $[0,T]$.

The coefficients (1) are the parameters of the system and their numerical values defines a specific law of motion of the load.

The coefficients μ_0,μ_1,μ_2 are the parameters of the system and their specific numerical values defines a specific law of motion of the towing load.

Coefficient values can only be determined from the experimental data on the movement of the towing system thus natural to assume that the true value of the coefficient of friction should provide a minimum standard deviation of analytical solutions of the experimental data. That is the problem of determining the values of the coefficient of friction is reduced to a variational problem of finding the minimum of the functional

$$F_1(\mu_0,\mu_1,\mu_2)=\sqrt{\int_0^T\left(\dot{l}(\mu_0,\mu_1,\mu_2,t)-\dot{l}_э(t)\right)^2 dt}$$

and

$$F_2(\mu_0,\mu_1,\mu_2)=\sqrt{\int_0^T\left(\ddot{l}(\mu_0,\mu_1,\mu_2,t)-\ddot{l}_э(t)\right)^2 dt}\,.$$

So, first we need to find an analytical solution $l(\mu_0,\mu_1,\mu_2,t)$ or that equivalent, changes of analytical laws of the generalized coordinates $\varphi(\mu_0,\mu_1,\mu_2,t),\alpha(\mu_0,\mu_1,\mu_2,t)$ of the towing system.

The complexity of the kinematic scheme of the towing systems as well as some of the technical difficulties associated with the determination of geometric and inertial parameters of its components determine the choice of a simple in a kinematic sense, but an equivalent effect of a mechanical system, which simulates the operation of the towing system.

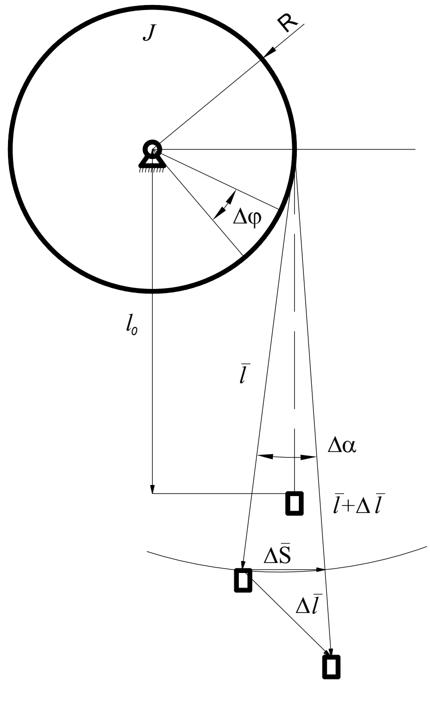

Fig. 3. The kinematic scheme of simulation system

The model of the towing system. The simulation system consists of a pulley rotating around a horizontal and fixed axis of the pulley coiled by inextensible and weightless thread and unknown moments of inertia and friction. Its scheme is shown in Figure 3. At the time of the system are forces of gravity, moment of friction forces on the axis of the pulley, as well as the centrifugal force of inertia of the oscillating load on the suspension. Since the towing system consists of many different elements, moving rotationally and progressively, the friction moment was adopted in the form of a polynomial of degree 2 of the angular velocity of the pulley.

The independent parameters of movement adopted the angles of rotation of the pulley φ and the suspension load α.

The system of equations in dimensional form, describing the dynamics of the system is as follows

$$\begin{cases} \left(J + mR^2\right)\ddot{\varphi} - mR\left(l_0 + R\varphi\right)\dot{\alpha}^2 + \left(\mu_2\dot{\varphi}^2 + \mu_1\dot{\varphi} + \mu_0\right) - mRg = 0; \\ m\left(l_0 + R\varphi\right)^2\ddot{\alpha} + 2mR\left(l_0 + R\varphi\right)\dot{\varphi}\dot{\alpha} + mg\left(l_0 + R\varphi\right)\sin\left(\alpha\right) = 0, \end{cases} \tag{1}$$

where l_0 − initial length of the suspension load;

m − mass of the load adopted for mass scale;

R − the radius of the pulley, taken as a linear scale of the task.

As the value of the selected time scale $T = \sqrt{\dfrac{2\pi R}{g}}$. Then the non-dimensional system has the form

$$\begin{cases} \ddot{\gamma} - \dfrac{mR}{\left(J + mR^2\right)}\gamma\dot{\alpha}^2 + \left(\dfrac{\mu_2}{\left(J + mR^2\right)}\dot{\gamma}^2 + \sqrt{\dfrac{2\pi R}{g}}\dfrac{\mu_1}{\left(J + mR^2\right)}\dot{\gamma} + \dfrac{2\pi R}{g}\dfrac{\mu_0}{\left(J + mR^2\right)}\right) - \\ \qquad - \dfrac{2\pi mR^2}{\left(J + mR^2\right)} = 0; \\ \ddot{\alpha} + \dfrac{2R\dot{\gamma}\dot{\alpha}}{\gamma} + \dfrac{2\pi R\sin(\alpha)}{\gamma} = 0, \ \gamma \neq 0, \end{cases} \tag{2}$$

where $\gamma = \left(l_0 + R\varphi\right)$. Equation $\gamma = 0$ is performed only in the state of rest of the system.

In the first equation of the system (2) the term of the square of the angular velocity of the oscillation of the load is the moment of the centrifugal force acting on the load. Discard this term based on the smallness of the angle α unreasonable, especially as the load oscillates in the shaft at a depth of 10-12 meters, and the angle of oscillation is difficult to assess. It is important to evaluate the velocity of oscillations, since the meaning of $R\dot{\alpha}^2$ may have a finite value.

To estimate the quantity $R\dot{\alpha}^2$ consider the second equation (2). Isocline equation $\ddot{\alpha} = 0$ in the phase plane $(\alpha, \dot{\alpha})$ is given $\dot{\gamma}\dot{\alpha} + \pi\sin(\alpha) = 0$. The value of $\dot{\gamma} = \dot{\varphi}$ increases monotonically to a finite limit, so the amplitude isocline will decrease with the change of α in time to a finite value. We find regions of positive and negative values of $\ddot{\alpha}$, provided $\gamma > 0,\ \dot{\gamma} \geq 0$. These conditions mean that the speed of rotation of the pulley does not decrease, and the starting position of the pulley characterized by the value of the generalized coordinates of $\varphi = 0$. Then from the second equation (2)

$$\frac{l_0\ddot{\alpha} + 2R\dot{\gamma}\dot{\alpha}}{2\pi R} = -\sin(\alpha). \qquad (3)$$

a) Let $\dot{\alpha} > 0, \alpha > 0$. If $\ddot{\alpha} \geq 0$ the left-hand side of the expression (3) has a positive value and the equation is not satisfied. For equal left and right sides (3) prerequisites $\ddot{\alpha} < 0$, then $\dot{\alpha}$ has reduced on the interval $\left(0, \frac{\pi}{2}\right)$;

b) Let $\dot{\alpha} > 0, \alpha < 0$. If $\ddot{\alpha} \geq 0$ at any point in time, it will cause the left side of the unlimited growth of the expression (3). So there must be a time interval where $\ddot{\alpha} < 0$, and therefore the moment of time at which the sign change of $\ddot{\alpha}$ occur from positive to negative;

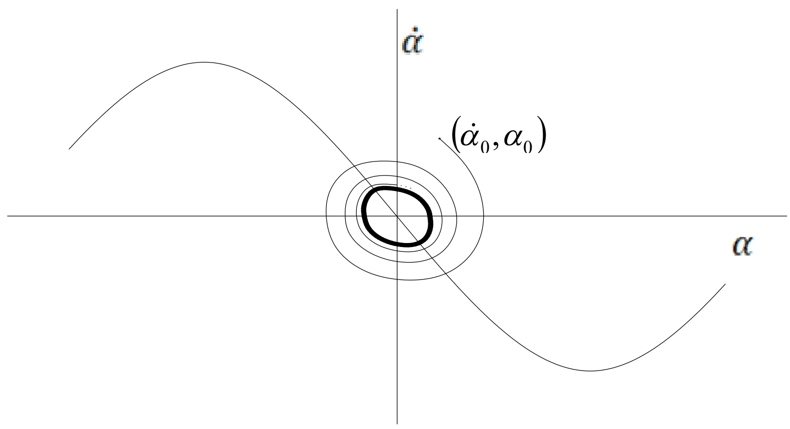

Fig. 4

c) The behavior of the phase trajectories for conditions $\dot{\alpha} < 0, \alpha < 0$ symmetrical condition a), and provided $\dot{\alpha} > 0, \alpha < 0$ − symmetrical to condition b) with respect to the coordinate system reference. It is seen that in the region above isocline acceleration $\ddot{\alpha}$ is negative, and under isocline – positive.

Consider the situation in the phase plane $(\alpha, \dot{\alpha})$ to the initial values of α_0 and $\dot{\alpha}_0$ (see Figure 4). When $\dot{\alpha}_0 > 0, \alpha_0 > 0$ and α grows, $\dot{\alpha}$ values decreases to zero. Further, the conditions $\dot{\alpha} < 0, \alpha > 0$. That is, there is a decrease in the value of α, and the point of intersection of the phase trajectory isocline $\ddot{\alpha} = 0$ growth begins values of $\dot{\alpha}$. If $\dot{\alpha} = 0$ begin growth of α, and after crossing the phase trajectory with isocline decrease – $\dot{\alpha}$.

The process will be repeated cyclically, but the maximum values of $|\dot{\alpha}|$ at each new cycle will decrease. This follows from the following considerations.

Let $\alpha^* > 0$ value at which $\dot{\alpha} = 0$. From equation isocline find $\dot{\alpha}^* < 0$. Further, since the α^*, α values will decrease and, consequently, the intersection of the phase trajectory with isocline happens at $\alpha < \alpha^*$. And since isocline monotonically decreases on the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, then the value of $\dot{\alpha}$, corresponding to the point of intersection, there will be greater than $\dot{\alpha}^*$. Similarly, we find that in case of $\alpha^* < 0$ is executed $\dot{\alpha}^* > 0$, and at the intersection with isocline phase trajectory $\dot{\alpha} < \dot{\alpha}^*$. Thus phase trajectory is a spiral converging to a closed path, or to the point, with some $|\dot{\alpha}|$ limited to values $|\dot{\alpha}|_{\max}$. From the second equation of the system (2) for $\dot{\gamma}_0 = 0$ implies that $|\dot{\alpha}|_{\max} = |\dot{\alpha}_0|$. In the experiment for measurement of the characteristics of motion of the system is possible to limit the amplitude of the initial suspension deviation value $|\alpha_{\max}| = 5^\circ$. Then the average velocity of the oscillations in the dimensional form equals $|\dot{\alpha}_{cp}^{p}| = \frac{\pi}{36}\sqrt{\frac{g}{l_0}}$, and the dimensionless average velocity of the oscillations – $|\dot{\alpha}_{cp}| = \frac{\pi}{36}\sqrt{2\pi\frac{R}{l_0}}$. When $l_0 \approx R$ $|\dot{\alpha}_{cp}| \approx 0.22$. From this we can estimate the order of the moment of inertia of the pulley, which value is unknown and will be determined by solving the variational problem.

Considering The above assessment, the value of $\frac{mR}{(J + mR^2)}\gamma\dot{\alpha}^2$ first term of equation (2) is limited to values of $\frac{mR}{(J + mR^2)}(l_0 + R\varphi_{\max})|\dot{\alpha}_{cp}|^2$. In the case where the value is of the order of 10^{-3} system of equations (2) can be linearized. Then the moment of inertia of the pulley must satisfy

$mR\left[2(l_0+R\varphi_{\max})|\dot{\alpha}_{cp}|^2\times 10^3-R\right]\approx J$. After substituting $l_0\approx R$, $R\varphi_{\max}\approx 15$, get $J\approx 240mR^2$.

The linearized system of equations. The linearized in the variables α and $\dot{\alpha}$ equations (2) has the form

$$\begin{cases}\ddot{\gamma}+\left(\dfrac{\mu_2}{(J+mR^2)}\dot{\gamma}^2+\sqrt{\dfrac{2\pi R}{g}}\dfrac{\mu_1}{(J+mR^2)}\dot{\gamma}+\dfrac{2\pi R}{g}\dfrac{\mu_0}{(J+mR^2)}\right)-\dfrac{2\pi mR^2}{(J+mR^2)}=0\\ \ddot{\alpha}+\dfrac{2R\dot{\gamma}\dot{\alpha}}{\gamma}+\dfrac{2\pi R\alpha}{\gamma}=0,\ \gamma\neq 0\end{cases}\tag{4}$$

The first equation of this system does not depend on α and $\dot{\alpha}$. Let us refer to: $\dfrac{\mu_2}{(J+mR^2)}=a_2\geq 0$; $\sqrt{\dfrac{2\pi R}{g}}\dfrac{\mu_1}{(J+mR^2)}=a_1\geq 0$; $\dfrac{2\pi R}{g}\dfrac{\mu_0}{(J+mR^2)}=a_0\geq 0$; $\dfrac{2\pi mR^2}{(J+mR^2)}=b\geq 0$; $\dot{\gamma}=y$. Then the first equation takes the form of

$$\begin{cases}\dot{y}+a_2y^2+a_1y+a_0-b=0\\ \dot{\gamma}=y\end{cases}\tag{5}$$

It is clear that the dynamic equilibrium of the system is possible with $\dot{y}=0$. From the first equation (5) it follows that this equation is satisfied for $y_{1,2}=\dfrac{-a_1\pm\sqrt{a_1^2-4a_2(a_0-b)}}{2a_2}$. If $a_0>\dfrac{a_1^2}{4a_2}+b$, then the system does not have a point of dynamic equilibrium and $|y|$ increases indefinitely. This means that the velocity of rotation of the pulley will increase indefinitely. Therefore, you must perform the opposite condition $a_0\leq\dfrac{a_1^2}{4a_2}+b$. Further, if you take the coefficients of friction and the moment of inertia of the pulley such that will satisfy the condition $b\leq a_0\leq\dfrac{a_1^2}{4a_2}+b$, then $\left|\sqrt{a_1^2-4a_2(a_0-b)}<\left|\sqrt{a_1^2}\right|\right|=a_1$. In this case both have a negative balance point coordinates y_1 and y_2. From a physical standpoint, this means that under the influence of gravity and frictional force load will be raised, not lowered. Therefore, the coefficients of friction and moment of inertia of the drum must necessarily satisfy the condition $a_0<b$.

Then $\left|\sqrt{a_1^2-4a_2(a_0-b)}\right| \geq \left|\sqrt{a_1^2}\right| = a_1$ and $y_1 = \dfrac{\left|\sqrt{a_1^2-4a_2(a_0-b)}\right|-a_1}{2a_2} \geq 0$, $y_1 = \dfrac{-\left|\sqrt{a_1^2-4a_2(a_0-b)}\right|-a_1}{2a_2} \leq 0$. In the (y,t) find the area of the positive and negative acceleration $\dot{y}$. If $\dot{y}=c\neq 0$ we have $a_2y^2+a_1y+a_0-b=-c$, and therefore $y=\dfrac{-a_1\pm\sqrt{a_1^2-4a_2(a_0+c-b)}}{2a_2}$. At the points of dynamic equilibrium $c=b-a>0$. Let $y>y_1$.

Then $\left|\sqrt{a_1^2-4a_2(a_0+c-b)}\right| > \left|\sqrt{a_1^2-4a_2(a_0-b)}\right|$, which implies $(a_0+c-b)>(a_0-b)$ and $c<0$. That is, above the point y_1 arranged isocline negative acceleration $\dot{y}$. A similar analysis at $y<y_2$ reveals that the lower point y_2 are also isocline negative acceleration $\dot{y}$. When $y_1<y<y_2$, $\left|\sqrt{a_1^2-4a_2(a_0+c-b)}\right| < \left|\sqrt{a_1^2-4a_2(a_0-b)}\right|$ which implies $(a_0+c-b)<(a_0-b)$ and $c>0$. Consequently, the point y_1 is a stable dynamic equilibrium of the system, as can be seen in Figure 5.

The solution of system (5) to be found in the class of functions monotonically converging to the point y_1 and satisfying the initial conditions of the problem.

Let

$$\alpha\beta = -\left(a_0\alpha\cdot sh^2(\beta t)+a_1\alpha\cdot sh(\beta t)ch(\beta t)+(a_0-b)\cdot ch^2(\beta t)\right)=$$
$$=-\frac{1}{4}\left(a_0\alpha\left(e^{2\beta t}+e^{-2\beta t}-2\right)+a_1\alpha\left(e^{2\beta t}-e^{-2\beta t}\right)+(a_0-b)\cdot\left(e^{2\beta t}+e^{-2\beta t}+2\right)\right).$$

$y=\alpha\cdot th(\beta t)$.

Then $\dot{y}=\dfrac{\alpha\beta}{ch^2(\beta t)}=-\left(a_0\alpha\dfrac{sh^2(\beta t)}{ch^2(\beta t)}+a_1\alpha\dfrac{sh(\beta t)ch(\beta t)}{ch^2(\beta t)}+(a_0-b)\right)$,

where

$$\alpha\beta = -\left(a_0\alpha\cdot sh^2(\beta t)+a_1\alpha\cdot sh(\beta t)ch(\beta t)+(a_0-b)\cdot ch^2(\beta t)\right)=$$
$$=-\frac{1}{4}\left(a_0\alpha\left(e^{2\beta t}+e^{-2\beta t}-2\right)+a_1\alpha\left(e^{2\beta t}-e^{-2\beta t}\right)+(a_0-b)\cdot\left(e^{2\beta t}+e^{-2\beta t}+2\right)\right).$$

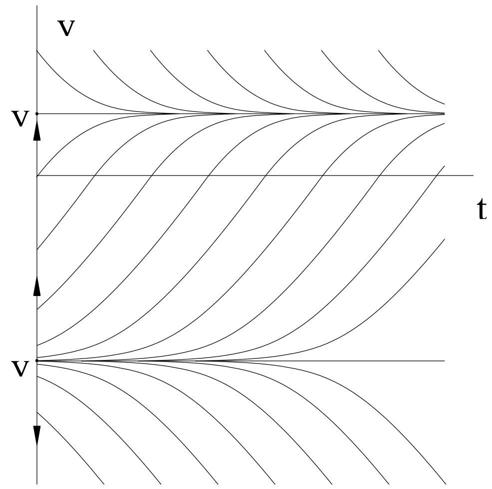

Fig. 5

When $t=0$ $\dot{y}_0=(b-a_0)=\alpha\beta$, while $t\to\infty$ $\dot{y}\to o$, $y\to\alpha$. Using equation $y_{1,2}=\dfrac{-a_1\pm\sqrt{a_1^2-4a_2(a_0-b+\dot{y})}}{2a_2}$, justified above, we obtain $\alpha_{1,2}=\dfrac{-a_1\pm\sqrt{a_1^2-4a_2(a_0-b)}}{2a_2}$ and therefore, $\beta_{1,2}=\dfrac{a_0-b}{\alpha_{1,2}}$. Thus we find a function that satisfies the system (5) with given initial conditions. Given that $y=\dot{\gamma}=\dot{\varphi}$, we can write this function in the generalized coordinates of the problem:

$$\dot{\varphi}(J,\mu_0,\mu_1,\mu_2)=\frac{\sqrt{a_1^2-4a_2(a_0-b)}-a_1}{2a_2}th\left(\frac{2(b-a_0)a_2t}{\sqrt{a_1^2-4a_2(a_0-b)}-a_1}\right)-$$
$$-\frac{\sqrt{a_1^2-4a_2(a_0-b)}+a_1}{2a_2}th\left(-\frac{2(b-a_0)a_2t}{\sqrt{a_1^2-4a_2(a_0-b)}+a_1}\right);$$

$$\ddot{\varphi}(J,\mu_0,\mu_1,\mu_2)=-\left[\frac{(b-a_0)}{ch^2\left(\dfrac{2(b-a_0)a_2t}{\sqrt{a_1^2-4a_2(a_0-b)}-a_1}\right)}-\right.$$
$$\left.-\frac{(b-a_0)}{ch^2\left(-\dfrac{2(b-a_0)a_2t}{\sqrt{a_1^2-4a_2(a_0-b)}+a_1}\right)}\right].$$

Returning to the set at the beginning of the article the problem of finding the coefficients of friction, functional variational problem can be written as

$$\begin{cases} F_1(J,\mu_0,\mu_1,\mu_2) = \sqrt{\int_0^T (\dot{\varphi}(J,\mu_0,\mu_1,\mu_2,t) - \dot{\varphi}_э(t))^2\, dt}; \\ F_2(J,\mu_0,\mu_1,\mu_2) = \sqrt{\int_0^T (\ddot{\varphi}(J,\mu_0,\mu_1,\mu_2,t) - \ddot{\varphi}_э(t))^2\, dt}, \end{cases} \quad (7)$$

where $\dot{\varphi}_э(t)$ and $\ddot{\varphi}_э(t)$ derived from the processing of experimental data. Conditions of minimum values $F_1(J,\mu_0,\mu_1,\mu_2)$ and $F_2(J,\mu_0,\mu_1,\mu_2)$

$$\begin{cases} grad\, F_n = 0; \ \left\| \dfrac{\partial^2 F_n}{\partial x_i \partial x_j} \right\| > 0, \\ n = \overline{1,2}; \ x_i = \overline{J,\mu_0,\mu_1,\mu_2}. \end{cases} \quad (8)$$

Verification of the data obtained from the solution of the system (8) is made by comparing the calculated and experimental data. Below is a graphs of the experimental and calculated curves of velocity and acceleration of the rotation of the pulley, the system simulates the towing system operation of three options for pulling loads.

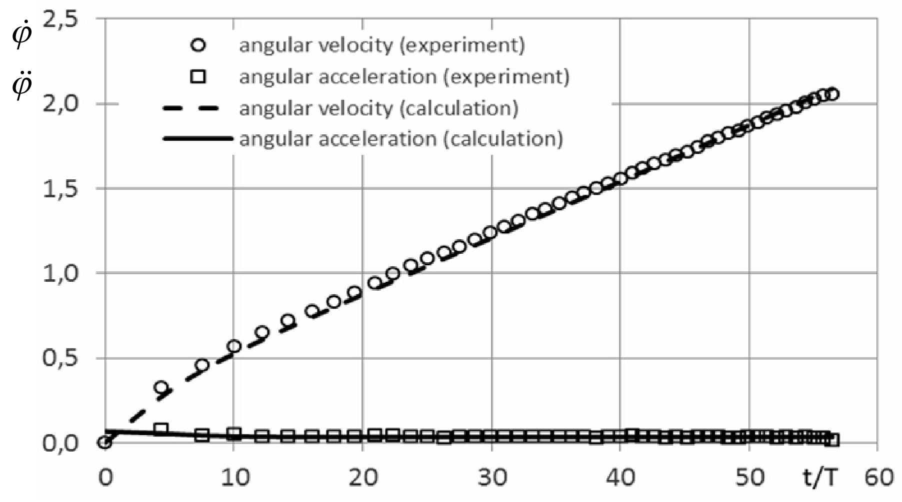

Fig. 6. Comparison of the calculated and experimental data, m = 0.158 kg

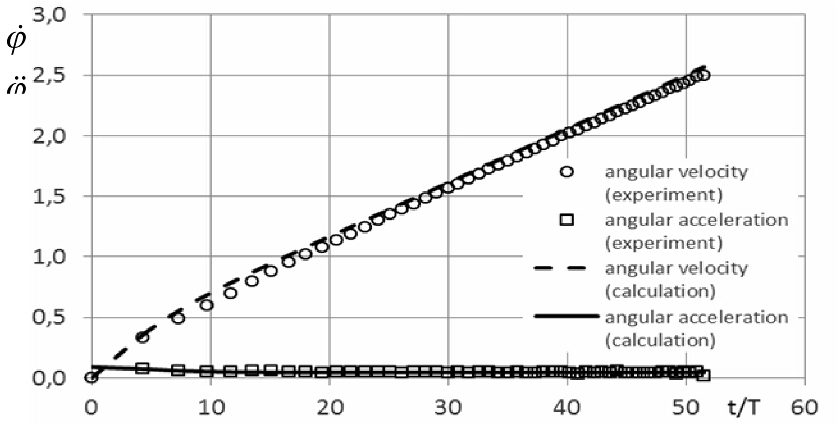

Fig. 7. Comparison of the calculated and experimental data, m = 0.208 kg

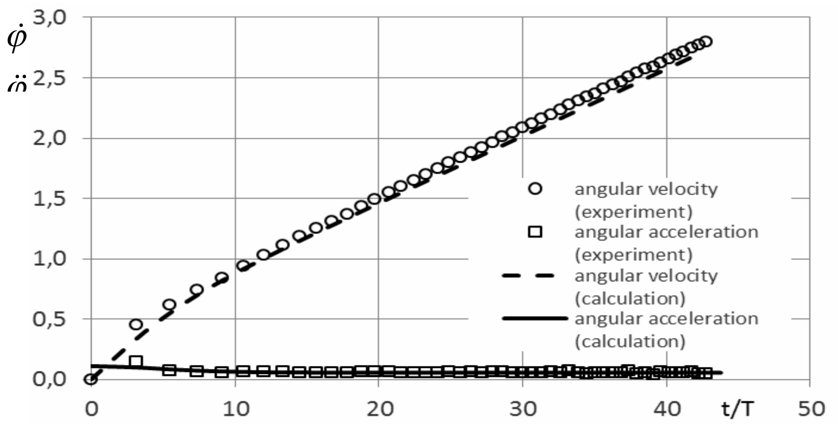

Fig. 8. Comparison of the calculated and experimental data, m = 0.258 kg

The developed method was tested during the model experiment. Friction coefficients were obtained towing system: $\mu_0 = 3 \cdot 10^{-3}$ Nm, $\mu_1 = 9{,}6 \cdot 10^{-2}$ Nms, $\mu_2 = 2 \cdot 10^{-4}$ Nms^2, and frictional forces arising therein during model experimental studies defined (see Table 1).

Table 1

The velocity of main rope *V*, m/s	Friction moment *Mfr*, Nm	*P = Mfr/gR*, kg
0.570	0.0578	0.034
0.685	0.0688	0.040
0.858	0.0855	0.050
0.958	0.0951	0.055
1.034	0.1025	0.060
1.088	0.1077	0.063

It was established that in the range of speeds of 1.0 m/s, the friction moment *Mfr* equal to the product *R.g* by weight, equal to 58 g. According to the previously used method the friction moment compensated by the moment of gravity of suspension for towing loads, with a mass equal to 58.5 g.

Conclusions. The method of determination of the coefficients of friction forces in the towing system was developed. It allows to obtain an analytical representation of these forces depending on the velocity of the pulling load.

It was found that the results of calculations of the frictional forces of the old and new method are in good agreement. At the same time, the new method allows to determine the friction force at any speed range cargo.

In addition, the proposed method requires less input information and allows to avoid using non-friction phenomena manifested in the areas of braking system − effect related to the change of coordinates the point of suspension load in the final phase of the movement.

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