УДК 629.113

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HEAVE AND PITCH MOTIONS OF A TANKER IN SHALLOW WATER BY STRIP THEORY, ASYMPTOTIC AND THREE-DIMENSIONAL DIFFRACTION ANALYSES

Hydrodynamic and kinematic properties of heave and pitch motions of a tanker in shallow water without forward speed were calculated by two methods: a classical strip theory and a three-dimensional diffraction analysis. Three water depths were considered with the depth to draft ratios of 2.0, 1.5 and 1.3. Comparison shows significant differences between the two groups of results with the strip theory incapable of accurately capturing the three-dimensional effects of the shallow water flow, especially the increase in added masses. Further comparison with the earlier work of Y.L. Vorobyov indicates that this deficiency of the strip theory approach could be resolved by applying the method of matched asymptotic expansions. Calculations of heave and pitch motions of by the strip theory method with the «corrected» added masses produced results which were very close to those obtained by the three-dimensional diffraction analysis.

Keywords: *ship motions, shallow water, strip theory, three-dimensional diffraction analysis.*

Гидродинамические и кинематические характеристики продольной качки танкера без хода на мелководье рассчитаны двумя методами: классическим методом плоских сечений и трехмерным дифракционным методом. Рассмотрены три глубины водоема, соответствующие отношениям глубины к осадке 2.0, 1.5 и 1.3. Сравнение показывает значительную разницу между двумя группами результатов, основная причина которых заключается в неспособности метода плоских сечений описать трехмерные эффекты мелководья с достаточной точностью, и в особенности, увеличение присоединенных масс. Сопоставление с более ранними результатами Ю.Л. Воробьева показывает, что этот недостаток метода плоских сечений может быть преодолен методом сращиваемых асимптотических разложений. Расчет вертикальной и килевой качки по методу плоских сечений с использованием «правильных» присоединенных масс дает результаты весьма близкие к полученным по трехмерному дифракционному расчету.

Ключевые слова: *качка, мелководье, метод плоских сечений, трехмерная дифракция.*

Гідродинамічні і кінематичні характеристики подовжньої хитавиці танкера без ходу на мілководді розраховані двома методами: класич-ним методом пласких перерізів і тривимірним дифракційним методом.

Розглянуті три глибини водойму які відповідають відношенням глибини до осідання 2.0, 1.5 і 1.3. Порівняння показує значну різницю між двома групами результатів, головна причина яких полягає в нездатності методу плоских перерізів описати тривимірні ефекти мілководдя з достатньою точністю, і особливо, збільшення приєднаних мас. Зіставлення з більш ранніми результатами Ю.Л. Воробйова вказує що цей недолік методу плоских перерізів може бути здоланий методом асимптотичних розкладань, що зрощуються. Розрахунок вертикальної і кільової хитавиці по методу плоских перерізів з використанням «правильних» приєднаних мас дає результати дуже близькі до отриманих по тривимірному дифракційному розрахунку.

Ключові слова: *хитавиця, мілководдя, метод плоских перерізів, тривимірна дифракція.*

Study objective. The objective of this study is to investigate the heave and pitch motions of a ship in shallow water using comparative analysis by several methods. The linear equations of ship motions with respect to six unknown displacements $X_i(t)$ in the frequency domain are known to be of the form [4; 5; 7]

$$\left\{-\sigma^2\left(M + A(\sigma)\right) + i\sigma\left(B(\sigma) + B_V(\sigma)\right) + C\right\} \cdot X(\sigma) = F(\sigma) \qquad (1)$$

Here M is the mass matrix of the ship; A denotes the generalized added masses, B and B_V are coefficients of the radiation and viscous damping; C are coefficients of restoring forces and F are the exciting forces. All coefficients of the equation (1) are matrices 6 x 6 and functions of the wave frequency σ. Solution of the system (1) in the form of a complex vector of six transfer functions $X(\sigma)$ defines the response amplitude and phase operators of motions. For a ship symmetrical about its centre plane, the system of equations (1) breaks into the two independent groups of equations describing motions in the longitudinal and transverse planes.

In this study, the attention is focused on ship motions in the longitudinal plane, primarily on its heave and pitch in shallow water without forward speed. The analysis was performed by the two main methods: the strip theory (ST) method and the boundary element method (BEM). Comparison of the hydrodynamic properties and response amplitude operators (RAO) computed by the two methods and their assessment against the results obtained by Y.L. Vorobyov [3] by the matched asymptotic expansion method explains the differences between the results and demonstrates the extent of the shallow water hydrodynamic effects on ship motions.

Review of the analysis methods. Methods for the prediction of the hydrodynamic properties of ship motions (added masses, damping coefficients and exciting forces) are described in a large number of studies. Only a brief review of the methods used in this study is given below.

Strip Theory. Strip theory, widely adopted in ship hydrodynamics over many years, is based on the assumption that the fluid flow past any cross-section of a slender ship hull is two-dimensional. The strip theory method, which was essentially developed in 1950s-70s, continues to be used till present days in the form of computer programs for seakeeping predictions. Good exposition of the method and further references can be found, for example, in texts [4; 5; 7]. The strip theory method was also applied for the prediction of hydrodynamic characteristics of ships in shallow water ([1; 2; 9]); however the vali-dity of such results, especially in the case of small water depths, has not been commonly accepted. At the same time, the strip theory applications for the analysis of ship motions and wave loads in shallow water are still attracting interest [11; 13].

In this study, the analysis of ship motions was performed by using the hydrodynamic coefficients of ship sections of Lewis form in shallow water, which were computed by Kokhanov [1] and presented in the catalogue [2]. The computation followed the Ursell method, in which the oscillations of a two-dimensional ship section were modelled by a series of pulsating singularities placed at the origin in the free surface: source, doublets and multipoles of increasing order. Potentials of these singularities satisfy boundary conditions on the free surface, on the bottom and the radiation condition at infinity. To determine the strengths of these singularities, the kinematic boundary condition on the section contour must be met at a number of points, thereby reducing the problem to a system of linear equations. For the purpose of this study, the hydrodynamic and kinematic characteristics of ship motions were predicted using a computer program developed by Kokhanov. In this program, the diffraction components of the exciting forces are computed using a known approach based on the hypothesis of relative velocities and «accelerations», which can be found in [5] and in several other texts.

Boundary Element Method. Application of the boundary element method (BEM) to ship hydrodynamics enables three-dimensional problems of wave diffraction and radiation to be solved for arbitrary ship forms in deep or shallow water. In the BEM approach, the hull surface is modelled by the distribution of pulsating sources. The potential of a unit source (Green function) satisfies boundary conditions on the free surface, on the bottom and the radiation condition at infinity. To determine the strength of the source distribution, the kinematic boundary condition on the hull surface must be met, which reduces the problem to an integral equation and further to a system of linear equations. BEM based programs are now commonly used for motion analyses of ships and offshore structures. Subject to correct modelling, results obtained by this method usually correlate well with model tests, and they can be consi-dered as the most accurate.

In this study, the analysis of ship motions was performed by using the program WADAM, part of the program package SESAM [12] developed by

DNV. The program component that performs the hydrodynamic computations is essentially identical to the code WAMIT, a program developed at MIT under the guidance of Newman [8], which is widely used in offshore industry.

Asymptotic Methods. The reasons for the limited applicability of the strip theory to shallow water problems were explained in works of Tuck [6], Newman [7] and several works of Vorobyov, which were consolidated in his book [3]. As opposed to adopting an intuitive assumption of two-dimensional flow past ship sections, these studies employed systematically the slender body theory and the method of matched asymptotic expansions (MAE). The strip theory was demonstrated to be rigorously applicable for high frequency oscillations only whereas for low frequencies and in shallow water the three-dimensional flow effects may be significant. Works of Vorobyov [3] and several other researches (refer to [10], for example) aimed at the development of a unified analysis method that would be applicable to ship motions in shallow water.

In this study, the results obtained earlier by Vorobyov and presented in his book [3] were used for comparison. Being independent of the main two numerical methods, these results enable a qualitative estimate to be made of the hydrodynamic shallow water effects and their influence on ship motions.

Input Data. Motion analyses were performed for a tanker of AFRAMAX size whose particulars are presented in Table 1. The partial loading condition is assumed with the draft of 15.0 m and zero trim. The original lines plan was used for the development of geometrical models for the two methods: hull cross sections and the three-dimensional panel model. The analyses were performed for three water depths, which are given in Table 2.

Table 1

General Particulars of the Vessel

Description	*Notation*	*Dimension*	*Value*
Length between perpendiculars	*Lpp*	*m*	*232.00*
Beam	*B*	*m*	*41.60*
Depth	*H*	*m*	*23.50*
Draft	*T*	*m*	*15.00*
Volume displacement	*V*	m^3	*115920*
Block coefficient	*Cb*	--	*0.801*

Table 2

Absolute and relative water depths

Description	Absolute depth	Relative depth
Water depth #1	H = 19.24 m	H/T = 1.28 = 1.3
Water depth #2	H = 22.20 m	
Water depth #3	H = 29.60 m	H/T = 1.48 = 1.5

Hydrodynamic characteristics of motions. The coefficients of added mass A_{33}, A_{55} and radiation damping B_{33}, B_{55} for heave and pitch motions were computed and presented in the following non-dimensional form

$$a_{33} = \frac{A_{33}}{\rho V} \quad ; \quad a_{55} = \frac{A_{55}}{\rho V L_C^2} \quad ; \tag{2}$$

$$b_{33} = \frac{B_{33}}{\rho V \sigma} \quad ; \quad b_{55} = \frac{B_{55}}{\rho V L_C^2 \sigma} \quad . \tag{3}$$

Here $\rho = 1025$ kg/m^3 is density of sea water; V is volume displacement of the vessel, m^3; L_C = 100.00 m is the reference length.

Heave motions. Coefficients of heave added mass and radiation damping computed by the strip theory (ST) and three-dimensional diffraction (3D) methods are presented in Figures 1 and 2. Both methods show increase in the added mass with reduction in relative water depth. However there are obvious discrepancies between the two groups of results.

The three-dimensional diffraction analysis predicts significant increase of the added mass when water depth reduces: for example, reduction of the water depth from H/T = 2.0 to H/T = 1.3 leads to about twofold increase of the added mass, whereas the increase predicted by the strip theory is more moderate. As a result, the difference between the added masses predicted by the two methods for the relative depth H/T = 1.3 is of the order of 100 % over the full frequency range considered. The three-dimensional diffraction analysis also shows significant increase of the added mass when the motion frequency reduces. This corresponds to a known fact that the heave added mass of a three-dimensional body in shallow water increases logarithmically when the oscillation frequency approaches zero [6]. Calculations by the strip theory, however, show weak dependence of the added mass on frequency; the added mass tends apparently to some finite limit, similar to that for a two-dimensional contour in shallow water.

For the heave damping coefficient, the correlation between the two methods is generally better than that for the added mass, especially for the water depth H/T = 2.0. However, the discrepancies persist when the water depth reduces, in particular for the frequency range 0.35-0.70 1/s and for low frequencies (less than 0.30 1/s), where the differences are seen to build up when the water depth reduces.

The added mass and damping coefficients computed by the two methods become close when the oscillation frequency increases, as they tend to some limit, possibly common, for the infinitely large frequency. Such behaviour is apparent for water depths H/T = 1.5 and 2.0.

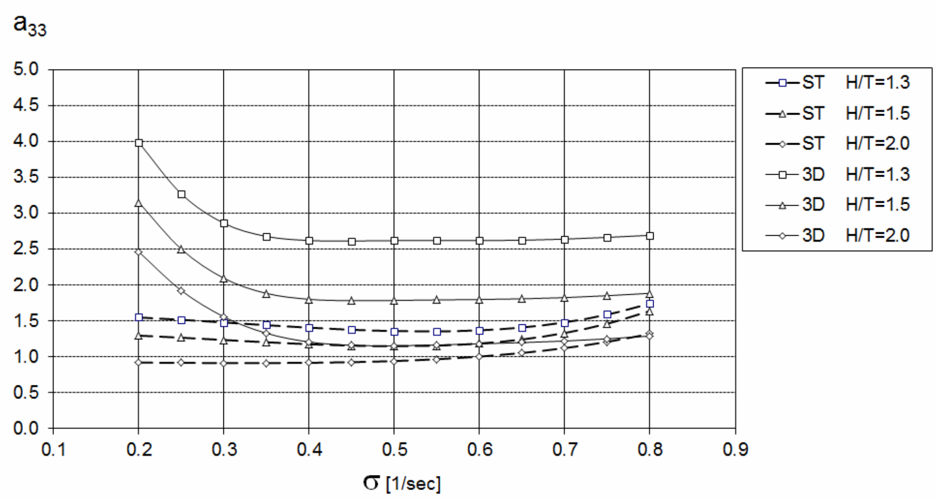

Fig. 1. Heave added mass (a_{33})

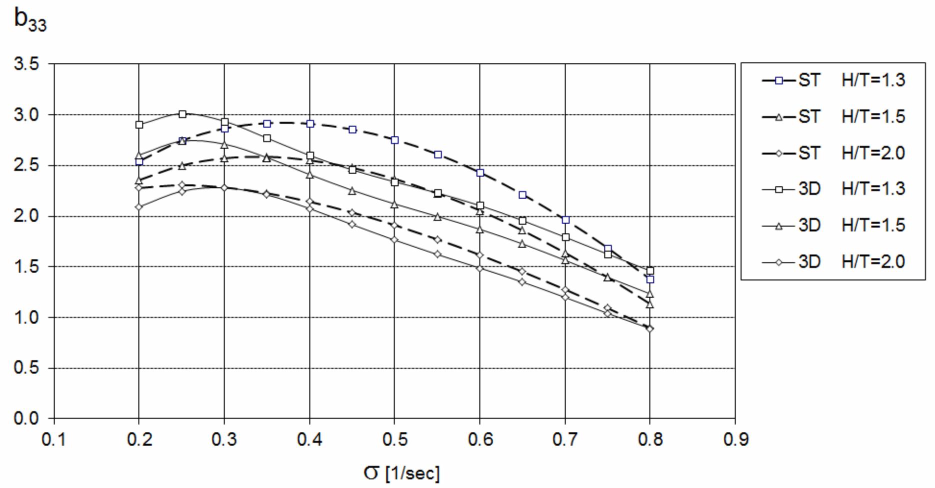

Fig. 2. Heave radiation damping (b_{33})

Pitch Motions. Coefficients of pitch added mass and radiation damping computed by the strip theory and three-dimensional diffraction methods are presented in Figures 3 and 4.

These results are qualitatively similar to those obtained for heave motion, including significant increase in the added mass for smaller water depths and discrepancies observed between the two analysis methods. Similar to heave motion, both methods predict qualitatively different behaviour of the added mass in the low frequency range, the differences for pitch hydrodynamic characteristics being even more pronounced than those for heave. When the frequency increases, the agreement between the two methods also appears to improve.

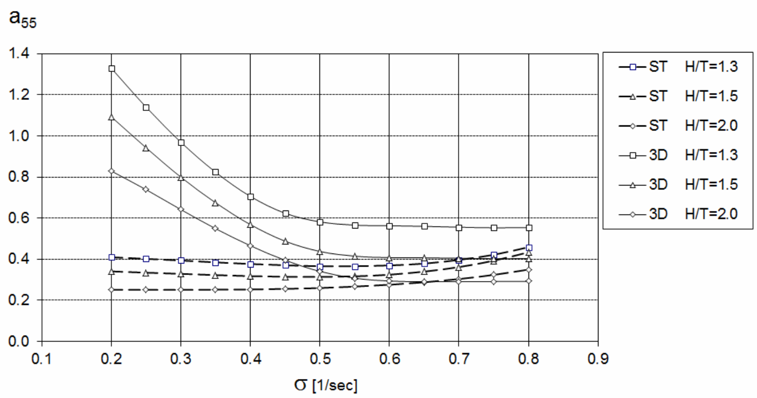

Fig. 3. Pitch added mass (a_{55})

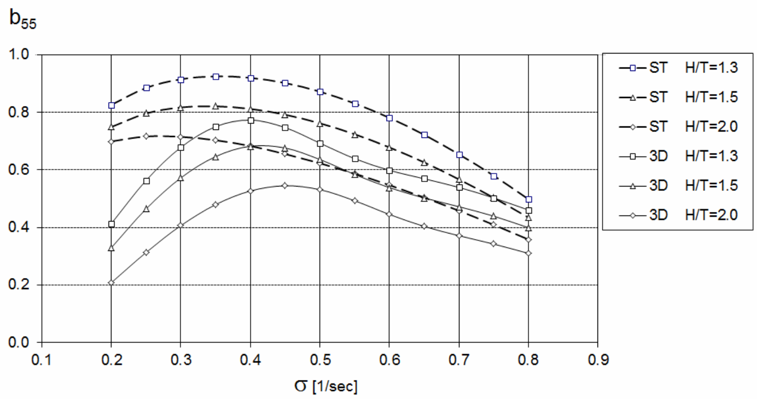

Fig. 4. Pitch radiation damping (b_{55})

Comparison with asymptotic methods. The reasons for the discrepancies observed between the strip theory and BEM numerical results, in particular in the values of the added masses, can be explained by comparing them with the results obtained by the method of matched asymptotic expansions (MAE).

Figure 5 and 6 compare the computed heave added mass of the tanker with the earlier results obtained by Vorobyov [3] for a ship of 60-series with the block coefficient 0.80. Instead of pressure integration over the hull surface, as it is usually done in standard algorithms, Vorobyov computed the added masses via the radiation damping coefficients by applying Kramers-Kronig's relations

$$A(\sigma) = A(\infty) + \frac{2}{\pi} \cdot \int_0^{\infty} \frac{B(x) - B(\infty)}{x^2 - \sigma^2} dx \,. \tag{4}$$

The added mass at the infinitely high frequency $A(\infty)$ was determined by a separate strip theory analysis, which was justified for a slender body in this limiting case. The damping coefficients for any frequency $B(\sigma)$ were determined by using «thin hull» or «slender» hull theoretical models, which enabled the three-dimensional wave effects in shallow water to be described analytically.

Because of the differences between the hull forms of ships considered in this study and in [3], one can't expect close agreement between the results. However, it is obvious from Figures 5 and 6 that the agreement between the earlier MAE results and those by the three-dimensional diffraction analysis in this study is in fact very good. It is also apparent that MAE «thin» and «slender» hull models produced results which were close to each other. Similar conclusion can be drawn by comparing results for the pitch added mass, which are shown in Figure 7 for the water depth H/T = 1.3; comparison for the water depth H/T = 1.5 (not presented here) also supports these findings.

Figure 5 shows also the estimate of the added mass according to the asymptotic solution of Tuck [6], which was obtained by MAE for motions of a slender ship in shallow water under the assumption of low wave frequencies. Calculations by simple formulae [6] show that the added mass increases markedly for small frequencies and tends theoretically to infinity. Tuck's solution captures correctly the low frequency limit, where the three-dimensional wave interaction is especially important, but becomes quickly unusable for moderate and high frequencies of motion.

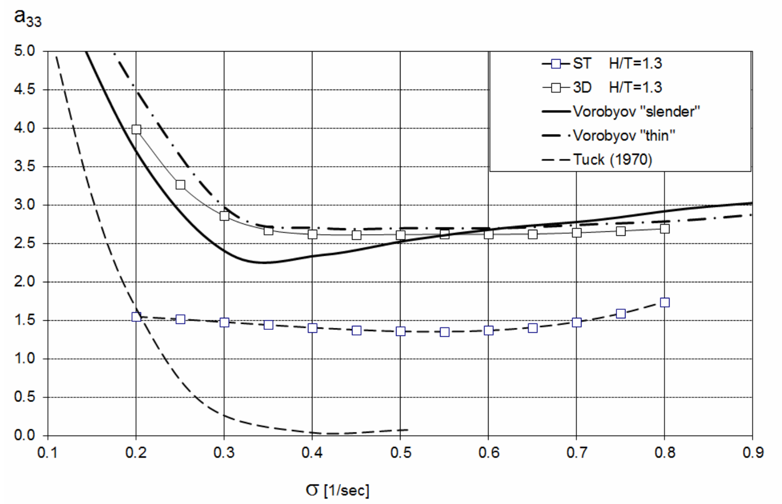

Fig. 5. Heave added mass (a_{33}). Water depth H/T = 1.30

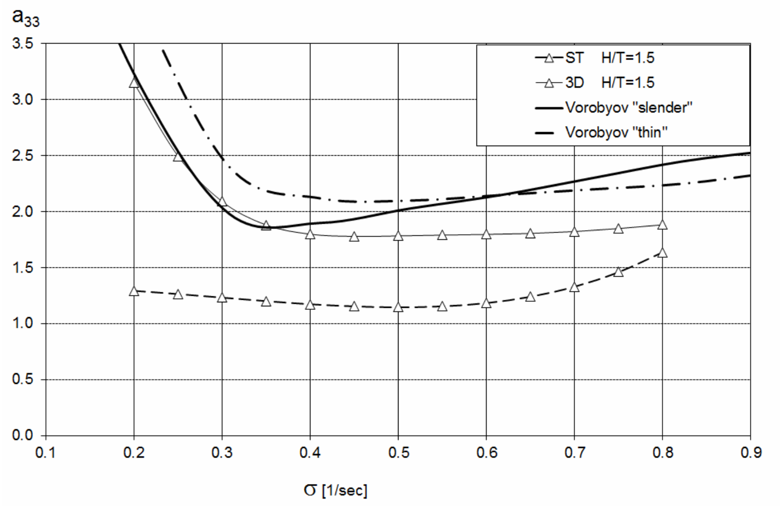

Fig. 6. Heave added mass (a_{33}). Water depth H/T = 1.50

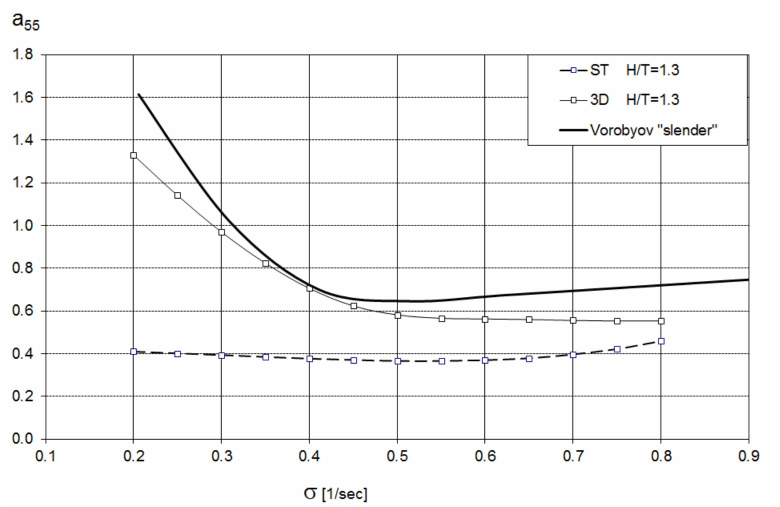

Fig. 7. Pitch added mass (a_{55}). Water depth H/T = 1.30

Response amplitude operators. Response amplitude operators (RAO) of heave and pitch motions computed by the strip theory and the three-dimensional diffraction methods are presented in Figures 8 and 9, 10 and 11, and 12 and 13, for the relative water depths H/T = 2.0, 1.5 and 1.3, respectively. The analyses were performed for the incidence angles of 150 and 180 degrees, the latter corresponding to head seas.

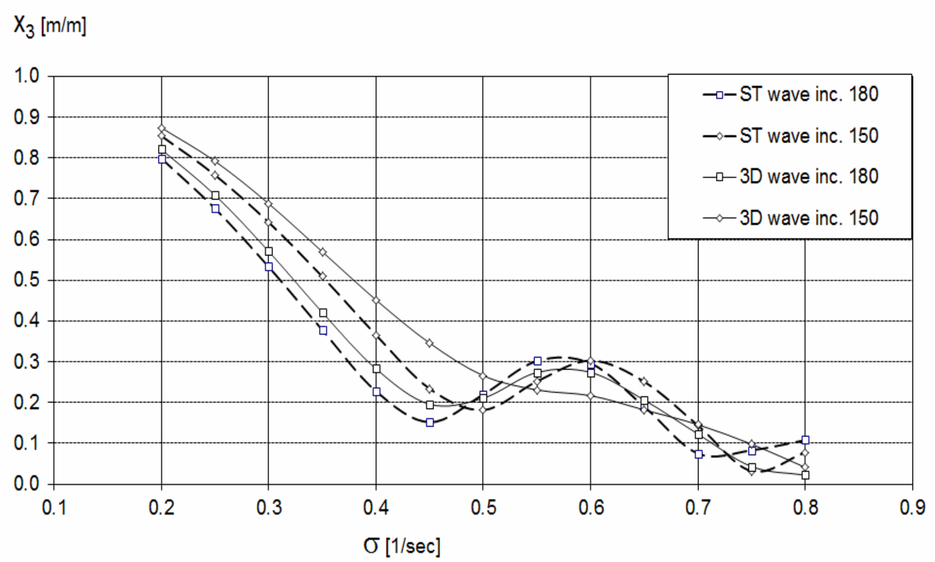

Рис. 8. Heave RAO (X_3) in shallow water H/T = 2.0

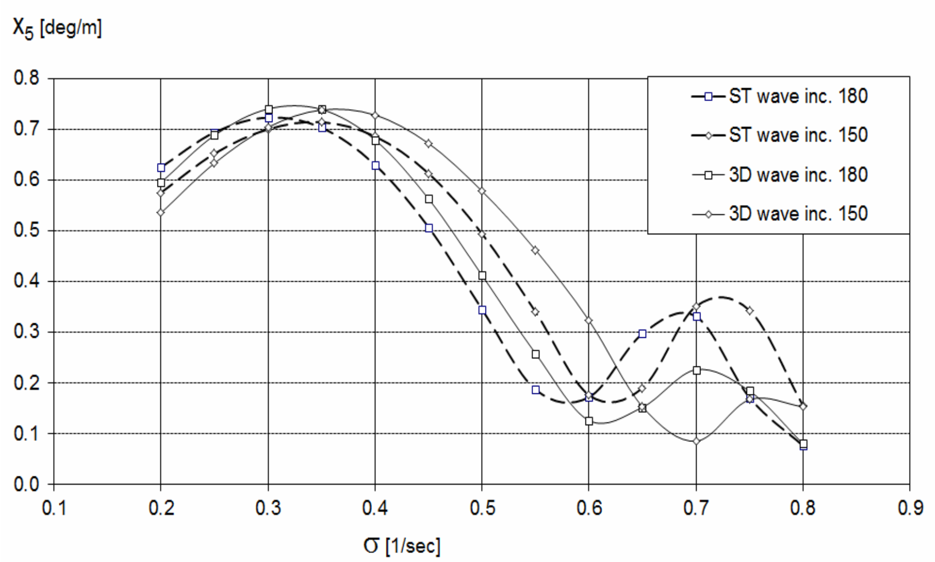

Fig. 9. Pitch RAO (X_5) in shallow water H/T = 2.0

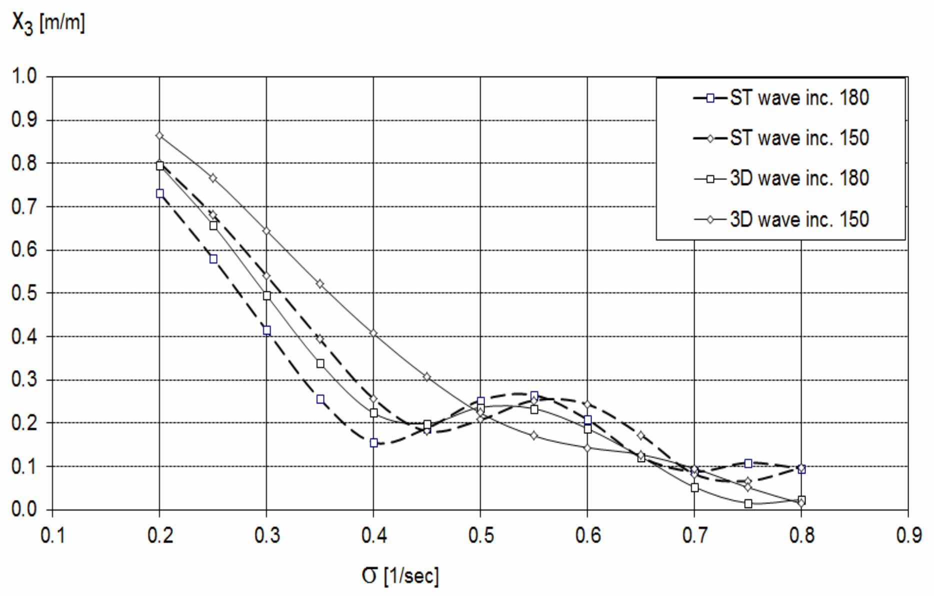

Fig. 10. Heave RAO (X_3) in shallow water H/T = 1.5

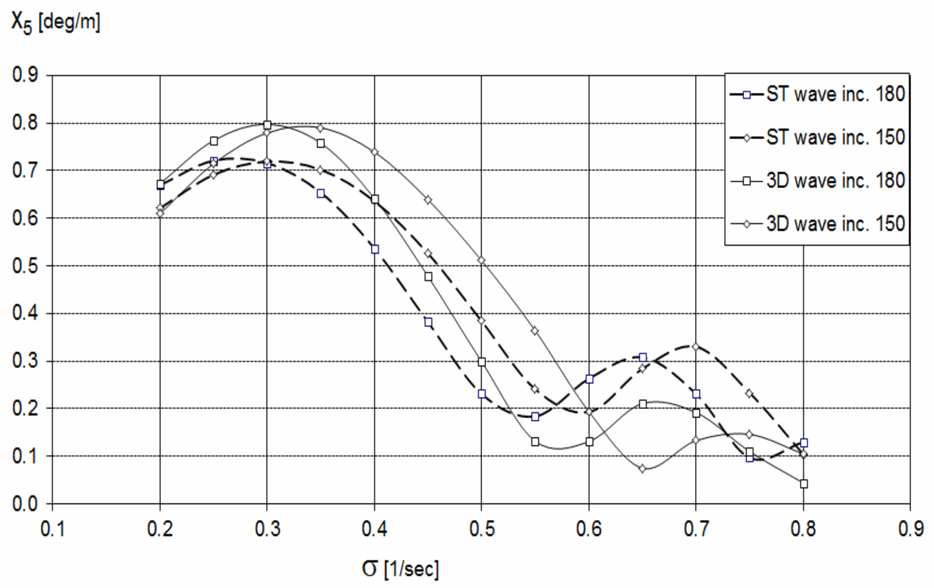

Fig. 11. Pitch RAO (X_5) in shallow water H/T = 1.5

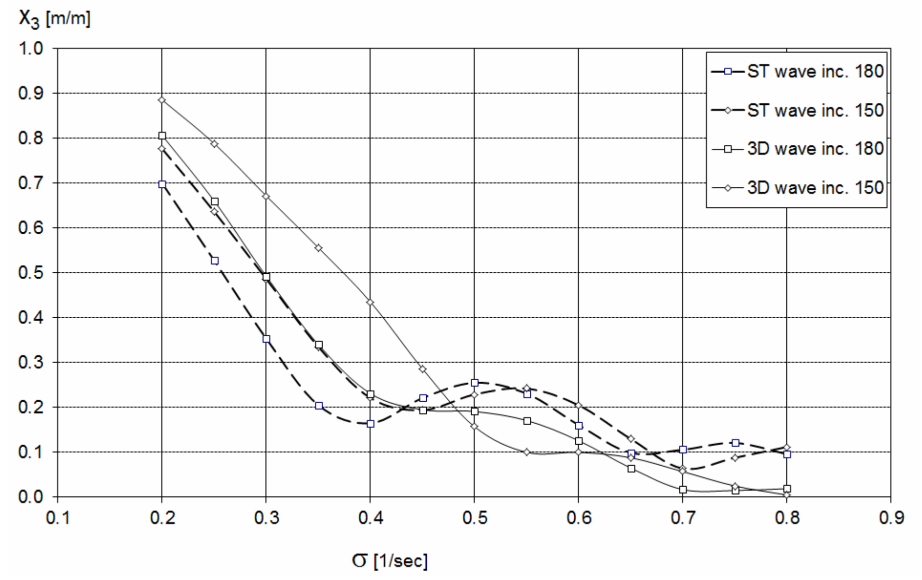

Fig. 12. Heave RAO (X_3) in shallow water H/T = 1.3

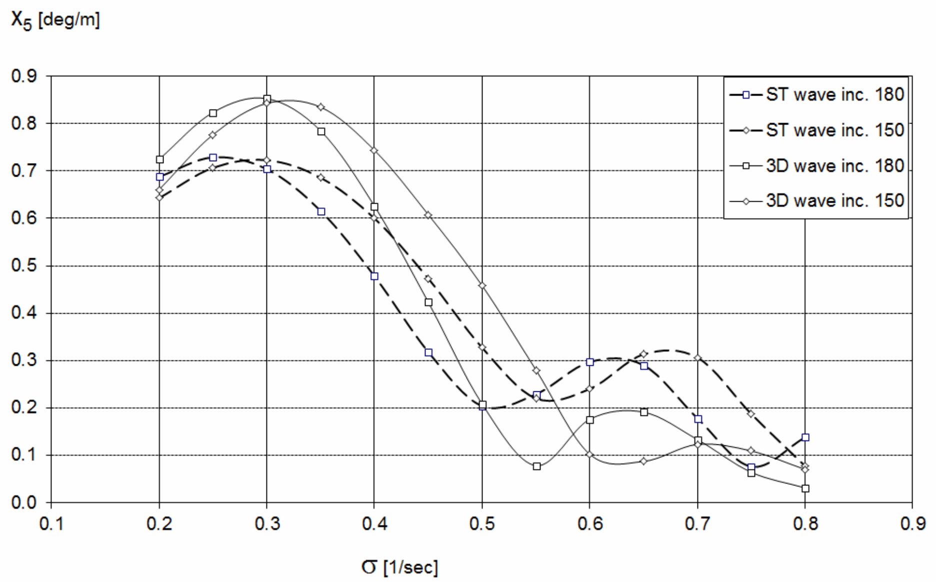

Fig. 13. Pitch RAO (X_5) in shallow water H/T = 1.3

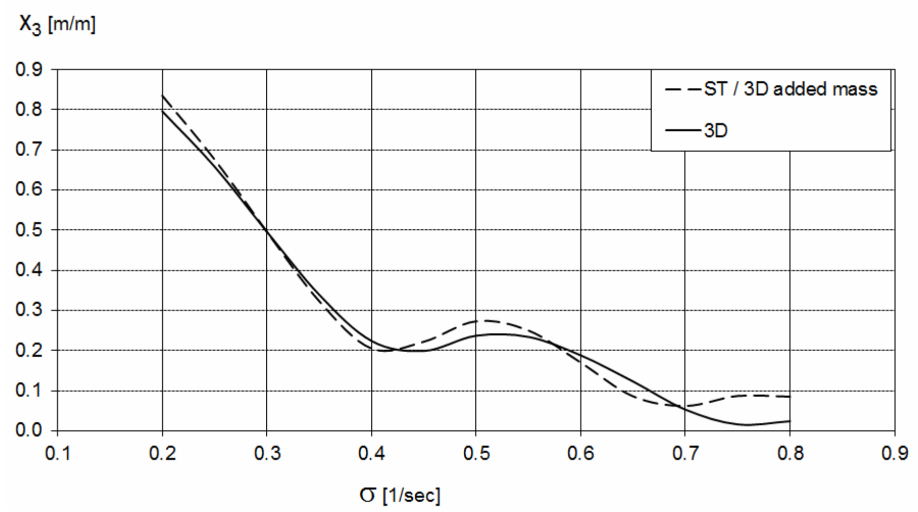

Fig. 14. Heave RAO (X_3) in shallow water H/T = 1.5 in head seas by combined method: strip theory with added masses according to the three-dimensional diffraction method

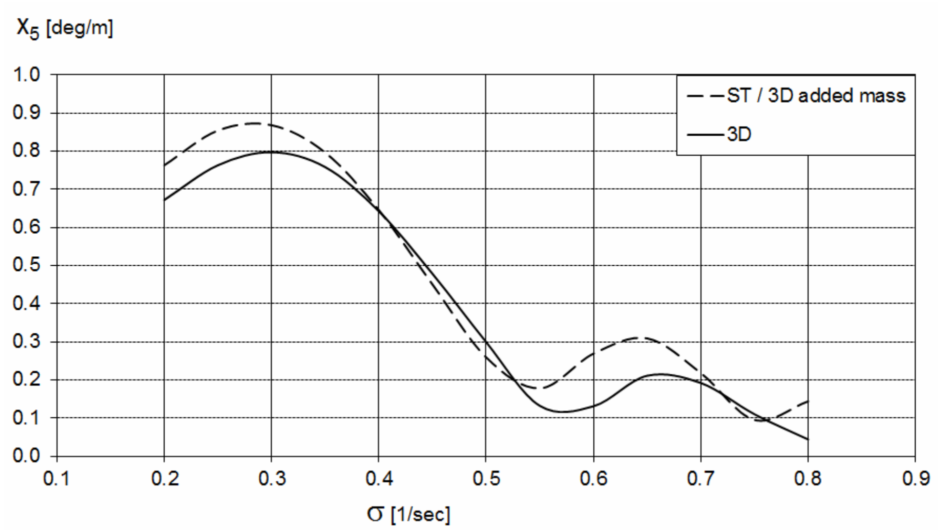

Fig. 15. Pitch RAO (X_5) in shallow water H/T = 1.5 in head seas by combined method: strip theory with added masses according to the three-dimensional diffraction method

Generally, the qualitative behaviour of the two sets of RAOs is similar. However, the differences are seen to increase as the water depth reduces. For the water depth H/T = 2.0 in head seas, the difference between the two RAO groups is estimated at about 10 %-20 % for wave frequencies of 0.35-0.70 1/s.

However, this difference increases to 50 % and more for the water depth H/T = 1.3. For the wave incidence angle of 150 degrees the differences between the two RAOs are seen to be considerably higher than those for head seas.

The discrepancies between the RAOs computed by the two methods are obviously related to differences in hydrodynamic properties. As the previous comparison shown, these differences were the largest for the added mass coefficients. Therefore, as a numerical experiment, additional RAOs were computed by the combined method whereby the added masses were taken from the three-dimensional diffraction analysis while the remaining hydrodynamic characteristics (damping coefficients and exciting forces) were still calculated by the strip theory method. The results of this combined method presented in Figures 14 and 15 for head sea and the water depth H/T = 1.5 show very good agreement for heave and reasonable agreement for pitch motions.

This qualitatively confirms the conjecture that the shallow water effect on ship motions is the most substantial through its influence on the added masses, at least in head sea case, when the diffraction components of wave exciting forces are reasonably small.

Conclusions. Findings of this study indicate that the strip theory method, which is used for seakeeping predictions till present days, can't be applied for the analysis of heave and pitch motions of ships in shallow water in its original form. This is especially true for relatively small depths («substantial shallow water» according to terminology introduced by Y.L Vorobyov) and for low and moderate frequencies of motions, which fall in the range of wave regimes frequently encountered in practice. In such conditions, the influence of the three-dimensional flow effects becomes significant even for hull forms which are deemed to be well represented by the slender body model.

The effects of shallow water on the added masses (and, perhaps, diffraction components of the wave exciting forces at oblique wave angles) are especially pronounced. In relatively small water depths and for low wave frequencies, the strip theory method does not describe the increase in the added mass with sufficient accuracy. This may lead to incorrect predictions of motion amplitudes with un-conservative errors (underestimated motions), especially for water depths less than H/T = 2.0. When the correct added masses are included, the difference between the «corrected» strip theory and the three-dimensional BEM method diminishes and becomes insignificant, at least for head sea condition.

These conclusions don't intend to diminish the known advantages of the strip theory and opportunities for its development, especially for solving non-linear problems of seakeeping and prediction of wave loads.

Acknowledgement. The authors thank Matthew White, naval architect of INTECSEA Pty Ltd, for performing motion analyses using the program WADAM.

REFERENCES

1. *Kokhanov E.V. Hydrodynamic Characteristics of Motions of Ship Sections in a Single-layer and Two-layer Fluid. PhD thesis. − Odessa: Institute of Marine Engineers, 1977 (in Russian).*
2. *Kokhanov E.V. Catalogue of Added Mass and Damping Coefficients of Ship Sections in Shallow Water. − Odessa: Institute of Marine Engineers, 1977. http://catalogue.nla.gov.au/Record/6303254*
3. *Vorobyov Y.L. Ship Hydrodynamics of a Restricted Waterway. SPb.: Sudostroenie, 1992 (in Russian).*
4. *Haskind M.D. Hydrodynamic Theory of Ship Motions. −M.: Nauka, 1973 (in Russian).*
5. *Remez Y.V. Ship Motions. − Leningrad: Sudostroenie, 1983 (in Russian).*
6. *Tuck E.O. Ship Motions in Shallow Water // Journal of Ship research. − December. − 1970.*
7. *Newman J.N. The Theory of Ship Motions, Advances in Applied // Mechanics. − 1978. − Volume 18. − P. 221-283.*
8. *Lee C.-H. WAMIT Theory Manual. − Massachusetts Institute of Technology, 1995*
9. *Andersen P., Wuzhou H. On the Calculation of Two-Dimensional Added Mass and Damping Coefficients by Simple Green's Function technique // Ocean Engineering. − Vol. 12. − №. 5. − P. 425-451, 1985.*
10. *Kim Y. Sclavounos. A Finite-Depth Unified theory for the Linear and Second-Order Problems of Slender Ships // Journal of Ship Research. − Vol. 42. − Dec. 1998. − P. 297-309.*
11. *Purinovic J.V., Jensen J.J. Wave Loads on Ships Sailing in restricted water depth // Marine Structures. − Vol. 16. − 2003. − 469-485.*
12. *Det Norske Veritas (DNV). SESAM User Manual // WADAM Wave Analysis by Diffraction and Morison Theory. − January 2. − 2010.*
13. *Gourlay T., Von Graefe A., Shigunov V., Lataire E. Comparison of AQWA, GL Rankine, Moses, Octopus, PDSTRIP and WAMIT with Model Test Results for Cargo Ship Wave-Induced Motions in Shallow Water. Proceedings of the ASME 2015 34th international Conference on Ocean, offshore and Arctic Engineering, OMAE2015-41691.*

Стаття надійшла до редакції 11.11.2015