ситуаций; фильтрационная модель и модели диффузионного заражения предложено использовать для изучения состояния информированности общества о путях решения проблем и для их привлечения к процессу управления; модель клеточного автомата предложено использовать, как обратную связь, на этапе мониторинга удовлетворенности граждан состоянием управления территорией. Сделан вывод о возможности использования указанных моделей на различных этапах концепции.
Ключевые слова: поведенческие модели, самоорганизация общества, информационное пространство, самоуправление территории, информация.

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# THE CONCEPT OF BEHAVIORAL MODELS USE TO ANALYZE THE EFFECT OF INFORMATION'S INFLUENCE ON THE PROCESSES OF SELF-ORGANIZATION IN THE TERRITORIAL COMMUNITY 


#### Abstract

Summary The article realizes the concept of the analysis of the development and involving processes of self-organization of the population to the management of socio-economic development of the territory. The concept generalizes the processes of information interaction of "government $\leftrightarrow$ society" and allows to draw conclusions about the mutual influence of these components of public administration in the territorial communities of the cities. The management of the territory development is presented as cooperation in the work of the two components of local governance: government and civil society bodies that represent the views and interests of the society. Considerable attention is paid to information exchange between branches of government. The stages of awareness of the problems of its origin and significance, the size of the problem, the solutions occur in the informational space by information exchange between stakeholders in solving social actors, this exchange of information may take several iterations of interaction. To explore the current condition and predict future development of the territorial community, at various stages of interaction, in this paper we propose the use of behavioral models, namely the model of Granovetter on the stage for the identification of the problem; the Axelrod's culture model at the stage of forecasting of development of problem situations; the cooperation model of John von Neumann at the stage of decision making about alternative solutions to problem situations; filtration model and the diffusion model of contamination proposed to be used for analyzing the state of awareness of the ways of solving problems and for their involvement in the management process; cellular automaton model is proposed to use, as feedback, on the stage of monitoring of satisfaction of citizens by the state government of the territory. Key words: behavioral models, self-organization of society, information space, government areas, information.


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## VALUE-AT-RISK AS A PRINCIPAL METHODOLOGY OF RISK ESTIMATION

The paper examines the current and historic literature to give an overview of various methodologies employed to determine Value-at-Risk. The estimation methods Value-at Risk employed to determine the significance of the fundamental variables $\mathrm{P} / \mathrm{E}$ ratio and Debt/Equity ratio with regards to the Value-at-Risk model are given. Key words: systematically important financial institution (SIFI), Price-to-Earnings (P/E), Debt/Equity, Value-at-Risk (VaR), ranking procedure,financial risk management.

Introduction. One of the main tasks for the researchers in the field of Finance remains the stock returns predictability. Obviously, there is no surprise in it, because a great portion of the academic research focuses on practicality of the findings. In finance the concept on capitalisation of knowledge is nothing else but its essence. And that is precisely why the practicality and accuracy of the modelling are such a key aspects.

Value-at Risk is a tool that is employed primarily in financial risk management as it can be classified as a risk measure of the risk of a loss occurring in a given timeframe on a specific portfolio of financial assets. All of the including globally operating banks, insurance companies and any other financial institution that enjoys the global interconnectivity to the extent that its failure could trigger a financial crises are indeed employing VaR as a risk measurement tool in order to quantify the risk exposure to
the market. This fact illustrates the extraordinary significance of the VaR model. Yet, despite the significance of the VaR tool and popularity amongst financial practitioner the level of sophistication to construct the VaR is far from high.

Aim. The aim is to determine if there is a measurable relation between the two aforementioned fundamentals and equity risk measured by Value-at-Risk.

Literature review. In fact, the most employed method to determine the VaR is through historic simulation and Risk Matrics which gives raise to limitations of the real world applicability of the tool. It is hence the attempt of many academics to estimate the VaR with a continuously increasing level of sophistication, where most of the risk is explained in an autoregressive manner such as the GARCH model. However, recently the stream of research is devoted to a more structural approach in which mar-
ket and firm specific factors are incorporated into the VaR model and this paper will be tied to this stream by attempting to expose the functionality of fundamental values such as $\mathrm{P} / \mathrm{E}$ ratio and Debt/ Equity Leverage ratio in the VaR model. Research conducted by Banz [1], Stattman [25], Rosengerg et al. [23] and Fama and French [14] has yielded that firm specific and market related variables can help explain the expected stock returns.

Some academics consider the mean returns predictability like Cochrane [10], others work on the volatility predictability, for instance, Dichev and Tang [12], however all of them, basically, work directly or indirectly with the problem of returns distribution predictability as mean and variance are only the moments of the distribution [19]. It is worth mentioning for practicality purposes that the mean returns and volatility, as one of the measures of risk, go hand in hand in the eyes of practitioners. In other words it is extremely important to be able to predict, at least a good part of the variation, of both of the variables. The reasoning behind it is that practitioners will always want to understand the risk/return ratio. Thus, the lack of robust forecasts for any of the two decreases the value of the prediction immensely.

This piece of academic work is focused on adding value to the risk predictability part of this research stream. Therefore, it is important to highlight the prime risk measures that might be used for the purposes of predicting the equity returns. The first approach is to go for the most straightforward measure of risk that is variance or standard deviation, which are directly incorporated into the returns distribution. The other approach that appeared in the academic literature is to create more complex measures of risk that may give better predictability options. Probably, there was a good reason for the approach to appear, for example, it may be the case that the variance/standard deviation models were not progressing in terms of the predictability. One of those measures is Value-at-Risk (VaR) that stands in line with other sophistications, such as: Expected Shortfall, which is a conditional VaR model well presented in the work by Rockafellar and Uryasev [22], Tail Conditional Expectation or Tail VaR, which was described in the paper by Barges, Cassette and Marceau [2], Entropic Risk Measures, which are based upon the risk aversion coefficients and touched upon by Rudloff, Sass and Wunderlich [24], etc. Furthermore, it is useful to mention, that the Basel Committee of Banking Supervision [3] imposed a regulatory requirement (contained in Basel II Accord) for financial institutions to use VaR figures and even designed a specific "three colour scheme" to test them.

Cristoffersen describes in detail several VaR models as well as specifies the main advantages of using VaR, such as the ease in calculation and applicability to market conditions [9]. The market conditions are also explicitly explained in his research, they are: almost no conditional mean predictability on the daily basis, the variance of daily returns greatly exceeds the mean, daily returns are not normally distributed, time-varying correlation between assets, etc. It is vital to understand that for any VaR model, which basically estimates the distribution quantiles, the estimation of the returns distribution is needed. The only exception is the regression quantile method highlighted by Chernozhukov and Umatsev [7] and Engle and Manganelli [13], where they estimate the quantile directly. The rest of the models do actually
estimate the returns distribution. Those approaches may be classified as follows: Historical Simulation introduced by Boudoukh et al. [5], which estimates the empirical quantile from the empirical returns distribution based on past data; Fully parametric methods, which model the complete returns distribution, such as RiskMetrics [21] and GARCH from Bollerslev [4]; Semi-parametric models, which incorporate the empirical distribution partly and the rest is modelled using the parametric tools, for example, the Extreme Value Theory (EVT) by McNeil and Frey [18]. On top of that various types of filtering may be used in the modelling, for instance, Filtered Historical Simulation (FHS) with the residual filtering using a GARCH type model or the filtered parametric VaR used by Chavez-Demoulin et al. [6].

Furthermore, the researchescreated a substantial support for the firm specific (or fundamental) and market specific variables are important to predict the equity returns distribution. From this point onwards academics created another sub-stream of research by introducing fundamental and market variables into the VaR models.

Consequently, there is a need for further research of the fundamental and market variables power with respect to better VaR estimations. To the best of our knowledge the fundamental and market variables were not assessed with respect to the VaR models performance. Thus, there is a substantial field for further research in this area.

## Value-atRiskMethodology.

## 1. Portfolio formation

The first step in estimating the value of such fundamental variables as the $\mathrm{P} / \mathrm{E}$ ratio and the leverage ratio for the calculation of VaR is to form the appropriate portfolios. These portfolios are made in a way to compare the fit of a VaR model for the overall portfolio and for a sorted portfolio with high and low $\mathrm{P} / \mathrm{E}$ or high and low leverage ratio stock. That procedure is called back testing and it will be described in further sections. Therefore, a good approach towards back testing may be dividing a ranked portfolio into percentile parts and testing the VaR model across the parts and the overall portfolio separately. The main issue at this stage is the ranking methodology. In this study two different ranking methodologies are applied to benefit from the pros of both of them. The rankings are: percentile ranking and simple (usual) ranking.

## 2. Ranking procedure

The ranking procedure is applied in this academic work to actually distinguish between the high and low $\mathrm{P} / \mathrm{E}$ ratio or high and low leverage ratio stocks following by the formation of separate portfolios accordingly. The portfolios are created for the top and bottom $25 \%$ quartiles $\mathrm{P} / \mathrm{E}$ and leverage ratio firms to represent a high and low $\mathrm{P} / \mathrm{E}$ and leverage portfolios, respectively.

## a. Simple ranking

The simple ranking methodology is based upon sorting the stocks according to the fundamental variable value assigning the highest rank (1) to the stock with the highest fundamental variable value and the lowest rank (e.g. 101) to the stock with the lowest one. This process is carried out daily, which results in every stock in every time $t$ getting the rank according to its fundamental variable value at that time in comparison with the same values of other stocks in the portfolio at the same time t .

## b. Percentile ranking

In the percentile ranking the values for the fundamental variable ( $\mathrm{P} / \mathrm{E}$ or leverage ratio) are cal-
culated for every stock at each point in time first. Then every stock is assigned a percentage quartile, in which its fundamental variable is across the time t using the formula below:

$$
\begin{equation*}
\% \operatorname{Rank}_{t}^{s}=\frac{F_{t}^{s}-F_{t}^{\min }}{F_{t}^{\max }-F_{t}^{\min }} * 100 \tag{1}
\end{equation*}
$$

where, the $\% \operatorname{Rank}_{t}^{s}$ is, essentially, the percentile rank for the stock $s$ at time $t$, the $F_{t}^{s}$ is the value of the fundamental variable for the stock $s$ at time $t$ and $F_{t}^{\text {max }} / F_{t}^{\text {min }}$ are the maximum and minimum values of the fundamental variable of all the stocks at time t, respectively. That basically estimates in which percentage quartile across all the stocks in the portfolio lies the fundamental variable value for each individual time $t$.

## 3. VaR estimation methods used

The Value-at-Risk itself, which is usually specified as $100 \alpha \% \mathrm{VaR}$, stands for the negative of the quartile of probability $1-\alpha$ of the returns distribution. The values of the probability $\alpha$ may vary between 0 and $100 \%$, however, usually they are in the range from $95 \%$ to $99 \%$. In some special cases it can also be set to $99.9 \%$, for example, as the Basel II Accord requires for operational risk. Thus, the $100 \alpha \% \mathrm{VaR}$ for the period $\mathrm{t}+\mathrm{k}$ conditionally on the information set $\Theta \mathrm{t}$, that is available at time t , is estimated as:

$$
\begin{gather*}
\operatorname{VaR}_{t+k}^{\alpha}=-Q_{1-\alpha}\left(R_{t+k} \mid \Theta_{t}\right)= \\
-\inf \left\{r \in \quad P\left(R_{t+k} \leq r \mid \Theta_{t}\right) \geq 1-\alpha\right\} \tag{2}
\end{gather*}
$$

where $\mathrm{Q}_{a}()$ represents the quantile of probability $\alpha, R_{t}$ stands for the random variable of returns at time $\mathrm{t}, \Theta_{\mathrm{t}}$ is the information set available at time t . The whole VaR estimation methodology is about estimating the appropriate quantile of an unknown returns distribution. Furthermore, as it can be seen from the formula, VaR is an absolute term that represents effectively the most possible amount one will loose on a portfolio with a certain probability.

Generally, the returns are assumed to be a location scale process conditional on the set of available information at time $t$ as it is shown in the next formula:

$$
\begin{equation*}
r_{t+k}=E\left(R_{t+k} \mid \Theta_{t}\right)+\varepsilon_{t+k}=\mu_{t+k}+\sigma_{t+k} z_{t+k} \tag{3}
\end{equation*}
$$

where $\mu_{\mathrm{t}+\mathrm{k}}$ and $\sigma_{\mathrm{t}+\mathrm{k}}$ are the expected return and the conditional scale, respectively, for the period $\mathrm{t}+\mathrm{k}$ given the information set $\Theta_{\mathrm{t}}$ available at time $\mathrm{t}, \varepsilon_{t+k}$ is an error term and $\mathrm{z}_{\mathrm{t}+\mathrm{k}}$ has a unit scale, zero location and a probability density function $\mathrm{F}_{\mathrm{z}}(\mathrm{O}$. The $100 \alpha \% \operatorname{VaR}$ forecast for the time $\mathrm{t}+\mathrm{k}$ conditional on the information set available at time t may be estimated using the formula below:

$$
\begin{equation*}
V a R_{t+k}^{\alpha}=-\left(\mu_{t+k}+\sigma_{t+k} Q_{1-\alpha}(Z)\right) \tag{4}
\end{equation*}
$$

where $Q_{a}$ is defined as the quantile of the probability density function $\mathrm{F}_{z}()$.

The main differences that vary throughout the VaR models are the specifications for the three parameters: conditional location, conditional scale and the probability density function or $\mu_{t+k}, \sigma_{t+k}$ and $\mathrm{F}_{\mathrm{z}}()$, respectively.

## 4. Historical Simulation VaR model

The most basic method to use for VaR estimation is the Historical Simulation model. It incorporates the usage of the empirical quantiles of the returns distribution. According to Kuester et. al [16] it is defined as naive historical simulation. The author justifies the employability of the method by assuming the stationarity of returns distribution, what implies that the empirical distribution is a consistent estimator of the unobserved future distribution.

Firstly, it is important to obtain the ordered statistics, which is just an ordered sample of returns. If we consider the sample of returns ( $r_{t}, r_{t-1}, \ldots, r_{t-\infty+1}$ ), the ordered statistic will be $\left(r_{(1)}, r_{(2)}, \ldots, r_{(9)}\right)$, where $\mathrm{r}_{(1)} \leq \mathrm{r}_{(2)} \leq \ldots . \leq \mathrm{r}_{(0)}$. Then the historical simulation $100 \alpha \% \mathrm{VaR}$ estimator for period $\mathrm{t}+1$ is defined as:
$V \widehat{a R_{t+1}^{\alpha}}=-\widehat{Q_{1-\alpha}}\left(r_{t} r_{t-1}, \ldots, r_{t-\omega+1}\right)=-r_{([(1-\alpha) * \omega])}$, (5)
where the expression in the square brackets stands for the integer part of the real number. For instance, if the sample size $\omega=100$, the $90 \%$ VaR estimate is the negative of the $10^{\text {th }}$ sample statistic: $V a R_{t+1}^{0.9}=-r_{(10)}$.

The simplicity of the historical simulation method brings up concerns about its performance. Further analysis of the weaknesses of this model and possible improvements was made by Pritsker [20].
5. Unconditional Parametric VaR model

The Unconditional Parametric VaR is a fully parametric method. That means that it assumes a family of probability distributions $\mathrm{F}_{\mathrm{z}}(\mathrm{z})$ of $\mathrm{z}_{\mathrm{t}}$ in Eq. (2). The model is called unconditional because it makes the assumption that the expected return and the scale are not changing along the timeframe: $\mu_{t+\mathrm{k}} \equiv \mu$ and $\sigma_{t+\mathrm{k}} \equiv \sigma$. Consequently, the probability density function of the returns is $F_{z}\left(\sigma^{-1}\left(r_{t}-\mu\right)\right)$. Therefore, the unconditional estimator of the $100 \alpha \% \mathrm{VaR}$ at time $t$ takes the values of:

$$
\begin{equation*}
V \widehat{a R_{t+1}^{\alpha}}=-\left(\mu+\sigma \widehat{Q_{1-\alpha}}(Z)\right), \tag{6}
\end{equation*}
$$

In other words the distribution of Z is adjusted for location and scale followed by the VaR estimation as the $1-\alpha$ quantile of that distribution. The quantile of z is estimated as follows:

$$
\begin{equation*}
\widehat{Q_{1-\alpha}}(z)=\mathrm{F}_{Z}^{-1}(1-\alpha), \tag{7}
\end{equation*}
$$

where $F_{z}$ stands for the cumulative distribution function corresponding to the $\mathrm{F}_{\mathrm{z}}$. Usually a continuous distribution function is used for $F_{z}$ corresponding to the family of $F_{z}$, however if $F_{Z}$ is not a continuous function the generalized inverse function may be used instead of the $\mathrm{F}_{\mathrm{z}}{ }^{-1}$. The distribution of Z is assumed to be Normal or Student-t in this piece of academic work. A lot of researchers such as Mandelbrot (2005) argue in favour of non-symmetric returns distribution. They state that the returns distribution has a mean close to zero, negative skewness and excess kurtosis. Regardless, in this study we shall account for kurtosis and skewness using a symmetric distribution with the Modified VaR model later on, what will allow us to deal with this kind of counterargument.

## 6. Modified VaR model

The Modified VaR (MVaR) is a method developed by Favre and Galeano (2002) on the basis of the normal VaR method. This method includes the property to adjust the risk, measured by volatility, for skewness and kurtosis of the empirical returns distribution, which gives the advantage of dealing with non-symmetric distributions.

Modified VaR is considered to be an extension to a simple unconditional parametric model. The formula for calculating MVaR is as follows:

$$
\begin{gather*}
M V a R_{t}=\mu+\left[z_{t}+\frac{1}{6}\left(z_{t}^{2}-1\right) S+\right. \\
\left.+\frac{1}{24}\left(z_{t}^{3}-3 z_{t}\right) K-\frac{1}{36}\left(2 z_{t}^{3}-5 z_{t}\right) S^{2}\right] \sigma, \tag{8}
\end{gather*}
$$

where $S$ and $K$ stand for the skewness and the excess kurtosis of the distribution, $\mathrm{z}_{\mathrm{t}}$ is defined as the distance between the returns and their mean in terms of the number of standard deviations. Nevertheless as the higher moments do in fact exist, the

MVaR model accounts for more of them in comparison to the usual Unconditional Parametric model and that, clearly, gives a much better estimation for the distribution.

## 7. Conditional Parametric VaR model

The Conditional Parametric model is different from an unconditional one because of the time varying location and scale ( $\mu$ and $\sigma$ from Eq. (2).

In this research the Conditional Parametric model considers a mean function of the past information. This function is an ARMA ( $\mathrm{p}, \mathrm{q}$ ) process model as follows:

$$
\begin{equation*}
\mu_{t}=\mu+\sum_{i=1}^{p} \varphi_{i}\left(r_{t-i}-\mu\right)+\sum_{j=1}^{q} \theta_{j} \epsilon_{t-j} \tag{9}
\end{equation*}
$$

where the equations $\phi(z)=1-\phi_{1} z-\ldots-\phi_{\mathrm{p}} \mathrm{z}^{\mathrm{p}}$ and $\Theta(z)=1-\Theta_{1} z-\ldots-\Theta_{q} z^{q}$ do not have common roots as well as neither of the roots is inside the unit circle.

The function for the varying scale parameter used in this academic work follows the GARCH $(\mathrm{r}, \mathrm{s})$ process:

$$
\begin{equation*}
\sigma_{t}^{2}=c_{o}+\sum_{i=1}^{r} c_{i} \epsilon_{t-i}^{2}+\sum_{j=1}^{s} d_{j} \sigma_{t-j}^{2} \tag{10}
\end{equation*}
$$

where $c_{0}>0, c_{i} \geq 0$ and $d_{j} \geq 0$ as it was considered by Bollerslev [4].

## 8. Testing the fit of VaR models

For the purposes of comparison of the different VaR methods used in this paper, the methods are applied to a rolling window of observations of size $\omega$. Consequently, the out-of-sample VaR estimates $\left\{{\left.\widehat{V a R_{t}^{\alpha}}\right\}_{t=\omega+1, \ldots T} \text {, are calculated. The subsequent step is }}\right.$ to measure the quality of those forecasts, what is done by comparing the ex-ante Value-at-Risk forecasts with the ex post realised returns. As it was mentioned before, this procedure is called backtesting. This academic study follows the backtesting procedure described by Christoffersen [8].

Firstly, the number of violations is defined as the number of times when the return loses exceeds the corresponding VaR forecasts. The model is supposed to be a good fit if the difference between the actual portion of losses is lower than the forecasted VaR value and the forecasted VaR probability 1- $\alpha$ is lower.

Secondly, introducing the sequence of VaR forecasts as $\left\{V_{V a R_{t}^{\alpha}}\right\}_{t=\omega+1, \ldots, T}$ and a sequence of realised returns as $\{r\}_{t=0+1, \ldots, T}$, there is a need to calculate the hit sequence $\left\{I_{t}\right\}_{t=\omega+1, \ldots, T}$ of VaR violations, where:

$$
I_{t}= \begin{cases}1 & \text { if } r_{t}<-V a R_{t}  \tag{11}\\ 0 & \text { if } \\ r_{t} \geq-V_{t}\end{cases}
$$

The important assumption made here is that the VaR violations are uniformly distributed and independent throughout the sample. This assumption is made to support the fact that $I_{t}$ will follow the Bernoulli distribution with parameter (1- $\alpha$ ). Moreover, $\alpha$ will vary from $95 \%$ to $99 \%$ in the vast majority of cases depending on the required confidence level.

## 9. Testing the unconditional coverage

The easiest way to test the fit of the VaR model having the hit sequence is to test the difference between number of hits and the required number of hits by the probability ( $1-\alpha$ ) of the VaR forecast model. Basically, it is the unconditional coverage test.

This process starts with denoting the number of ones and zeros in the hit sequence by $\mathrm{T}_{1}$ and $\mathrm{T}_{0}$ respectively. The total number of observations is made equal to T. Following by the estimation of the
violations ratio $\hat{\pi}=T_{1} / T$. Afterwards the likelihood ratio test is:

$$
\begin{equation*}
L R_{U C}=-2 \ln [L(1-\alpha) / L(\hat{\pi})] \tag{12}
\end{equation*}
$$

where $L()$ stands for the likelihood function of an iid Bernoulli sequence. By replacing that with an appropriate function the likelihood ratio test becomes:

$$
\begin{equation*}
L R_{U C}=-2 \ln \left[\frac{\alpha^{T_{0}}(1-\alpha)^{T_{1}}}{\left(1-T_{1} / T\right)^{T_{0}}\left(T_{1} / T\right)^{T_{1}}}\right] \stackrel{a}{\sim} \chi_{1}^{2} \tag{13}
\end{equation*}
$$

which is asymptotically distributed with a chisquare distribution with 1 degree of freedom.
10. Testing the independence of the violation

The problem with unconditional coverage test is that it does not test the frequency of the violations within the period. It means that for a given level of $\alpha$ of a VaR setting the unconditional coverage will just compare the total number of violations with the expected one, but will not go into further details, i.e. how the violations were distributed within the sample. That is an important issue to investigate because of the proven presence of volatility clustering. For example, the occurrence of violations subsequently within a short timeframe delivers a much higher risk to an institution than the same number of violations during a longer timeframe. Consequently, there is a strong requirement to test for the independence of violations or, in other word, to test whether the likelihood of a violation is changing given the information of past violations. Furthermore, if one would take a broader look at the volatility clustering issue it would become clear that this issue brings a huge systemic risk for the regulators to deal with.

The test for the independence of VaR violations follows the methodology employed by Christoffersen [8], where the author assumes, under the dependence of violations, that the hit sequence may be described by a first-order Markov process with transition probability matrix:

$$
\Pi_{1}=\left[\begin{array}{ll}
1-\pi_{01} & \pi_{01}  \tag{14}\\
1-\pi_{11} & \pi_{11}
\end{array}\right]
$$

where $\pi_{01}$ and $\pi_{11}$ are the probabilities that the next observation of the return is going to be a violation given that current observation of the return violated or not.

Introducing the sample size of T , the likelihood function of the first-order Markov process is:

$$
L\left(\Pi_{1}\right)=\left(1-\pi_{01}\right)^{T_{00}} \pi_{01}{ }^{T_{01}}\left(1-\pi_{11}\right)^{T_{10}} \pi_{11}{ }^{T_{11}},(15)
$$

where $\mathrm{T}_{\mathrm{ij}}$ is the number of observations in the hit sequence of a violation, if $i=1$, or with no violation, if $i=0$, that precedes a violation, if $j=1$, or a no violation value, if $\mathrm{j}=0$. Thus, the maximum likelihood estimates of the transition probabilities are as follows:

$$
\begin{equation*}
\hat{\pi}_{01}=\frac{T_{01}}{T_{00}+T_{01}} \text { and } \hat{\pi}_{11}=\frac{T_{11}}{T_{10}+T_{11}} . \tag{16}
\end{equation*}
$$

There is a possibility that for some sample of returns the value of $\mathrm{T}_{11}$ may be zero. This scenario will lead to a likelihood function:

$$
\begin{equation*}
L\left(\widehat{\Pi_{1}}\right)=\left(1-\widehat{\pi_{01}}\right)^{T_{00}} \widehat{\pi_{01}} T_{01}^{T_{01}} \tag{18}
\end{equation*}
$$

The null hypothesis of the test for independence of the violations is $\pi_{01}=\pi_{11}=\pi$, which will give a transition matrix as the following:

$$
\widehat{\Pi}=\left[\begin{array}{ll}
1-\hat{\pi} & \hat{\pi}  \tag{19}\\
1-\hat{\pi} & \hat{\pi}
\end{array}\right]
$$

where $\hat{\pi}=T_{1} / T$ is the estimator for the ratio of violations in the same way it was in the unconditional coverage test. In the scenario of independence the likelihood function is:

$$
\begin{equation*}
L(\widehat{\Pi})=(1-\hat{\pi})^{T_{00}+T_{10}} \hat{\pi}^{T_{01}+T_{11}} \tag{20}
\end{equation*}
$$

Therefore, the likelihood ratio for a null hypothesis $\pi_{01}=\pi_{11}$ is given by:

$$
\begin{equation*}
L R_{\text {ind }}=-2 \ln \left[L(\widehat{\Pi}) / L\left(\widehat{\Pi_{1}}\right)\right] \underset{\sim}{a} \chi_{1}^{2} . \tag{21}
\end{equation*}
$$

In addition, it is worth mentioning that this likelihood ratio has an asymptotic chi-square distribution with 1 degree of freedom.

## 11. Testing the conditional coverage

Both the unconditional coverage test and the test for independence of the violations are important in order to measure the fit of the VaR model. Thus, a joint test should be executed. Following the procedure of Christoffersen [8] to simultaneously test whether the violations occur independently and whether the model predicts them correctly, the null hypothesis for the test is given by: $\pi_{01}=\pi_{11}=1-\alpha$. The academic employs the likelihood ratio test as:

$$
\begin{equation*}
L R_{C C}=-2 \ln \left[L(1-\alpha) / L\left(\widehat{\Pi_{1}}\right)\right] \underset{\sim}{a} \chi_{2}^{2}, \tag{22}
\end{equation*}
$$

which follows the asymptotic chi-square distribution with two degrees of freedom. The value of $\mathrm{L}(1-\alpha)$ are exactly the same as the ones used in the Eq. (12). Using simple transformation it can be shown that:

$$
\begin{equation*}
L R_{C C}=L R_{U C}+L R_{\text {ind }} . \tag{23}
\end{equation*}
$$

The likelihood ratio for the conditional coverage test is nothing more than the sum of the likelihood ratio for the unconditional coverage test plus the likelihood ratio for the independence of the violations test.
12. The three color scheme from Basel II

In the real world there are official classifications of the VaR models. The Basel Committee on Banking

Supervision [3] developed a so-called "three color scheme" concept, where the VaR models that fall into the green zone are acceptable, the yellow zone VaR models are supposed to be disputable and the red zone ones are substantially faulty.

Methodologically, the color zone depends on the number of violations. The green zone contains the VaR models that at a $99 \%$ significance level produce the number of violation that is lower than the $95 \%$ quantile of a binominal distribution with the success chance of 0.01 . The yellow zone calculations are the same but for the number of violations being between the $95 \%$ and $99.99 \%$ quantiles of the same binominal distribution. Lastly, the ones that give the number of violations that falls into higher than $99.99 \%$ quantile of the above mentioned distribution are classified into red zone.

Conclusion. The $100 \alpha \%$ VaR provides a value such that the probability of observing a loss greater than VaR is smaller the confidence level 1- $\alpha$ for a given timeframe, where the time frame can vary from very frequent such of one day for example and up to 10 days in other cases for market risk and up to one year for credit risk or operational risk. In essence the VaR gives an indication of the tails of the profit/loss distribution bell-shaped curve. In probabilistic terms this may seem quite trivial as it is merely the negative of the $1-\alpha$ probability quantile of the returns distribution; however in practice because of this definition the actual estimation of VaR becomes quite sophisticated.According to our findings, introduction of $\mathrm{P} / \mathrm{E}$ and Debt/Equity ratios as the basis for ranking yields further accuracy for the risk estimation.

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## VALUE-AT-RISK ЯК ОСНОВНА МЕТОДОДОГІЯ ОЦІНКИ РИЗИКІВ

## Резюме

У статті представлено короткий огляд методології, яка застосовується для оцінки Value-at Risk. Наведено та проаналізовано методи оцінки Value-at Risk, що використовуються для визначення значущості фундаментальних змінних Price-to-Earnings (P/E) iDebt/Equity по відношенню до моделі Value-at-Risk. Ключові слова: системно-важливий фінансовий інститут (СВФI), Price-to-Earnings (P/E), Debt/Equity, Value-atRisk (VaR), процедура ранжування, фінансовий ризик-менеджмент

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## VALUE-AT-RISK KАК ОСНОВНАЯ МЕТОДОЛОГИЯ ОЦЕНКИ РИСКОВ

## Резюме

В статье представлен краткий обзор методологии, которая применяется для оценки Value-at Risk. Представлены и проанализированы методы оценки Value-at Risk, которые используются для определения значимости фундаментальных переменных Price-to-Earnings (P/E) iDebt/Equity по отношению к модели Value-at Risk.
Ключевые слова: системно-важный финансовый институт (СВФИ), Price-to-Earnings (P/E), Debt/Equity, Value-atRisk (VaR), процедура ранжирования, финансовый риск-менеджмент.

