UDC 539.3

Z. Yu. Zhuravlyova
I. I. Mechnikov Odesa National University

ON A STRESS–STATE OF AN ELASTIC SEMI–STRIP UNDER MECHANICAL AND THERMAL STRESSES

The new methodic of plane elasticity problems' solving for the semi-infinite strip under the mechanical and thermal pressures is considered in the article. The sense of it is the applying of the integral Fourier transformations directly to the Lame's equations. It leads the initial problems to the one-dimensional ones which are solved with the help of the matrix differential calculation and Green's matrix apparatus. The problems' solving is reduced to one integral and one integro-differentional equations with regard to the unknown function of the displacement correspondingly. These equations are solved approximately by the orthogonal scheme. The normal stresses' absolute values are analyzed.
MSC: 74B05, 42A38.
Key words: semi-strip, fixing, first main problem of elasticity, the Green's matrix, Fourier transformation.

Introduction. The plain elasticity problems for a semi-strip have been solved by many authors, this can be explained by the importance of these problems as the model example for different engineering applications. The methods, proposed in works G. V. Kolosov [1], N. L. Muskchelishvili [2], are based on the use of the complex variable functions' theory and Koshy-type integrals. The wide rewiev, dedicated to the integral transformation method, is given in Ya. S. Uflyand's monograph [3]. I. I. Vorovich, V. V. Kopasenko [4] considered the problem on the symmetrically loaded semi-strip fixed by the short edge. The solving is reduced to the Fredholm's integral equation of the first kind with regard to the normal stress at the fixed edge. With the help of the stress function, in [10] the problem was solved for the semi-strip with the free longitudinal sides, when the selfequilibrium loading influences at the short edge. The application of the sin-transformation to the equation relatively a stress function reduced the problem of the semi-strip's strain with the free longitudinal edges and fixed short edge to the infinite system of the linear equations in [11]. Often, the stress function is performed as the combination of the Fourier's integrals and series. The analogical approach is used by C. B. Ling, F. H. Cheng [12], G. Pickett, K. T. S. Jyengar [13] for the problems on a semi-strip. In [14] the problem for a semi-strip with the free lateral edges and short edge loaded by the displacements is considered. It is supposed that the displacements have the form of the polynomials. When the lateral edges of the semi-strip are free and short edge is under the concentrated force, the problem solving can be based on the energetic method, which is proposed by P. Thecaris [15]. The variation method is used for the analogical problem in L. P. Trapeznikov's work [16]. V. G. Suchevan solved the mixed elasticity problem with help of the variation Castigliano method [17]. Another big class of problems on the solving of elasticity problem for a semi-strip is based on the use of Fadle-Papkovich functions [18, 19]. With the aim to avoid the different shortcoming's of the known methods, the new approach, which was proposed by G. Ya. Popov [5],

Received 28.08.2015
© Zhuravlyova Z. Yu., 2015

is used. Accordantly to it, the integral transformations are applied directly to the equilibrium equations and boundary conditions, which leads to the one-dimensional boundary problem in the transformation's domain. The last one is formulated as the vector boundary valued problem and solved with the apparatuses of the matrix differential calculations and Green's matrix function. The following solving is based on the approximate solving of the singular integral or integro-differential equations.

Main Results

1. The statement of the problem. The elastic (G is a share module, μ is a Poison's coefficient) semi-strip, $0 < x < a$, $0 < y < \infty$ is loaded at the edge $y = 0$, $0 < x < a$

$$\sigma_y|_{y=0} = -p(x), \tau_{yx}|_{y=0} = 0, 0 < x < a \tag{1}$$

where $p(x)$ is the known function. At the edges $x = 0$, $0 < y < \infty$ and $x = a$, $0 < y < \infty$ the two variants of boundary conditions can be considered. The first one is the condition of fixing on the both lateral sides:

$$u_x\,(0, y) = 0, u_x\,(a, y) = 0, u_y\,(0, y) = 0, u_y\,(a, y) = 0. \tag{2}$$

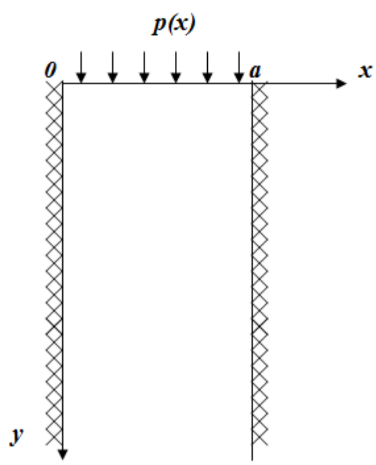

Fig. 1. Statement of the problem

The another one is the fixing condition at the lateral side $x = 0$, $0 < y < \infty$ and condition of the first main elasticity problem at side $x = a$, $0 < y < \infty$:

$$u_x\,(0, y) = 0, u_y\,(0, y) = 0, \sigma_x\,(a, y) = 0, \tau_{xy}\,(a, y) = 0. \tag{3}$$

The equilibrium equations are written in form

$$\begin{array}{l} \mu_* \frac{\partial^2 u(x,y)}{\partial x^2} + \frac{\partial^2 u(x,y)}{\partial y^2} + \mu_0 \frac{\partial^2 v(x,y)}{\partial x \partial y} + \rho \frac{\partial T}{\partial x} = 0, \\ \frac{\partial^2 v(x,y)}{\partial x^2} + \mu_* \frac{\partial^2 v(x,y)}{\partial y^2} + \mu_0 \frac{\partial^2 u(x,y)}{\partial x \partial y} + \rho \frac{\partial T}{\partial y} = 0, \end{array} \tag{4}$$

$\mu_0 = 1/(1 - 2\mu), \mu_* = 1 + \mu_0, u_x(x, y) \equiv u(x, y), v_x(x, y) \equiv v(x, y)$, $T(x, y)$ is temperature, which is the solution of the thermal conductivity problem for a semi-strip, which was found in [9]. After the expression of the constants μ_0, μ_* through the Muskchelishvili constant $\kappa = 3 - 4\mu$, one obtains the system (4)

$$\begin{array}{c} \frac{\partial^2 u(x,y)}{\partial x^2} + \frac{\kappa-1}{\kappa+1}\frac{\partial^2 u(x,y)}{\partial y^2} + \frac{2}{\kappa+1}\frac{\partial^2 v(x,y)}{\partial x \partial y} + \frac{\kappa-1}{\kappa+1}\rho\frac{\partial T}{\partial x} = 0, \\ \frac{\partial^2 v(x,y)}{\partial x^2} + \frac{\kappa+1}{\kappa-1}\frac{\partial^2 v(x,y)}{\partial y^2} - \frac{2}{\kappa-1}\frac{\partial^2 u(x,y)}{\partial x \partial y} + \frac{\kappa+1}{\kappa-1}\rho\frac{\partial T}{\partial y} = 0, \end{array} \tag{5}$$

where $\rho = 2\frac{\mu+1}{1-2\mu}\alpha_t$, α_t is a linear expansion coefficient. The boundary conditions (1) are reformulated in the form

$$\begin{array}{c} 2G\mu_0\left(\mu\frac{\partial u(x,0)}{\partial x} + (1-\mu)\frac{\partial v(x,0)}{\partial y}\right) = -p(x) \\ \frac{\partial u(x,0)}{\partial y} + \frac{\partial v(x,0)}{\partial x} = 0 \end{array} \tag{6}$$

The stress state of the semi-strip should be found.

2. The obtaining and solving of the one-dimensional boundary problem. The Fourier's transformation is applied to the system of Lame's equation and to the boundary conditions by the scheme

$$\begin{bmatrix} u_\beta(x), T_\beta(x) \\ v_\beta(x) \end{bmatrix} = \int_0^\infty \begin{bmatrix} u(x,y), T(x,y) \\ v(x,y) \end{bmatrix} \begin{bmatrix} \cos\beta y \\ \sin\beta y \end{bmatrix} dy \tag{7}$$

with the inverse formula

$$\begin{bmatrix} u(x,y) \\ v(x,y) \end{bmatrix} = \frac{2}{\pi}\int_0^\infty \begin{array}{c} u_\beta(x)\cos\beta y d\beta \\ v_\beta(x)\sin\beta y d\beta \end{array}. \tag{8}$$

For the first case of the conditions on the lateral sides (2) equations (5) have the form:

$$\begin{array}{c} \frac{d^2 u_\beta(x)}{dx^2} - \frac{\beta^2(\kappa-1)u_\beta(x)}{\kappa+1} + \frac{2\beta}{\kappa+1}\frac{dv_\beta(x)}{dx} = \frac{3-\kappa}{\kappa+1}\chi'(x) + \frac{\tilde{\rho}}{\kappa+1}\frac{dT_\beta}{dx}(x), \\ \frac{d^2 v_\beta(x)}{dx^2} - \frac{\beta^2(\kappa+1)v_\beta(x)}{\kappa-1} - \frac{2\beta}{\kappa-1}\frac{du_\beta(x)}{dx} = -\beta\frac{\kappa+1}{\kappa-1}\chi(x) - \frac{\beta\rho}{\kappa-1}T_\beta(x) \\ u_\beta(0) = 0, u_\beta(a) = 0, \\ v_\beta(0) = 0, v'_\beta(a) = 0 \end{array} \tag{9}$$

where $\tilde{\rho} = \rho\frac{\kappa-1}{\kappa+1}$. Here the new unknown function is inputted $\chi(x) = v(x,0), \chi'(x) = v'(x,0)$. As it is seen from the second boundary condition (6) $\frac{\partial u(x,0)}{\partial y} = -\chi'(x)$, so the second condition is satisfied automatically.

With the aim to reduce the problem to the vector boundary problem one must input the vectors and the matrixes

$$\vec{y}_\beta(x) = \begin{pmatrix} u_\beta(x) \\ v_\beta(x) \end{pmatrix}, \vec{f}(x) = \begin{pmatrix} \frac{3-\kappa}{\kappa+1}\chi'(x) + \frac{\tilde{\rho}}{\kappa+1}\frac{dT_\beta}{dx}(x) \\ -\beta\frac{\kappa+1}{\kappa-1}\chi(x) - \frac{\beta\rho}{\kappa-1}T_\beta(x) \end{pmatrix},$$

$$P = \begin{pmatrix} \frac{\kappa-1}{\kappa+1} & 0 \\ 0 & \frac{\kappa+1}{\kappa-1} \end{pmatrix}, Q = \begin{pmatrix} 0 & \frac{1}{\kappa+1} \\ -\frac{1}{\kappa-1} & 0 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

As a result the vector boundary problem is constructed:

$$\begin{array}{c} L_2\vec{y}_\beta(x) = \vec{f}(x) \\ I\vec{y}_\beta(0) = 0, I\vec{y}_\beta(a) = 0 \end{array} \tag{10}$$

$L_2\vec{y}_\beta(x) = I\vec{y}''_\beta(x) + 2\beta Q\vec{y}'_\beta(x) - \beta^2 P\vec{y}_\beta(x)$, I is an identity matrix and

$$L_2\vec{y}_\beta(x) = \vec{f}(x) \\ I\vec{y}_\beta\,(0) = 0, A\vec{y}_\beta'(a) + B\vec{y}_\beta(a) = \vec{g}(a) \tag{11}$$

here $A = \begin{pmatrix} 1-\mu & 0 \\ 0 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 0 & \mu\beta \\ \beta & 0 \end{pmatrix}$, $\vec{g} = \begin{pmatrix} \mu\chi(x) \\ 0 \end{pmatrix}$.

The solution of the vector boundary problem will be searched as the superposition of a homogenous vector equation's general solution $\vec{y}_\beta^0(x)$ and a particular solution of the inhomogeneous one $\vec{y}_\beta^1(x)$

$$\vec{y}_\beta(x) = \vec{y}_\beta^0(x) + \vec{y}_\beta^1(x) \tag{12}$$

These solutions will be constructed with the help of the matrix differential calculation apparatus as it shown earlier [9].

The solution of the matrix homogenous equation is constructed in the formula

$$Y\,(x) = \frac{1}{2\pi i}\oint_C e^{\xi x} M^{-1}\,(\xi)\,d\xi \tag{14}$$

where M^{-1} is the inverse matrix to the matrix $M\,(\xi)$. The closed contour C covers all singularity points of the matrix M^{-1}. With the help of the residual theorem and after the calculations of the residuals, one obtains the following matrix system of the fundamental matrix solutions

$$Y_1\,(x) = \frac{e^{\beta x}}{2}\begin{pmatrix} \frac{\kappa-\beta x}{\beta(\kappa-1)} & -\frac{x}{\kappa+1} \\ \frac{x}{\kappa-1} & \frac{\kappa+\beta x}{\beta(\kappa+1)} \end{pmatrix}, \tag{15}$$

$$Y_2\,(x) = \frac{e^{-\beta x}}{2}\begin{pmatrix} \frac{-\kappa+\beta x}{\beta(\kappa-1)} & \frac{x}{\kappa+1} \\ -\frac{x}{\kappa-1} & -\frac{\kappa-\beta x}{\beta(\kappa+1)} \end{pmatrix}. \tag{16}$$

The solution of the vector equation (10) can be rewritten

$$\vec{y}_\beta\,(x) = Y_1\,(x)\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} + Y_2\,(x)\begin{pmatrix} c_3 \\ c_4 \end{pmatrix} + \vec{y}_\beta^1\,(x)\,, \tag{17}$$

where constants $c_i, i = \overline{1,4}$ are founded from the boundary conditions.

For the obtaining of the vector boundary problem's particular solution, one needs to construct the Green's matrix function. It can be done with the help of the matrix integral transformations' method [7] The construction of the Green's matrix function is done by the method which is proposed in [9]:

$$G\,(x,\xi) = \frac{2}{a}\sum_{n=0}^{\infty}{}' H\,(x,\alpha_n)\,\omega_\beta^{-1} H\,(\xi,\alpha_n) \tag{19}$$

where

$$H\,(x,\alpha_n) = \begin{pmatrix} \sin\alpha_n x & 0 \\ 0 & \cos\alpha_n x \end{pmatrix}, \alpha_n = \tfrac{n\pi}{a}, n = 0, 1, 2\ldots$$

The representation (19) is the bilinear expansion for the Green's matrix function. One can be sure that all properties of Green's function are executed.

The solutions of the inhomogeneous boundary problems are constructed in the form

$$\vec{y}_\beta(x) = Y_1(x)\begin{pmatrix} c_1 \\ c_1 \end{pmatrix} + Y_2(x)\begin{pmatrix} c_3 \\ c_4 \end{pmatrix} + \int\limits_0^a G(x,\xi)\vec{f}(\xi)d\xi \tag{20}$$

Accordingly to the constructed solution (20) the displacements' transformations formulae for the first case of the boundary conditions (2) will be the following

$$\begin{array}{c} u_\beta(x) = Y_1^{11}(x)\,c_1 + Y_1^{12}(x)\,c_2 + Y_2^{11}(x)\,c_3 + Y_2^{12}(x)\,c_4, \\ v_\beta(x) = Y_1^{21}(x)\,c_1 + Y_1^{22}(x)\,c_2 + Y_2^{21}(x)\,c_3 + Y_2^{22}(x)\,c_4 + \\ +\frac{3-\kappa}{\kappa+1}\int\limits_0^a G^{21}(x,\xi)\,\chi'(\xi)\,d\xi - \tilde{\rho}\int\limits_0^a \frac{\partial G^{21}}{\partial\xi}(x,\xi)T_\beta(\xi)d\xi + \\ +\beta\frac{\kappa+1}{\kappa-1}\int\limits_0^a \phi^{22}(x,\xi)\,\chi(\xi)\,d\xi - \beta\rho\int\limits_0^a G^{22}(x,\xi)T_\beta(\xi)d\xi. \end{array} \tag{21}$$

where $G^{i,j}(x,\xi)$ is the Green's matrix function element in a i row and j column, $\phi^{22}(x,\xi) = \int G^{22}(x,\xi)d\xi$.

The expressions of the displacements' transformations for the second case of the boundary conditions (3) have the next form

$$\begin{array}{c} u_\beta(x) = Y_1^{11}(x)\,c_1 + Y_1^{12}(x)\,c_2 + Y_2^{11}(x)\,c_3 + Y_2^{12}(x)\,c_4 \\ +\frac{3-\kappa}{\kappa+1}\int\limits_0^a G^{11}(x,\xi)\,\chi'(\xi)\,d\xi - \tilde{\rho}\int\limits_0^a \frac{\partial G^{11}}{\partial\xi}(x,\xi)T_\beta(\xi)d\xi \\ -\beta\frac{\kappa+1}{\kappa-1}\int\limits_0^a G^{12}(x,\xi)\,\chi(\xi)\,d\xi - \beta\rho\int\limits_0^a G^{12}(x,\xi)T_\beta(\xi)d\xi, \\ v_\beta(x) = Y_1^{21}(x)\,c_1 + Y_1^{22}(x)\,c_2 + Y_2^{21}(x)\,c_3 + Y_2^{22}(x)\,c_4 + \\ +\frac{3-\kappa}{\kappa+1}\int\limits_0^a G^{21}(x,\xi)\,\chi'(\xi)\,d\xi - \tilde{\rho}\int\limits_0^a \frac{\partial G^{21}}{\partial\xi}(x,\xi)T_\beta(\xi)d\xi + \\ +\beta\frac{\kappa+1}{\kappa-1}\int\limits_0^a \phi^{22}(x,\xi)\,\chi(\xi)\,d\xi - \beta\rho\int\limits_0^a G^{22}(x,\xi)T_\beta(\xi)d\xi. \end{array} \tag{22}$$

The formulae (21), (22) would be the final ones if the unknown function $\chi'(\xi)$ is known. For its finding one must satisfy the boundary condition (6) which is unsatisfied yet. With this aim the inverse integral transformations' formulae should be applied to the displacements' transformations (8). The final formulae for the displacements in the case of the conditions (2) will be

$$\begin{array}{c} u(x,y) = \int\limits_0^a \left[\chi'(\xi)\int\limits_0^\infty f_1(x,\xi,\beta)\cos(\beta y)d\beta + \right. \\ \left. + \int\limits_0^\infty T(\xi,\eta)\int\limits_0^\infty f_2(x,\xi,\beta)\frac{(\cos(\beta(y+\eta)) + \cos(\beta(y-\eta)))}{2}d\beta d\eta\right] d\xi \\ v(x,y) = \int\limits_0^a \left[\chi'(\xi)\int\limits_0^\infty g_1(x,\xi,\beta)\sin(\beta y)d\beta + \right. \\ \left. + \int\limits_0^\infty T(\xi,\eta)\int\limits_0^\infty g_2(x,\xi,\beta)\frac{(\sin(\beta(y+\eta)) + \sin(\beta(y-\eta)))}{2}d\beta d\eta\right] d\xi \end{array} \tag{23}$$

The formulae for the conditions (3) have the analogical structure where functions $f_i(x,\xi,\beta), g_i(x,\xi,\beta), i = 1,2$ should be changed by another ones.

It should be taken into consideration that integrals in these correspondences are conditionally convergent integrals. So, before the differentiating of the displacements' expressions, at first one must extract the weakly convergence parts at these integrals [9].

3. The solving of the singular integral equations. The singular integral equation for the first problem, for example, can be written as

$$\int_0^1 \tilde{\chi}(\xi)\left[\frac{1}{\xi-x}+f(\xi,x)\right]d\xi = r(x)-\int_0^a\int_0^\infty T(\xi,\eta)g(\xi,\eta,x)d\eta d\xi, x\in[0,1] \tag{24}$$

here $\tilde{\chi}(\xi) = \chi'(a\xi)$, $r(x), f(\xi,x), g(\xi,\eta,x)$ are the known regular functions. The integral equation is solved approximately by the orthogonal scheme [6]. This method allows taking into consideration the real singularities of the solution at the ends of the integration interval. The order of singularities is found from the known solution for an edge with the angle of openness $\pi/2$ [3], [20]. With regard of it, the function is expanded in the series by the Jacobi's polynomials

$$\tilde{\chi}\left(\xi\right) = \sum_{n=0}^{\infty} \tilde{c}_n \xi^\alpha \left(1-\xi\right)^\beta P_n^{\alpha,\beta}\left(1-2\xi\right) \tag{25}$$

The infinite system of the linear algebraic equations relatively to the unknown coefficients $\tilde{c}_i, i = 0,1,2,\ldots$

$$\sum_{n=0}^{\infty} \tilde{c}_n d_{mn} = f_m, m = 0,1,2\ldots \tag{26}$$

is obtained and solved by the reduction method. The substitution of the founded constants in the formula (25) and following using of the formulae (23) completes the construction of the first problem's solution.

For the case of the boundary conditions (3) the integro-differential equation is constructed

$$\begin{gathered}\frac{d}{dx}\int_0^1 \tilde{\chi}(\xi)\left[\frac{1}{\xi-x}+f(\xi,x)\right]d\xi = \\ = r(x)-\int_0^a\int_0^\infty T(\xi,\eta)g(\xi,\eta,x)d\eta d\xi - \chi(a)h(x), x\in[0,1]\end{gathered} \tag{27}$$

here $\tilde{\chi}(\xi) = \chi'(a\xi)$, $r(x), f(\xi,x), g(\xi,\eta,x), h(x)$ are the known regular functions. The function $\tilde{\chi}(\xi)$ is presented as $\tilde{\chi}(\xi) = \chi_1(\xi)+\chi(a)\chi_2(\xi)$, where the functions $\chi_i(\xi), i = 1,2$ are expanded in the series by the Jacobi's polynomials

$$\tilde{\chi}_i\left(\xi\right) = \sum_{n=0}^{\infty} \tilde{c}_n^i \xi^\alpha \left(1-\xi\right)^\beta P_n^{\alpha,\beta}\left(1-2\xi\right), i = 1,2 \tag{28}$$

The infinite systems of the linear algebraic equations type (26) relatively to the unknown coefficients $\tilde{c}_n^i, n = 0,1,2,\ldots, i = 1,2$ is obtained. The substitution of the founded constants in the formula (28) and following using of the formulae for the displacements complete the construction of the second problem's solution.

4. The results of the numerical analyses. The calculations were done for the elastic half-strip ($G = 82.03125 * 109$ Pa, $\mu = 0.28$) with the length of the edge

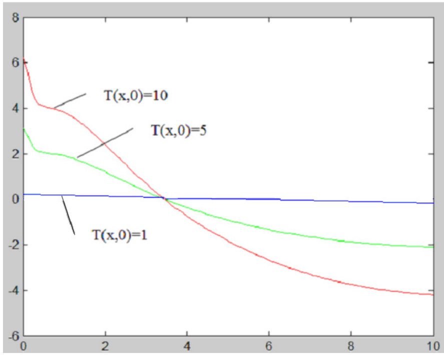

Fig. 2. Normal stresses $\sigma_x(5, y)$ for the first problem

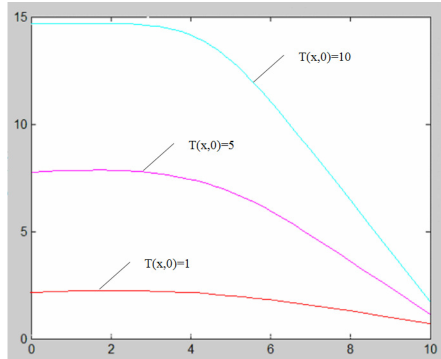

Fig. 3. Normal stresses $\sigma_x(5, y)$ for the second problem

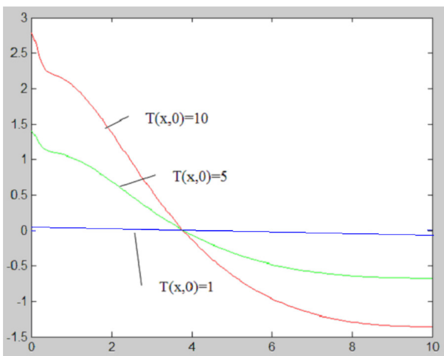

Fig. 4. Normal stresses $\sigma_y(5, y)$ for the first problem

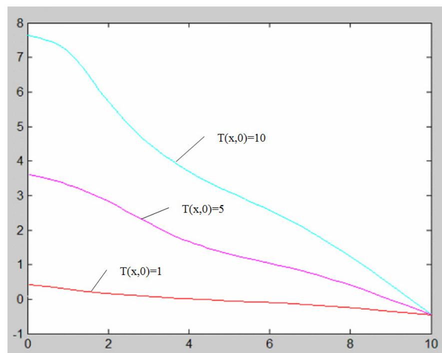

Fig. 5. Normal stresses $\sigma_y(5, y)$ for the second problem

$a = 10$ and mechanical load $p(x) = 1$. For the first and second problem the normal stresses $\sigma_x(5, y)$ (Fig. 2, Fig. 3) and $\sigma_y(5, y)$ (Fig. 4, Fig. 5) were analyzed at the line $x = 5, 0 < y < \infty$ of the semi-strip.

At the Fig. 2, Fig. 3 the stresses $\sigma_x(5, y)$ are shown in dependence of the applied temperature different values. As it seen, in the case when the conditions of the first main elasticity problems are given the values of stresses are greater. With the temperature increasing, the values of stresses are also increasing. The analogical behavior one can note at the Fig. 4, Fig. 5 when the stresses $\sigma_y(5, y)$ are shown at the line $x = 5, 0 < y < \infty$.

Conclusion.

1. The problem for the elastic semi-strip with two types of the boundary conditions on the lateral sides (the conditions of "fixing/fixing" or "fixing/first main elasticity problems") is solved. The method allows the reduction of the problem to singular integral or integro-differential equations in dependence of the boundary conditions' type at the lateral sides.

2. The investigation of the normal stresses' absolute values showed that temperature influence is more significant while the given conditions of the main elasticity boundary problem on the lateral side in comparing with the condition of the fixing.

1. **Kolosov G. V.** The use of complex diagram and theory of functions of the complex variable to the elasticity theory (in Russian) / Kolosov G. V. – M. : ONTI, 1935. – 224 p.
2. **Muskhelishvili N. I.** Some main problems of the mathematical elasticity theory (in Russian) / Muskhelishvili N. I. – M. : Nauka, 1966. – 707 p.
3. **Uflyand Ya. S.** Integral transformations in the problems of the elasticity theory (in Russian) / Uflyand Ya. S. – L. : Nauka, 1968. – 402 p.
4. **Vorovich I. I.** Some problems of elasticity theory for the semi-strip. (in Russian)/ I. I. Vorovich, V. V. Kopasenko// Prikladnaya matematica i mekchanica – 1966. v.30 – No. 1. – P. 128–136.
5. **Popov G. Ya.** About new transformations of the elasticity resolving equations and the new integral transformations with their application to the boundary problems of mechanics / G. Ya. Popov// Appl. Mech. – 2003. v.39 – No. 12. – P. 46–73.
6. **Popov G. Ya.** The elastic stress' concentration around dies, cuts, thin inclusions and reinforcements(in Russian) / G. Ya. Popov – M. : Nauka, 1966. – 707 p.
7. **Popov G. Ya.** Green's functions and matrixes of the one-dimensional boundary problems (in Russian) / G. Ya. Popov, S. A. Abdimanov, V. V. Ephimov – Almati : Raczah, 1999.
8. **Gradshtein I. S.** The tables of integrals, series and products(in Russian) / Gradshtein I. S., Rygik I. M. – M. : Nauka, 1963. – 1108 p.
9. **Vaysfel'd N. D.** On one new approach to the solving of an elasticity mixed plane problem for the semi-strip/ N. Vaysfel'd, Z. Zhuravlova// Acta Mechanica – 2015. DOI: 10.1007/s00707-015-1452-x
10. **Horvay G.** The end problem of rectangular strips/ I. I. Vorovich, V. V. Kopasenko// J. Appl. Mech. – 1953. v.20 – - P. 87–94.
11. **Koiter W.** On the bending of cantilever rectangular Plates/ W. Koiter, J. Alblas// Proc. Koninke Nederl. Acad. wet. B. – 1954. v.57 – No. 2.
12. **Ling C. B.** Stresses in a semi-infinite strip/ C. B. Ling, F. H. Cheng// Int. J. Eng. Sci. – 1967. v.5 – No. 2. – p. 155.
13. **Pickett G.** Stress concentrations in post-tensioned prestressed concrete beams / G. Pickett, K. T. S. Jyengar // J. Technol. – 1956. v.1 – No. 2.
14. **Aglovyan L. A.** About some mixed problems of elasticity theory for the semi-strip (in Russian)/ L. A. Aglovyan, R. S. Gevorkyan// News of academy of science Armenian SSR, Mechanics – 1970. v.23 – No. 3. – P. 3-*13.
15. **Thecaris P.** The stress distribution in a semi-infinite strip subjected to a concentrated load/ P. Thecaris// Trans. J. Appl. Mech. – 1959. v.26 – No. 3. – P. 401–406.
16. **Trapeznikov L. P.** Influence lines for the normal tensions in semi-strip (in Russian)/ L. P. Trapeznikov// News of USSR n.-i. of the hydromechanical institute – 1963. – V. 73.

17. **Suchevan V. G.** The tensioned state of the elastic semi-strip with fixed edges (in Russian)/ V. G. Suchevan// Matematiceskie issledovaniya – 1976. v.40 – P. 122–135.
18. **Gogoleva O. S.** The examples of solutions of the first main boundary problem of elasticity theory in the semi-strip (symmetrical problem) (in Russian)/ O. S. Gogoleva// Journal Omskiy gosudarstvenniy universitet – 2012. v.145 – No. 9. – P. 138–142.
19. **Menshova I. V.** The semi-strip with lateral edges rigidity, working for tension-compression (in Russian)/ I. V. Menshova, E. S. Lapikova// Journal ChGPU named I. Ya. Yakovlev, series: Mechanic of the limited state – 2014. v.20 – No. 2. – P. 106–118.
20. **Zhuravlova Z. Yu.** The plane mixed elastical problem for the semi-infinite strip (in Russian)/ Z. Yu. Zhuravlova //The Odessa's national university vestnik – 2014. v. 19 – No. 3(23). – P. 66–75.

Журавльова З. Ю.
НАПРУЖЕНИЙ СТАН ПРУЖНОЇ ПІВСМУГИ ПІД ДІЄЮ МЕХАНІЧНОГО ТА ТЕМПЕРАТУРНОГО НАВАНТАЖЕНЬ

Резюме

У роботі розглянута нова методика розв'язання плоских задач теорії пружності для півсмуги під дією механічного та температурного навантажень. Вона полягає у застосуванні інтегрального перетворення Фур'є безпосередньо до рівнянь Ламе. Це приводить вихідні задачі до одновимірних, які розв'язані за допомогою апаратів диференціального числення та матриці-функції Гріна. Розв'язування двох задач зведено до одного інтегрального та одного інтегро-диференціального рівнянь відносно невідомої функції зміщень відповідно.
Ключові слова: півсмуга, защемлення, перша основна задача теорії пружності, матриця Грина, перетворення Фур'е.

Журавлёва З. Ю.
НАПРЯЖЁННОЕ СОСТОЯНИЕ УПРУГОЙ ПОЛУПОЛОСЫ ПОД ДЕЙСТВИЕМ МЕХАНИЧЕСКОЙ И ТЕМПЕРАТУРНОЙ НАГРУЗОК

Резюме

В работе рассмотрена новая методика решения плоских задач теории упругости для полуполосы под действием механической и температурной нагрузок. Она состоит в применении интегрального преобразования Фурье непосредственно к уравнениям Ламе. Это приводит исходные задачи к одномерным, которые решены с помощью аппаратов дифференциального исчисления и матрицы-функции Грина. Решение двух задач сведено к одному интегральному и одному интегро-дифференциальному уравнениям относительно неизвестной функции перемещений соотвественно.
Ключевые слова: полуполоса, защемление, первая основная задача теории упругости, матрица Грина, преобразование Фурье.

REFERENCES

1. Kolosov, G. V. (1935). *Primenenie kompleksnih diagram i teorii funkcii kompleksnoy peremennoy k teorii uprugusti [The use of complex variable to the elasticity theory].* Moscow: ONTI [in Russian].
2. Muskhelishvili, N. S. (1966). *Nekotorie osnovnie zadachi matematicheskoy teorii uprugosti [Some main problem of the mathematical elasticity theory].* Moscow: Nauka [in Russian].

3. Uflyand, Ya. S. (1968). *Integral'nie preobrazovaniya v zadachah teorii uprugosti [Integral transformations in the problems of the elasticity theory].* Leningrad: Nauka [in Russian].

4. Vorovich, I. I., & Kopasenko, V. V. (1966). Nekotorie zadachi teorii uprugosti dlya polupolosi [Some problems of elasticity theory for the semi-strip]. *Prikladnaya matematica i mekcanica – Applied mathematics and mechanics, Vol. 30*(1), 128–136 [in Russian].

5. Popov, G. Ya. (2003). About new transformations of the elasticity resolving equations and the new integral transformations with their application to the boundary problems of mechanics. *Applied Mechanics, Vol. 38*(12), 46–73.

6. Popov, G. Ya. (1966). *Concentraciya uprugih napryageniy vozle shtampov razrezov tonkih vklyucheniy i podkrepleniy [The elastic stress' concentration around dies, cuts, thin inclusions and reinforcements].* Moscow: Nauka [in Russian].

7. Popov, G. Ya., Abdimanov, S. A., & Ephimov, V. V. (1999). *Funkcii i matrici Grina odnomernih kraevih zadach [Green's functions and matrixes of the one-dimensional boundary problems].* Almati: Raczah [in Russian]. / G. Ya. Popov, S. A. Abdimanov, V. V. Ephimov – Almati : Raczah, 1999.

8. Granshtein, I. S., & Rygik, I. M. (1963). *Tablici integralov, sum, ryadov i proizvedeniy [The tables of integrals, series and products].* Moscow: Nauka [in Russian].

9. Vaysfel'd, N. D., & Zhuravlova, Z. Yu. (2015). On one new approach to the solving of an elasticity mixed plane problem for the semi-strip. *Acta Mechanica, Vol. 226*(12), 4159–4172.

10. Horvay, G. (1953). The end problem of rectangular strips. *Journal of Applied Mechanics, Vol. 23*, 87–94.

11. Koiter, W., & Alblas, J. (1954). On the bending of cantilever rectangular plates. *Proc. Koninke Nederl. Acad. wet. B., Vol. 57*(2).

12. Ling, C. B., & Cheng, F. H. (1967). Stresses in a semi-infinite strip. *Int. J. Eng. Sci., Vol. 5*(2), 155.

13. Pickett, G., & Jyengar, K. T. S. (1956). Stress concentrations in post-tensioned prestressed concrete beams.*J. Technol., Vol. 1*(2).

14. Aglovyan, L. A., & Gevorkyan, R. S. (1970). O nekotorih smeshannih zadachah teorii uprugosti dlya polupolosi [About some mixed problems of elasticity theory for the semi-strip]. *Izvestiya akademii nauk armyanskoy SSR - New of academy of science Armenian SSR, Vol. 23*(3), 3–13 [in Russian].

15. Thecaris, P. (1959). The stress distribution in a semi-infinite strip subjected to a concentrated load. *Trans. J. Appl. Mech., Vol. 26*(3), 401–406.

16. Trapeznikov, L. P. (1963). Linii vliyaniya dlya normal'nih napryazheniy v polupolosy [Influence lines for the normal tensions in semi-strip]. *Izvestiya nauchno-issledovatel'skogo institute gidromehaniki – New of USSR scientific-research of the hydromechanical institute, Vol. 73* [in Russian].

17. Suchevan, V. G. (1976). Napryagennoe sostoyanie uprugoy polupolosi s zadelannimi krayami [The tensioned state of the elastic semi-strip with fixed edges]. *Metematiceskie issledovanija – Mathematical researches, Vol. 40*, 122–135 [in Russian].

18. Gogoleva, O. S. (2012). Primeri resheniya pervoy osnovnoy kraevoy zadachi teorii uprugosti v polupolose (simmetrichnaya zadacha) [The examples of solutions of the first main boundary problem of elasticity theory in the semi-strip (symmetrical problem)]. *Vestnik OGU – Journal of Omskiy state university, Vol. 145*(9), 138–142 [in Russian].

19. Menshova, I. V., & Lapikova, E. S. (2014). Polupolosa s prodol'nimi rebrami, rabotayushimi na rastyazhenie-szhatie [The semi-strip with lateral edges rigidity, working for tension-compression]. *Vestnik ChGPU im. I. Ya. Yakovleva, seriya: Mehanica predel'nogo sostoyaniya — Journal ChGPU named I. Ya. Yakovlev, series: Mechanic of the limited state, Vol. 20*(2), 106–118 [in Russian].

20. Zhuravlova, Z. Yu. (2014). Ploskaya smeshannaya zadacha teorii uprugosti dlya polubeskonechnoy polosi [The plane mixed elastic problem for the semi-infinite strip]. *Vestnik Odesskogo nacional'nogo universiteta – The Odessa's national university vestnik, Vol. 19,* 3(23), 66–75 [in Russian].