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STRESSES IN HOLLOW AXLES WITH ELLYPTICAL CRACKS UNDER ROTATING BENDING

This paper presents a model for the calculation of nominal stresses (not talking into account stress concentrations) in characteristic points of a fatigue crack on the surface of a hollow axle under rotating bending. A study of fractographical evidence offers an explanation for the decline of crack growth speed in the centre of the crack, thus changing its geometrical shape. The results of the study facilitate the calculation of the strength of hollow axles employed in transport, as well as in industrial equipment with long service life.

Keywords: *geometrical characteristics of strength, fractographs, kinetics of failure.*

Introduction. Macrofractographical studies of surface fatigue cracks show that during their growth they endure a series of changes. For example, for steel axles under rotating bending, in the initial stages of the crack, when its depth is under 0,5 mm, the nominal stresses along the crack are quite similar, thus advancing at the same place into the material, taking a semicircular shape. Afterwards, crack growth speed at the ends of the crack increases, taking an elliptical form, and sometimes even becoming straight-lined [1].

As a result of the study of the geometrical strength characteristics of the section of a hollow axle under rotating bending with a straight crack, several analytical equations were obtained, relating the axle strength with its wall thickness and crack size [2, 3]. These calculations showed axle endurance dramatically drops when its diameter ratio surpasses a certain value $k_h = d/D > 0{,}75$ (fig. 1).

Cracks deeper than 2…5 mm take an elliptical shape and its geometrical characteristics (axis a and b) have been analytically defined [4, 5]. The centre of the ellipse (point E) is considered to lie on the crack initiation point, thus making it not possible to fulfill the known condition of perpendicularity of the crack end to the axle surface [6].

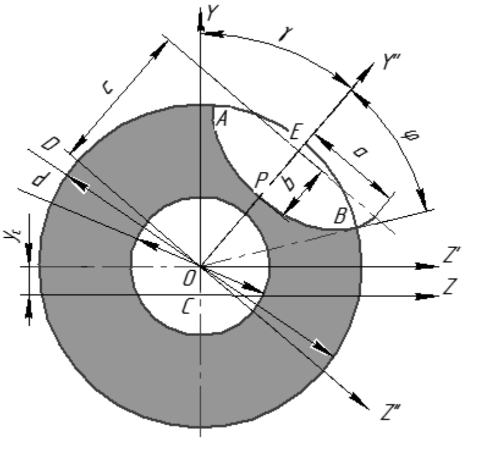


Figure 1 – Section of a hollow axle with a surface elliptical crack

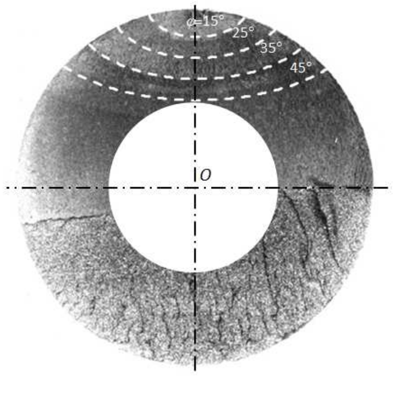


Figure 2 – Results of crack model placed on a fractograph of a fractured railway axle

These inaccuracies observed in crack models lead to lower exactness in calculations used for the prediction of crack growth in hollow axles. It is therefore more difficult to estimate the residual life of machine parts in machinery with a long expected life, as in transport, energy, metallurgy, etc.

Problem exposition. Observing the section of a hollow axle with a circumferential surface elliptical crack (fig. 1), the crack's growth under rotating bending is limited to the point when it breaks into the inner surface

$$b < c - 0{,}5d \,. \qquad (1)$$

Problem's aim is to build an analytical model for the calculation of nominal stresses in the characteristic points of the crack front, as well as to study them through dimensionless strength characteristics of the hollow axle.

Solution. The characteristics of a hollow axle with a crack (semiaxis a, b, position c of point E, angle φ, fig. 1) correspond to a growing crack observed in fractographs of fractured axles, hollow and solid.

Assuming that the centre of the ellipse (point E) stays on the axle surface ($c = D/2$), the relative axis $\bar{b}$ is defined as

$$\bar{b} = b/D = \varphi/\pi \ . \tag{2}$$

The relative length of the second semiaxis can as well be defined as function of φ [4]

$$\bar{a} = \frac{a}{D} = \frac{\sin\varphi}{2}\left[1-\left(\frac{\pi}{2}\cdot\frac{1-\cos\varphi}{\varphi}\right)^2\right]^{-0,5} . \tag{3}$$

The geometrical model proposed by (2) and (3) does not satisfy the known condition of perpendicularity of the surface ends with the axle surface [6], which can be solved by defining the ordinate c as a function $c(\varphi)>D/2$. From (3), new values for the semiaxis $\bar{b}$ and $\bar{c}$ can be obtained, which will satisfy the ellipse equation $\left(\frac{y_{Y''}-c}{b}\right)^2+\left(\frac{z_{Z''}}{a}\right)^2=1$ in the system of axis $OY''Z''$, as

$$\left.\begin{aligned} \bar{b} &= \frac{b}{D} = \frac{\varphi}{\pi}\left(1+\frac{2\varphi}{\pi\cos\varphi}\right)-\frac{1-\cos\varphi}{2\cos\varphi}, \\ \bar{c} &= \frac{c}{D} = \left[2\left(\frac{\varphi}{\pi}\right)^2+\cos\varphi-\frac{1}{2}\right]\cdot\frac{1}{\cos\varphi} \end{aligned}\right\} \tag{4}$$

The calculation of geometrical characteristics of the axle strength with a crack under rotating bending is carried out in the following manner:

$$A=\pi D^2(1-k_h^2)/4-D^2\Phi_2(\varphi)/8-ab\Phi_2(\delta)/2 \ , \tag{5}$$

where A – the area of cross section; $\Phi_n(\delta)$ – auxiliary function

$$\Phi_n(\delta)=n\delta-\sin(n\delta), \ n\in\mathbf{N}; \tag{6}$$

N – natural number.

For the definition of the area A the argument α in equation (6) takes the values of angle φ or angle $\delta=\arccos\left(\frac{2\bar{c}-\cos\varphi}{2\bar{b}}\right)$.

It is considered that the bending moment M_Z stays on the vertical plane. The symmetry axis OY turns an arbitrary value of angle γ to its new position OY'' (fig. 1).

The relative ordinate of the centre of gravity of the area A can be represented as

$$\bar{y}_C=\frac{2}{3}\cdot\frac{(1-8\bar{a}\bar{b})\sin^3\varphi+6\bar{a}\bar{b}\bar{c}\Phi_2(\delta)}{2\pi(1-k_h^2)-\Phi_2(\varphi)-4\bar{a}\bar{b}\Phi_2(\delta)} \ . \tag{7}$$

The moments of inertia related to the rotating axis OZ'' and OY'' are carried out in the following manner:

$$I_{Z''}=\pi D^4(1-k_h^4)/64-D^4\Phi_4(\varphi)/256-ab^3\Phi_4(\delta)/16-abc^2\Phi_2(\delta)/2+4ab^2c\sin^3\delta/3 \ , \tag{8}$$

$$I_{Y''}=\pi D^4(1-k_h^4)/64-D^4\Phi_4(\varphi)/256-a^3b\Phi_4(\delta)/16+D^4\sin^3\delta/24 \ . \tag{9}$$

The moments of inertia related to the axis OZ' and OZ can be calculated thus:

$$I_{Z'}=\sqrt{(I_{Z''}\cos\gamma)^2+(I_{Y''}\sin\gamma)^2} \ , \tag{10}$$

$$I_Z=I_{Z'}-y_c^2\cdot A \ . \tag{11}$$

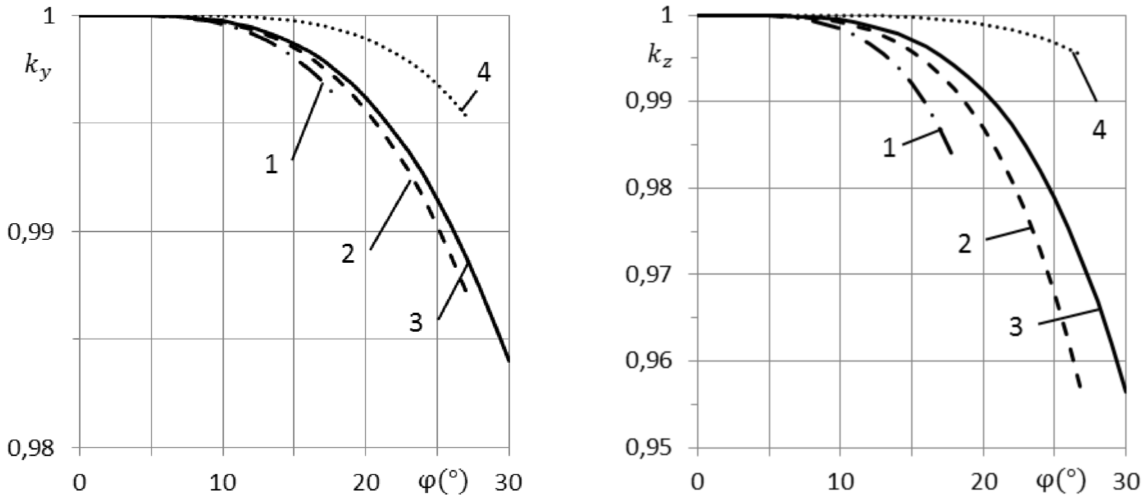
And from the exposed equations (3) – (11) the main moments of inertia can be expressed as

$$I_{Z'}=k_z\cdot I_{Z0} \ , \tag{12}$$

$$I_{Y''}=k_y\cdot I_{Y0} \ , \tag{13}$$

where $I_{Z0} = I_{Y0} = \pi D^4(1-k_h^4)/64$ – own moment of inertia of cross section of the hollow axle, prior to the crack.

The proportional coefficients k_y, k_z in (12) and (13) characterize the decrease of the main moments of inertia of the axle section during the crack growth (fig. 3). Lines 1, 2 and 4 do not go beyond $\varphi = 30°$ because of condition (1).



a *b*

Figure 3 – Plots of functions $k_y(\varphi)$ (*a*) and $k_z(\varphi)$ (*b*) (lines correspond to: 1 – $k_h = 0{,}8$; 2 – $k_h = 0{,}7$; 3 – $k_h = 0{,}6$; 4 – axle with $k_h = 0{,}7$ and flat crack [2])

The analysis of these graphs show that the character of the axle deformation changes during the crack growth: bending becomes eccentric and the loads asymmetric, and therefore the characteristics of the cycle vary. If until the crack appears the mean value of stresses is $y_m = 0$ and its asymmetry coefficient $R = -1$, for the cracked axle this coefficient R grows (staying negative). Calculations show, that for different points of the elliptical crack front the value of R differs – being higher in points A and B than in the centre P.

For the study of the relative nominal maximum stresses in characteristic points of the elliptical crack front the following expressions have been used:

$$\bar{\sigma}_A = \frac{\sigma_A}{\sigma_0} = \frac{y_A}{I_Z}\cdot\frac{2I_{Z0}}{D} = I_{Z0}\cdot\frac{2\bar{y}_c\cos\gamma + \cos\gamma}{I_Z}, \qquad (14)$$

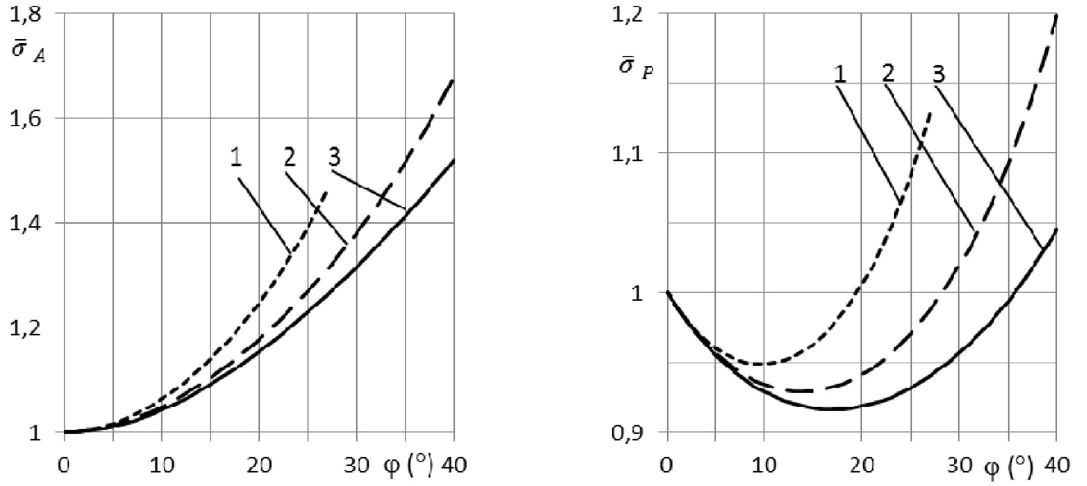
$$\bar{\sigma}_P = \frac{\sigma_P}{\sigma_0} = \frac{y_P}{I_Z}\cdot\frac{2I_{Z0}}{D} = I_{Z0}\cdot\frac{2(\bar{y}_c + \bar{c} - \bar{b})\cdot\cos\gamma}{I_Z}. \qquad (15)$$

Based on the graphs for functions (14) and (15) the following can be observed about the kinetics of surface crack growth.

Under rotating bending the crack opens and closes periodically. When it is closed (i.e. when it goes through the lower half of the axle, fig. 1), its moments of inertia increase up to $I_{Z0} = I_{Y0}$, which correspond to the crackles section. The analysis of the process of crack growth show that when the crack goes through the values $-\varphi < \gamma < \varphi$ all points on its surface reach their maximum values (fig. 5).

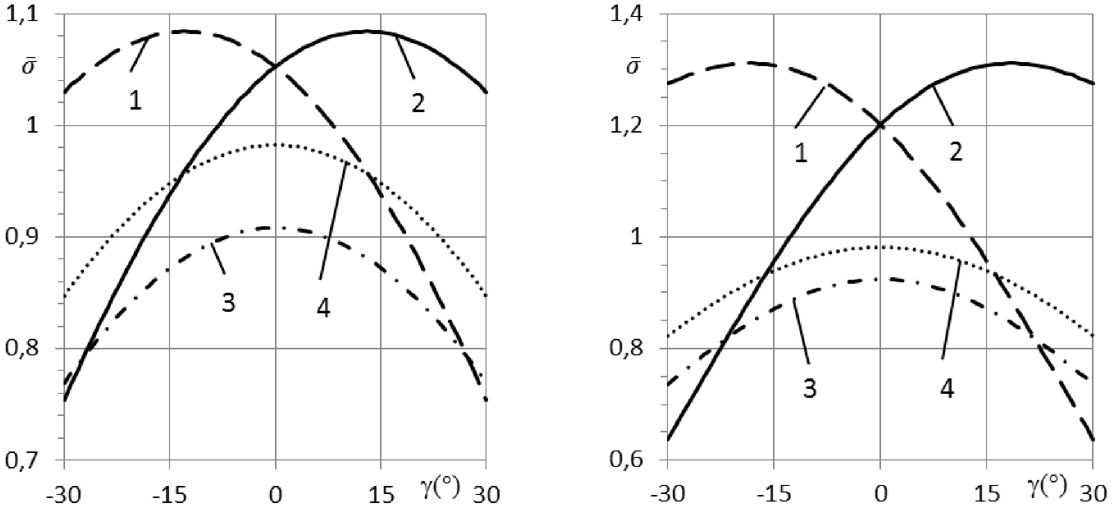
As the axle rotates clockwise from its initial position ($\gamma = 0$) point A goes through its highest position, where its stress attain its maximum value, and stresses in points P and B decreases (fig. 1). Once gone through the lower side of the axle, point B goes as well through its highest position, where it attains its maximum stress. Calculations show that for axles with a diameter ratio of $k_h = 0{,}75\ldots0{,}80$ the nominal stresses for points A and B for a crack with size $2\varphi = 30°$ grows up to 16…25 % as compared with the crackles axle. Maximum stresses in points A and B (symmetrical points) are a 18 % higher than in point P, and 40 % when $2\varphi = 60°$.

As a result of the exposed analysis it can be deduced that, the longer the crack front line, the more the stresses in the centre and ends of the crack differ. This leads to the higher crack growth speed in the crack ends as compared to the centre, and therefore to its flattening.



a *b*

Figure 4 – Graphs of functions $\overline{\sigma}_A(\varphi)$ (*a*) and $\overline{\sigma}_P(\varphi)$ (*b*) (lines correspond to: 1 – $k_h = 0{,}7$; 2 – $k_h = 0{,}5$; 3 – $k_h = 0$)



a *b*

Figure 5 – Relative maximum stresses in the elliptical crack front characteristic points under rotation, with $k_h = 0{,}6$ being $\varphi = 15°$ (*a*) and $\varphi = 30°$ (*b*)
(lines correspond to points: 1 – point *B*, 2 – point *A*, 3 – point *P*, line 4 correspond to the flat crack [2])

Conclusions

1. The developed model for the calculation of nominal stresses (not taking into account stress concentrations) in points of elliptical fatigue cracks in hollow axles under rotating bending stresses enable to complete the analysis of fracture kinetics and clarify the parameters of the cyclical stress variations for the calculation of fatigue strength of hollow axles.

2. The peculiarities of the variation of cyclical stresses for axles with elliptical cracks under rotating bending is established – the decrease of the moment of inertia I_z leads to a decrease of stress in the deepest area of the crack (point *P*).

3. The proposed model and analytical method, based on dimensionless relative characteristics of hollow axle strength provide a way to the study of fracture kinetics in axles for transport machinery, metallurgy, energy, etc. calculated for long service life.

4. The exposed model can be further developed by taking into account the influence of loads, kind of stress concentrations, material properties, and other factors into the geometrical characteristics of the growing crack, which implies an analysis and extension of relevant fractographs.

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Гутиря С.С., Хом'як Ю.М., Йон Аріза де Мігель. Напруження у трубчастих осях з утомними еліптичними тріщинами при згині з обертанням

Запропонована аналітична модель для розрахунку номінальних напружень (без врахування концентрації) у характерних точках поверхневої утомної тріщини для трубчастої осі при згині з обертанням. На підставі фрактографічного аналізу кінетики тріщин встановлені причини зниження швидкості їх поширення у центрі й зміни геометричної форми. Результати досліджень дозволяють уточнити розрахунки на утомну міцність трубчастих осей у складі транспортних машин, енергетичного, металургійного й ін. устаткування із тривалим строком експлуатації.

Ключові слова: геометричні характеристики міцності, фрактограми, кінетика руйнування.

Гутыря С.С., Хомяк Ю.М., Йон Ариза де Мигель. Напряжения в трубчатых осях с усталостными эллиптическими трещинами при изгибе с вращением

Предложена аналитическая модель для расчета номинальных напряжений (без учета концентрации) в характерных точках поверхностной усталостной трещины для трубчатой оси при изгибе с вращением. На основании фрактографического анализа кинетики трещин установлены причины снижения скорости их распространения в центре и изменения геометрической формы. Результаты исследований позволяют уточнить расчеты на усталостную прочность трубчатых осей в составе транспортных машин, энергетического, металлургического и др. оборудования с длительным сроком эксплуатации.

Ключевые слова: геометрические характеристики прочности, фрактограммы, кинетика разрушения.