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COMPARATIVE ANALYSIS OF STATISTICAL CHARACTERISTICS FOR TIME-SERIES OF INVESTMENT INSTRUMENT PRICES

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This paper is devoted to the study of statistical characteristics of market dynamics, which is an important task for making effective investment decisions based on understanding and predicting the behaviour of these markets. One of the basic methods is to estimate the statistical characteristics for the price and profitability time series of the investment tools at the market. In this paper, a comparative analysis of the statistical characteristics of price time-series of investment vehicles on the example of three markets: the gold market, the EUR / USD currency pair and the Bitcoin cryptocurrency are conducted. The most informative and indicative statistical characteristics are revealed, and frequency distribution histograms of time series are examined in detail. On this basis, it is concluded that the conformity of each market with the principles of existing hypotheses describing the dynamics of the market: Efficient Market Hypothesis, Fractal Market Hypothesis and Coherent Market Hypothesis. The paper finds that the foreign exchange market best meets the requirements of an efficient market. This indicates a concentration of rational investors in the market and a low probability of making a profit higher than the average market profitability. The cryptocurrency Bitcoin is a coherent market. It is the least mature market and is constantly influenced by crowd mood rather than rational investor decisions. Given the rapid changes in phases and mood in the cryptocurrency market, this investment vehicle is only recommended for the short-term investment horizon. Due to the fact that the gold market is more in line with the fractal market hypothesis, it is better to use fractal analysis methods for investments in exchange gold. Based on the calculations, practical recommendations were made regarding the investment prospects for each of the three markets.

ПОРІВНЯЛЬНИЙ АНАЛІЗ СТАТИСТИЧНИХ ХАРАКТЕРИСТИК ЧАСОВИХ РЯДІВ ЦІНИ ІНВЕСТИЦІЙНИХ ІНСТРУМЕНТІВ

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Ключові слова:
часові ряди, статистичні характеристики, гістограма розподілу частот, гіпотеза ефективного ринку, гіпотеза фрактального ринку, гіпотеза когерентного ринку.

Статтю присвячено дослідженню статистичних характеристик динаміки ринків, що є важливим завданням для прийняття ефективних інвестиційних рішень на основі розуміння та передбачення поведінки цих ринків. Одним із базових методів дослідження є оцінка статистичних характеристик часових рядів ціни та прибутковості інвестиційних інструментів на ринку. Проведено порівняльний аналіз статистичних характеристик часових рядів ціни інвестиційних інструментів на прикладі трьох ринків: ринку золота, валютної пари EUR/USD та криптовалюти Біткоїн. Виявлено найбільш інформативні та показові статистичні характеристики та детально розглянуто гістограми розподілу частот часових рядів. На підставі цього зроблено висновок щодо відповідності кожного ринку засадам існуючих гіпотез, що описують динаміку ринку: гіпотезі ефективного ринку, гіпотезі

фрактального ринку чи гіпотезі когерентного ринку. Установлено, що валютний ринок найкраще відповідає передумовам ефективного ринку. Це свідчить про зосередження на ринку раціональних інвесторів та низьку ймовірність отримання прибутку вище ніж середня прибутковість ринку. Ринок криптовалюти Біткоїн є когерентним. Він є найменш зрілим ринком і постійно перебуває під впливом настроїв натовпу, а не раціональних рішень інвесторів. Враховуючи швидкі зміни у фазах та настроях на ринку криптовалюти, цей інвестиційний інструмент може бути рекомендованим лише на короткостроковому горизонті інвестування. У зв'язку з тим, що ринок золота більш відповідає гіпотезі фрактального ринку, для інвестицій в біржове золото краще застосовувати методи фрактального аналізу. На підставі проведених розрахунків надано практичні рекомендації стосовно перспектив інвестування на кожному з трьох ринків.

Statement of the problem

To understand and predict market behaviour, and to make effective investment decisions, an important task is to study the dynamics of markets which selected for investing. One of the basic methods of the study is to evaluate the statistical characteristics of the time series of the investment tool price. Another important indicator that characterizes the type of dynamics is the shape of price and profitability, frequency histograms and such important estimates as volatility, asymmetry, and excess.

According to shape of time series distribution, an appropriate hypothesis is established that best characterizes a given market, and, having decided on the hypothesis, and, having decided on the hypothesis, it would be much easier for an investor to predict the price behaviour of a financial instrument. It should be noted that one of the prerequisites for the Efficient Market Hypothesis is the normal (Gaussian) distribution for the price and profitability of the investment instrument. Although the fractal market hypothesis does not imply any particular kind of distribution, it does highlight the characteristic features that are present in the frequency distributions of real financial markets. And the Coherent Market Hypothesis classifies the state of the market into five phases according to the type of distribution. In addition, statistical characteristics are important valuation parameters for comparative market analysis.

Analysis of recent studies and publications

For today, the following theories describing the dynamics of financial markets have become the most widespread: the Efficient Market Hypothesis (EMN) [1] – [2], the Fractal Market Hypothesis (FMH) [3] – [4] and the Coherent Market Hypothesis (CMH) [5].

Despite the fact that the Efficient Market Hypothesis was formulated in the 1970s, its ideas remain in demand and are relevant today: in 2013, Eugene Fama, the founder of the efficient market hypothesis, received the Nobel Prize for "empirical analysis of asset prices" [6]. According to the hypothesis of efficient markets, the prices and profitability of financial instruments don't have memory, that is, independent random variables with equally distributed increments. It follows from the central limit theorem that the distribution of a large number of random independent quantities with approximately the same size will be approached to normal distribution. Therefore, two approaches are used to diagnose the market dynamics of EMH preconditions: the first is to perform statistical tests to verify the independence of events (reviewed by the author in [7]); the second is to check that the profitable distribution of a particular market complies with the conditions of normal distribution [8] – [9].

The shape of the distribution for the profitability time series plays an important role in the Coherent Market Hypothesis. The coherent market hypothesis is based on the construction and study of nonlinear statistical models, where in terms of the probability density function of profitability distribution concludes about one of the five phases in the market: the phase of the random walk market, the unstable transition market, the chaos market, the coherence market and the phase of the anti-persistent market [5].

Although the prerequisites for the fractal market hypothesis are not based on the investigated time series distributions, the source [3] highlights the main features inherent in financial market density distributions (high value of excesses, "fat tails," etc.). A detailed study, evaluation and comparison of these parameters is an additional tool in making investment decisions.

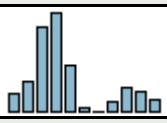
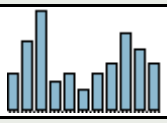
Objectives of the article

The purpose of the study is to compare the statistical characteristics of the price time series for the investment instruments on the example of three investment markets (the gold market, the EUR / USD currency market and Bitcoin cryptocurrency market) to determine the type of dynamics, the internal laws of operation, predictability of each market, and provide practical recommendations on the prospects for investing in each of the three markets.

The main material of the research

We calculate the main statistical indicators [10] of the time series (TS) for the selected investment instruments and check their compliance with the normal distribution. From the use of statistical analysis tools, the point estimates values are calculated (Table 1).

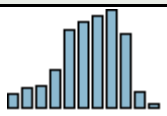
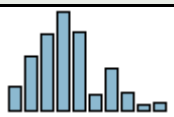
Table 1 - The main point estimates for the price time series of gold, EUR / USD and Bitcoin from January 2012 to March 2019

Dynamics evaluation indicators	Gold	Currency pair EUR/USD	Bitcoin
1. Mean	1335,9	1,215	1212,8
2. Median	1285,5	1,2195	390,3
3. Standard deviation	181,25	0,106	2720,5
4. Range	742,4	0,3547	19187
5. Coefficient of variation, %	**13,6%**	8,7%	**224%**
6. Coefficient of oscillation%	**55,6%**	29,2%	**1582%**
7. Frequency distribution histogram			
8. Skewness (asymmetry)	0,99	0,05	3,8
9. Excess kurtosis	-0,1	-1,5	15,25

From the distribution histogram and the point estimates of the time series, we can conclude that none of the given time series from January 2012 to March 2019 can be described as a normal distribution, Bitcoin prices are much more volatile (the variations and oscillations are ten times higher than the corresponding figures for the price of gold and currency pair) and Bitcoin price distribution has an extremely high kurtosis.

The dynamics of gold and currency prices are characterized by separate periods with different behaviour of the system (two "heads" of distribution). In this regard, the separate periods are investigated: for time series of the gold price is the period from June 2013 to March 2019; for currency pair – from January 2015 to March 2019. The main statistical indicators are summarized in Table 2.

Table 2 - The main point estimates for the price time series of gold and EUR / USD currency pair during the periods with similar dynamics

Dynamics evaluation indicators	Gold from June 2013 to March 2019	EUR/USD from January 2015 to March 2019
1. Mean	1246,9	1,1215
2. Median	1254,9	1,1177
3. Standard deviation	74,37	0,045
4. Range	370	0,212
5. Coefficient of variation, %	6%	4%
6. Coefficient of oscillation%	29,7%	18,9%
7. Frequency distribution histogram		
8. Skewness (asymmetry)	-0,48	0,63
9. Excess kurtosis	-0,22	-0,01

Since the shape of the distribution and the point estimates of the given time series cannot uniquely confirm or deny the hypothesis of normal distribution, a number of statistical tests are conducted.

Normality testing is performed by several criteria:

1) Comparison of the mean and median. The module difference between the mean and the median should not exceed three standard errors of the mean.

2) Comparison of the calculated skewness (asymmetry) coefficient with the three standard skewness errors.

3) Comparison of the calculated excess kurtosis coefficient with the three standard kurtosis errors.

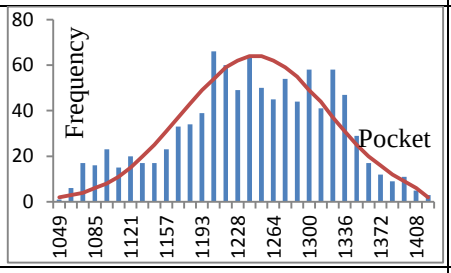
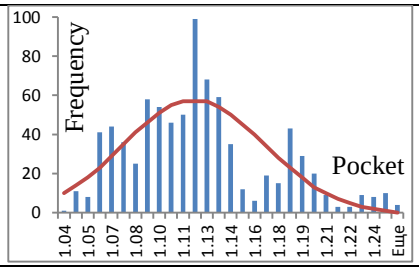
4) The Kolmogorov-Smirnov test (K-S test or KS test).

5) The Shapiro-Wilk test.

The calculation results are shown in Table 3.

The analysis revealed that, despite the stochastic nature of the dynamics, neither the TS for the gold price nor the currency pair TS can be described by a normal distribution.

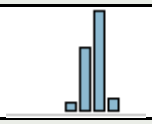
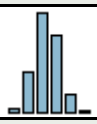
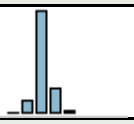
Table 3 - Results of the check for compliance with the normal law of distribution for gold price and EUR / USD time series for periods with similar dynamics

	Gold from June 2013 to March 2019	EUR/USD from January 2015 to March 2019
Frequency distribution histogram		
1. Comparison of mean and median	8 > 6,33	0,0038 < 0,0048 *
2. Checking skewness (asymmetry)	|-0,48| > 0,207	|0,63|>0,225
3. Checking kurtosis (excess)	|-0,217| < 0,417 *	|-0,014|<0,51 *
4. Kolmogorov-Smirnov test	p<0,01	p<0,01
5. Shapiro-Wilk test	p=0,000	p=0,000

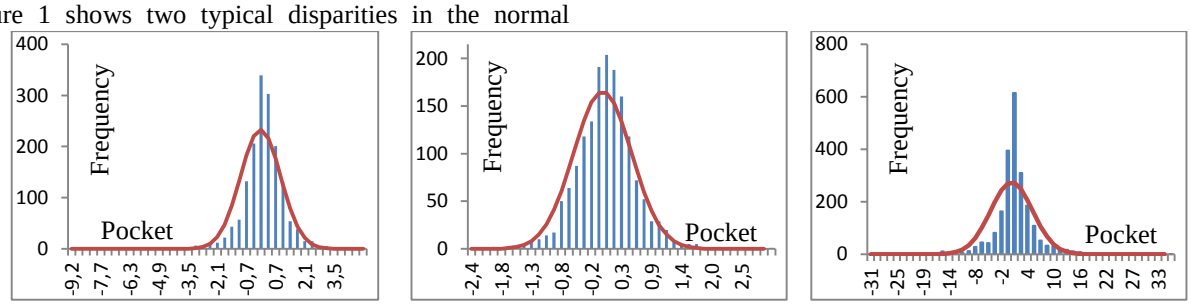
** - the null hypothesis about normal distribution is not rejected*

We conduct similar studies for the profitability time series of the respective investment instruments. The calculation results of point statistical estimates are shown in Table 4:

Table 4 - The main point estimates for the profitability time series of gold, EUR / USD and Bitcoin from January 2012 to March 2019

Dynamics evaluation indicators	Profitability of:		
	gold	currency pair EUR/USD	Bitcoin
1. Mean	-0,00426	-0,00224	0,449
2. Median	0	0	0,259
3. Standard deviation	0,9746	0,529	4,69
4. Range	14,1	5,459	67,2
7. Frequency distribution histogram			
8. Skewness (asymmetry)	-0,39	0,181	0,194
9. Kurtosis (excess)	7,85	2,088	9,28

For a more detailed study, the profitability distribution histograms of the respective investment instruments are compared with the normal distribution curve (Figure 1). Figure 1 shows two typical disparities in the normal distribution characteristics that researchers are faced with when studying real economic series. Firstly, it is a high kurtosis, and secondly, the so-called "fat tails."



a) b) c)

Fig. 1. Frequency histogram in comparison with the normal distribution curve for time-series profitability of: a) gold; b) EUR / USD currency pair; c) Bitcoin

According to the law of normal distribution, 99.7% of all events are in the interval $[\mu - 3\sigma; \mu + 3\sigma]$. Table 5 shows the actual number of events that are outside the interval $[\mu - 3\sigma; \mu + 3\sigma]$ compared to the allowed number of such events n in the total number of events N of each time series according to the normal distribution.

Table 5 - Comparison of the actual number for events beyond the interval [μ-3σ; μ + 3σ] with the maximum allowable number of such events according normal distribution

	n1 < μ-3σ	n2 > μ+3σ	n1+n2	n
Gold	10	13	23	≈5
EUR/USD currency pair	6	11	17	≈5
Bitcoin	27	20	47	≈7

where n1, n2, n is the number of actual events, n is the maximum allowed number of events for normal distribution in intervals $\begin{cases} < \mu - 3\sigma \\ > \mu + 3\sigma \end{cases}$.

According to the five mentioned above criteria, the TS of profitability are checked for compliance with the normal distribution. The results are shown in Table 6.

Table 6 - Results of the check for compliance with the normal law of distribution for gold price, EUR / USD and Bitcoin time series for periods from January 2012 to March 2019

	Profitability of:		
	currency pair EUR/USD	currency pair EUR/USD	currency pair EUR/USD
1. Comparison of mean and median	|-0,00426| < 0,072 *	|-0,00224|<0,039 *	|0,449| > 0,3
2. Checking skewness (asymmetry)	|-0,39| > 0,183	|0,1806| < 0,183 *	|0,19| > 0,156
3. Checking excess kurtosis	|7,845| > 0,366	|2,088| > 0,366	|9,28| > 0,312
4. Kolmogorov-Smirnov test	p<0,01	p<0,01	p<0,01
5. Shapiro-Wilk test	p=0,000	p=0,000	p=0,000

* - *the null hypothesis about normal distribution is not rejected*

As can be seen from the table, no time series fully meets the conditions of normal distribution, and the main statistical indicators of investment instruments differ significantly from each other.

Conclusions

Comparing the results obtained from the analysis of point statistical characteristics and the estimation of conformity to the normal distribution, it can be noted that the Bitcoin time series is characterized by a very high value of variation, moreover, this series is the least consistent with the properties of normal distribution, and such anomalies as high excess kurtosis and "fat tails" are most clearly expressed. Bitcoin cryptocurrency dynamics are most consistent with the Coherent Market Hypothesis. There are several completely different periods (phases) in this market: before 2017, the rate of change was minimal, 2017 was a year of incredible growth and excitement, and in 2018, overvalued assets began to decline. The Bitcoin market is the least mature market and is influenced by crowd moods rather than rational investor decisions. Given the rapid changes in phases and market sentiment, this investment tool is only recommended in the short-term investment horizon. In addition, it should be noted that, on the one hand, this instrument is risky for the investor (high volatility of the market and rapid changes in trends), and on the other hand, gives the opportunity to earn additional profit in the short-term investment.

The EUR / USD currency pair is best described by the Efficient Market Hypothesis. Despite the significant inconsistencies and deviations, the time series of the currency is the closest to a normal distribution compared to the time series of gold and Bitcoin. In addition, this time series is characterized by the lowest volatility, the excess kurtosis and the smallest "fat tails." All this

indicates a concentration of rational investors in the market and a low probability of making a profit higher than the average market profitability. Therefore, it is inappropriate to use technical analysis tools to operate in this market (according to the EMH, the time series does not have memory and previous prices are unrelated to future ones), attention should be paid to the study of long-term trends and the analysis of fundamental factors.

For the gold market, there is no such dramatic change in mood and crowd effect as for Bitcoin, however, deviations from the normal distribution are also much larger than in the currency market. According to [3], high excess kurtosis and "fat tails" are an indication of a fractal market that focuses on investors with different investment horizons, therefore, it is recommended to use nonlinear dynamics and fractal analysis tools [10] to work in the gold market.

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