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EFFECTIVE ELASTIC MODULUS DETERMINATION OF UNIDIRECTIONAL COMPOSITE FOR STOCHASTIC GEOMETRIC CHARACTERISTICS OF FIBER

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This paper proposes a method of determining the effective longitudinal elastic modulus for the composite material in the problem of longitudinal tensile composite. Object of study is unidirectional composite material with a hexagonal arrangement of fibers. The fiber composite element is presented as a whole cylinder, to the matrix corresponds a hollow cylinder. Included in the composite the matrix and the fiber are assumed to be isotropic. The radius of the fiber is considered as a continuous random quantity, normally distributed, respectively, the volume content of the fiber in the composite is also a random quantity. Its mathematical expectation is expressed through the mathematical expectation and variance of the fiber radius.

To determine the effective elastic modulus of the composite is supposed to use the kinematic matching conditions. First, to solve the boundary value problem for joint deformation of an isotropic matrix and isotropic fiber. For this, previously founding a general solution of the problem on axisymmetric deformation of isotropic cylinder under longitudinal tension, herewith uses the basic equations of elasticity theory in a cylindrical coordinate system. The obtained solution is used for determining the constituent elements of the tension and deformation of the fiber and the matrix. It is assumed, that at the contact surface of the matrix and the fiber is the normal displacement and tension of the fiber and the matrix are the same, axial movement of the fiber and the matrix are also equal for all values of the axial coordinate. The normal tension on the outer surface of the cylinder modeling the matrix are equal to zero. Found using these conditions, axial and radial displacements and tensions are functions of the fiber volume content in the cell matrix, as well as technical elastic constants of the matrix and the fiber.

Next, obtained a solution corresponding to the boundary value problem for the composite. Here is a model of composite solid homogeneous transversely isotropic cylinder. This solution depends on the effective elastic constants of transversely isotropic material. For both tasks, axial tension is assumed to be constant. The ratio between the axial tension of the matrix and the fiber, and the axial tension in the transversely isotropic material model of the composite is determined by the equilibrium conditions. As the matching conditions for considering the problem of longitudinal tensile homogeneous transversely isotropic composite and joint longitudinal tensile of the isotropic matrix and the isotropic fiber are selected conditions of the axial displacement at arbitrary axial coordinate and equality of radial displacements on an outer surface of the cylinder, modeling composite. The use of such matching conditions allowed to determine the effective longitudinal elastic modulus as a function of the determined values – the elastic constants of the matrix and the fiber, as well as random argument – the volumetric fiber content in the composite cell. Found mathematical expectation of this indicator. Proposed in the paper methodology also allows to determine the effective elastic constants for composites, having random characteristics with different distribution laws.

Key words: composite material, matrix, fiber, effective modulus of elasticity, normal distribution, matching conditions.

ВИЗНАЧЕННЯ ЕФЕКТИВНОГО МОДУЛЯ ПРУЖНОСТІ ОДНОСПРЯМОВАНОГО КОМПОЗИТУ ПРИ СТОХАСТИЧНИХ ГЕОМЕТРИЧНИХ ХАРАКТЕРИСТИКАХ ВОЛОКНА

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У роботі пропонується підхід до визначення ефективного модуля пружності для односпрямованого композиційного матеріалу. Композит, що складається з ізотропної матриці та ізотропного волокна, моделюється суцільним однорідним трансверсально-ізотропним матеріалом. При цьому волокно розглядається як циліндр, радіус якого є випадковою величиною, розподіленою за нормальним законом.

Ключові слова: композиційний матеріал, матриця, волокно, ефективний модуль пружності, нормальний закон розподілу, умови узгодженості.

ОПРЕДЕЛЕНИЕ ЭФФЕКТИВНОГО МОДУЛЯ УПРУГОСТИ ОДНОНАПРАВЛЕННОГО КОМПОЗИТА ПРИ СТОХАСТИЧЕСКИХ ГЕОМЕТРИЧЕСКИХ ХАРАКТЕРИСТИКАХ ВОЛОКНА

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В работе предлагается подход к определению эффективного модуля упругости для однонаправленного композиционного материала. Композит, состоящий из изотропной матрицы и изотропного волокна, моделируется сплошным однородным трансверсально-изотропным материалом. При этом волокно рассматривается как цилиндр, радиус которого является случайной величиной, распределенной по нормальному закону.

Ключевые слова: композиционный материал, матрица, волокно, эффективный модуль упругости, нормальный закон распределения, условия согласованности.

INTRODUCTION

One of the most common modeling method in the mechanics of composite materials is to build a model of the composite as a homogeneous continuous medium with elastic constants, which is adequately reflected the most important mechanical properties of the material. These steels, which are called effective, are defined as coefficients, that connect average by value components of the stress tensor and deformation under certain boundary conditions. Definition of constants for transverse elastic matrix and fiber with deterministic geometrical characteristics studied in several works. The most widespread is the ratio obtained in [1, 2] for an isotropic matrix and fiber. In the work [3] for the plane mechanics problem of rubber cord materials, offered values, taking into account the properties of transversely isotropic values of the fiber. The same ratio but for the spatial mechanics problem of composites obtained in the work [4], based on kinematic conditions of approval. For transversely isotropic matrix and fiber obtained elastic characteristics of the composite material in the three-dimensional case, based on kinematic [5] and energy [6] approval conditions. Asymptotic approach to determine elastic-plastic, thermoelastic and other characteristics of fibrous composites described in [7]. Based on asymptotic approaches obtained effective elastic constants for composite material with transversely isotropic properties of the matrix and the fiber in the hexagonal reinforcement's structure [8]. Determination of effective constants for the composite material with a number of stochastic characteristics suggested in these works. In the work [9] defined the effective elastic characteristics of the fibrous composite with randomly oriented and randomly arranged fibers. The paper [10] is based on homogenization methods obtained transversely isotropic properties of the composite material reinforced by randomly distributed unidirectional circular fibers based on their interaction.

The current state of the composite materials production technologies allows to draw conclusions, about the mathematical modeling relevance of their properties in view of some stochastic composites characteristics. The aim of this paper is to determine the dependence of the effective longitudinal elastic modulus of the composite under volume content of fibers with a radius, by the normal distribution.

PROBLEM STATEMENT AND ITS GENERAL SOLUTION SCHEME

Research object is the one directional composite material with hexagonal arrangement of fibers, where the matrix and the fiber are considered to be isotropic. The diameter of the fiber is a random variable, by the normal distribution. It is needed to find effective longitudinal elastic modulus of the composite during its presentation in the form of homogeneous transversely isotropic materials, for the case of axially symmetric longitudinal tension.

An element of the fibrous composite material represented as a combination of two infinite isotropic cylinders, radial section scheme of which is shown in Fig. 1.

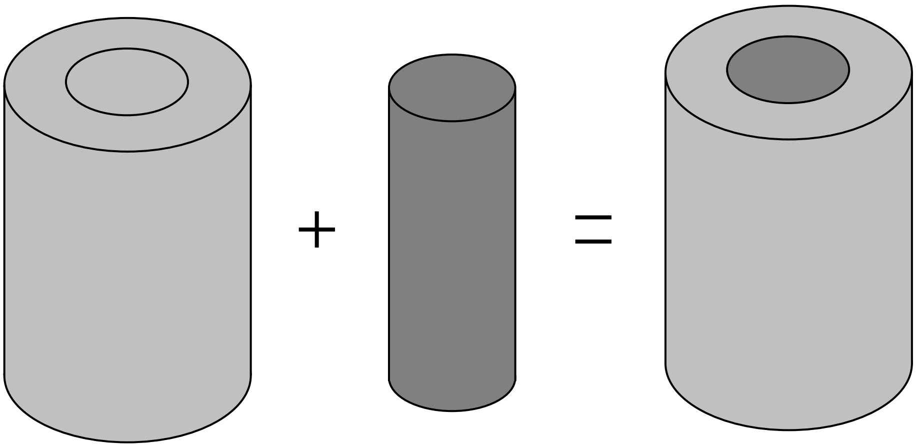

Fig. 1. The fibrous composite element

Fiber is considered as a solid cylinder with the radius a, an elementary hexagonal cell of the matrix approximates a hollow cylinder, whose radius is equal to b (Fig. 2).

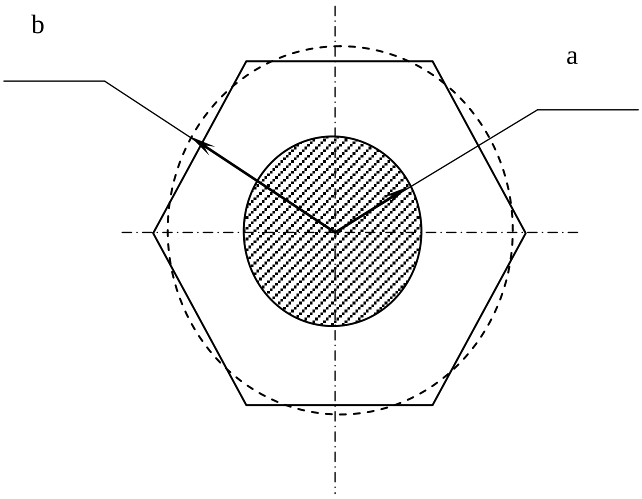

Fig. 2. Hexagonal cell

The value b is selected in order the fiber f volume content in the hexagonal cell and the cylinder was the same. In this case carried out the following equality

$$f = \frac{a^2}{b^2}. \quad (1)$$

Technological processes of composite materials production in many cases determine the random nature of certain composites parameters. In particular, the absence of systematic deviations in the application of industrial technologies, the following parameters can be considered as random variables with normal distribution. Let the radius a of the fiber is a random variable, normally distributed with known distribution parameters: the mathematical expectation a_0 and standard deviation s (these parameters can be determined on the basis of statistical quality control data of composite materials production). Then the f fiber's volume content in the cell is also a random variable, f_0 mathematical expectation of which is equal to:

$$f_0 = m(f) = \frac{m\left(a^2\right)}{b^2} = \frac{a_0^2 + s^2}{b^2}. \quad (2)$$

To determine the effective longitudinal elastic modulus of the composite, it is necessary to solve two boundary value problems. First, we solve the boundary value problem of joint deformation of the isotropic matrix and the isotropic fiber, where we will find components of the composite stress-strain state as a function of elastic constants of matrix and fiber materials as well as their volume fraction in the composite. Then solve the same boundary value problem for the composite, represented as homogeneous transversely isotropic materials with unknown elastic constants. In this case we obtain stress-strain state components as a function of elastic constants of a homogeneous material, which modeling the composite, i.e. effective elastic constants. These unknown values we will define using consistency conditions. These conditions, in particular, are the equality component of the displacement vector for the first and the second boundary value problems [5].

DETERMINATION OF THE COMPOSITE's EFFECTIVE ELASTIC MODULUS FOR LONGITUDINAL TENSION

Find the general solution of the problem on axisymmetrical deformation of an isotropic infinite cylinder with longitudinal tension in the cylindrical coordinate system, i.e. find components of the stress-strain state of this object.

These include radial ($u_r\left(r\right)$) and axial ($u_z\left(z\right)$) displacements of its points as well as radial (σ_{rr}), tangential ($\sigma_{\theta\theta}$) and axial (σ_{zz}) tensions and corresponding deformations. Axial stress σ_{zz} is constant: $\sigma_{zz} = \sigma_0 = \text{const}$.

From the basic relations of elasticity theory [11] we obtain the equation, relative to the radial displacement of u_r:

$$\frac{d^2u_r}{dr^2} + \frac{1}{r}\cdot\frac{du_r}{dr} - \frac{u_r}{r^2} = 0. \tag{3}$$

The general solution of this equation is:

$$u_r = C_1 r + \frac{C_2}{r}, \tag{4}$$

where C_1 and C_2 – arbitrary constants.

Obtained expression for the displacement and the main value of the elasticity theory for axisymmetrical problem allow to get the following expressions for the components of the stress-strain state:

$$\varepsilon_{rr} = \frac{du_r}{dr} = C_1 - \frac{C_2}{r^2}; \tag{5}$$

$$\varepsilon_{\theta\theta} = \frac{u_r}{r} = C_1 + \frac{C_2}{r^2}; \tag{6}$$

$$\sigma_{rr} = \frac{E\left(\nu\cdot\varepsilon_{zz} + C_1 + \left(2\nu - 1\right)\frac{C_2}{r^2}\right)}{\left(1-2\nu\right)\left(1+\nu\right)}; \tag{7}$$

$$\sigma_{\theta\theta} = \frac{E\left(\nu\cdot\varepsilon_{zz} + C_1 + \left(1-2\nu\right)\frac{C_2}{r^2}\right)}{\left(1-2\nu\right)\left(1+\nu\right)}. \tag{8}$$

Given these correlations and equality $\sigma_{zz} = \sigma_0$, we get an expression for the axial deformation of ε_{zz}:

$$\varepsilon_{zz} = \frac{\sigma_0 (1-2\nu)(1+\nu) - 2\nu E C_1}{E(1-\nu)}. \tag{9}$$

As $\varepsilon_{zz} = \frac{\partial u_z}{\partial z} = \text{const}$ and while $u_z(0) = 0$, the axial movement $u_z(z)$ is as follows:

$$u_z(z) = \int_0^z \varepsilon_{zz} dz_1 = \frac{\sigma_0 (1-2\nu)(1+\nu) - 2\nu E C_1}{E(1-\nu)} \cdot z. \tag{10}$$

Due to equation (10), the expressions for tensions σ_{rr} and $\sigma_{\theta\theta}$ is written in the following form:

$$\sigma_{rr} = E\left[\frac{\nu\sigma_0}{E(1-\nu)} + \frac{C_1}{1-\nu} - \frac{C_2}{r^2(1+\nu)}\right]; \tag{11}$$

$$\sigma_{\theta\theta} = E\left[\frac{\nu\sigma_0}{E(1-\nu)} + \frac{C_1}{1-\nu} + \frac{C_2}{r^2(1+\nu)}\right]. \tag{12}$$

Thus, we got the tension, deformations and displacements as functions of elastic properties of materials and arbitrary constants of integration, determined with the boundary conditions. Then we use them to determine the components of the stress-strain state of the solid cylinder ($0 \le r \le a$) that simulates the fiber and the hollow cylinder ($a \le r \le b$), which simulates matrix.

In the following designations we use the symbol $^\circ$ to denote quantities, characterizing the fiber and the symbol * to denote variables, related to the matrix.

On the surface of the fiber-matrix combination, radial displacements and tension are continuous, axial displacements with a fixed $z = h$ are matching. The outer surface of the cylinder, which modeling the matrix, the radial tension is missing. Thus, we have the boundary conditions:

$$\begin{cases} \sigma_{rr}^\circ(a) = \sigma_{rr}^*(a), \\ u_r^\circ(a) = u_r^*(a), \\ u_z^\circ(h) = u_z^*(h), \\ \sigma_{rr}^*(b) = 0. \end{cases} \tag{13}$$

Radial displacement of an isotropic fiber (4), taking into account the equality $u_r^\circ(0) = 0$, take the form of $u_r^\circ(r) = Cr$. Given this equality, of the relations (7), (8) and (10) for the fiber, we get:

$$u_z^\circ = \frac{1}{1-\nu^\circ}\left(\frac{\sigma_0^\circ(1-\nu^\circ - 2\nu^{\circ 2})}{E^\circ} - 2C\nu^\circ\right) z, \tag{14}$$

$$\sigma_{rr}^\circ = \frac{E^\circ}{1-\nu^\circ}\left(\frac{\sigma_0^\circ \nu^\circ}{E^\circ} + C\right), \tag{15}$$

$$\sigma_{\theta\theta}^\circ = \frac{E^\circ}{1-\nu^\circ}\left(\frac{\sigma^\circ \nu^\circ}{E^\circ} + C\right). \tag{16}$$

Similarly, we can write the ratio, describing the stress-strain state of the matrix:

$$u_r^*(r) = Ar + \frac{B}{r}, \tag{15}$$

$$u_z^* = \frac{1}{1-\nu^*}\left(\frac{\sigma_0^*\left(1-\nu^* - 2\nu^{*2}\right)}{E^*} - 2A\nu^*\right)z\,, \tag{16}$$

$$\sigma_{rr}^* = E^*\left(\frac{\sigma_0{}^*\nu^*}{E^*\left(1-\nu^*\right)} + \frac{A}{1-\nu^*} - \frac{B}{r^2\left(1+\nu *\right)}\right), \tag{17}$$

$$\sigma_{\theta\theta}^* = E^*\left(\frac{\sigma_0{}^*\nu^*}{E^\circ\left(1-\nu^*\right)} + \frac{A}{1-\nu^*} + \frac{B}{r^2\left(1+\nu^*\right)}\right). \tag{18}$$

A, B and C constants values, as well as the relationship between the axial tensions σ_0° and σ_0° we find, using the boundary conditions (13). From the second of these equations it follows, that

$$C = A + \frac{B}{a^2}\,. \tag{19}$$

From $\sigma_{rr}^*\left(b\right) = 0$ we receive:

$$A = \frac{B\left(1-\nu^*\right)}{b^2\left(1+\nu^*\right)} - \frac{\sigma_0{}^*\nu^*}{E^*}\,. \tag{20}$$

Then the expression (19) can be written as:

$$C = B\left[\frac{f\left(1-\nu^*\right)+1+\nu^*}{a^2\left(1+\nu^*\right)} - \frac{\sigma_0^*\nu^*}{E^*}\right]. \tag{21}$$

Using the first of equations (13), we find an expression for the constant B as:

$$B = \left(\frac{\sigma_0^\circ\nu^\circ}{E^\circ} - \frac{\sigma_0^*\nu^*}{E^*}\right)\cdot\frac{a^2E^\circ\left(1+\nu^*\right)}{E^*\left(f-1\right)\left(1-\nu^\circ\right) - E^\circ\left(f\left(1-\nu^*\right)+1+\nu^*\right)}.$$

Designate $d_1 = E^*\left(f-1\right)\left(1-\nu^\circ\right)$, $d_2 = E^\circ\left(f\left(1-\nu^*\right)+1+\nu^*\right)$. After transformations, we get A, B and C constant values as follows:

$$A = \frac{f\nu^\circ\left(1-\nu^*\right)}{d_1-d_2}\sigma_0^\circ - \frac{\nu^*}{E^*}\frac{fE^\circ\left(1-\nu^*\right)+d_1-d_2}{d_1-d_2}\sigma_0^*, \tag{22}$$

$$B = \frac{\nu^\circ a^2\left(1+\nu^*\right)}{d_1-d_2}\sigma_0^\circ - \frac{a^2E^\circ\left(1+\nu^*\right)}{d_1-d_2}\frac{\nu^*}{E^*}\sigma_0^*, \tag{23}$$

$$C = \frac{d_2}{d_1-d_2}\frac{\nu^\circ}{E^\circ}\sigma_0^\circ - \frac{\nu^*}{E^*}\frac{d_1}{d_1-d_2}\sigma_0^*. \tag{24}$$

From the condition of $u_z^\circ\left(h\right) = u_z^*\left(h\right)$ find a ratio between σ_0^* and σ_0°:

$$\sigma_0^\circ d^\circ = \sigma_0^* d^*, \tag{25}$$

$$d^\circ = \frac{E^*\left(f-1\right)\left(1-\nu^\circ - 2\nu^{\circ 2}\right) - E^\circ\left(f\left(1-\nu^* - 2\nu^\circ\nu^*\right)+1+\nu^*\right)}{E^\circ}, \tag{26}$$

$$d^{*}=\frac{E^{*}\left(f-1\right)\left(1-\nu^{\circ}-2\nu^{\circ}\nu^{*}\right)-E^{\circ}\left(f\left(1-\nu^{*}-2\nu^{*2}\right)+1+\nu^{*}\right)}{E^{*}}. \qquad (27)$$

Let’s consider a similar problem for a homogeneous transversely isotropic materials, modeling characteristics of the composite. In this case, the stress field will be determined by the following relations:

$$\sigma_{zz}=\sigma_0=\text{const},\ \sigma_{rr}=0,\ \sigma_{\theta\theta}=0,\ \sigma_{\text{zr}}=\sigma_{z\theta}=\sigma_{r\theta}=0. \qquad (28)$$

Under this, equilibrium condition for both problems must match and in this case, carried out the following equality $\pi a^2\sigma_0^{\circ}+\pi\left(b^2-a^2\right)\sigma_0^{*}=\pi b^2\sigma_0$, which can be written as:

$$f\sigma_0^{\circ}+\left(1-f\right)\sigma_0^{*}=\sigma_0. \qquad (29)$$

Due to equation (25), from the last formula we get:

$$\sigma_0^{*}=\frac{\sigma_0 d^{\circ}}{d^{\circ}+f\left(d^{*}-d^{\circ}\right)}, \qquad (30)$$

$$\sigma_0^{\circ}=\frac{\sigma_0 d^{*}}{d^{\circ}+f\left(d^{*}-d^{\circ}\right)}. \qquad (31)$$

Using equations (30) and (31), from the formulas of Hooke’s law for transversely isotropic materials, we obtain expressions for the deformation: $\varepsilon_{rr}=-\frac{\nu_{21}}{E_2}\sigma_0$, $\varepsilon_{zz}=-\frac{\sigma_0}{E_1}$. This equalities allow us to determine the displacement $u_r\left(r\right)$ and $u_z\left(z\right)$:

$$u_r\left(r\right)=-\frac{\nu_{21}}{E_2}\sigma_0 r+C_1,$$

$$u_z\left(z\right)=\frac{\sigma_0 z}{E_1}+C_2.$$

From the conditions of $u_r\left(0\right)=0$ and $u_z\left(0\right)=0$ it follows that $C_1=C_2=0$, i.e. expressions for the displacements take the form of:

$$u_r\left(r\right)=-\frac{\nu_{21}}{E_2}\sigma_0 r, \qquad (32)$$

$$u_z\left(z\right)=\frac{\sigma_0 z}{E_1}. \qquad (33)$$

Let us choose agreeing conditions to the problem of longitudinal tensile of homogeneous transversely isotropic composite, and the problem of compatible longitudinal tensile isotropic matrix and fiber conditions of equality axial displacement for an arbitrary $z=h$:

$$u_z\left(h\right)=u_z^{\circ}\left(h\right)=u_z^{*}\left(h\right). \qquad (34)$$

Due to the obtained expression (33) for moving $u_z\left(z\right)$, from equation (34) we obtain:

$$\frac{\sigma_0^{*}\left(1-\nu^{*}-2\nu^{*2}\right)}{E^{*}\left(1-\nu^{*}\right)}-\frac{2A\nu^{*}}{1-\nu^{*}}=\frac{1}{E_1}\sigma_0.$$

Hence, using formula (22), (30) and (31), we obtain an expression for the effective longitudinal elastic modulus E_1 of the composite material:

$$E_1 = \frac{\left(d^\circ(1-f)+d^* f\right)\left(E^*(f-1)\left(1-\nu^\circ\right)+E^\circ\left(\nu^*(f-1)-1-f\right)\right)E^*}{\left(E^*(f-1)\left(1-\nu^\circ\right)-E^\circ\left(1+\nu^*+f\left(1-\nu^*-2\nu^{*2}\right)\right)\right)d^\circ-2f\nu^*\nu^\circ E^* d^*}. \qquad (35)$$

The formula (35) of effective longitudinal elastic modulus of the composite coincides with a similar formula obtained in the study [5] for the case of transversely isotropic matrix and fiber, if we take the matrix and the fiber as isotropic materials.

As the radius of the fiber a is a random variable having a normal distribution of the mathematical expectation a_0 and standard deviation s, the volume content $f(a)=\frac{a^2}{b^2}$ of the fiber in the cell is also a random variable, the distribution density of $\varphi(f)$ is determined by taking into account the fact that the function $f(a)$ is not monotonous:

$$\varphi(f)=\frac{b}{2s\sqrt{2\pi f}}\left(e^{-\frac{\left(b\sqrt{f}+a_0\right)^2}{2s^2}}+e^{-\frac{\left(b\sqrt{f}-a_0\right)^2}{2s^2}}\right)$$

if $f>0$.

Then, the mathematical expectation value of a random variable E_1 is determined by convergent improper integral:

$$M\left(E_1\right)=\int_0^{+\infty} E_1(f)\cdot\varphi(f)\,df, \qquad (36)$$

where the random function argument $E_1(f)$ determined by the equality (35).

Thus, we have identified the mathematical expectation of effective longitudinal modulus of the composite material that allows to use this indicator in the study of mechanical properties of composites, matrix and fiber of which are isotropic.

CONCLUSIONS

Proposed approach for determining the effective longitudinal elastic modulus for composite material composed of an isotropic matrix and fiber, on the basis of kinematic conditions of approval. The radius of the fiber was considered as a continuous random variable, normally distributed, so the volume content of fibers is also a random variable, the mathematical expectation of which is expressed through the mathematical expectation of the fiber's radius. To determine the effective longitudinal elastic modulus of the composite initially is solved boundary value problem for joint deformation of the isotropic matrix and the isotropic fiber, which determine the components of stress and strain as a function of technical elastic constants of the matrix and fiber as well as fiber volume content in the cell of the matrix. And then, the acquired solution of the corresponding boundary-value problem for a composite model of which considers homogeneous transversely isotropic cylinder. Using kinematic coordination conditions allowed us to determine the effective longitudinal modulus of the composite, as a function of the determined values - elastic constants of the matrix and the fiber, as well as occasional argument – fiber volume content. It is defined the mathematical expectation of this indicator.

Proposed in the paper methodology allows to carry out further research towards the determining of effective elastic constants for composites with properties, that are random variables with different distributions of these quantities.

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НЕЛІНІЙНИЙ АНАЛІЗ ДИНАМІКИ МЕХАНІЧНОЇ СИСТЕМИ ІЗ ЗОСЕРЕДЖЕНИМ АБСОРБЕРОМ

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У роботі досліджено коливання літального апарату з абсорбером поблизу збуреної поверхні. Показано, що цей процес моделюється системою диференціальних рівнянь у частинних похідних. Цю систему розв'язано у двох випадках: геометрично лінійному та нелінійному, що характеризує собою можливі великі деформації балки при вимушених коливаннях. Для розв'язання задачі в лінійному випадку застосовуються метод Фур'є, метод варіації довільних сталих, а в нелінійному – метод малого параметра та вище зазначені методи. Числові результати візуалізовано графічно.

Ключові слова: метод Фур'є, метод малого параметра, метод варіації довільних сталих, система диференціальних рівнянь, поперечне навантаження, прогин балки, зосереджена маса, демпферна система, нелінійні коливання.