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UDK 539.3

JUSTIFICATION OF THE GENERALIZED FOURIER METHOD FOR THE MIXED PROBLEM OF ELASTICITY THEORY IN THE HALF-SPACE WITH THE CYLINDRICAL CAVITY

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The mixed problem of the elasticity theory for the homogeneous isotropic half-space with the infinite circular cylindrical cavity, parallel to the boundary of the half-space, is considered. These investigations are of practical interest in connection with problems of geomechanics and geotechnical engineering. The aim of the work is to substantiate and to apply the research method of the stress-strain state of elastic half-space with a circular cylindrical cavity in the case when the stresses are set on a half-space boundary and the displacements are set on a cavity surface.

A boundary value problem for the Lamé equation with the appropriate boundary conditions in the given domain is solved by the generalized Fourier method. The general solution of the boundary value problem is presented as a superposition of the external basis solutions of the Lamé equation for the cylinder and the internal basis solutions for the half-space. The addition theorems of the basis solutions of the Lamé equations for the cylinder and the half-space allow to satisfy the boundary conditions and to reduce the problem to the infinite system of linear algebraic equations which is solved by the reduction method. It is proved that the operator of the system is quite continuous in space l_2 . The results of numerical calculations have been presented.

Key words: generalized Fourier method; elastic half-space; cylindrical cavity; basis solutions of the Lamé equation; addition theorems; reduction method.

ОБОСНОВАНИЕ ОБОБЩЕННОГО МЕТОДА ФУРЬЕ ДЛЯ СМЕШАННОЙ ЗАДАЧИ ТЕОРИИ УПРУГОСТИ В ПОЛУПРОСТРАНСТВЕ С ЦИЛИНДРИЧЕСКОЙ ПОЛОСТЬЮ

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Рассматривается смешанная задача теории упругости для однородного изотропного полупространства с бесконечной круговой цилиндрической полостью, параллельной его границе. Эта задача представляет практический интерес в связи с проблемами геомеханики и геотехнической инженерии. Цель работы – обоснование и применение метода исследования напряженно-деформированного состояния упругого полупространства с круговой цилиндрической полостью в случае, когда на границе полупространства заданы напряжения, а на поверхности полости – перемещения. Решение однородного уравнения Ламе с соответствующими граничными условиями в этой области найдено обобщенным методом Фурье. Общее решение задачи представляется в виде суперпозиции базисных решений для цилиндра и полупространства. С помощью теорем сложения этих решений удовлетворяются граничные условия и задача сводится к бесконечной системе линейных алгебраических уравнений. Доказана теорема о том, что оператор системы является вполне непрерывным в пространстве l_2 . Это позволяет решать систему методом редукции. Приведены результаты численных расчетов.

Ключевые слова: обобщенный метод Фурье; упругое полупространство; цилиндрическая полость; базисные решения уравнения Ламе; теоремы сложения; метод редукции.

ОБҐРУНТУВАННЯ УЗАГАЛЬНЕНОГО МЕТОДУ ФУР'Є ДЛЯ МІШАНОЇ ЗАДАЧІ ТЕОРІЇ ПРУЖНОСТІ В НАПІВПРОСТОРІ З ЦИЛИНДРИЧНОЮ ПОРОЖНИНОЮ

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У статті розглядається мішана задача теорії пружності для однорідного ізотропного напівпростору з нескінченною круговою циліндричною порожниною, паралельною до його поверхні. Ця задача становить практичний інтерес у зв'язку з проблемами геомеханіки і геотехнічної інженерії. Метою дослідження є обґрунтування і застосування методу дослідження напружено-деформованого стану пружного напівпростору з круговою циліндричною порожниною у разі, коли на поверхні напівпростору задані напруження, а на поверхні порожнини – переміщення. Розв'язок однорідного рівняння Ламе з відповідними граничними умовами в цій області знайдено за допомогою узагальненого методу Фур'є. Загальний розв'язок задачі представляється у вигляді суперпозиції базисних розв'язків для циліндра і напівпростору. За допомогою теорем додавання цих розв'язків задовольняються граничні умови і задача зводиться до нескінченної системи лінійних алгебраїчних рівнянь. Доведено теорему про те, що оператор системи є цілком неперервним у просторі l_2 . Це дозволяє розв'язати систему методом редукції. Представлені результати чисельних розрахунків.

Ключові слова: узагальнений метод Фур'є; пружний напівпростір; циліндрична порожнина; базисні розв'язки рівняння Ламе; теорема додавання; метод редукції.

INTRODUCTION

Questions of calculation of strength and reliability arise in the design of underground tunnels, mines and mining [1, p.24]. As a model of such objects we can use an infinite hollow cylinder in an elastic half-space. We can consider this domain as a multiply connected elastic body and solve the main problems of the elastic theory for this body, in particular, determine the stress and the strain near a cylindrical surface.

The stress-strain state of an elastic half-space having a cavity of finite size has been studied intensively by many authors. For an infinite half-space with the cylindrical cavity the main problems of the potential theory and the elastic theory have been considered in the papers [2, p.52;

3, p.102; 4, p.17; 5, p.189; 6, p.192]. The generalized Fourier method [7, p.83] was used for the solution of these problems. However justification of this method to the solution of the mixed problem of the elastic theory was not given.

The aim of this work is justification and applying of the generalized Fourier method for investigation of the stress-strain state of an elastic half-space with a circular cylindrical cavity in a case when the stresses are set on a half-space boundary and the displacements are set on a cavity surface. This problem is of special interest for practice.

FORMULATION OF THE PROBLEM

Denote by Ω a half-space with an infinite circular cylindrical cavity. Suppose the cylinder is parallel to the boundary of the half-space. The domain Ω is filled with a homogeneous isotropic elastic medium. We introduce a rectangular Cartesian coordinate system $\{x, y, z\}$ and a cylindrical coordinate system $\{\rho, \varphi, z\}$. The z -axis coincides with the cylinder axis. The y -axis is directed vertically upward. Let S_1 ($y = h$) be the boundary of the half-space, and S_2 ($\rho = a$) the surface of the cylinder. Here, a is radius of the cylinder, and h is the distance between the cylinder axis and the boundary of the half-space, with $a < h$. The domain Ω is defined by the system of inequalities $\{y < h, \rho > a\}$.

Consider the boundary value problem for the Lamé equation

$$\Omega: \Delta \vec{u} + \frac{1}{1-2\sigma} \nabla \operatorname{div} \vec{u} = 0, \tag{1}$$

$$F\vec{u}|_{S_1} = F\vec{u}_{01}(x, z), \tag{2}$$

$$\vec{u}|_{S_2} = \vec{u}_{02}(\varphi, z), \tag{3}$$

where σ is the Poisson's ratio. We use the generalized Fourier method for the solution of the problem (1)-(3).

GENERALIZED FOURIER METHOD

According to the generalized Fourier method, the system of basis solutions of the Lamé equation is introduced for each boundary surface of the domain Ω . These solutions are written in the coordinate system associated with the corresponding surface.

We seek the general solution of the boundary value problem as superposition of the basis solutions with unknown integral densities and unknown series coefficients. These integral densities and series coefficients are determined by the boundary conditions.

The basis solutions of the Lamé equation for the half-space and the cylinder are constructed in the papers [2, p.53; 3, p.103]. These solutions are written in the Cartesian and the cylindrical coordinate systems respectively. By $\vec{u}_k^{(\pm)}(x, y, z; \lambda, \mu)$ denote the internal (external) basis solutions for the half-space. They are the vector functions regular in $\{y < h\}$ ($\{y > h\}$). By $\vec{R}_{k,m}(\rho, \varphi, z; \lambda)$ ($\vec{S}_{k,m}(\rho, \varphi, z; \lambda)$) denote the internal (external) basis solutions for the cylinder. They are the vector functions regular in $\{\rho < a\}$ ($\{\rho > a\}$). These basis solutions have the form

$$\vec{u}_i^{(\pm)}(x, y, z; \lambda, \mu) = N_i^{(1)} u^\pm(x, y, z; \lambda, \mu),$$

$$\vec{u}_2^{(\pm)}(x, y, z; \lambda, \mu) = \frac{4(\sigma-1)}{\lambda} u^\pm(x, y, z; \lambda, \mu) \vec{e}_2^{(1)} + \frac{1}{\lambda} \nabla (y u^\pm(x, y, z; \lambda, \mu)),$$

$$\begin{aligned} \vec{R}_{k,m}(\rho, \varphi, z; \lambda) &= N_k^{(2)} r_m(\rho, \varphi, z; \lambda), \\ \vec{S}_{k,m}(\rho, \varphi, z; \lambda) &= N_k^{(2)} s_m(\rho, \varphi, z; \lambda). \end{aligned}$$

Here

$$\begin{aligned} N_1^{(j)} &= \frac{1}{\lambda} \nabla, \quad N_3^{(j)} = \frac{i}{\lambda} \text{rot}(\vec{e}_3^{(j)} \cdot), \\ N_2^{(2)} &= \frac{1}{\lambda} \left[\nabla \left(\rho \frac{\partial}{\partial \rho} \right) + (4\sigma - 4) \left(\nabla - \vec{e}_3^{(2)} \frac{\partial}{\partial z} \right) \right], \\ i &= 1, 3, \quad j = 1, 2, \quad k = 1..3, \quad m = 0, \pm 1, \pm 2, \dots \end{aligned}$$

In these expressions the vectors $\vec{e}_j^{(k)}$ are the orts of the Cartesian ($k=1$) and cylindrical ($k=2$) coordinate system, and the functions $u^\pm(x, y, z; \lambda, \mu)$, $r_m(\rho, \varphi, z; \lambda)$, $s_m(\rho, \varphi, z; \lambda)$ represent the Cartesian and cylindrical basis solutions of the Laplace equation [8, p.58, p.73]

$$\begin{aligned} u^\pm(x, y, z; \lambda, \mu) &= e^{i\mu x + i\lambda z \pm \gamma y}, \\ r_m(\rho, \varphi, z; \lambda) &= e^{im\varphi + i\lambda z} I_m(\lambda\rho), \\ s_m(\rho, \varphi, z; \lambda) &= (\text{sign}\lambda)^m e^{im\varphi + i\lambda z} K_m(|\lambda|\rho), \end{aligned}$$

where $I_m(\lambda\rho)$, $K_m(|\lambda|\rho)$ are the modified Bessel functions of the first and second kind of order m respectively, $\gamma = \sqrt{\lambda^2 + \mu^2}$, $\lambda, \mu \in (-\infty, \infty)$.

The general solution of the boundary value problem (1)-(3) can be represented as a superposition of the external basis solutions of the Lamé equation for the cylinder $\vec{S}_{k,m}(\rho, \varphi, z; \lambda)$ and the internal basis solutions for the half-space $\vec{u}_p^{(+)}(x, y, z; \lambda, \mu)$

$$\vec{u} = \sum_{k=1}^3 \sum_{m=-\infty}^{\infty} \int B_{km}(\lambda) \vec{S}_{k,m}(\rho, \varphi, z; \lambda) d\lambda + \sum_{p=1}^3 \int_{-\infty}^{\infty} \int H_p(\lambda, \mu) \vec{u}_p^{(+)}(x, y, z; \lambda, \mu) d\mu d\lambda. \tag{4}$$

In our case $B_{km}(\lambda)$ and $H_p(\lambda, \mu)$ are unknown integral densities.

Consider the boundary conditions (2) and (3). We write the second term of the expression (4) in the cylindrical coordinate system. Then, to satisfy the boundary condition (3) we apply the re-expansion formulas of the internal basis solutions for the half-space $\vec{u}_p^{(+)}(x, y, z; \lambda, \mu)$ on the internal cylindrical basis solutions $\vec{R}_{k,m}(\rho, \varphi, z; \lambda)$ [2, p.53]. We obtain a system of linear algebraic equations with respect to the integral densities $B_{km}(\lambda)$ ($k=1..3, m=0, \pm 1, \pm 2, \dots$). Let $D_1^m(|\lambda|a, \sigma)$ be the determinant of this system. This determinant is not equal to zero and bounded below [4, p.20]

$$D_1^m(|\lambda|a, \sigma) \geq 4(1-2\sigma) \prod_{\alpha=-1}^1 K_{m+\alpha}(|\lambda|a). \tag{5}$$

To satisfy the boundary condition (2), we write the first term of the expression (4) using the re-expansion formulas of the external cylindrical basis solutions $\vec{S}_{k,m}(\rho, \varphi, z; \lambda)$ on the external basis solutions for the half-space $\vec{u}_l^{(-)}(x, y, z; \lambda, \mu)$ [2, p.54]. We obtain a system of linear algebraic

equations defining the functions $H_p(\lambda, \mu)$ ($p=1..3$). Let $D_2(\lambda, \mu)$ be the determinant of this system. This determinant is given by $D_2(\lambda, \mu) = -4G^3\gamma^4 e^{3\gamma h} / \lambda^2$, where G is the shear modulus. This determinant is not equal to zero. Solving this system, we express the integral densities $H_p(\lambda, \mu)$ in terms $B_{km}(\lambda)$. As a result the first system of linear algebraic equations reduces to the infinite system of linear algebraic equations defining $B_{km}(\lambda)$

$$B_{km}(\lambda) = \sum_{j=1}^3 \sum_{n=-\infty}^{\infty} G_{kj}^{mn}(\lambda) B_{jn}(\lambda) + Q_k^m(\lambda),$$

$$k = 1..3, m = 0, \pm 1, \pm 2, \dots \tag{6}$$

This system can be rewritten as $(I + G)\vec{b} = \vec{q}$, where G is an operator of the system, I the unit operator, \vec{b} the column vector of the unknowns $B_{jn}(\lambda)$, and \vec{q} the column vector of right part $Q_k^m(\lambda)$.

Theorem The operator G of the system (6) is quite continuous in l_2 if boundary surfaces S_1 and S_2 are not intersect ($a < h$).

Proof. Consider a double series

$$\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} |G_{kj}^{mn}(\lambda)|, \tag{7}$$

where $k, j = 1..3$. Let's prove the convergence of this series. The functions $G_{kj}^{mn}(\lambda)$ include the integrals reducible to the Laplace integral. The Laplace integral may be given by $2K_{m+n+\alpha}(2\lambda h)$, where $\alpha \in N$. The series (7) contains expressions

$$\sum_{m=-\infty}^{\infty} \left| \frac{f_1^{mp}(|\lambda|a) f_2^{mr}(|\lambda|a) f_3^{ms}(\lambda a)}{(m^2 + 1) I_m(\lambda a) D_1^m(|\lambda|a, \sigma)} \right| \times \sum_{n=-\infty}^{\infty} (n^2 + 1) |I_n(\lambda a) K_{m+n+\alpha}(2\lambda h)|. \tag{8}$$

Here $f_1^{mp}(|\lambda|a)$ is one of the functions $K_{m+p}(|\lambda|a)$ ($p \neq 0$), $\left(\rho \frac{\partial K_{m+p}(|\lambda|\rho)}{\partial \rho} \right) \Big|_{\rho=a}$,

$f_2^{mr}(|\lambda|a) = K_{m+r}(|\lambda|a)$, $f_3^{ms}(\lambda a)$ is one of the functions $I_{m+s}(\lambda a)$, $\left(\rho \frac{\partial I_{m+s}(\lambda\rho)}{\partial \rho} \right) \Big|_{\rho=a}$, with

$$p = 0, \pm 1, r = \pm 1, s = 0, \pm 1.$$

Assume that $\lambda > 0$. We write the addition theorem for the modified Bessel functions

$$\sum_{n=-\infty}^{\infty} K_{m+n}(\lambda y) I_n(\lambda x) \cos n\varphi = K_m(\lambda z) \cos m\phi, \tag{9}$$

where $z = \sqrt{x^2 + y^2 - 2xy \cos \varphi}$, $\sin \phi = \frac{x}{z} \sin \varphi$. Differentiating this equality on a variable φ twice, we will put $\varphi = 0$, $\phi = 0$, $x = a$, $y = 2h$, then

$$\sum_{n=-\infty}^{\infty} n^2 K_{m+n}(2\lambda h) I_n(\lambda a) = \frac{\lambda ah}{\xi} [K_{m+1}(\lambda \xi) + K_{m-1}(\lambda \xi)], \tag{10}$$

where $\xi = 2h - a$. On applying the equalities (9), (10), we summarize the series (8) over an index n and obtain

$$\sum_{m=-\infty}^{\infty} \frac{f_1^{mp}(\lambda a) f_2^{mr}(\lambda a) f_3^{ms}(\lambda a)}{(m^2 + 1) I_m(\lambda a) D_1^m(|\lambda| a, \sigma)} \times \left(K_{m+\alpha}(\lambda \xi) + \frac{\lambda a h}{\xi} (K_{m+\alpha+1}(\lambda \xi) + K_{m+\alpha-1}(\lambda \xi)) \right). \quad (11)$$

By $L(\lambda, a, h)$ denote the expression (11). Taking into account the estimate (5), we get

$$L(\lambda, a, h) < \frac{1}{4(1-2\sigma)} \times \sum_{m=-\infty}^{\infty} \frac{f_1^{mp}(\lambda a) f_2^{mr}(\lambda a) f_3^{ms}(\lambda a)}{K_m(\lambda a) K_{m-1}(\lambda a) K_{m+1}(\lambda a)} \times \left(\frac{K_{m+\alpha}(\lambda(\xi))}{(m^2 + 1) I_m(\lambda a)} + \frac{\lambda a h}{\xi} \frac{K_{m+\alpha+1}(\lambda \xi) + K_{m+\alpha-1}(\lambda \xi)}{(m^2 + 1) I_m(\lambda a)} \right). \quad (12)$$

By $L_{prs}^\alpha(\lambda, a, h)$ denote a series contained in right part of this inequality. For example,

$$L_{-110}^0(\lambda, a, h) = \sum_{m=-\infty}^{\infty} \frac{1}{(m^2 + 1) K_m(\lambda a)} \left((K_m(\lambda \xi) + \frac{\lambda a h}{\xi} (K_{m-1}(\lambda \xi) + K_{m+1}(\lambda \xi))) \right).$$

Using the addition theorem (9) and the inequality [9, p.426]

$$\frac{1}{K_m(\nu)} \leq 4\sqrt{\pi} e(1+\nu) m^2 I_m(\nu),$$

with $\nu > 0$, $|m| \geq 1$, it can be shown that $L_{-110}^0(\lambda, a, h)$ is bounded above by a continuous positive function of λ $L_{-110}^0(\lambda, a, h) \leq g(\lambda)$. The function $g(\lambda)$ has a finite limit at the point $\lambda = 0$, and $g(\lambda) \rightarrow 0$ as $\lambda \rightarrow \infty$. The function $L_{-110}^0(\lambda, a, h)$ is positive and bounded for all $\lambda \in [0, \infty)$

$$L_{-110}^0(\lambda, a, h) \leq \max_{\lambda \in [0, \infty)} L_{-110}^0(\lambda, a, h) = \tilde{L}_{-110}^0,$$

where $\tilde{L}_{-110}^0 = const$.

Similarly, it can be proved that the function $L_{prs}^\alpha(\lambda, a, h)$ is positive and bounded above for all values p, r, s and α . It follows that series (7) converges for positive values of the λ under the condition $a < h$. Under the same condition the series made from the squares of the modules $|G_{kj}^{mn}(\lambda)|^2$ also converges.

The case of the negative values λ is reduced to previous by means of replacement $\lambda = -\kappa$, with $\kappa > 0$.

Hence the series $\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} |G_{kj}^{mn}(\lambda)|^2$ converges for all $\lambda \in (-\infty, \infty)$ under the condition $a < h$, and the operator G of the system (6) is quite continuous in l_2 . The **Theorem** is proved.

In the same way, it can be proved that under the condition $a < h$ the series made from the modules of the right parts $|Q_k^m(\lambda)|$ of the system (6) converges for all $\lambda \in (-\infty, \infty)$. Thus, the right parts $Q_k^m(\lambda)$ of the system (6) belong to the space l_2 .

From Gilbert's alternative and belonging the right parts of (6) to the space l_2 , it follows that the system of equations (6) is solvable and has a unique solution in the space l_2 . An approximate solution of the problem can be obtained by the reduction method.

RESULTS

According to a reduction method, the infinite system of the linear algebraic equations for the unknown integral densities $B_{km}(\lambda)$ was replaced by the finite system of the linear algebraic equations. To estimate the rate of convergence for the method we calculate an approximation of the function \vec{u} on the cylinder surface and of the function $F\vec{u}$ on the boundary of the half-space. We consider that the solution is found if an error of approximation of the boundary conditions does not exceed 10^{-6} .

We compute the solution of the problem (1)-(3) for the functions $F\vec{u}_{01}(x, z)/(2G) = (0, -\cos(\lambda z)/(1+(x/l)^2), 0)$ and $\vec{u}_{02}(\varphi, z) = \vec{0}$ and for various values of a geometrical parameter $\varepsilon = a/h$. Calculations show that the stresses in the body concentrate near the cavity surface. The stress distribution on the cylinder is presented in Fig. 1, 2.

Fig. 1 shows the normal stresses σ_ρ , σ_φ and the tangential stress $\tau_{\rho\varphi}$ on the cylinder surface in a plane $z=0$ for $\varepsilon=0.3$, $\sigma=0.25$, $\lambda=1$ and $l=1$. The dimensionless stresses are presented. They are non-dimensionalized by division by Young's modulus. The σ_ρ component reaches the greatest values. At $\varphi \in [0, \pi]$ σ_ρ is compressive stress and at $\varphi \in [\pi, 2\pi]$ is tensile stress.

Fig. 2 gives the σ_ρ on the cylinder surface for various values of the parameter ε . The stress in the body significantly depends on the geometrical parameter ε . They sharply increase as $\varepsilon \rightarrow 1$. The largest compressive stresses act in the domain between the boundary of the half-space and the cavity. The largest tensile stresses arise in symmetrically located areas under the cylinder at $\varphi \approx 7\pi/6$ and $\varphi \approx 11\pi/6$. The occurrence of the tensile stresses can lead to destruction of the elastic body.

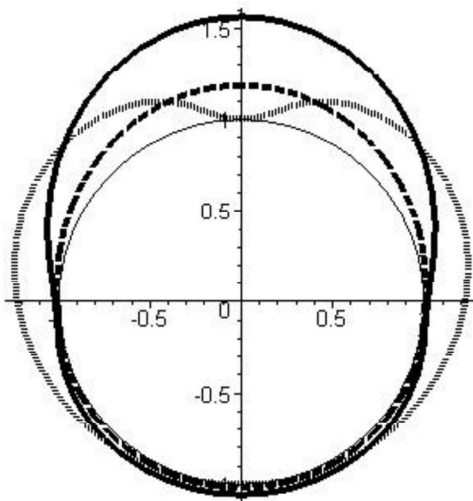


Fig. 1. The stress distribution on the cylinder surface: σ_ρ (solid line), σ_φ (dashed line) and $\tau_{\rho\varphi}$ (dotted line)

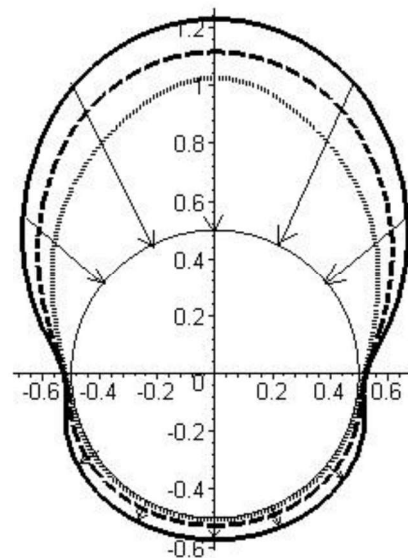


Fig. 2. The normal stress σ_ρ on the cylinder surface for $\varepsilon = 0.3$ (dotted line), $\varepsilon = 0.5$ (dashed line) and $\varepsilon = 0.9$ (solid line)

In conclusion we note that the known numerical methods of solution of the space problems for the multiply connected elastic body, for example, a finite element method, are not applicable for the infinite domains and are ineffective in case of close located boundary surfaces. The generalized Fourier method, used in the present paper, allows to obtain the solution in case of close located boundary surfaces by rather small increase in an order of the system of the linear algebraic equations.

CONCLUSIONS

The mixed problem of the elastic theory for a half-space with a circular cylindrical cavity is considered. The generalized Fourier method is used for the solution of the problem. Application of addition theorems of the basis solutions of the Lamé equation for the half-space and for the cylinder allows to satisfy the boundary conditions and to reduce the problem to the infinite system of the linear algebraic equations. The theorem that the operator of the system is quite continuous in space l_2 was proved. The system was solved by a reduction method. Results of calculations have been discussed.

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УДК 539.3

ОСОБЕННОСТИ ДЕФОРМИРОВАННОГО СОСТОЯНИЯ ЭЛАСТОМЕРНОГО ВИБРОИЗОЛЯТОРА ПРИ РАЗЛИЧНЫХ МЕХАНИЧЕСКИХ ХАРАКТЕРИСТИКАХ

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Рассматривается задача определения деформированного состояния эластомерного элемента сложной геометрической формы при различных механических характеристиках резины. Эластомерный материал имеет ряд уникальных свойств – высокую механическую прочность, эластичность и слабую сжимаемость. В связи с этим для адекватного описания поведения конструкций из эластомеров в условиях эксплуатации нужны специальные приемы и методы решения поставленных задач. Приведен обзор различных подходов к решению проблемы нахождения напряженно-деформированного состояния эластомерных элементов методом конечных элементов. Все рассмотренные методы основаны либо на системе упрощающих гипотез, либо имеют вид, не удобный для использования в расчетах, либо позволяют производить расчеты лишь для частных случаев конкретных постановок задач.

Для расчетов была применена схема конечного элемента, которая заключается в тройной аппроксимации полей перемещений, деформаций и функции изменения объема. Причем порядок разложения деформации и функции изменения объема должен находиться в строгом соответствии с порядком разложения перемещений. Данная схема получила название моментной схемы конечного элемента для слабосжимаемого материала.

При выборе рациональных параметров резиновых деталей машин большое значение имеет правильная оценка технических свойств резин, применяемых в современном машиностроении. Проведен расчет деформированного состояния эластомерного виброизолятора при различных механических характеристиках. Приведенные результаты численных расчетов могут быть применены при выборе марки резины для практического применения.

Ключевые слова: эластомеры, виброизоляторы, моментная схема конечного элемента, деформированное состояние.