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AN ARC CRACK AT THE INTERFACE BETWEEN TWO ELECTROSTRICTIVE MATERIALS

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Exact analytical solution for an electrostrictive plane with circular electrostrictive inclusion and an arc crack at the materials interface under the influence of general mechanical and electrical loadings at infinity is obtained. It is assumed that both materials are isotropic and linear elastic, the crack faces don't interact with each other and are permeable to an electric field. The problem is considered as an uncoupled problem of electroelasticity. Solution of electrostatics problem is obtained by complex potentials method. Boundary problem of electroelasticity for four complex potentials that are analogues of Kolosov-Muskhelishvili potentials is reduced to the problem of linear relationship at the crack. Unknown constants in general solution of this problem are determined from the boundary conditions at infinity and the restrictions imposed on stresses and displacements. Analytical expressions for the stress-strain state in the whole plane, in particular for the crack opening, normal and shear stresses at materials interface and the stress intensity factors at the crack tips, are found.

Key words: electrostriction, arc crack, problem of linear relationship, stress intensity factor.

ДУГОВАЯ ТРЕЩИНА НА ГРАНИЦЕ РАЗДЕЛА ДВУХ ЭЛЕКТРОСТРИКЦИОННЫХ МАТЕРИАЛОВ

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Получено точное аналитическое решение для электрострикционной плоскости с круговым электрострикционным включением и дуговой трещиной на границе раздела материалов под действием произвольных механических и электрических нагрузок на бесконечности. Принимается, что оба материала являются изотропными и линейно упругими, а берега трещины не взаимодействуют друг с другом и являются проницаемыми для электрического поля. Задача рассматривается как несвязанная задача электроупругости. Решение задачи электростатики получено с помощью метода комплексных потенциалов. Граничная задача электроупругости для четырех комплексных потенциалов, являющихся аналогами потенциалов Колосова-Мусхелишвили, сведена к задаче линейного сопряжения на трещине. Неизвестные константы из общего решения этой задачи определены из граничных условий на бесконечности и ограничений, наложенных на напряжения и перемещения. Найдены аналитические выражения для напряженно-деформированного состояния всей плоскости, в частности для раскрытия трещины, нормальных и касательных напряжений на границе раздела сред и коэффициентов интенсивности напряжений в вершинах трещины.

Ключевые слова: электрострикция, дуговая трещина, задача линейного сопряжения, коэффициент интенсивности напряжений.

ДУГОВА ТРІЩИНА НА МЕЖІ ПОДІЛУ ДВОХ ЕЛЕКТРОСТРИКЦІЙНИХ МАТЕРІАЛІВ

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Отримано точний аналітичний розв'язок для електрострикційної площини з круговим електрострикційним включенням і дуговою тріщиною на межі поділу матеріалів під дією довільних механічних та електричних навантажень на нескінченності. Вважається, що обидва матеріали є ізотропними та лінійно пружними, а береги тріщини не взаємодіють один з одним та є проникними для електричного поля. Задача розглядається як незв'язана задача електропружності. Розв'язок задачі електростатики отримано за допомогою методу комплексних потенціалів. Граничну задачу електропружності для чотирьох комплексних потенціалів, які є аналогами потенціалів Колосова-Мусхелішвілі, зведено до задачі лінійного спряження на тріщині. Невідомі константи з загального розв'язку цієї задачі визначено з граничних умов на нескінченності та обмежень, що накладено на напруження і переміщення. Знайдено аналітичні вирази для напружено-деформованого стану всієї площини, зокрема для розкриття тріщини, нормальних і дотичних напружень на границі поділу середовищ та коефіцієнтів інтенсивності напружень у вершинах тріщини.

Ключові слова: електрострикція, дугова тріщина, задача лінійного спряження, коефіцієнт інтенсивності напружень.

INTRODUCTION

Electrostrictive materials, in particular ferroelectric relaxors, become widespread in modern technologies because for these relaxors electrostrictive effect is close to the piezoelectric one. As described in [1], cracks may appear in electrostrictive materials under the action of large electrical and mechanical stresses. This causes the importance of studying of cracked electrostrictive materials behavior under the action of electrical and mechanical loads.

In general case constitutive equations of electrostrictive materials are quite complex and require solving of the coupled electroelasticity problem that is associated with considerable mathematical difficulties. However, in the case of small deformations the constitutive equations can be simplified so that the electroelasticity problem becomes uncoupled. For this case an analogue of Kolosov-Muskhelishvili equations [2] that takes into account electrostriction was developed in [3]. The electrostrictive body with an arc crack under the action of electrical load at infinity parallel to the crack axis of symmetry is analyzed in the article [4]. The homogeneous electrostrictive plane with an arc crack under the action of arbitrary electrical and mechanical loads at infinity is considered in [5] and [6]. Stress intensity factors for electrostrictive fibrous composite with an arc-shaped permeable interface crack under electric loadings are found in [7]. Nevertheless, general stress-strain state, especially crack opening, of electrostrictive composite with an arc interface crack under the action of arbitrary electrical and mechanical loadings has not been considered yet. Thereby important point related to the possible appearance of the crack faces contact zones has not been also investigated.

Much more works are devoted to investigation of arc cracks in electrically passive materials. Firstly, an arc crack in elastic plane was considered by Muskhelishvili [2]. His method was extended to the case of different materials by England [8]. Method designed by England was used for investigation of interfacial arc crack under the action of arbitrary loading at infinity [9] and at the crack [10]. Partially debonded circular inclusion was also considered by means of finite elements method in [11]. Stress intensity factors of arc crack between homogeneous cylinder and its coating are obtained from system of singular integral equations in [12]. A plane containing the system of partially debonded circular inclusions is considered in [13] using superposition principle and general displacement solution.

A contact problem for the crack in a homogeneous plane [14] and for the crack between matrix and inclusion [15] was firstly considered by Chao and Laws. A contact problem for interfacial arc crack under the action of arbitrary loading at infinity was resolved using singular integral equations in [16]. Closure of an arc cracks in homogeneous material and its influence on stress intensity factors are analyzed in [17] using boundary elements method. Contact zones that arise in vicinity of interfacial arc crack tips are investigated also in [18] and [19] using boundary elements method.

In the present article an electrostrictive plane with circular electrostrictive inclusion and an arc crack at materials interface under the influence of arbitrary mechanical and electrical loadings at infinity is considered. Electrostatics boundary problem for three unknown complex potentials is resolved by expanding these functions in Laurent series. Boundary problem of electroelasticity for four complex potentials that are analogues of Kolosov-Muskhelishvili potentials is reduced to the problem of linear relationship using the method developed by England [8]. Solution of this problem is obtained by well-known methods of analytical function theory described in [2] and [20]. The unknown constants in general solution of the problem of linear relationship are found from boundary conditions at infinity, displacements uniqueness condition and finiteness of displacements and stresses at origin.

Analytical expressions for stresses and displacements at the whole plane are obtained, and also the formulas for crack opening and stress intensity factors at the crack tips are found. Crack opening, normal and shear stresses at materials interface and stress intensity factors at the crack tips are found for various material constants and loading at infinity. The figures that demonstrate the influence of different parameters on the crack opening, stresses and the stress intensity factors are presented.

FORMULATION OF THE PROBLEM

Infinite plane with a circular inclusion of radius R bonded along the whole interface except of the arc $r = R$, $|\theta| < \beta$ is considered. We assume that crack faces cannot interact with each other and are permeable to electric field. Mechanical properties of inclusion and matrix are characterized by shear modules μ_1, μ_2 and Poisson's ratios ν_1, ν_2 respectively. Electrostrictive properties of inclusion are determined by constants $a_1^{(1)}$ and $a_2^{(1)}$, and the matrix electrostrictive properties are determined by constants $a_1^{(2)}$ and $a_2^{(2)}$ [21]. The dielectric permittivities of inclusion, matrix and crack filler are denoted as ε_1, ε_2 and ε_c, accordingly. Principal stresses N_1 and N_2 act at infinity; the angle between the direction of N_1 and the abscissa axis is α_N. Also the electric field with intensity vector of magnitude E_0 that forms the angle α with abscissa axis is applied at infinity (Fig. 1).

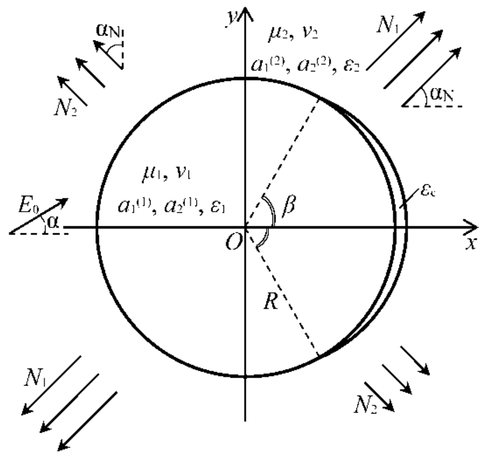

Fig. 1

Analogues of Kolosov-Muskhelishvili equations for electrostrictive materials are [3, 21]

$$2\mu_j\left(u_r^{(j)}+iu_\theta^{(j)}\right)=e^{-i\theta}\left(\kappa_j\varphi_j(z)-z\varphi_j'(z)-\psi_j(z)+\chi_j\overline{W_j(z)}-\frac{k_j}{2}w_j(z)\overline{w_j'(z)}\right),\tag{1}$$

$$\tilde{\sigma}_{rr}^{(j)}+i\tilde{\sigma}_{r\theta}^{(j)}=\varphi_j'(z)+\overline{\varphi_j'(z)}-\bar{z}\overline{\varphi_j''(z)}-\frac{\bar{z}}{z}\overline{\psi_j'(z)}+\frac{k_j}{2}\left(w_j'(z)\overline{w_j'(z)}-\frac{\bar{z}}{z}w_j(z)\overline{w_j''(z)}\right),\tag{2}$$

$$\sigma_{rr}^{M(j)}+i\sigma_{r\theta}^{M(j)}=\frac{\varepsilon_j}{2}\frac{\bar{z}}{z}\overline{W_j'(z)},\tag{3}$$

where $j=1$ refers to area $r<R$ and $j=2$ refers to area $r>R$; $\tilde{\sigma}_{ks}^{(j)}=\sigma_{ks}^{(j)}+\sigma_{ks}^{M(j)}$ are pseudo total stresses [4]; $w_j'(z)$ are electrostatic complex potentials; $W_j'(z)=\left[w_j'(z)\right]^2$; $\chi_j=\dfrac{a_1^{(j)}-2\varepsilon_j}{4}$; $\kappa_j=3-4\nu_j$, $k_j=-\dfrac{\left(1-2\nu_j\right)\left(a_1^{(j)}+2a_2^{(j)}\right)}{4\left(1-\nu_j\right)}$ for plane strain and $\kappa_j=\dfrac{3-\nu_j}{1+\nu_j}$, $k_j=-\dfrac{a_1^{(j)}\left(1-\nu_j\right)+2a_2^{(j)}\left(1-2\nu_j\right)}{4}$ for plane stress [21].

Boundary conditions for displacement and stresses at the interface are the following [4]:

$$\tilde{\sigma}_{rr}^{(1)}+i\tilde{\sigma}_{r\theta}^{(1)}=\tilde{\sigma}_{rr}^{(2)}+i\tilde{\sigma}_{r\theta}^{(2)},\ u_r^{(1)}+iu_\theta^{(1)}=u_r^{(2)}+iu_\theta^{(2)}\quad \text{for}\quad r=R,\quad \beta<|\theta|\le\pi,\tag{4}$$

$$\tilde{\sigma}_{rr}^{(1)}+i\tilde{\sigma}_{r\theta}^{(1)}=\tilde{\sigma}_{rr}^{(2)}+i\tilde{\sigma}_{r\theta}^{(2)}=\frac{\varepsilon_c}{2}\frac{\bar{z}}{z}\overline{W_c'(z)}\quad \text{for}\quad r=R,\quad |\theta|<\beta,\tag{5}$$

where $w_c'(z)$ is an electrostatic complex potential of the crack, $W_c'(z)=\left[w_c'(z)\right]^2$. The boundary conditions at infinity can be presented as

$$\sigma_{rr}^{(2)}+i\sigma_{r\theta}^{(2)}=\frac{N_1+N_2}{2}+\frac{N_1-N_2}{2}e^{2i(\alpha_N-\theta)}\quad \text{for}\quad r\to\infty.\tag{6}$$

Electrostatic complex potentials $w_1'(z)$, $w_2'(z)$ and $w_c'(z)$ are determined by boundary problem of electrostatics which solution is given in the next section.

BOUNDARY PROBLEM OF ELECTROSTATICS

The equations of electrostatics are as follows [22, 23]:

$$\Delta\varphi_1 = 0 \quad \text{for} \quad r < R, \quad \Delta\varphi_2 = 0 \quad \text{for} \quad r > R, \quad \Delta\varphi_c = 0 \quad \text{for} \quad r = R, \quad |\theta| \le \beta, \tag{7}$$

$$\mathbf{E}^{(1)} = -\boldsymbol{\nabla}\varphi_1, \quad \mathbf{E}^{(2)} = -\boldsymbol{\nabla}\varphi_2, \quad \mathbf{E}^{(c)} = -\boldsymbol{\nabla}\varphi_c, \tag{8}$$

$$\mathbf{D}^{(1)} = \varepsilon_1 \mathbf{E}^{(1)}, \quad \mathbf{D}^{(2)} = \varepsilon_2 \mathbf{E}^{(2)}, \quad \mathbf{D}^{(c)} = \varepsilon_c \mathbf{E}^{(c)}, \tag{9}$$

where φ_1, φ_2, φ_c are the potentials of electrostatic field, $\mathbf{E}^{(1)}$, $\mathbf{E}^{(2)}$, $\mathbf{E}^{(c)}$ are the intensities of electrostatic field, $\mathbf{D}^{(1)}$, $\mathbf{D}^{(2)}$, $\mathbf{D}^{(c)}$ are the electric displacements in inclusion, matrix and crack filler respectively.

Electrostatic boundary condition at infinity are

$$\mathbf{E}^{(2)} = \mathbf{i_1} E_0 \cos\alpha + \mathbf{i_2} E_0 \sin\alpha \quad \text{for} \quad r \to \infty, \tag{10}$$

and electrostatic boundary conditions on the interface have the form [22, 1–4]

$$\mathbf{D}^{(1)} \cdot \mathbf{n} = \mathbf{D}^{(2)} \cdot \mathbf{n}, \quad \mathbf{E}^{(1)} \cdot \mathbf{t} = \mathbf{E}^{(2)} \cdot \mathbf{t} \quad \text{for} \quad r = R, \quad \beta < |\theta| \le \pi, \tag{11}$$

$$\mathbf{D}^{(1)} \cdot \mathbf{n} = \mathbf{D}^{(2)} \cdot \mathbf{n} = \mathbf{D}^{(c)} \cdot \mathbf{n}, \quad \mathbf{E}^{(1)} \cdot \mathbf{t} = \mathbf{E}^{(2)} \cdot \mathbf{t} = \mathbf{E}^{(c)} \cdot \mathbf{t} \quad \text{for} \quad r = R, \quad |\theta| \le \beta, \tag{12}$$

where $\mathbf{n} = \mathbf{i_1} \cos\theta + \mathbf{i_2} \sin\theta$ is the vector of outward unit normal to the circle $r = R$ and $\mathbf{t} = -\mathbf{i_1} \sin\theta + \mathbf{i_2} \cos\theta$ is unit vector tangent to this circle.

Complex potentials $w_1'(z)$, $w_2'(z)$ and $w_c'(z)$ are determined as

$$E_x^{(1)} + iE_y^{(1)} = \overline{w_1'(z)} \quad \text{for} \quad r < R, \quad E_x^{(2)} + iE_y^{(2)} = \overline{w_2'(z)} \quad \text{for} \quad r > R,$$

$$E_x^{(c)} + iE_y^{(c)} = \overline{w_c'(z)} \quad \text{for} \quad r = R, \quad |\theta| \le \beta,$$

where functions $w_1'(z)$, $w_2'(z)$ and $w_c'(z)$ are analytical in the correspondent areas. This choice of unknown functions allows satisfy Laplace equations (7) completely.

As the boundary conditions (11)-(12) are formulated for $z = \mathrm{Re}^{i\theta}$, they may be written by presenting of complex potentials in the following way:

$$\varepsilon_2 \,\mathrm{Re}[zw_2'(z)] = \varepsilon_1 \,\mathrm{Re}[zw_1'(z)], \quad \mathrm{Im}[zw_2'(z)] = \mathrm{Im}[zw_1'(z)] \quad \text{for} \quad r = R, \quad |\theta| \le \pi. \tag{13}$$

$$\varepsilon_c \,\mathrm{Re}[zw_c'(z)] = \varepsilon_1 \,\mathrm{Re}[zw_1'(z)], \quad \mathrm{Im}[zw_c'(z)] = \mathrm{Im}[zw_1'(z)] \quad \text{for} \quad r = R, \quad |\theta| \le \beta. \tag{14}$$

It follows from boundary condition at infinity (10) that

$$w_2'(z) \xrightarrow[z \to \infty]{} E_0 e^{-i\alpha}. \tag{15}$$

Thereby boundary problem of electrostatics is reduced to determination of three unknown complex potentials $w_1'(z)$, $w_2'(z)$ and $w_c'(z)$ that are analytical in correspondent areas from boundary conditions (13)-(15). Unknown coefficients of Laurent series for functions $w_1'(z)$, $w_2'(z)$ and $w_c'(z)$ are determined from these boundary conditions. Thus, the electrostatic potentials are given in the following way:

$$w_1'(z) = \frac{2\varepsilon_2}{\varepsilon_1 + \varepsilon_2} E_0 e^{-i\alpha}, \quad w_2'(z) = E_0 \left(e^{-i\alpha} + \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2} e^{i\alpha} \frac{R^2}{z^2} \right),$$

$$w_c'(z) = E_0 \frac{\varepsilon_2}{\varepsilon_c}\left(\frac{\varepsilon_1+\varepsilon_c}{\varepsilon_1+\varepsilon_2}e^{-i\alpha} + \frac{\varepsilon_1-\varepsilon_c}{\varepsilon_1+\varepsilon_2}e^{i\alpha}\frac{R^2}{z^2}\right). \tag{16}$$

PROBLEM OF LINEAR RELATIONSHIP

Functions $\phi(z)$ and $\omega(z)$ are introduced to satisfy boundary conditions at electrostrictive materials interface (4) by the formulas

$$\phi(z) = \begin{cases} \mu_2\kappa_1\varphi_1(z) + \mu_1 z\overline{\varphi}_2'\left(\frac{R^2}{z}\right) + \mu_1\overline{\psi}_2\left(\frac{R^2}{z}\right) - \mu_1\chi_2\overline{W}_2\left(\frac{R^2}{z}\right) + \frac{1}{2}\mu_1 k_2 w_2(z)\overline{w}_2'\left(\frac{R^2}{z}\right), & |z| < R, \\ \mu_1\kappa_2\varphi_2(z) + \mu_2 z\overline{\varphi}_1'\left(\frac{R^2}{z}\right) + \mu_2\overline{\psi}_1\left(\frac{R^2}{z}\right) - \mu_2\chi_1\overline{W}_1\left(\frac{R^2}{z}\right) + \frac{1}{2}\mu_2 k_1 w_1(z)\overline{w}_1'\left(\frac{R^2}{z}\right), & |z| > R, \end{cases} \tag{17}$$

$$\omega(z) = \begin{cases} \varphi_1(z) - z\overline{\varphi}_2'\left(\frac{R^2}{z}\right) - \overline{\psi}_2\left(\frac{R^2}{z}\right) - \frac{1}{2}k_2 w_2(z)\overline{w}_2'\left(\frac{R^2}{z}\right), & |z| < R, \\ \varphi_2(z) - z\overline{\varphi}_1'\left(\frac{R^2}{z}\right) - \overline{\psi}_1\left(\frac{R^2}{z}\right) - \frac{1}{2}k_1 w_1(z)\overline{w}_1'\left(\frac{R^2}{z}\right), & |z| > R. \end{cases} \tag{18}$$

The derivatives of these functions are analytical at the complex plane with cut along the arc $r = R$, $|\theta| \le \beta$ except of infinity and zero points. Further the function $F'(z) = \phi'(z) - K\omega'(z)$, $K = -\frac{\mu_1(1-\lambda\kappa_2)}{1+\lambda}$, $\lambda = \frac{\mu_1+\mu_2\kappa_1}{\mu_2+\mu_1\kappa_2}$ is used instead of $\phi'(z)$.

Boundary condition (5) specifies that $\tilde{\sigma}_{rr}^{(1)} + i\tilde{\sigma}_{r\theta}^{(1)} = \tilde{\sigma}_{rr}^{(2)} + i\tilde{\sigma}_{r\theta}^{(2)}$ at the crack. It follows from this condition and equations (2) and (18) that $\omega'^{+}(z) = \omega'^{-}(z)$. Also formulas (17) – (18) specify that the functions $\omega'(z)$ and $F'(z)$ are finite at infinity and have second-order poles at zero point. Since the function $\omega'(z)$ is analytical at the whole complex plane except of zero point it is given by expression

$$\omega'(z) = A_0 + \frac{A_1}{z} + \frac{A_2}{z^2}. \tag{19}$$

It follows from the boundary conditions (5) that the function $F'(z)$ should satisfy the following problem of linear relationship at the crack:

$$F'^{+}(z) + \lambda F'^{-}(z) = \frac{R^2}{z^2}\left(\mu_1\chi_2\overline{W}_2'\left(\frac{R^2}{z}\right) + \lambda\mu_2\chi_1\overline{W}_1'\left(\frac{R^2}{z}\right) + \right.$$

$$\left. + (\mu_1 + \mu_2\kappa_1)\frac{\varepsilon_c}{2}\overline{W}_c'\left(\frac{R^2}{z}\right)\right) \quad \text{for} \quad r = R,\ |\theta| \le \beta. \tag{20}$$

Using formulas (16) equation (20) is transformed to the following form:

$$F'^{+}(z) + \lambda F'^{-}(z) = f(z) \quad \text{for} \quad r = R,\ |\theta| \le \beta, \tag{21}$$

where $f(z) = \frac{C_1}{z^2} + C_2 + C_3 z^2$.

The stresses and displacements of the inclusion are expressed in terms of functions $F(z)$ and $\omega(z)$ by the following way:

$$\tilde{\sigma}_{rr}^{(1)}+i\tilde{\sigma}_{r\theta}^{(1)}=\frac{1}{\mu_1+\mu_2\kappa_1}\left(\left(\mu_1+K\right)\omega'\left(z\right)+F'\left(z\right)-\mu_1\chi_2\frac{R^2}{z^2}\overline{W}_2'\left(\frac{R^2}{z}\right)+\right.$$

$$+\left(1-\frac{R^2}{z\overline{z}}\right)\left(\left(\mu_1+K\right)\overline{\omega}'\left(\overline{z}\right)+\overline{F}'\left(\overline{z}\right)-\mu_1\chi_2\frac{R^2}{\overline{z}^2}W_2'\left(\frac{R^2}{\overline{z}}\right)\right)+$$

$$+\left(\frac{R^2}{z}-\overline{z}\right)\left(\left(\mu_1+K\right)\overline{\omega}''\left(\overline{z}\right)+\overline{F}''\left(\overline{z}\right)+\mu_1\chi_2\frac{R^4}{\overline{z}^4}\left(2\frac{\overline{z}}{R^2}W_2'\left(\frac{R^2}{\overline{z}}\right)+W_2''\left(\frac{R^2}{\overline{z}}\right)\right)\right)$$

$$+\frac{R^2}{z\overline{z}}\left(-\left(\mu_1+K\right)\omega'\left(\frac{R^2}{\overline{z}}\right)+\lambda F'\left(\frac{R^2}{\overline{z}}\right)-\lambda\mu_2\chi_1\frac{\overline{z}^2}{R^2}\overline{W}_1'(\overline{z})\right)+$$

$$\left.+\frac{k_1}{2}\left(\overline{w}_1'\left(\overline{z}\right)\left(w_1'\left(z\right)-\frac{R^2}{z\overline{z}}w_1'\left(\frac{R^2}{\overline{z}}\right)\right)-\frac{\overline{z}}{z}\overline{w}_1''(\overline{z})\left(w_1\left(z\right)-w_1\left(\frac{R^2}{\overline{z}}\right)\right)\right)\right), \qquad (22)$$

$$u_r^{(1)}+iu_\theta^{(1)}=\frac{e^{-i\theta}}{2\mu_1\left(\mu_1+\mu_2\kappa_1\right)}\left(\kappa_1\left(\left(\mu_1+K\right)\omega\left(z\right)+F\left(z\right)+\mu_1\chi_2\overline{W}_2\left(\frac{R^2}{z}\right)\right)+\right.$$

$$+\left(\frac{R^2}{\overline{z}}-z\right)\left(\left(\mu_1+K\right)\overline{\omega}'\left(\overline{z}\right)+\overline{F}'\left(\overline{z}\right)-\mu_1\chi_2\frac{R^2}{\overline{z}^2}W_2'\left(\frac{R^2}{\overline{z}}\right)\right)+\left(\mu_1+K\right)\omega\left(\frac{R^2}{\overline{z}}\right)-$$

$$\left.-\lambda F\left(\frac{R^2}{\overline{z}}\right)-\lambda\mu_2\chi_1\overline{W}_1\left(\overline{z}\right)+\left(\mu_1+\mu_2\kappa_1\right)\left(\frac{k_1}{2}\overline{w}_1'\left(\overline{z}\right)\left(w_1\left(\frac{R^2}{\overline{z}}\right)-w_1\left(z\right)\right)+\chi_1\overline{W}_1\left(\overline{z}\right)\right)\right). \qquad (23)$$

Stresses and displacements of matrix may be obtained from expressions (22) and (23) by replacing suffix 1 by 2, 2 by 1 and λ by $1/\lambda$.

SOLUTION OF PROBLEM OF LINEAR RELATIONSHIP

General solution of the problem of linear relationship (21) has the form [2, 20]

$$F'(z)=\frac{X_0(z)}{2\pi i}\int_L\frac{f(t)dt}{X_0^+(t)(t-z)}+X_0(z)P_0(z),$$

$$X_0(z)=\frac{1}{\sqrt{\left(z-Re^{-i\beta}\right)\left(z-Re^{i\beta}\right)}}\left(\frac{z-Re^{-i\beta}}{z-Re^{i\beta}}\right)^{i\gamma},\quad P_0(z)=B_1z+B_0+\frac{D_1}{z}+\frac{D_2}{z^2}, \qquad (24)$$

where $\gamma=\dfrac{\ln\lambda}{2\pi}$, L is the arc $r=R$, $|\theta|\le\beta$ that is bypassed counterclockwise. It should be noted that the branch of the function $X_0(z)$ that satisfies conditions $\left(\dfrac{z-Re^{-i\beta}}{z-Re^{i\beta}}\right)^{i\gamma}\xrightarrow[|z|\to\infty]{}1$ and $\dfrac{z}{\sqrt{\left(z-Re^{-i\beta}\right)\left(z-Re^{i\beta}\right)}}\xrightarrow[|z|\to\infty]{}1$ is selected.

Contour integral from (24) is given by the following expression [2]:

$$\frac{1}{2\pi i}\int_L\frac{f(t)dt}{X_0^+(t)(t-z)}=\frac{1}{1+\lambda}\left(\frac{f(z)}{X_0(z)}-\sum_{j=0}^{3}b_jz^j\right),$$

therefore

$$F'(z)=\frac{1}{1+\lambda}f(z)+X_0(z)P(z),\quad P(z)=B_3z^3+B_2z^2+B_1z+B_0+\frac{D_1}{z}+\frac{D_2}{z^2}. \tag{25}$$

Expressions (19) and (25) contain seven unknown constants A_0, A_1, A_2, B_0, B_1, D_1 and D_2 that need to be determined.

The function $X_0(z)$ is expanded in the following series near zero and a point at infinity:

$$X_0(z)\xrightarrow[z\to 0]{}-\frac{e^{2\gamma\beta}}{R}\left(1+X_1^{(0)}z+X_2^{(0)}z^2+\ldots\right),\quad X_0(z)\xrightarrow[z\to\infty]{}\frac{1}{z}\left(1+\frac{X_1^{(\infty)}}{z}+\frac{X_2^{(\infty)}}{z^2}+\frac{X_3^{(\infty)}}{z^3}+\ldots\right).$$

Because the function $F'(z)$ is limited at infinity, $B_3=-\frac{1}{1+\lambda}C_3$, $B_2=\frac{X_1^{(\infty)}}{1+\lambda}C_3$. Therefore, this function has the following representations near zero point and point at infinity:

$$F'(z)\xrightarrow[z\to 0]{}\left(\frac{1}{1+\lambda}C_1-\frac{e^{2\gamma\beta}}{R}D_2\right)\frac{1}{z^2}-\frac{e^{2\gamma\beta}}{R}\left(D_1+X_1^{(0)}D_2\right)\frac{1}{z}+$$

$$+\left(\frac{1}{1+\lambda}C_2-\frac{e^{2\gamma\beta}}{R}\left(B_0+X_1^{(0)}D_1+X_2^{(0)}D_2\right)\right)+\ldots, \tag{26}$$

$$F'(z)\xrightarrow[z\to\infty]{}\left(\frac{1}{1+\lambda}C_2+X_1'C_3+B_1\right)+\left(B_0+B_1X_1^{(\infty)}+X_2'C_3\right)\frac{1}{z}+\ldots, \tag{27}$$

where $X_1'=\frac{1}{1+\lambda}\left(\left(X_1^{(\infty)}\right)^2-X_2^{(\infty)}\right)$, $X_2'=\frac{1}{1+\lambda}\left(X_1^{(\infty)}X_2^{(\infty)}-X_3^{(\infty)}\right)$.

It follows from finiteness of stresses and displacements at zero point, boundary condition at infinity (6) and condition of displacements uniqueness that

$$D_1+X_1^{(0)}D_2=Re^{-2\gamma\beta}(\mu_1+K)A_1,\quad B_0+B_1X_1^{(\infty)}+X_2'C_3=\frac{1}{\lambda}(\mu_1+K)A_1,$$

$$(\mu_1+K)A_2-\frac{e^{2\gamma\beta}}{R}D_2=\mu_1\chi_2R^2E_0^2e^{2i\alpha}-\frac{1}{1+\lambda}C_1,\quad A_1=0,$$

$$-(\mu_1+K)\left(A_0+\overline{A}_0\right)+\lambda\overline{B}_1+\frac{e^{2\gamma\beta}}{R}\left(B_0+X_1^{(0)}D_1+X_2^{(0)}D_2\right)=$$

$$=(\mu_1+\mu_2\kappa_1)\frac{2k_1\varepsilon_2^2E_0^2}{(\varepsilon_1+\varepsilon_2)^2}-2\mu_1\chi_2\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}E_0^2+\frac{1-\lambda}{1+\lambda}C_2-\lambda X_1'\overline{C}_3.$$

$$(\mu_2+K)A_0+B_1=(\mu_2+\mu_1\kappa_2)\left(\frac{N_1+N_2}{4}-\frac{k_2E_0^2}{4}\right)-\frac{1}{1+\lambda}C_2-X_1'C_3,$$

$$(\mu_2+K)A_2+\frac{e^{2\gamma\beta}}{\lambda R}D_2=\frac{1}{\lambda(1+\lambda)}C_1-\frac{1}{\lambda}\mu_1\chi_2R^2E_0^2e^{2i\alpha}-$$

$$-R^2(\mu_2+\mu_1\kappa_2)\left(\frac{N_1-N_2}{2}e^{2i\alpha_N}+\left(\frac{\varepsilon_2}{2}+\frac{k_2}{2}\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}\right)E_0^2e^{2i\alpha}\right). \tag{28}$$

As one can see, system of linear algebraic equations (28) fully determines the unknown constants.

STRESSES, CRACK OPENING AND STRESS INTENSITY FACTORS

As it follows from (23), crack opening $\Delta = u_r^{(2)} - u_r^{(1)}$ is given by expression

$$\Delta = \frac{1}{2\mu_1\mu_2}\mathrm{Re}\left(e^{-i\theta}\left(F^-(z) - F^+(z)\right)\right), \quad z = Re^{i\theta}, \quad |\theta| \le \beta . \tag{29}$$

Using equations (21) and (25) the expression (29) is transformed to the form

$$\Delta = \frac{1+\lambda}{2\mu_1\mu_2}\mathrm{Re}\left(e^{-i\theta}\int X_0^-(z)P(z)dz\right) = \frac{1+\lambda}{2\mu_1\mu_2}\mathrm{Re}\left(e^{-i\theta}F_0(z)\right), \quad z = Re^{i\theta}, \quad |\theta| \le \beta , \tag{30}$$

where

$$F_0(z) = \sqrt{\left(z - Re^{-i\beta}\right)\left(z - Re^{i\beta}\right)}\left(\frac{z - Re^{-i\beta}}{z - Re^{i\beta}}\right)^{i\gamma}\left(p_1 z^2 + p_2 z + p_3 + \frac{p_4}{z}\right), \quad p_1 = -\frac{1}{3(1+\lambda)}C_3 ,$$

$$p_2 = -\frac{1}{3(1+\lambda)}R^2 X_1^{(0)} C_3, \quad p_3 = B_1 - \left(2R^2\cos 2\beta - \left(X_1^{(\infty)}\right)^2\right)\frac{1}{3(1+\lambda)}C_3, \quad p_4 = -R^2 D_2 .$$

Stresses on the bonded part of the interface are given by the following equation:

$$\sigma_{rr}^{(1)} + i\sigma_{r\theta}^{(1)} = \frac{1}{\mu_1 + \mu_2\kappa_1}\left((1+\lambda)F'(z) - \right.$$

$$\left. -\frac{R^2}{z^2}\left(\mu_1\chi_2\overline{W}_2'(\overline{z}) + \lambda\mu_2\chi_1\overline{W}_1'(\overline{z})\right) - \frac{\varepsilon_1}{2}\frac{R^2}{z^2}\overline{W}_1'(\overline{z}) \right), \quad z = Re^{i\theta}, \quad \beta < |\theta| \le \pi . \tag{31}$$

Crack opening (30) and stresses on the bonded part of the interface (31) have a physically unreal oscillation near crack tips in case of different materials of matrix and inclusion. Such oscillation of stresses and displacements near the tips of an interfacial crack is a known limitation of the “open” crack model that was described, for example, in [8]. However, in most cases the oscillation zones are negligible small and the use of the “open” crack model can be approved by Rice [24] approach. Thereby stress intensity factors at the crack tips are introduced as

$$K_1^{\pm} + iK_2^{\pm} = \lim_{\theta \to \pm\beta \pm 0}\sqrt{2\pi R}\left(\pm\theta - \beta\right)^{\frac{1}{2} \pm i\gamma}\left(\tilde{\sigma}_{rr}^{(1)} + i\tilde{\sigma}_{r\theta}^{(1)}\right). \tag{32}$$

After calculation of the limits they get the following form:

$$K_1^{\pm} + iK_2^{\pm} = \mp\frac{1+\lambda}{\mu_1 + \mu_2\kappa_1}\sqrt{\frac{\pi}{R\sin\beta}}\, ie^{\mp\frac{i\beta}{2}}\left(2e^{\mp i\beta}\sin\beta\right)^{\pm i\gamma} P\left(Re^{\pm i\beta}\right). \tag{33}$$

NUMERICAL RESULTS

All results presented in this section are obtained for plane stress state, Poisson’s ratios $\nu_1 = \nu_2 = 0{,}26$ and uniaxial tension at infinity $N_2 / N_1 = 0$, $N_1 > 0$.

Fig. 2-4 show the crack opening, normal and shear stresses at the interface, respectively, for the ratios of the intensities of electrical and mechanical loads at infinity $\varepsilon_1 E_0^2 / N_1 = 0$, $\varepsilon_1 E_0^2 / N_1 = 0{,}5 \cdot 10^{-4}$ and $\varepsilon_1 E_0^2 / N_1 = 10^{-4}$. These Figures are obtained for $\beta = 60^\circ$, $\frac{\mu_2}{\mu_1} = 2$, $\frac{\varepsilon_c}{\varepsilon_1} = 10^{-4}$, $\frac{\varepsilon_2}{\varepsilon_1} = 2$, $\frac{a_1^{(1)}}{\varepsilon_1} = 200$, $\frac{a_2^{(1)}}{\varepsilon_1} = -45$, $\frac{a_1^{(2)}}{\varepsilon_1} = 400$, $\frac{a_2^{(2)}}{\varepsilon_1} = -75$, $\alpha = -45^\circ$, $\alpha_N = 0$. As it is shown in Fig. 2, an increasing of electrical load intensity leads to decrease of the crack opening.

Also it should be noted from Fig. 3-4, that intensity of the electrical load influences the normal stresses at the interface much more essentially than the shear ones.

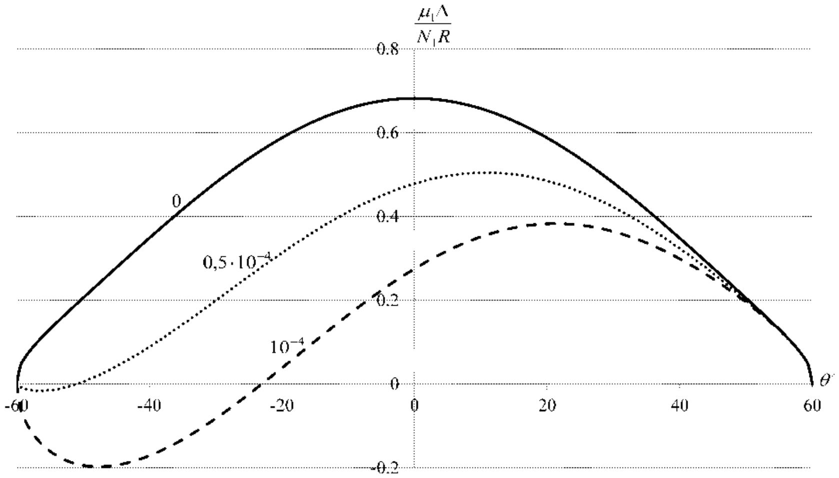

Fig. 2

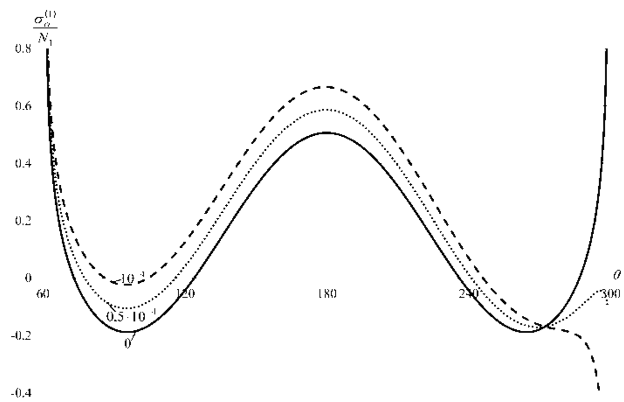

Fig. 3

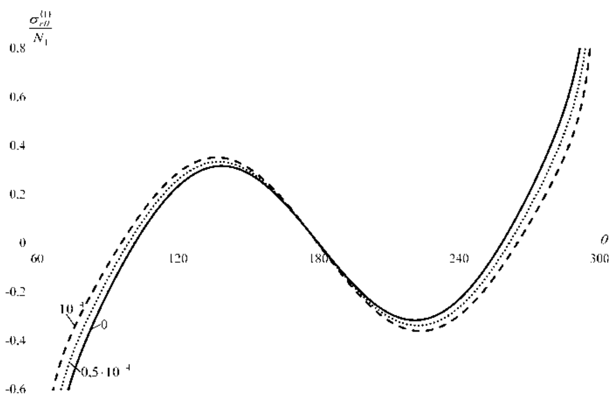

Fig. 4

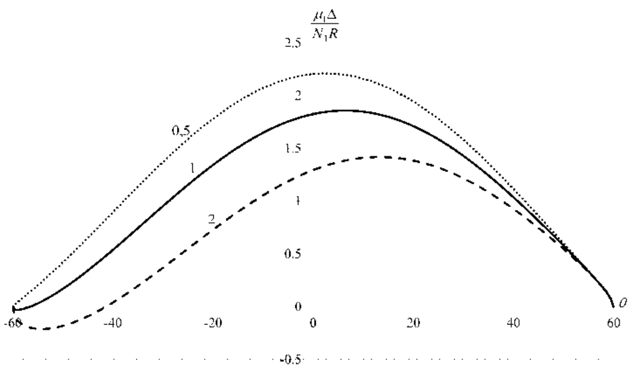

Fig. 5

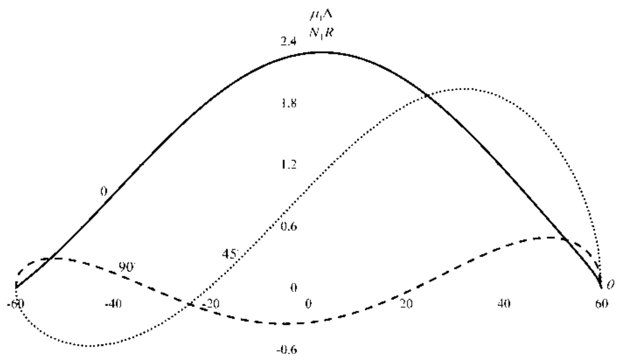

Fig. 6

The crack opening for various parameters of materials and applied loads is shown in Fig. 5-8. All these results are obtained for $\beta = 60^{\circ}$, $\frac{\varepsilon_c}{\varepsilon_1} = 10^{-4}$, $\varepsilon_1 E_0^2 / N_1 = 10^{-4}$. Particularly, Fig. 5 shows the crack opening for the ratios of materials dielectric permittivities $\frac{\varepsilon_2}{\varepsilon_1} = 0{,}5$, $\frac{\varepsilon_2}{\varepsilon_1} = 1$ and $\frac{\varepsilon_2}{\varepsilon_1} = 2$, and also $\frac{\mu_2}{\mu_1}=0{,}5$, $\frac{a_1^{(1)}}{\varepsilon_1}=400$, $\frac{a_2^{(1)}}{\varepsilon_1}=-75$, $\frac{a_1^{(2)}}{\varepsilon_1}=0$, $\frac{a_2^{(2)}}{\varepsilon_1}=0$, $\alpha=-30^\circ$, $\alpha_N=0$. As it is shown in this figure, increasing of the matrix dielectric permittivity decreases the crack opening. Fig. 6 shows the crack opening for the direction angles of mechanical load at infinity $\alpha_N=0$, $\alpha_N=45^\circ$ and $\alpha_N=90^\circ$, and also $\frac{\mu_2}{\mu_1}=0{,}5$, $\frac{\varepsilon_2}{\varepsilon_1}=0{,}5$, $\frac{a_1^{(1)}}{\varepsilon_1}=400$, $\frac{a_2^{(1)}}{\varepsilon_1}=-75$, $\frac{a_1^{(2)}}{\varepsilon_1}=200$, $\frac{a_2^{(2)}}{\varepsilon_1}=-45$, $\alpha=-45^\circ$.

Fig. 7 shows crack opening for the direction angles of electrical load at infinity $\alpha=0$, $\alpha=45^\circ$ and $\alpha=90^\circ$, and also $\frac{\mu_2}{\mu_1}=0{,}5$, $\frac{\varepsilon_2}{\varepsilon_1}=2$, $\frac{a_1^{(1)}}{\varepsilon_1}=200$, $\frac{a_2^{(1)}}{\varepsilon_1}=-45$, $\frac{a_1^{(2)}}{\varepsilon_1}=400$, $\frac{a_2^{(2)}}{\varepsilon_1}=-75$, $\alpha_N=0$. As one can see, angles α and α_N have a great influence on the crack faces intersection zones appearing. The cases of none intersection zones, one zone and two zones are presented. Fig. 8 shows the crack opening for ratios of elastic modules $\frac{\mu_2}{\mu_1}=0{,}5$, $\frac{\mu_2}{\mu_1}=1$ and $\frac{\mu_2}{\mu_1}=2$. These results are obtained for $\frac{\varepsilon_2}{\varepsilon_1}=0{,}5$, $\frac{a_1^{(1)}}{\varepsilon_1}=400$, $\frac{a_2^{(1)}}{\varepsilon_1}=-75$, $\frac{a_1^{(2)}}{\varepsilon_1}=200$, $\frac{a_2^{(2)}}{\varepsilon_1}=-45$, $\alpha=0$, $\alpha_N=15^\circ$. As it is shown in this figure, decreasing of the matrix elastic modulus increases the crack opening.

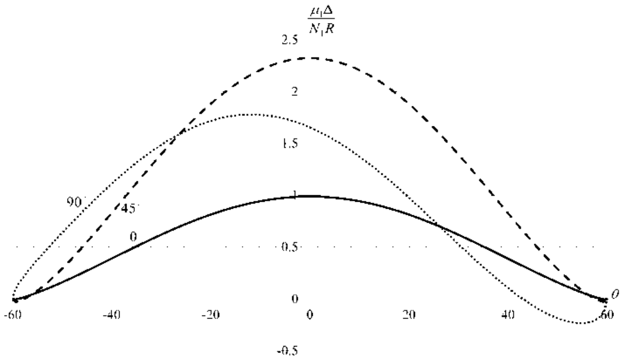

Fig. 7

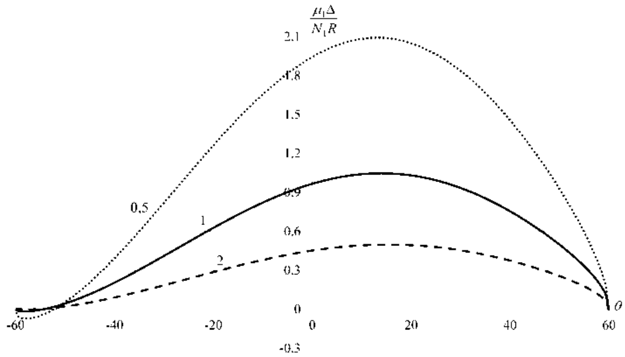

Fig. 8

Fig. 9-14 show the dependencies of the normal and shear stress intensity factors at the upper crack tip from different variables. All these results are obtained for $\frac{\varepsilon_2}{\varepsilon_1}=0{,}5$, $\frac{a_1^{(1)}}{\varepsilon_1}=400$, $\frac{a_2^{(1)}}{\varepsilon_1}=-75$, $\frac{a_1^{(2)}}{\varepsilon_1}=200$, $\frac{a_2^{(2)}}{\varepsilon_1}=-45$. Particularly, Fig. 9-10 show variation of stress intensity factors from crack angle β for ratios of the intensities of electrical and mechanical loads at infinity $\varepsilon_1 E_0^2/N_1=0$, $\varepsilon_1 E_0^2/N_1=10^{-4}$ and $\varepsilon_1 E_0^2/N_1=2\cdot 10^{-4}$. These results are obtained for $\frac{\varepsilon_c}{\varepsilon_1}=10^{-4}$, $\frac{\mu_2}{\mu_1}=0{,}5$, $\alpha=0$, $\alpha_N=0$. As one can see, increasing of the electrical load intensity decreases the absolute values of both normal and shear stress intensity factors.

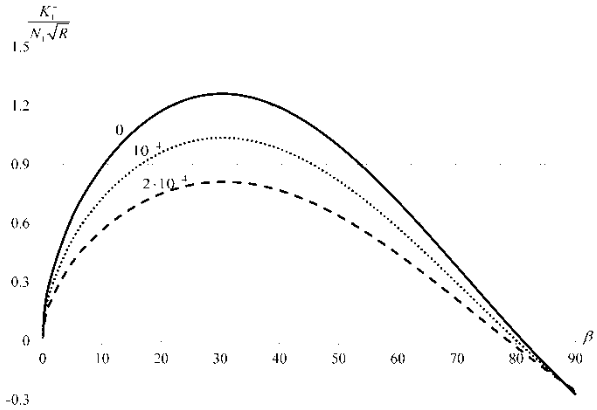

Fig. 9

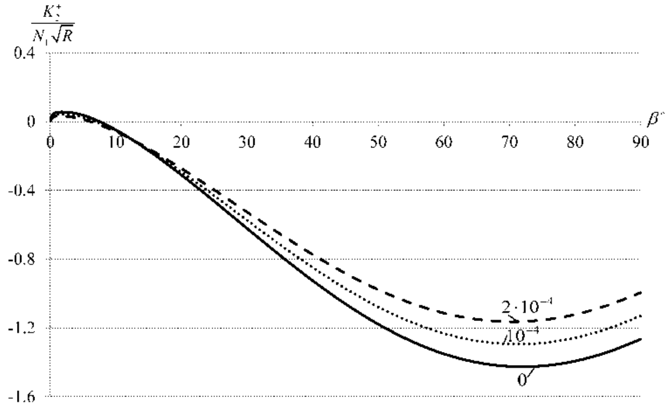

Fig. 10

Fig. 11-12 show variation of stress intensity factors with respect to the angle of mechanical load applying at infinity α_N for ratios of elastic modules $\frac{\mu_2}{\mu_1}=0,5$, $\frac{\mu_2}{\mu_1}=1$ and $\frac{\mu_2}{\mu_1}=2$. These results are obtained for $\beta=60^\circ$, $\frac{\varepsilon_c}{\varepsilon_1}=10^{-4}$, $\varepsilon_1 E_0^2/N_1=10^{-4}$, $\alpha=0$. As it can be seen from these figures, increasing of the matrix elastic modulus decreases the absolute values of the stress intensity factors. Fig. 13-14 show variation of stress intensity factors on the angle α of electrical load applying at infinity for the ratios of dielectric permittivities $\frac{\varepsilon_c}{\varepsilon_1}=0,5\cdot10^{-4}$, $\frac{\varepsilon_c}{\varepsilon_1}=10^{-4}$ and $\frac{\varepsilon_c}{\varepsilon_1}=2\cdot10^{-4}$. These results are obtained for $\beta=60^\circ$, $\frac{\mu_2}{\mu_1}=2$, $\varepsilon_1 E_0^2/N_1=10^{-4}$, $\alpha_N=0$.

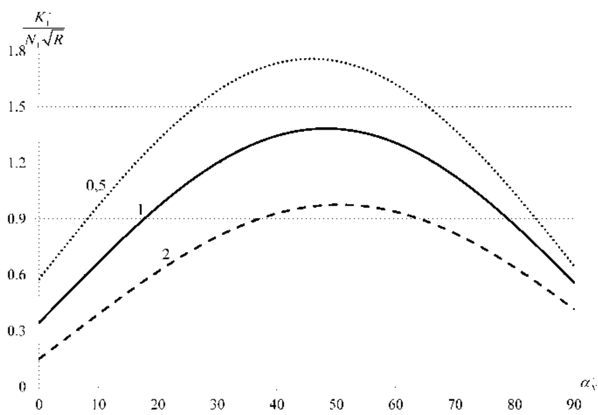

Fig. 11

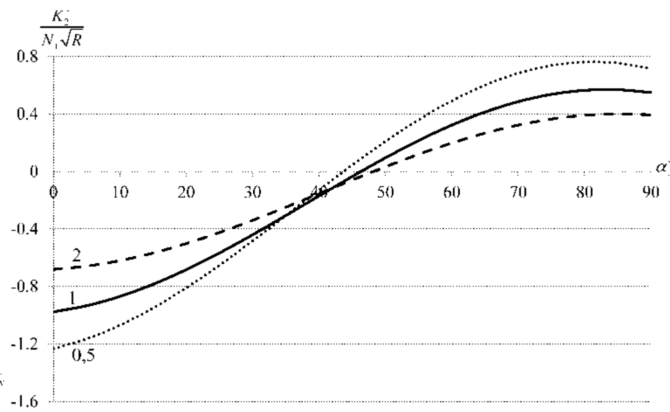

Fig. 12

As it is shown in above figures, various combinations of materials and parameters of electrical and mechanical loadings may cause intersection of crack faces. It is obvious that appearance of the intersection zones in the "open" crack model solution means that in reality the crack faces contact with each other and the crack model, which takes into account the crack faces contact should be applied. Nevertheless, "open" crack model solution can be used to predict number and configuration of the contact zones for such cases. Furthermore, the obtained results are precise enough at a certain distance from the contact zones; in particular, they are reliably applicable for the determination of the fracture parameter at the most dangerous crack tips, where the crack is completely open except small zones of oscillation.

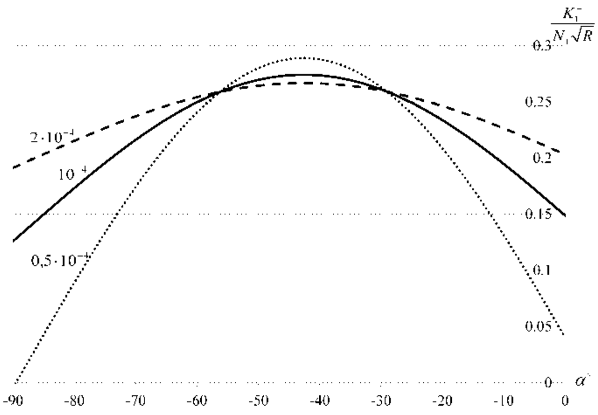

Fig. 13

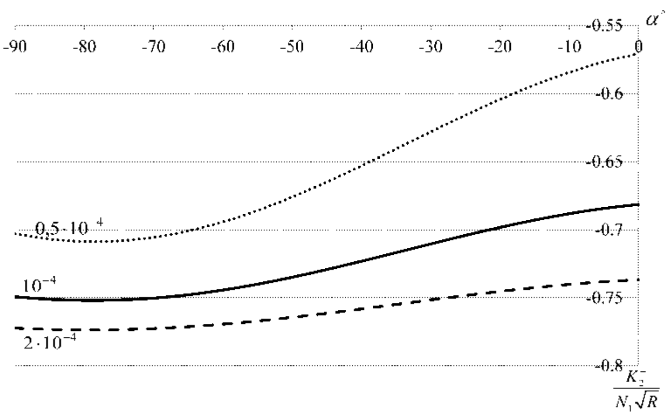

Fig. 14

SUMMARY

An electrostrictive plane with a circular electrostrictive inclusion made of another material and having an arc crack at the material interface under the influence of arbitrary mechanical and electrical loadings at infinity is considered. Crack faces are assumed to be non-interacting and permeable to an electric field. Specified problem is resolved within the framework of uncoupled problem of electroelasticity.

At the beginning the boundary problem of electrostatics is resolved by expanding three unknown complex potentials in Laurent series. Further, the problem of electroelasticity is resolved taking into account the obtained solution of electrostatics problem. Boundary problem of electroelasticity is formulated for four complex potentials that are analogues of Kolosov-Muskhelishvili potentials. It is reduced to the problem of linear relationship at the crack, which is resolved by the methods of analytical function theory. Unknown constants in the general solution of the problem of linear relationship are found from the boundary conditions at infinity and from the limitations imposed on stresses and displacements at origin, at infinity and near the crack.

The obtained solution determines completely the stress-strain state of the plane with circular inclusion and an interface arc crack under arbitrary electro-mechanical loading at infinity. Particularly, the formulas determining stress-strain state at any point of the plane are found and also the exact analytical expressions for the crack opening and the main fracture mechanical parameters are obtained. Dependencies of the crack opening and the stress intensity factors near crack tips from mechanical and dielectric properties of materials and from applied mechanical and electrical loadings are analyzed. The scope of applicability of the "open" crack model is defined and the importance of this model for the determination of the fracture parameter at the most dangerous crack tips is emphasized.

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ЧИСЕЛЬНИЙ РОЗРАХУНОК ЧАСТОТ ВІЛЬНИХ КОЛИВАНЬ НЕКРУГОВОЇ ЦИЛІНДРИЧНОЇ ОБОЛОНКИ З ЖОРСТКО ЗАКРІПЛЕНИМИ ТОРЦЯМИ

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Визначаються частоти та форми вільних коливань тонкої циліндричної оболонки еліптичного поперечного перерізу сталої товщини з жорстко закріпленими торцями. Дослідження проводились методом скінченних елементів, який реалізовано на ліцензійному програмному засобі FEMAP з розв'язувачем NX Nastran. Достовірність отриманих результатів забезпечується використанням обґрунтованої математичної моделі, коректністю постановки задачі, розв'язком тестових задач та практичною збіжністю розрахованих частот при застосуванні методу