UDC 355.233.1.005

K.S. Smelyakov, E.M. Drob

*Ivan Kozhedub Kharkov Air Force University, Kharkov*

# UNIFORM MODELS AND METHODS FOR GRAY-SCALE TRANSFORMATION OF DIGITAL IMAGES

*For attaining a required level of digital image pre-processing efficacy it is proposed a series of unified models and methods of gray-scale transformation of images which enables to perform tone transformation, sharpening and normalization, but simultaneously and for optional brightness ranges of input and output images.*

***Keywords:*** *model, unification, method, efficiency, gray-scale transformation, image.*

## Introduction

At present, the models and methods of gray-scale transformation (gradation correction) are used for solving of a series of important problems of digital image pre-processing. As a rule, the gray-scale transformation methods are used for a tone correction of an image with the aim to lighten it, or to darken [1 – 4].

Most often, for solving this problem these are used a polynomial, sinusoidal, exponential, or logarithmic gray-scale transformation model of the following type

$$p(x) = x\,; \tag{1}$$

$$s(x) = (255/2)\cdot\sin((\pi/255)\cdot x - \pi/2) + 255/2\,; \tag{2}$$

$$ef(x) = e^{k\cdot x} - 1;\ lf(x) = k^{-1}\cdot\ln(x+1);\ k = \frac{8\cdot\ln(2)}{255}, \tag{3}$$

the parameters of which are chosen so that the values of brightness $x$ of the image being specified within the range $[0, \ldots, 255]$ are transformed onto the same brightness value range [1, 2].

Meanwhile, when using such models it is often not taken into consideration the necessity of simultaneous conducting of tone correction [1, 2], image sharpening [5, 6] and image brightness normalization [7]. Besides, the existing gray-scale transformation models are not oriented towards localization of image informative brightness interval, thus stipulating for inefficiency of tone transformation and image sharpening.

In that way, for providing the required level of digital image pre-processing efficacy the topical question arises that consists in solving of the problem of development of a series of practically effectual uniform models and methods of gray-scale transformation, which allow us to perform, efficiently and simultaneously, tone correction, image sharpening and image brightness normalization.

## Main part

### 1. Gray-scale transformation models

For obtaining a smooth increase of sharpness, as well as to provide tone correction and brightness $x$ normalization within the specified range ( $x \in [a, \ldots, b]$, $0 \le a < b \le 255$ ), it is proposed to use the following basic models of gray-scale transformation

$$p(x) = k\cdot(x-a)\,;\quad k = 255/(b-a)\,; \tag{4}$$

$$s(x) = (255/2)\cdot\sin\left(\frac{\pi}{b-a}\cdot x - \frac{\pi}{2} - \frac{a\cdot\pi}{b-a}\right) + 255/2\,; \tag{5}$$

$$t(x) = p(x) + (p(x) - s(x))\,, \tag{6}$$

which transform the brightness values of image from an optional range $[a, \ldots, b]$ into the standard output range $[0, \ldots, 255]$. The shortage of the gray-scale transformation models (4) – (6) consists in worsening of visual perception of image fragments with the values of brightness being close to range bounds $[a, \ldots, b]$, since these values will be transformed either to black, or to white colour. In order to compensate for this shortage, it is proposed to do the following.

At first, set up the sensitivity thresholds $c$ and $d$, ( $0 \le c < d \le 255$ ), for the output brightness range. After this, the brightness $x$ is transformed from the initial range $[a, \ldots, b]$ into the output range $[c, \ldots, d]$ with the use of the following modified models

$$p(x) = k\cdot(x-a) + c\,,\ k = (d-c)/(b-a)\,; \tag{7}$$

$$s(x) = \frac{d-c}{2}\cdot\sin\left(\frac{\pi}{b-a}\cdot x - \frac{\pi}{2} - \frac{a\cdot\pi}{b-a}\right) + \frac{d+c}{2}\,; \tag{8}$$

$$t(x) = p(x) + (p(x) - s(x))\,. \tag{9}$$

Meanwhile, for applications it is proposed to use a single uniform model – the sinusoidal gray-scale transformation of the form

$$F_B(\lambda, x) = \lfloor \lambda\cdot s(x) + (1-\lambda)\cdot t(x) \rfloor,\ 0 \le \lambda \le 1\,, \tag{10}$$

where $\lfloor\cdot\rfloor$ – is the rounding operator.

At the expense of selection of coefficient $\lambda$ the model (10) may be pliably adjusted to the peculiarities of the unique application problem. At $\lambda = 1$ the model $F_B(1, x)$ is presented by the basic sinusoidal model $s(x)$ in the form of (8); this model serves for increasing an image pixel brightness the more the closer the brightness to the middle of the range $[a, \ldots, b]$. At $\lambda = 0.5$ the model $F_B(1, x)$ is presented by the basic polynomial model $p(x)$ in the form of (7) which is used for proportional linear transformation of image pixel brightness. At $\lambda = 0$ the model $F_B(1, x)$ is presented by the basic model $t(x)$ in the form of (9); this model serves for increasing an image pixel brightness the more the closer the brightness to the bounds of the range $[a, ..., b]$.

For setting the series of functions of considered model $F_B(\lambda, x)$ onto the optional brightness ranges $[a, ..., b]$ and $[c, ..., d]$ they are scaled automatically as it is shown in Fig. 1.

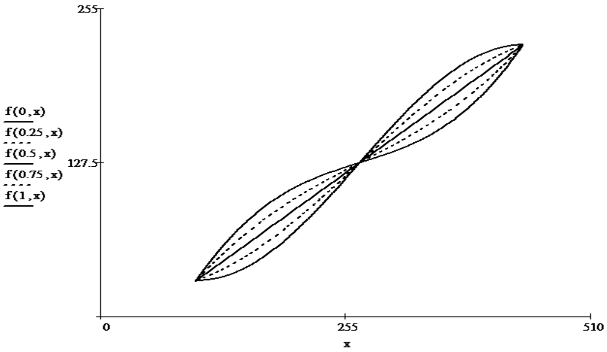



Fig. 1. The series of functions of the model $F_B(\lambda, x)$ obtained for $\lambda$ with a step $\Delta\lambda = 0.25$, where: $a = 100, b = 440, c = 30, d = 225$

The conducted experimental study of image histogram shows [8] that the brightness of the most part of real images varies in such a range $[a, ..., b]$ the width of which $b - a$, as a rule, is less than the width of the range $[0, ..., 255]$. On this condition the evaluation of the range $[a, ..., b]$ bounds may be automated as follows:

$$a = \min_{i,j}\{f_{i,j}\}_{i,j}, \quad b = \max_{i,j}\{f_{i,j}\}_{i,j}, \qquad (11)$$

where $\{f_{i,j}\}_{i,j}$ – are the image pixel brightness values.

Due to adjustment of bounds $[c, ..., d]$ to the peculiarities of the processed image, making use of gray-scale transformation functions (7) – (9) allows us to obtain such an improved image where all the objects are seen regardless of that to which range their brightness values belong. At the same time one should remember that for the computer program to obtain a maximal increase of image bound contrast relative to the background, the values $c$ and $d$ are advisable to set up as follows: $c = 0$, $d = 255$ [8]. Meanwhile, in some applications, for example – in medicine where an improvement of X-ray images [1] is demanded, it is required to significantly increase the brightness and contrast of definite objects presented on the image in a specified range of values.

In this situation for the gray-scale transformation these are currently used the exponential and logarithmic models of form (2) and (3). For solving of such problems of gray-scale transformation instead of model (10) it is proposed to use the following uniform exponentially-logarithmic model of gray-scale transformation

$$F_{ELC}(\lambda, x) = \begin{cases} \lfloor Z1 \rfloor & \text{if } \lambda \geq 1; \\ \lfloor Z2 \rfloor & \text{else}, \quad 0 \leq \lambda \leq 2; \end{cases} \qquad (12)$$

$$Z1 = (\lambda - 1) \cdot lf(x) + [1 - (\lambda - 1)] \cdot p(x); \qquad (13)$$

$$Z2 = \lambda \cdot p(x) + (1 - \lambda) \cdot ef(x), \qquad (14)$$

which is based on use of polynomial model (7) being complemented by the following exponential and logarithmic models

$$ef(x) = k3 \cdot \left[e^{k1 \cdot k2 \cdot (x-a)} - 1\right] + c; \qquad (15)$$

$$lf(x) = k3 \cdot k1^{-1} \cdot \ln[(x - a) \cdot k2 + 1] + c, \qquad (16)$$

with coefficients

$$k1 = 8 \cdot \ln(2)/255, \; k2 = 255/(b - a), \; k3 = (d - c)/255, \qquad (17)$$

which transform the input image brightness values from the range $[a, ..., b]$ into the output range $[c, ..., d]$.

At $\lambda = 0$ the model $F_{ELC}(0, x)$ is presented by exponential model $ef(x)$ in the form of (15); this model serves for significant increase of brightness and contrast, the more the brightness is closer to the end $b$ of the range $[a, ..., b]$. At $\lambda = 1$ the model $F_{ELC}(1, x)$ is presented by the basic polynomial model $ef(x)$ in the form of (7) which is used for proportional linear transformation of image pixel brightness. At $\lambda = 2$ the model $F_{ELC}(2, x)$ is presented by the basic logarithmic model $\ln(x)$ in the form of (16); this model serves for significant increase of brightness and contrast, the more the brightness is closer to the end $a$ of the range $[a, ..., b]$.

For setting the series of functions of the considered model $F_{ELC}(\lambda, x)$ onto the optional brightness ranges $[a, ..., b]$ and $[c, ..., d]$, they are scaled automatically as it is shown in Fig. 2.

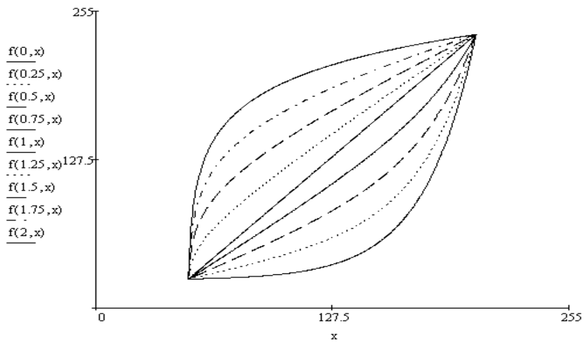



Fig. 2. The series of functions of the model $F_{ELC}(\lambda, x)$ obtained for $\lambda$ with a step $\Delta\lambda = 0.25$, where: $a = 50, b = 205, c = 25, d = 235$

At an effectual noise filtering and image smoothing, making use of the proposed models of gray-scale transformation enables an efficient tone transformation, brightness distribution normalization and image sharpening.

Consider now the structure of methods provided for gray-scale transformation of images.

## 2. Gray-scale transformation methods

So, assume that the gray-scale transformation model $F(x;a,b,c,d,\lambda)$ of form (10) or (12) be chosen, and its parameters are specified.

Method 1. On this assumption it is proposed the following basic method of gray-scale transformation of image ( $hc$ -method), the main steps of which are as follows.

Step 1. Construction of gray-scale transformation table function. By taking into account that the brightness $x$ is defined over an integer set of values, with the aim to minimize the volume of computations during the step of gray-scale transformation of the image, the table function $F(x;a,b,c,d,\lambda)$ of brightness $x$ for the gray-scale transformation of image is constructed as follows:

1) define a matrix-string $M_{1\times n}$ where $n$ – is the number of possible values of brightness $x$, $n = x^* + 1$, and $x^*$ – is the maximal brightness of the scale;

2) in the matrix cells $M[i]$ of the matrix $M$ put the values of brightness $F(i) = F(i;a,b,c,d,\lambda)$, $i = 0,1,..., x^*$, by following the rule

$$M[i]=\begin{cases} c, & \text{if } i<a,\\ F(i), & \text{if } a\le i\le b,\\ d, & \text{if } i>b. \end{cases} \qquad (18)$$

Step 2. Gray-scale transformation. During the per-string scanning of the image transform the brightness values $x(\xi,\eta)$ of pixels $d(\xi,\eta)$ by conforming to the rule: $x(\xi,\eta) = M[x(\xi,\eta)]$.

Step 3. End.

If the bounds $[a,b]$ were not specified a priory, they should be estimated at the initial stage of gray-scale transformation with respect to (11).

Method 2. In this situation the method is modified as follows.

Step 1. Construction of histogram of relative frequencies. Construct the histogram $H$ of relative frequencies $h_i$ at the range $[0, ..., x^*]$ of integer values of image brightness values

$$H = \{h_i\}_{i=0,...,x^*},\ h_i = k_i / N, \qquad (19)$$

where $k_i$ – is the number of image pixels with the brightness $i$, and $N$ – is the number of image pixels.

Formation of histogram $H$ of relative frequencies is advisable for reducing the time requirements when finding the bounds of the range $[a,..., b]$ as well as for the subsequent unification of gray-scale transformation method.

Step 2. Finding the bounds of brightness range. Within the range $[0, ..., x^*]$ find the first $a$ and the last $b$ brightness values with positive relative frequencies.

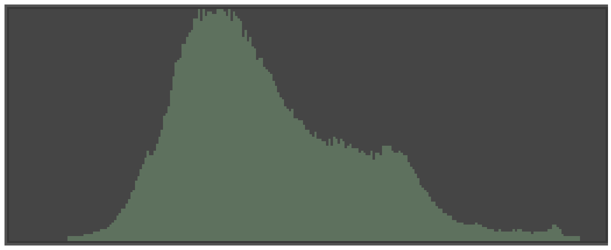
Step 3. Construction of gray-scale transformation table function. This function is formed as in the basic method.

Step 4. Gray-scale transformation. This operation is performed as in the basic method.

Step 5. End.

When solving various application problems we may see that a digital image brightness histogram is often characterized by evidently manifested tails (Fig. 3) which present those values of brightness the probabilities of which are vanishingly small.

a



b

Fig. 3. Airphoto by http://geographyofrussia.com/wp-content/uploads/2009/03/00.jpg (a) and its histogram (b)

Therefore, when these values of brightness do not specify the pixels of objects we are interested in, they may be truncated by zeroing their relative frequencies. So, if after that we find in the brightness histogram the minimal $a$ and the maximal $b$ image brightness with non-zero relative frequencies, the interval of informative area of brightness would become much more narrow. Moreover, this attributes to additional contrasting of the image.

Method 3. Realization of this approach for the specified parameters $(c,d,\lambda)$ and insignificant relative frequency truncation threshold $T$, it is proposed the following modified method of gray-scale transformation of image ( $hcc$ -method).

Step 1. Construction of histogram of relative frequencies – is performed as in the first step of the preceding gray-scale transformation method.

Step 2. Finding the brightness range bounds. Within the range $[0, ..., x^*]$ find the first $a$ and the last $b$ brightness with the relative frequency $h_i$ which exceeds the minimally allowable level $T$ : $h_i > T$.

Step 3. Construction of gray-scale transformation table function. This function is formed as in the basic method.

Step 4. Gray-scale transformation. This function is formed as in the basic method.

Step 5. End.

As far as in general case the brightness is estimated by 256 values, and the linear dimensions of modern images make several thousand pixels, the capital contribution to the total computational laboriousness yield the steps associated with the processing of the entire image. For this reason the proposed methods of gray-scale transformation are characterized, by an order of magnitude, by the same computational requirements as their known analogues.

## 3. Integral index of image sharpening by gray-scale transformation

Image sharpening may be estimated by an integral coefficient $c_s$ which describes stretching of range $[a, ..., b]$ onto the range $[c, ..., d]$

$$c_S = \nu / \mu\,,\ \nu = (d - c)/255\,,\ \mu = (b - a)/x^*\,, \quad (20)$$

with respect to inequalities $0 \le a < b \le x^*$, $0 \le c < d \le 255$, viz. Geometrically, it presents the tangent $tg(\alpha)$ of angle $\alpha$ of the diagonal in the rectangle with ranges, so that

$$c_S = tg(\alpha) = \nu/\mu\,. \quad (21)$$

If the initial interval $[0, ..., x^*]$ of brightness values is defined by the range $[0, ..., 255]$, then the relation (21) yields

$$c_S = \frac{\nu}{\mu} = \frac{d - c}{b - a}\,,\ \nu = (d - c)/255\,,\ \mu = (b - a)/255\,. \quad (22)$$

If the coefficient $c_s$ is greater than 1, a narrow range $[a, ..., b]$ is stretched into more wide interval $[c, ..., d]$. With the growth of coefficient $c_s$ the image sharpness will only be increasing. On the contrary, if a wide interval $[a, ..., b]$ is contracted onto more narrow range $[c, ..., d]$, the coefficient $c_s$ will be less than 1. With decreasing of the coefficient $c_s$ the image sharpness will only be decreasing.

For this reason, when carrying out a gray-scale transformation the coefficient $c_s$ may be used in a capacity of integral index of image sharpening. However, one must remember that making use of nonlinear transformation implies local nonlinear variation of sharpness at different subareas of the range $[a, ..., b]$.

Meanwhile, an increase in coefficient $c_s$ benefits not only to sharpening of objects and small details of the image; the amplitude of not-filtered noise is also increases; without exception, this disadvantage pertains to all gray-scale transformation methods. Therefore, for an effectual processing of gray-scale transformation of image, and for an adequate perception of the obtained result one must take this effect into account.

Therefore, provided that an efficacious preliminary noise filtering is undertaken, making use of the models and methods of gray-scale transformation, which are proposed in this article, allow us not only to perform a tone transformation and image brightness normalization, but also to constrict a range $[a, b]$ significantly and thus – to increase the image sharpness by $c_s$ times.

## Conclusion

The uniform models of gray-scale transformation of image brightness are proposed which enable to perform tone transformation, sharpening and image brightness normalization, and for the optional brightness ranges of input $[a, ..., b]$ and output $[c, ..., d]$ images. At this, the types of models and their parameters are chosen in such a way that they enable maximally flexible adjustment to the requirements of the most important applications.

The proposed models and methods are effectual in allowing us to cut off non-informative brightness areas on the image (including the noise) and, on this ground, to constrict the informative image brightness range adequately; besides, they provide sharpening of object and line bounds.

Therefore, by taking into account the above given estimates and characteristics we may conclude that the proposed models and methods ensure an efficacious solving of the problem of gray-scale transformation of digital image in respect of fulfilment of tone transformation, image sharpening, and image brightness normalization.

## References


*1. Rafael C. Gonzalez, Richard E. Woods Digital Image Processing / C. Rafael. – New Jersey (USA): Pearson Education, 2008. – 954 p.*

*2. Sonka Milan Image Processing, Analysis, and Machine Vision / Milan Sonka, Vaclav Hlavac, Roger Boyle. – 4th edition. – Australia: Cengage Learning, 2015. – 920 p.*

*3. Шапиро Л. Компьютерное зрение / Л. Шапиро, Дж. Стокман. – М.: БИНОМ, 2006. – 752 с.*

*4. Форсайт Д. Компьютерное зрение. Современный подход / Д. Форсайт, Ж. Понс. – М.: Вильямс, 2004. – 928 с.*

*5. Смеляков К.С. Модель и метод повышения резкости границ и линий / К.С. Смеляков, Е.М. Дроб // Наука і техніка Повітряних Сил Збройних Сил України. – № 1 (18). – 2015. – С. 139-141.*

*6. Яне Б. Цифровая обработка изображений / Б. Яне. – М.: Техносфера, 2007. – 584 с.*

*7. Смеляков К.С. Анализ эффективности фильтров резкости изображений / К.С. Смеляков, Е.М. Дроб // Системи обробки інформації. – № 8 (124). – 2014. – С. 137-143.*

*8. Smelyakov K.S. Models and methods of segmentation of irregular object images for off-line machine vision systems / K.S. Smelyakov – Thesis for Doctor's degree in specialty 01.05.02 – mathematical modelling and computing methods. – Karazin Kharkiv National University, Kharkov, 2012. – 306 p. (In Russian)*


*Поступила в редколлегию 17.09.2015*

**Рецензент:** д-р техн. наук, проф. И.В. Рубан, Харьковский национальный университет радиоэлектроники, Харьков.

## УНІФІКОВАНІ МОДЕЛІ ТА МЕТОДИ ГРАДАЦІЙНОЇ КОРЕКЦІЇ ЦИФРОВОГО ЗОБРАЖЕННЯ

К.С. Смеляков, Є.М. Дроб

*Для забезпечення ефективності попередньої обробки цифрового зображення в статті пропонується таке сімейство уніфікованих моделей і методів градаційної корекції яскравості зображень, яке дозволяє одночасно виконувати тонову корекцію, підвищення різкості і нормування, причому для довільних діапазонів яскравості вхідного і вихідного зображень.*

***Ключові слова:*** *модель, уніфікація, метод, ефективність, градаційна корекція, зображення.*

## УНИФИЦИРОВАННЫЕ МОДЕЛИ И МЕТОДЫ ГРАДАЦИОННОЙ КОРРЕКЦИИ ЦИФРОВОГО ИЗОБРАЖЕНИЯ

К.С. Смеляков, Е.М. Дроб

*Для обеспечения эффективности предварительной обработки цифрового изображения в статье предлагается такое семейство унифицированных моделей и методов градационной коррекции яркости изображения, которое позволяет одновременно выполнять тоновую коррекцию, повышение резкости и нормировку, причем для произвольных диапазонов яркости исходного и выходного изображений.*

***Ключевые слова:*** *модель, унификация, метод, эффективность, градационная коррекция, изображение.*