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Effective length of end plate in moment connections with four bolts in the row

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Анотація. У сталевих каркасних конструкціях часто використовуються торцеві болтові з'єднання, розраховані на сприйняття згинального моменту. Цей тип з'єднань широко застосовується балок і колон, а також для з'єднання елементів балки і колони між собою. У випадках впливу великих навантажень на сталеві каркаси застосовують торцеві з'єднання на чотирьох болтах в одному ряду. Використовуються різні методи розрахунку з'єднань балок із колонами на чотирьох болтах у кожному ряду, однак правилами проектування згідно з Єврокодом 3 передбачено болтові з'єднання тільки з двома болтами в кожному ряду.

У цій роботі представлено аналітичну модель двотаврового торця з чотирма болтами, яку можна використовувати з метою прогнозування стійкості з'єднання та початкової жорсткості. Розроблено формули для визначення ефективної довжини торцевої плити в момент з'єднання на чотирьох болтах в одному ряду та наведено порівняння з результатами випробувань.

Аннотация. В стальных каркасных конструкциях часто применяются торцевые болтовые соединения, рассчитанные на восприятие изгибающего момента. Этот тип соединений широко используется для соединения балок и колонн, а также для соединения элементов балки и колонны между собой. В случаях воздействия больших нагрузок на стальные каркасы применяются торцевые соединения на четырех болтах в одном ряду. Используются различные методы расчета соединений балок с колоннами на четырех болтах в каждом ряду, хотя правила проектирования согласно Еврокоду 3 предусматривают стыковые болтовые соединения только с двумя болтами в каждом ряду.

В данной работе представлена аналитическая модель двутаврового торца с четырьмя болтами, которую можно применять с целью прогнозирования устойчивости соединения и первоначальной жесткости. Разработаны формулы для определения эффективной длины торцевой плиты в момент соединения на четырех болтах в одном ряду и приведены сравнения с результатами испытаний.

Abstract. End-type bolted joints in order to perceive bending moment are often applied in steel framed structures. These types of joints are widely used to connect beams to columns as well as beam and column elements together. In case of heavy loads action on steel frames end-type joints with the use of four bolts in the row are applied. Different methods are employed for the calculation of beam-to-column joints with four bolts in each row, however design rules given in Eurocode 3 relate to bolted end-type joints with only two bolts in each bolt row.

In this paper an analytical model for T-stub with four bolts, which can be used for prediction of the joint resistance and initial stiffness, has been presented. A formulas for determination of effective length of end plate in moment connections with four bolts in the row were elaborated, and compared with tests results.

Key words: bolted extended connections, beam-to-column joints, T-stub, effective length.

Introduction. Steel framed structures are often used nowadays because of their advantage, like low self weight, quick erection time, easy to recycling. In case of unbraced steel framed structures, bolted end-plate moment connections are applied. These types of joints are widely applied to connect beams to columns and as beam and column splices. The most common end-plate connections utilize only two bolt rows in the tension zone with two bolts in each row. When the depth of the beam is large, the moment capacity of the joint with only two bolt rows in tension is not sufficient to carry the external moment. Increase of the resistance of the joint can be achieved by applying additional bolt rows; however additional bolt rows have a rather small participation in the resistance of the whole joint, because of their reduced associated lever arm. The triangular stiffeners connected beam flange with column are also used, but from the architectural point of view they are not accepted. When greater resistance of joints is required, designers apply joints with four bolts in each row (fig. 1). Design rules given in Eurocode 3 [1] relate to bolted end-plate joints with only two bolts in each bolt row.

The analytical models used to predict the resistance of the multiple rows end-plate joints are generally based on experimental tests. In practice, three main methods are applied: yield line theory, bolt force determination and mechanical models using component method [2].

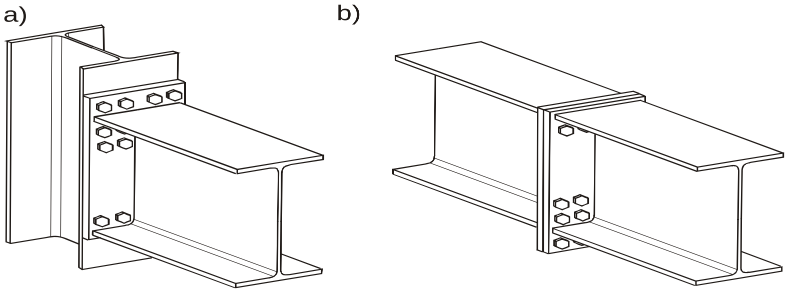

Fig. 1. Multiple rows extended end-plate joints with four bolts in each row:
a – beam to column joint, b – beam splices

Proposed analytical model. Analytical model for calculation the resistance and stiffness of end plate joint, presented in Eurocode [1], use component method. In component method, the moment resistance of the joints is based on the lowest resistance of the joint components such as column web, column flange, beam flange, bolts and end-plate. The resistance of such joints is determined by the resistance of the tension, compression and/or shear zones of the connection. Resistances of the shear and compression zones of joints with four bolts in each row can be predicted according to the European Standard [1] like for joints with two bolts in each row. Influence of the bolts number in the row is considered only in calculation of the column flange and end plate in bending. These components can be predicted by the use of the equivalent T-stub model. In case of joints with four bolts in the row, the T stub with four bolts should be applied.

Based on the observed behaviour of the test specimens, a model was developed to determine the behaviour of the T-stub. For this model, a simply beam with additional spring supports in place of the bolts was adopted. The resistance of the T-stub depends on the resistance of the bolts, thickness of the end-plate and geometrical dimensions of the specimen. The possible models of failure are presented in fig. 2.

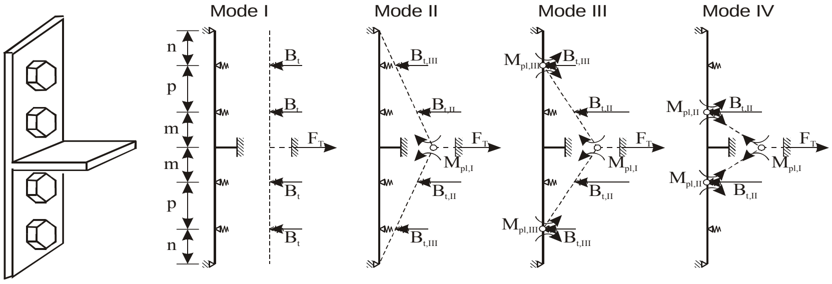

Fig. 2. T-stub model for end – plate with four bolts

Final resistance of the equivalent T-stub in tension is the minimum value of the resistances obtained for each failure modes. The bolt forces are determined by static conditions, admitting elastic distribution of internal forces to the beam.

The effective resistance of the equivalent T – stub can be received from:

$$F_{T.eff.Rd} = \min(F_{T.1.Rd}; F_{T.2.Rd}; F_{T.3.Rd}; F_{T.4.Rd}), \tag{1}$$

where: FT.1.Rd – the resistance due to bolt failure; FT.2.Rd - the resistance due to bolt failure with partial yielding of the T-stub flange. FT.3.Rd – the resistance due to bolt failure with yielding of the T-stub flange; and FT.4.Rd – the resistance due to the complete yielding of the T-stub flange.

The component method can also be applied to determine the rotational stiffness of the joint. The mechanical model for calculation of the initial stiffness is based on a flexibility of the components.

The stiffness of the joint can be predicted from [1], as:

$$S_j = \frac{E \cdot z^2}{\mu \sum \frac{1}{k_i}}, \tag{2}$$

where: E – the elastic modulus; z – the level arm; μ – the stiffness ratio and k_i – the stiffness coefficient for basic joint component i.

The stiffness coefficients for column web in tension, compression and in shear can be taken as for joint with two bolts in the row. The flexibilities of the column flange, end–plate in bending and bolts in tension should be modelled differently. In the component method procedure, as introduced by Eurocode 3 [1], the end–plate and bolts are considered separately. Because of different loading of the particular bolts in T-stub with four bolts, a modified model considering both bending of the end-plate and elongation of the bolts should be applied. The stiffness coefficients are obtained separately for each of the modes of failure [3]. The exemplary scheme for analysis of the T-stub deformation is shown below (fig. 3).

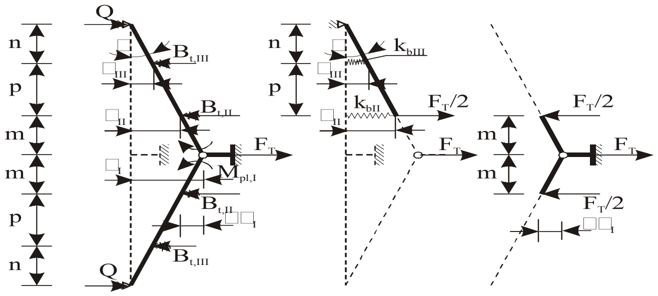

Fig. 3. The mechanical model for calculation of the T-stub deflection

The formulae for calculation of the resistance and stiffness coefficients of equivalent T-stub with four bolts are collected in table 1. Genesis of this formulae was presented in [3].

Table 1

Formulae for calculation of the resistance and stiffness coefficients

Mode of failure	resistance	stiffness coefficient
I	$F_{T,1,Rd} = \sum B_{t.Rd}$	$k_{T,1} = 3,2 \cdot \frac{A_s}{L_b^p}$
II	$F_{T,2,Rd} = \frac{2 \cdot M_{pl,I,Rd} + (4 \cdot n + 2 \cdot p) \cdot B_{t.Rd}}{m + p + n}$	$k_{T,2} = \frac{1}{\frac{(n+p) \cdot L_b^p}{1,6 \cdot (2 \cdot n + p) \cdot A_s} + \frac{m^3}{0,9 \cdot l_{eff} \cdot t^3}}$
III	$F_{T,3,Rd} = \frac{2 \cdot M_{pl,I,Rd} + 2 \cdot M_{pl,III,Rd} + 2 \cdot p \cdot B_{t.Rd}}{m + p}$	$k_{T,3} = \frac{1}{\frac{L_b^p}{1,6 \cdot A_s} + \frac{m^3}{0,9 \cdot l_{eff} \cdot t^3}}$
IV	$F_{T,4,Rd} = \frac{2 \cdot M_{pl,I,Rd} + 2 \cdot M_{pl,II,Rd}}{m}$	$k_{T,4} = \frac{0,9 \cdot l_{eff} \cdot t^3}{m^3}$

where Bt, Rd – the bolt tensile resistance, considering tensile resistance of the bolt shank and punching shear resistance of the bolt head, Mpl,I,Rd, Mpl,II,Rd, Mpl,III,Rd, – the yielding moment of T-stub flange; *m* – reduced distance between the bolts and the T-stub web; p – the bolt spacing; *n* – minimum distance between the bolt and the edge of the T-stub flange limited to 1,25·m; *As* – the tensile stress area of the bolt; *Lbp* – the bolt elongation length, taken as a sum of the thickness of plate and washer, plus half of the heights of the bolt head or nut; *t* – the thickness of the T-stub flange; *fy* – the yield resistance of T-stub; γM0 – partial safety factor; and l_{eff} – the effective length (assumed as the width of the T-stub flange for examined T-stubs);

The yielding moments can be determined as:

$$M_{pl,Rd} = 0,25 \cdot \sum l_{eff} \cdot t^2 \cdot f_y / \gamma_{M0}, \qquad (3)$$

where: l_{eff}; *t*; *fy*; γ*M*0 – as above.

Parametric study. On the basis of the developed model, a wide parametric study of the resistance and stiffness of the simply T-stub with four bolts in the row has been conducted [4].

Constant values of the following parameters were established:

- steel grade S235;
- bolts M20 grade 10.9;
- hickness of the flange plate of T-stub $t = 20$ mm;
- distance between center of inner bolts and web $m = 30$ mm;
- bolt spacing $p = 50$ mm;
- distance between center of end bolts and end of the T stub flange $n = 30$ mm;
- T-stub flange width $l_{eff} = 100$ mm.

To receive the influence of any parameters on the resistance and stiffness of the T-stub, only investigated parameter was changed. The example of the influence of the T-stub flange width on resistance of T-stub is presented on fig. 4. In this figure, the range of modes of the failure is presented too.

Main conclusions from parametric study are as follows:

- thickness of the T-stub flange t and its width l_{eff} have a big influence on the resistance of the T-stub;
- web spacing m influences the resistance less than end plate thickness;
- bolt grade have an influence on resistance and stiffness of the joint when the failure is govern by II, III and IV mode;
- bigger thickness of the end plate t and web distance m decrease the stiffness of the T-stub;

- stiffness of the joint is depended on the failure mode (leaps on the diagram).

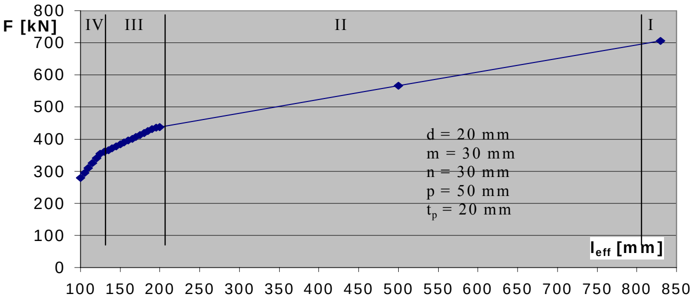

Fig. 4. Influence of the plate width l_{eff} on the resistance of the T-stub

Effective length for one bolt and group of bolt row

During calculations of the joint resistance, when an II, III and IV mode of failure is governing, the main role plays an effective length. According to Eurocode 3 [1], the effective length should be determined for one bolt and a bolts group. Based on the yield line pattern, the effective length, for T-stub with two bolts, can be obtained according to [5]. Based on these assumptions, and on the experimental tests for simply T-stubs, the effective length was predicted for two bolts on one side of the T-stub web [6].

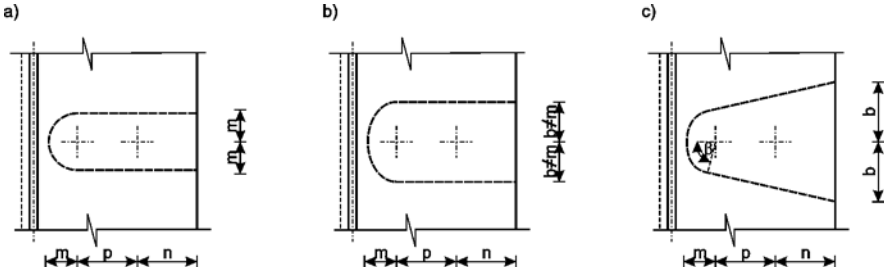

Fig. 5. Circular yield line pattern for two bolts

For the circular yield line pattern (fig. 5) the effective length can be obtain as follows:

$$l_{eff} = \pi \cdot m + 2 \cdot (p + n); \tag{4}$$

$$l_{eff} = \sqrt{\pi \cdot m \cdot (\pi \cdot m + 4 \cdot p + 4 \cdot n)}; \tag{5}$$

$$l_{eff} = 2 \cdot \left(m \cdot \beta + m \cdot ctg\beta + (p + n) / \sin\beta\right), \tag{6}$$

where: l_{eff}; m; n, p – as above and β –in [rad]. Usually, value of $\beta = 54^{\circ}$

For a non-circular yield line pattern (fig. 6) the effective length can be obtain from:

$$l_{eff} = 4 \cdot \sqrt{m^2 / 2 + p \cdot m + n \cdot m}\,; \tag{7}$$

$$l_{eff} = 2 \cdot \sqrt{3 \cdot m^2 + 4 \cdot p \cdot m + 4 \cdot n \cdot m}\,, \tag{8}$$

where: l_{eff}; m, n, p – as above.

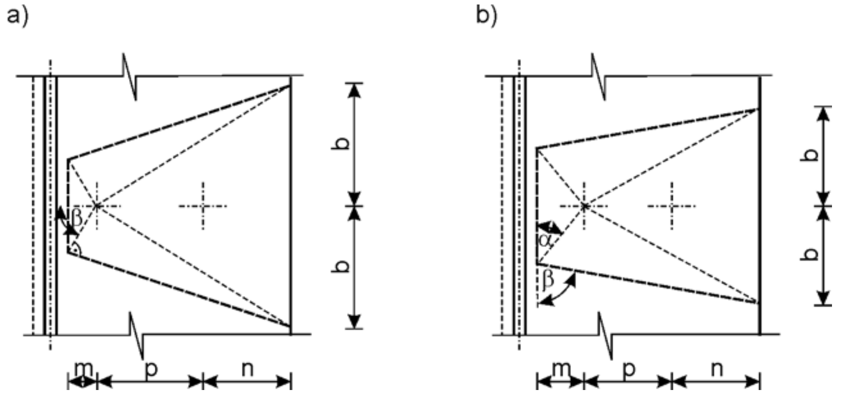

Fig. 6. Non-circular yield line pattern for two bolts

In case of the big spacing of the bolts in the row, yield line pattern can be taken as for bolt separately (fig. 7). It can happen only in III and IV mode of failure.

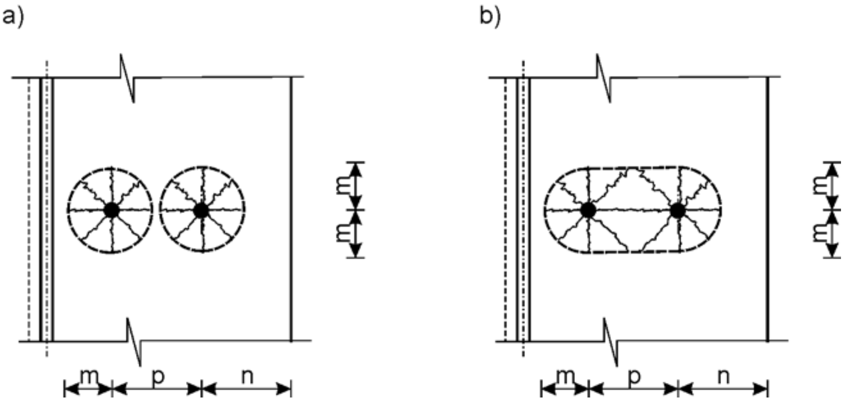

Fig. 7. Yield line pattern separately for each bolt

$$l_{eff} = 4 \cdot \pi \cdot m\,; \tag{9}$$

$$l_{eff} = 2 \cdot \pi \cdot m + 2 \cdot p \,. \tag{10}$$

In case of bolt row adjacent to the stiffener, the effective length can be obtained similarly, considering distances between bolt and perpendicular stiffeners. Yield line pattern for bolt row adjacent to the stiffener are presented in fig. 8. Appropriate effective length can be obtain according to formulae (11–16).

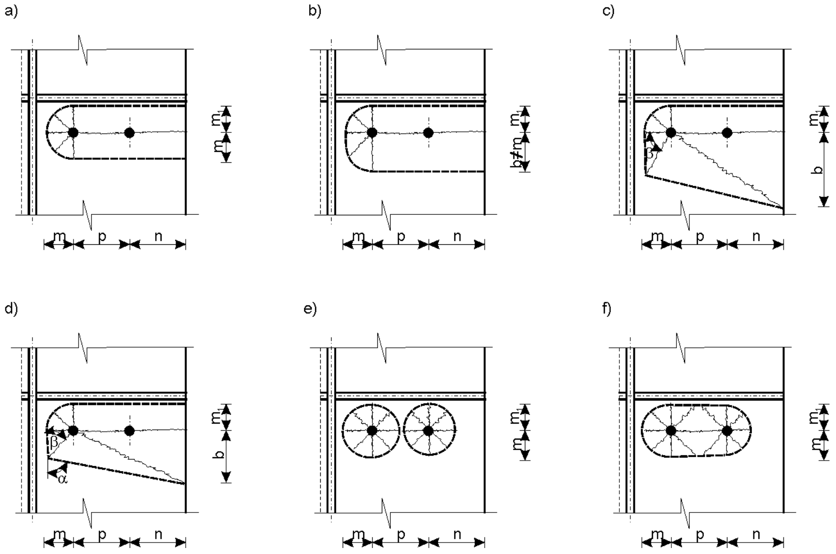

Fig. 8. Yield line pattern separately for bolt row adjacent to the stiffener

$$l_{eff} = \alpha \cdot m + 2 \cdot \left(p + n\right); \tag{11}$$

$$l_{eff} = \frac{\alpha \cdot m}{2} + 2 \cdot m \cdot \sqrt{\frac{\pi \cdot \left(p + n\right)}{4 \cdot m} + 1}\,; \tag{12}$$

$$l_{eff} = \frac{\alpha \cdot m}{2} + 2 \cdot m \cdot \sqrt{\frac{\left(p + n\right)}{m} + \frac{1}{2}}\,; \tag{13}$$

$$l_{eff} = \frac{\alpha \cdot m}{2} + 2 \cdot m \cdot \sqrt{\frac{4 \cdot \left(p + n\right)}{m} + 3}\,; \tag{14}$$

$$l_{eff} = \alpha \cdot m + 2 \cdot \pi \cdot m_1\,; \tag{15}$$

$$l_{eff} = \frac{\alpha \cdot m}{2} + \pi \cdot m + 2 \cdot p\,, \tag{16}$$

where l_{eff}, m, n, p – as above, and α –according to [1].

Verification of the analytical model. The verify proposed analytical model, comparison with experiment for B6 specimen was conducted [6]. The specimen was made of HEA 240 profile steel grade S235, and screwed by 8 bolts M12 grade 10.9. Geometrical dimensions of the specimen are presented in fig. 9. The ultimate strength of the specimen obtained from experimental test was *F* = 694,1 kN.

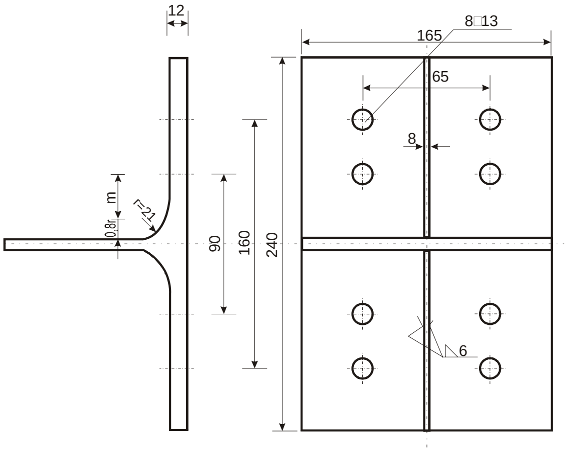

Fig. 9. Geometrical dimensions of the specimen [6]

Calculation according to proposed model gave the following results: the failure of the specimen was govern by III mode, minimum value of the effective length for bolts was obtained from equation (12) and equals $l_{eff} = 188,7\ \text{mm}$, resistance of the T-stub received from analytical model was $\mathrm{F}_{\mathrm{T,3,Rd}} = 449,25\ \text{kN}$.

The result obtained from experimental test and analytical model differ about 40 %. This difference can result of the material properties taken to calculation of resistance according to proposed model (nominal values) and influence of bolts head on effective length of the yield line pattern.

Conclusions

The design methods existing up to now for determination of the resistance of the joint with four bolts in each row give results which differ very much [2]. Design rules given in Eurocode 3 [1] relate to bolted end-plate joints with only two bolts in each bolt row.

The analytical model for assessment of the resistance and stiffness of T-stub with four bolts presented in the paper is based on the component method, and can be adopted in Eurocode procedures. Comparison of the resistance of the T-stub from proposed method and test results shows that analytical models rather well agree with experimental results.

The effective length for one bolt and a group bolts mainly depend on the joint configuration. In case of joints with four bolts in the row, perpendicular unstiffened T-stubs can be adopted for column flange and end plate and yield pattern line used for bolt rows apart form stiffeners. For bolt rows adjacent to stiffener or beam flange the interaction between perpendicular T-stub and T-stub not adjacent to the web should be considered in determination of the effective length of this bolt row.

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