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Features of probabilistic design of steel communication structures

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Анотація. В статті пропонується метод імовірнісного розрахунку сталевих ґратчастих опор зв'язку під дією статичної і поздовжньої пульсаційної складової швидкості вітру. При цьому враховується просторово-часова мінливість швидкості вітру і просторовий розподіл аеродинамічного коефіцієнту опор.

Аннотация. В статье предлагается метод вероятностного расчёта стальных решётчатых опор связи под действием статической и продольной пульсационной составляющей скорости ветра. При этом учитывается пространственно-временная изменчивость скорости ветра и пространственное распределение аэродинамического коэффициента опор.

Abstract. This paper proposes a consistent method for the analysis of lattice towers reliability under stochastic along-wind aerodynamic actions. Stochastic actions are related to spatial-temporal changeability of wind velocity and spatial dependence of aerodynamic coefficient.

Key works: wind turbulence, reliability, lattice towers, guyed masts.

Nomenclature

b_h	[*m*]	width of tower face
C_{aer}	[-]	aerodynamic coefficient
d_u	[-]	Solari's constant (6.868)
q	[*N/m*]	wind load on tower
k_r	[-]	terrain factor
n	[*Hz*]	frequency
z_0	[*m*]	roughness length
T_{ef}	[*yr*]	lifetime of the tower
U	[*m/s*]	stochastic processes of the mean wind velocity
u'	[*m/s*]	random processes of longitudinal nil mean turbulent fluctuations
w	[*Pa*]	wind pressure
β_u	[-]	turbulence intensity factor
φ	[rad.]	angle of wind attack
φh	[-]	function of wind velocity profile
Γ	[-]	Gamma-function
N+	[-]	outlier number of random process

Subscripts and Superscripts

m	mean wind

u turbulent wind
~ random variable or stochastic process
^ standard deviation
− mathematical expectation

Itroduction. Along-wind vibrations of structures present one of the most well-known subjects in the field of wind engineering. This is due, on the one hand, to the relative simplicity of the excitation mechanism the longitudinal turbulence, and, on the other hand, to the gust factor technique (Davenport 1961, 1964; Barshtein 1957, 1959, 1974), a method as simple as it is reliable.

Despite this fact, probabilistic character of wind turbulence is very often place emphasis only and probabilistic character of the mean wind is not considered. In building codes this approach seems justified. But in probabilistic design this approach leads to underestimations of construction reliability.

Steel lattice towers belong to a class of slender vertical structures. The accuracy of reliability estimation of these structures first of all depends on adequate description of stochastic model of wind load (mean wind and atmospheric turbulence). Besides, the method of reliability estimation should consider spatial variability of wind velocity and should be correct and simple.

Faced with the growing of the problem and the evident lack of engineering design criteria, this paper formulates a mathematical model of the reliability estimation of slender vertical structures (e.g. lattice towers and guyed masts) subjected to gust-excited along-wind vibrations.

1. Stochastic Along-wind Process. The stochastic process of instantaneous wind velocity $\tilde{U}(z, t, \tau)$ is given by the sum of a macro-meteorological component defined as the stochastic process of mean wind velocity $\tilde{U}_m(z, t)$ on average time interval, and micro-meteorological component $\tilde{u}(z, \tau)$ defined as stochastic process of atmospheric longitudinal turbulence.

Process of mean wind velocity is product of stochastic stationary processes $\tilde{U}_{m,10}(t)$ and function $\varphi_h(z)$ of wind vertical profile. The function $\varphi_h(z)$ can be described by logarithmic law:

$$\varphi_h(z) = k_r \ln[z / z_0] . \tag{1.1}$$

The experimental distributions of mean wind velocity at 10m height are well corresponded to the Weibull's law. Its density distribution is written as:

$$f_m(U) = \beta_U / \alpha_U U^{\beta_U - 1} \exp[-U^{\beta_U} / \alpha_U] . \tag{1.2}$$

Mathematic expectation $\bar{U}_{m,10}$, standard deviation $\hat{U}_{m,10}$ and coefficient of variation $V_{m,10}$ of process $\tilde{U}_{m,10}(t)$ is given by:

$$\bar{U}_{m,10} = \alpha_U^{1/\beta_U} \Gamma(1+\beta_U^{-1}) ; \tag{1.3}$$

$$\hat{U}_{m,10} = \sqrt{\alpha_U^{2/\beta_U} [\Gamma(1+2\beta_U^{-1}) - \Gamma^2(1+\beta_U^{-1})]} ; \tag{1.4}$$

$$V_m = \sqrt{\Gamma(1+2\beta_U^{-1}) / \Gamma(1+\beta_U^{-1})^2 - 1} . \tag{1.5}$$

Power spectrum $s_{\omega,m}(\omega)$, effective frequency $\omega_{e,m}$ and narrow-band factor $\beta_{\omega,m}$ are frequency characteristics of process $\tilde{U}_{m,10}(t)$. For practical use equations for resulted in table 1 are recommended.

Table 1

Frequency characteristics of stochastic process of mean wind velocity at 10m height

$s_{\omega,m}(\omega)$	$\omega_{e,m}$	$\beta_{\omega,m}$
$2\alpha /[\pi(\alpha^2 + \omega^2)]$	$\alpha\sqrt{2}$	$\sqrt{3}$ (1,732)
$4\alpha^3 /[\pi(\alpha^2 + \omega^2)^2]$	α	$2\sqrt{2} - 1$ (1,826)
$16\alpha^5 /[3\pi(\alpha^2 + \omega^2)^3]$	$\alpha / \sqrt{3}$	3.0

As is known, stochastic process of mean wind velocity at 10m height generates stochastic process of the mean wind pressure $\tilde{w}_{m,10}(t) = \rho\tilde{U}_{m,10}^2(t)/2$. Distribution density of this process will submit also to Weibull's law (1.2) with parameters α_W, β_W:

$$\beta_W = \beta_U / 2, \qquad \alpha_W = \alpha_U / 1,6^{\beta_W} . \tag{1.6}$$

The effective frequency of stochastic process of mean wind pressure can be described by means effective frequency $\omega_{e,m}$ as follows:

$$\omega_{e,w} = \omega_{e,m} \Big/ \sqrt{V_m} . \tag{1.7}$$

Thus it is possible to pass from consideration of probabilistic model of the mean wind velocity to probabilistic model of mean wind-excited pressure on lattice tower surfaces.

The turbulent component $\tilde{u}(z,\tau)$ along axis z is a nil mean Gaussian random stationary process described in the domain of the frequency n, by its cross-power spectrum:

$$\tilde{S}_u(z,z',n) = \sqrt{\tilde{S}_u(z,n)\tilde{S}_u(z',n)}\,\mathbf{c\tilde{o}h}(z,z',n). \tag{1.8}$$

Here $\tilde{S}_u(\bullet)$ is the power spectrum of $\tilde{u}(\bullet)$ and $\mathbf{c\tilde{o}h}(\bullet)$ is $\tilde{u}(\bullet)$ coherence function along z. The closed form solution derived in this paper is based on the model developed by G. Solari [1]:

$$\frac{n\tilde{S}_u(z,n)}{\tilde{\sigma}_u^2} = \frac{d_u\tilde{L}_u(z)n/\tilde{U}_m(z,t)}{[1+1.5d_u\tilde{L}_u(z)n/\tilde{U}_m(z,t)]^{5/3}}; \tag{1.9}$$

$$\mathbf{c\tilde{o}h}(z,z',n) = \exp\left[-\frac{2n\tilde{C}_{uz}\left|z-z'\right|}{\tilde{U}_m(z,t)+\tilde{U}_m(z',t)}\right]; \tag{1.10}$$

$$\begin{aligned}&\tilde{L}_u(z) = 300\tilde{\lambda}_u(z/200)^{\nu};\\ &\nu = 0.67+0.05\ln(z_0);\end{aligned} \tag{1.11}$$

$$\tilde{\sigma}_{u'} = \tilde{\varepsilon}_u\tilde{U}_{m,10}(t); \tag{1.12}$$

$$\tilde{\varepsilon}_u = \tilde{k}_{\varepsilon}/\ln(z/z_0), \tag{1.13}$$

where $\tilde{\sigma}_u$ is standard deviation of turbulence, assumed as independent of z; $\tilde{L}_u$ is integral length scale in a wind direction; $\tilde{C}_{uz}$ is exponential decay coefficient. Detailed discussion on the properties of Eqs. (1.9) – (1.11) and an introduction to advanced turbulence modelling are reported in [1, 2].

Let's notice, that turbulence characteristics $\tilde{\sigma}_u$, $\tilde{S}_u(\bullet)$, $\mathbf{c\tilde{o}h}(\bullet)$, $\tilde{L}_u$ and $\tilde{\varepsilon}_u$ should be examined as random variables. The probabilistic nature of these variables is caused by stochastic properties of three dimensionless factors: $\tilde{C}_{uz}$, $\tilde{\lambda}_u$, $\tilde{k}_{\varepsilon}$. This statement confirms, both our researches, and researches of the Italian scientists [1]. Statistical characteristics of mentioned above factors following: $\bar{C}_{uz}=10$, $\hat{C}_{uz}=2$; $\bar{\lambda}_u=1$, $\hat{\lambda}_u=0,25$; $\bar{k}_{\varepsilon}=0,4\sqrt{\bar{\beta}_{u'}}$, $\hat{k}_{\varepsilon}=0,05\sqrt{\bar{\beta}_{u'}}$:

$$\bar{\beta}_{u'} = 6-1,1arctg[\ln(z_0)+1,75]. \tag{1.14}$$

2. Along-wind Excited Response. Let's consider steel lattice tower, which schematised as slender cantilever vertical beam coaxial with *z*, of total height H. Tower breaks on N sites with current number $j = 1, 2, \ldots, k, \ldots, m, \ldots N$ and it has a linear elastic behaviour with viscous damping. The along-wind displacement of tower point at height z_j is expressed by [3, 4]:

$$y(z_j, t, \tau) = y_m(z_j, t) + y_u(z_j, \tau), \tag{2.1}$$

where y_m is the mean displacement caused by processes $\tilde{U}_m(z_j, t)$ and y_u is nil mean fluctuating displacement caused by processes $\tilde{u}(z_j, \tau)$.

For carrying out of calculations it is suitable to use equivalent static wind load which would cause the same displacement of lattice tower as from a gusty wind in a wind direction (Davenport, 1964):

$$\begin{aligned} &\tilde{q}_\Sigma(z,t) = \tilde{q}_m(z,t) + \tilde{q}_u(z) = \tilde{q}_m(z,t)\tilde{\psi}_G(z) = \\ &= \tilde{w}_{m,10}(t)\tilde{\psi}_G(z)\varphi_h^2(z)C_{aer}(z)b_h(z), \end{aligned} \tag{2.2}$$

where $\tilde{\psi}_G(z)$ is the gust factor along *z*, assumed as random variables; $b_h(z)$ is the size of tower orthogonal to the wind direction.

Solving the equation (2.2) by taking several vibrations modes into account in general calls for numerical analysis. The problem may be solved in closed form assuming that response depends only on the first mode. In this case the fluctuating wind load can be expressed by (formula $\alpha(z) = (z / H)^2$ is used for indicated mode shape):

$$\tilde{q}_u(z) = \tilde{w}_{m,10}\tilde{\eta}(z)m(z)\,; \tag{2.3}$$

$$\tilde{\eta}(z) = \alpha(z)\tilde{\Delta}_\eta / M = \tilde{\Delta}_\eta z^2 / (MH^2)\,; \tag{2.4}$$

$$M = \int_0^H m(z)\alpha^2(z)dz\,; \tag{2.5}$$

$$\tilde{\Delta}_\eta^2 = \frac{8k_r^4\tilde{k}_\varepsilon^2}{H^4}\int_0^\infty \frac{J^2(n)dn}{[1-(n/n_1)^2]^2 + \gamma^2(n/n_1)^2}\,; \tag{2.6}$$

$$J^2(n)=\int_0^H\int_0^H b_h(z_k)b_h(z_m)C_{aer}(z_k)C_{aer}(z_m)z_k^2z_m^2\ln\left(\frac{z_k}{z_0}\right)\ln\left(\frac{z_m}{z_0}\right)\times$$

$$\times\sqrt{\tilde{S}_{u'}(z_k,n)\tilde{S}_{u'}(z_m,n)}\cdot\tilde{\Lambda}_{u'}(\bullet)dz_kdz_m.$$

Let's consider relation of the power spectrum $\tilde{S}_u(\bullet)$ at heights z and H. Taking into account that $1,5d_{u'}L_{u'}(z)n/\tilde{U}_m(z,t)\gg 1$ it's possible to write

$$\sqrt{\frac{\tilde{S}_{u'}(z,n)}{\tilde{S}_{u'}(H,n)}}\approx k_S\left(\frac{1}{z^{\nu}}\ln\left(\frac{z}{z_0}\right)\right)^{1/3}. \tag{2.7}$$

The constant k_S is defined by formula

$$k_S=[H^{\nu}/\ln(H/z_0)]^{1/3}. \tag{2.8}$$

For further analysis tower has form of isosceles trapeze. Width of bottom is b_b, width of top is b_t. Subsequently width at any height of tower will be expressed by the equation

$$b_h(z)=\beta_b[H-(1-\alpha_b)z], \tag{2.9}$$

where $\alpha_b=b_t/b_b$, $\beta_b=b_b/H$ are non-dimensional constants of geometrical form of the tower.

Substituting equations (1.9), (2.6) and (2.8) into equation (2.5) provides salutation (Pichugin-Makhinko, 2008):

$$\tilde{\Delta}_{\eta}=\tilde{G}_S\cdot\tilde{\tilde{\xi}}_S\cdot J_S\cdot\tilde{\nu}_S; \tag{2.10}$$

$$\tilde{G}_S=2k_r^2\tilde{k}_{\varepsilon}k_S\beta_b; \tag{2.11}$$

$$\tilde{\tilde{\xi}}_S=\sqrt{\int_0^{\infty}\frac{2d_{u'}\varepsilon^2d\varepsilon}{[(\varepsilon^2-\tilde{\varepsilon}_S^2)^2+\gamma^2\tilde{\varepsilon}_S^2\varepsilon^2][1+1,5d_{u'}/\varepsilon]^{\frac{5}{3}}}}; \tag{2.12}$$

approximating formula:

$$\tilde{\tilde{\xi}}_S=0,67\cdot\ln\left(\tilde{\varepsilon}_S+0,105\right)+3,01; \tag{2.13}$$

$$J_S = \frac{1}{H^2}\sqrt{\int_0^H\int_0^H [H - z_k(1-\alpha_b)][H - z_m(1-\alpha_b)]\times}$$
$$\sqrt{\times C_{aer}(z_k)C_{aer}(z_m) z_k^{2-\frac{\nu}{3}} z_m^{2-\frac{\nu}{3}} \ln^{\frac{4}{3}}\left(\frac{z_k}{z_0}\right)\ln^{\frac{4}{3}}\left(\frac{z_m}{z_0}\right) dz_k dz_m},$$

where $\tilde{\varepsilon}_S = \dfrac{\tilde{U}_m(H,t)}{n_1 \tilde{L}_{u'}(H)}$ is the non-dimensional period of lattice tower fluctuation; $\tilde{\nu}_S \le 1$ is factor of spatial correlation. For steel lattice tower its value depends on terrain category, tower height and non-dimensional period. It is given by:

$$\tilde{\nu}_S^2 = \frac{\displaystyle\int_0^\infty \frac{\tilde{S}_{u'}(H,n)\int_0^H\int_0^H z_1^{2-\frac{\nu}{3}} z_2^{2-\frac{\nu}{3}} \ln^{\frac{4}{3}}\left(\frac{z_1}{z_0}\right)\ln^{\frac{4}{3}}\left(\frac{z_2}{z_0}\right)\Lambda_{u'}(z_1,z_2,n)dz_2 dz_1}{[1-(n/n_1)^2]^2+\gamma^2(n/n_1)^2}dn}{\displaystyle\int_0^\infty \frac{\tilde{S}_{u'}(H,n)\int_0^H\int_0^H z_1^{2-\frac{\nu}{3}} z_2^{2-\frac{\nu}{3}} \ln^{\frac{4}{3}}\left(\frac{z_1}{z_0}\right)\ln^{\frac{4}{3}}\left(\frac{z_2}{z_0}\right)dz_2 dz_1}{[1-(n/n_1)^2]^2+\gamma^2(n/n_1)^2}dn}.$$

On basis of formula (2.10) for gust factor it is possible to offer the equation, which is integrally in closed form and simple to apply:

$$\tilde{\psi}_D(z) = 1 + \frac{\tilde{G}_S \cdot \tilde{\xi}_S \cdot \tilde{\nu}_S \cdot J_S}{b_h(z)C_{aer}(z)\varphi_h^2(z)}\frac{H^2 z^2(z)m(z)}{\int_0^H z^4 m(z)dz}. \tag{2.14}$$

Probabilistic properties of the gust factor are formed under the influence of stochastic nature of three non-dimensional factors: dynamic coefficient $\tilde{\xi}_S$, factor of spatial correlation $\tilde{\nu}_S$ and factor $\tilde{G}_S$. In the meantime, probabilistic properties of these coefficients are defined by probabilistic properties of factors $\tilde{C}_{uz}$, $\tilde{\lambda}_u$, $\tilde{k}_\varepsilon$ (see section 1). Considering this fact, let's designate through $\tilde{\mathbf{M}}_S$ product of three random variables $\tilde{G}_S \cdot \tilde{\xi}_S \cdot \tilde{\nu}_S$, and designate through $\Pi(z)$ determinative variable

$$\Pi(z) = \frac{H^2 \cdot J_S \cdot z^2 m(z)}{b_h(z) \cdot C_{aer}(z) \cdot \varphi_h^2(z) \cdot \int_0^H z^4 m(z)}. \qquad (2.15)$$

Having entered the specified designations, the gust factor will be expressed as:

$$\tilde{\psi}_D(z) = 1 + \tilde{\mathrm{M}}_S \Pi. \qquad (2.16)$$

The Eq. (2.16) shows, that statistical properties of the gust factor will be defined by statistical properties of variable $\tilde{\mathrm{M}}_S$. The distribution law of variable $\tilde{\mathrm{M}}_S$ was searched by means of Monte-Carlo simulations. By modelling it was considered, that distribution law of factors $\tilde{C}_{uz}$, $\tilde{\lambda}_u$, $\tilde{k}_\varepsilon$ in the situation of an information lack can be accepted asymptotic normal. The study was carried out by generating 10^7 realisations, associated with different loading conditions. As a result of modelling it is found out, distribution law of random variable $\tilde{\mathrm{M}}_S$ a small differs from double Gumbel's distribution with parameters (see Fig. 1):

$$\gamma_{0,\mathrm{M}} = \bar{\mathrm{M}}_S - 0,45\hat{\mathrm{M}}; \quad \lambda_{0,\mathrm{M}} = 1,282 / \hat{\mathrm{M}}, \qquad (2.17)$$

where $\bar{\mathrm{M}}_S$, $\hat{\mathrm{M}}_S$ are mathematic expectation and standard deviation of variable $\tilde{\mathrm{M}}_S$.

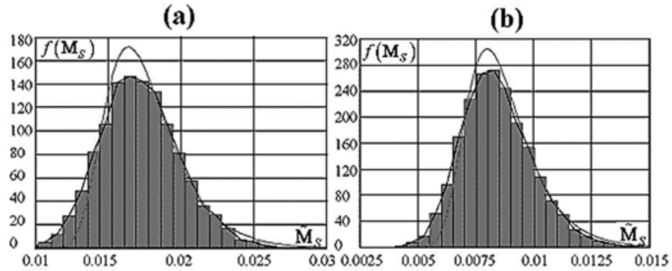

Fig. 1. Distribution density of variable $\bar{\mathrm{M}}_S$:
(a) H = 50m, n1 = 0.5Hz; (b) H = 150m, n1 = 1.0Hz

This conclusion does not depend on height and type of lattice tower, terrain category, mean wind pressure and turbulence intensity.

As a consequence, under Eq. (2.2) probabilistic properties of sum wind pressure $\tilde{w}_\Sigma(z,t)$ on lattice tower will be defined by product of stochastic process of mean wind pressure $\tilde{w}_{m,10}(t)$ distributed under Weibull's law and random variable of gust factor $\tilde{\psi}_G(z)$, distributed under Gumbel's law. The density distribution of sum wind pressure $\tilde{w}_\Sigma(z,t)$ is expressed as (Pichugin-Makhinko, 2007):

$$f_\Sigma(w_\Sigma) = \frac{w_\Sigma^{\beta_w - 1}\beta_w}{\alpha_w \varphi_h^{2\beta_w - 1}(z)} \int_0^1 \frac{\exp\left[\dfrac{-w_\Sigma^{\beta_w} / \varphi_h^{2\beta_w}(z)}{\alpha_w[\gamma_{0,\mathrm{M}} - \ln(-\ln Z)/\lambda_{0,\mathrm{M}}]^{\beta_w}}\right]}{[\gamma_{0,\mathrm{M}} - \ln(-\ln Z)/\lambda_{0,\mathrm{M}}]^{\beta_w}} dZ \,.$$

This formula is inconvenient and difficult for practical use. Hence possibility of approximation of the formula by Weibull's distribution (1.2) has been proved with parameters:

$$\alpha_{w,ref} = \left(\frac{\overline{w}_{m,10}\overline{\psi}_D(z)\varphi_h^2(z)}{\Gamma(1+\beta_w^{-1})} \right)^{\beta_w} = \alpha_w \psi_h^{\beta_w}(z) \,. \qquad (2.18)$$

Here $\psi_h(z) = \overline{\psi}_D(z)\varphi_h^2(z)$ is coefficient named in "factor of dynamic intensifying".

Equation (2.18) together with the formula (1.2) allows carrying out probabilistic dynamic designs of lattice towers in the quasi-static formulation.

3. Random Wind Directions on Tower. The wind flow on a tower is characterized not only by random change of wind velocity, but also by random change of wind direction. It leads to induce in tower elements of stretching stochastic stress and compression stochastic stress. For that reason it is necessary to know distribution law of these stresses at reliability estimation, both individual elements and towers. Thus probabilistic model for stochastic wind directions is examined. At the heart of model there is a hypothesis about equal probability of different wind directions. Hypothesis is fair because orientation of communication towers is not on wind rose and according to accepted directional diagram and terrain topography.

Aspects of this model are examined on an example of square and triangular lattice tower, schematically shown on Fig. 2. The unit load moves on circle from a chord with number 1 clockwise. Numbers of settlement situations are thus fixed. As a result there is a sequence from eight and six members accordingly. It's possible to present number of settlement situation as discrete random variable adopting values from 1 to 8 for square tower and from 1 to 6 − for triangular tower with probabilities $p_{1\text{-}8} = 0.125$, $p_{1\text{-}6} = 0.167$ accordingly. If these cases are plotted in Cartesian co-ordinates for one of tower elements (chord, girder, diagonal strut) as it is shown on Fig. 3 it will be obvious, that it is possible to unite cases 1,3; 4,8; 5,7 for square tower and to unite cases 1,3; 4,6 for triangular tower. This fact allows to replace discrete random variable with a sinusoid curve:

$$\tilde{N}_{\varphi}(z,t) = \tilde{N}_{\max}(z,t)\sin(\alpha_d - \tilde{\varphi}), \tag{3.1}$$

where $\tilde{N}_{\max}$ is maximum stretching stochastic stress in elements; α_d is constant depending on tower plan form, type (chord, girder, diagonal strut) and position of tower element. In Table 2 values of constant α_d for chords, girders and diagonal struts of square and triangular towers are resulted.

Let's notice, that values of constant α_d depend also on choice of altitude reference. In this paper altitude reference is shown on Fig. 2.

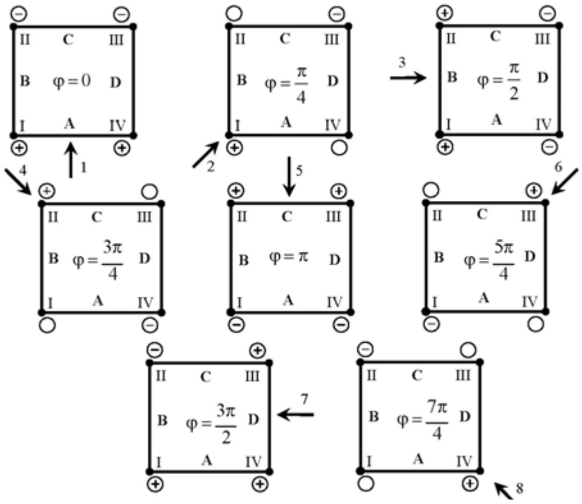

Fig. 2. Wind directions for square tower

Table 2

Values of constant α_d

Form plane	Marks of chord and faces		Type of tower elements		
			chord	girders	diagonal strut
Square	I	A	3π/4	π	2π
	II	B	5π/4	3π/2	5π
	III	C	7π/4	2π	π
	IV	D	9π/4	5π/2	3π
Triangular	I	A	5π/6	0	0
	II	B	9π/6	2π/3	2π/3
	III	C	13π/6	4π/3	4π/3

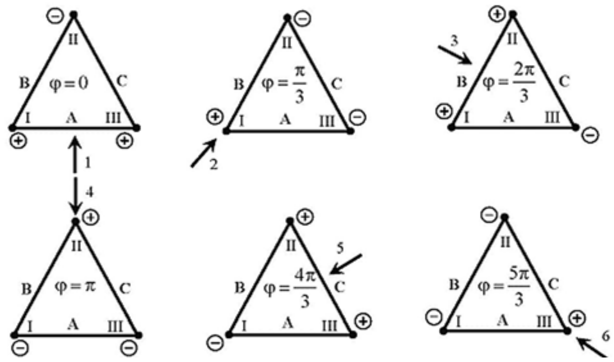

Fig. 3. Wind directions for triangular tower

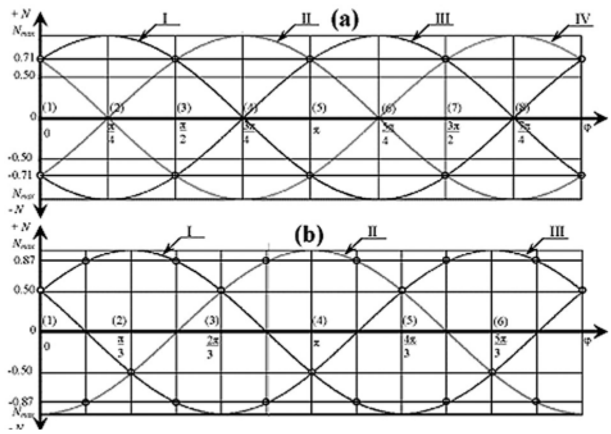

Fig. 4. Stress in chord from attack angle:
(a) for square tower; (b) for triangular tower

To analyse the structural behaviour with respect to reliability, response must be expressed in terms of stresses. The stochastic process of wind-excited vibrations of tower $\tilde{q}_{\Sigma}(z,t)$ produces fluctuating stresses $\tilde{S}_{\varphi}(z,t,\varphi)$ which determine failure of individual elements of the tower:

$$\tilde{S}_{\varphi}(z,t,\varphi) = \tilde{N}_{\varphi}(z,t)\tilde{C}_{aer}^{\varphi}(z,\varphi), \tag{3.2}$$

where $\tilde{C}_{aer}^{\varphi}(\bullet)$ is the functional dependence of aerodynamic coefficient on a wind direction.

Thus for reliability estimation of lattice tower elements it's necessary to know distribution law of stochastic process $\tilde{N}_{\varphi}(z,t)$ and random variable $\tilde{C}_{aer}^{\varphi}(\bullet)$. Due to structural linearity, of the process $\tilde{N}_{\max}(z,t)$ will have Weibull's law distribution (see section 2). Mathematic expectation and standard deviation of process $\tilde{N}_{\max}(z,t)$ can be expressed as:

$$\begin{aligned} \overline{N}_{\max}(z) &= \alpha_{\max}(z)\overline{w}_{\Sigma}(z)b_h(z); \\ \hat{N}_{\max}(z) &= \alpha_{\max}(z)\hat{w}_{\Sigma}(z)b_h(z), \end{aligned} \tag{3.3}$$

where $\alpha_{\max}(z)$ is influence factor of wind load for individual elements of the tower; $\overline{w}_{\Sigma}(z)$ and $\hat{w}_{\Sigma}(z)$ are mathematic expectation and standard deviation of process $\tilde{w}_{\Sigma}(z,t)$.

Random variable $\tilde{\varphi}$ is described by uniform distribution, therefore distribution density of random variables $\tilde{A}_{\varphi} = \sin(\alpha_d - \tilde{\varphi})$ is given by:

$$f_{A\varphi}(A_{\varphi}) = 1 \Big/ \pi \cdot \sqrt{1 - A_{\varphi}^2} \,. \tag{3.4}$$

It makes possible to express above distribution density of stochastic process $\tilde{S}_{\varphi}(z,t,\varphi)$ as product of three random quantities:

$$\tilde{S}_{\varphi}(z,t,\varphi) = \tilde{A}_{\varphi}(\varphi)\tilde{N}_{\max}(z,t)\tilde{C}_{aer}^{\varphi}(\varphi) \,. \tag{3.5}$$

For further decision of a problem it is necessary to investigate dependence $\tilde{C}_{aer}^{\varphi}(\bullet)$ in details.

4. Aerodynamic Coefficient. Aerodynamic coefficient characterizes aerodynamic properties of the lattice towers. It depends from the sizes, lengthening and relative positioning of plane lattice towers; from forms cross-section of individual elements towers and from sizes of these elements; from area of the gusset plate, from elements quantity in one node; from wind direction and solidity ratio. Load code narrows these frame-works and does dependence of aerodynamic coefficient from four factors: from quantity of sides of lattice tower, from the tower form in the plan, from wind direction and solidity ratio. Three first factors are considered in details by building codes. The fourth factor in building codes is reduced to consideration two or three most wind directions on a spatial lattice structures. In framework of method of safety partial factors such approach is quite justified, but in probabilistic design of towers it is not enough. In probabilistic design we should operate with functional dependence of aerodynamic coefficient on a wind direction and solidity ratio. Therefore in this section on the basis of numerous experiments [5] influence of the mentioned above two parameters on aerodynamic coefficient is studied. Results of this studying for triangular and square towers are illustrated by group of figures 5-7 and allow present following remarks.

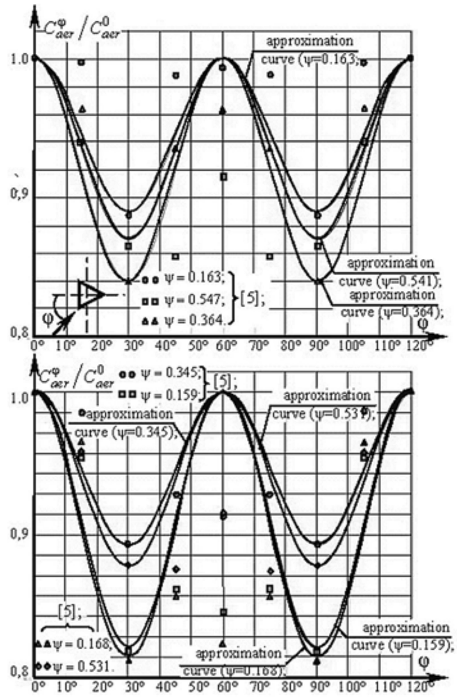

Fig. 5. Relation $C_{aer}^{\varphi} / C_{aer}^{0}$ for square towers (a) circular members; (b) flat-sided members

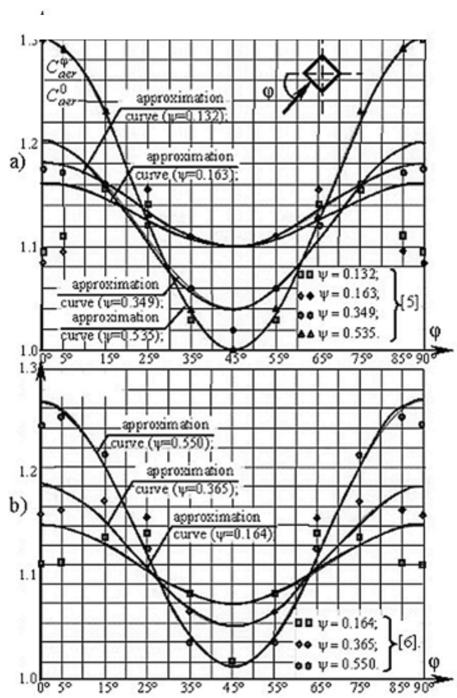

Fig. 6. Relation $C_{aer}^{\varphi} / C_{aer}^{0}$ for triangular towers (a) circular members; (b) flat-sided members

Dependence $C_{aer}(\varphi)$ has periodic character with period $T = \pi / 3$ and $T = 2\pi / 3$ for triangular lattice towers and $T = \pi / 2$ for square towers. Thus for dependence $C_{aer}(\varphi)$ it is possible to offer trigonometrically approximating formula:

$$C_{aer}^{\varphi} / C_{aer}^{0} = A_0 + A_1 \cos(A_2 \varphi), \tag{4.1}$$

where A_0, A_1, A_2 − constants which depend on the form of cross-section elements of lattice tower and solidity ratio of flat side.

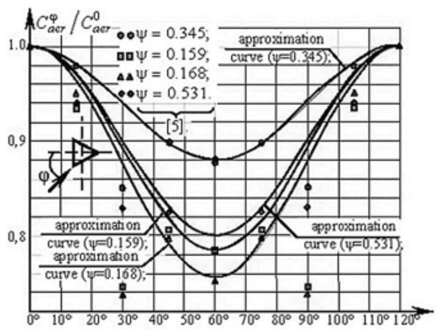

Fig. 7. Relation $C_{aer}^{\varphi}/C_{aer}^{0}$ for triangular tower with circular-section members

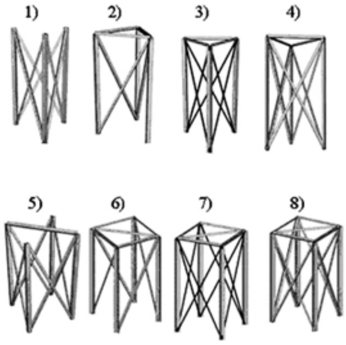

Fig. 8. Spatial section of lattice tower

The sense of constants A_0, A_1, A_2 is discussed further. Parameter A_2 characterises the period of function (4.1) and it is connected with period elementary relation $A_2 = 2\pi / T$. That's why for all square towers $A_2 = 4$ (Fig. 5), and for triangular towers $A_2 = 6$ or $A_2 = 3$ (Fig. 6, 7). Situations when it is necessary to accept for triangular towers $A_2 = 6$ and $A_2 = 3$ explains Figs. 6, 7. Figures show that formula (4.1) at $A_2 = 6$ (Fig. 6) allows to describe specific character of aerodynamic factor near to angle of wind attack $\varphi \approx 30°$ and $90°$. Formula (4.1) does not allow to describe this character at $A_2 = 3$. Equation (4.1) at argument $A_2 = 6$ will overestimate values of aerodynamic coefficient near to angle of wind attack $\varphi \approx 60°$ which will be adequately described by equation (4.1) at $A_2 = 3$. Therefore formula (4.1) at $A_2 = 3$ needs to be applied to spatial lattice structures with solidity ratio $\psi \geq 0{,}3$, accordingly, at $A_2 = 6$ equation (4.1) will be more correct to describe aerodynamic properties of spatial lattice structures with smaller values of solidity ratio. In probabilistic design of lattice towers it is necessary to consider two variants and to choose variant which gives results in a safety margin.

By means of constants A_0, A_1 maximum $C_{aer}^{\max}$ and minimum $C_{aer}^{\min}$ values of aerodynamic coefficient of lattice towers are defined:

$$\begin{aligned} C_{aer}^{\max} / C_{aer}^{0} &= A_0 + A_1; \\ C_{aer}^{\min} / C_{aer}^{0} &= A_0 - A_1. \end{aligned} \tag{4.2}$$

For constants A_0, A_1 it is possible to offer inverse relationships, on the basis of that values $C_{aer}^{\max}$, $C_{aer}^{\min}$ are resulted in building codes and wind engineering literature:

$$\begin{aligned} A_0 &= 0,5(C_{aer}^{\max} + C_{aer}^{\min}) / C_{aer}^{0}; \\ A_1 &= 0,5(C_{aer}^{\max} - C_{aer}^{\min}) / C_{aer}^{0}. \end{aligned} \tag{4.3}$$

For square towers aerodynamic coefficient is minimum when a wind is perpendicular one of tower sides (designated C_{aer}^{0}). In building codes, for example Eurocode 1, and many wind engineering manuals it is possible to find relation $C_{aer}^{\max} / C_{aer}^{0}$ which is designated as $k_{\max}$. Hence the equation (4.3) after simplification is given:

$$A_0 = (k_{\max} + 1) / 2, \quad A_1 = (k_{\max} - 1) / 2 . \tag{4.4}$$

For triangular towers relation of coefficients $C_{aer}^{\max} / C_{aer}^{0}$ to equally unit, and relation $C_{aer}^{\min} / C_{aer}^{0}$ is designated as $k_{\min}$. Values of factor $k_{\min}$ are resulted in wind engineering literature together with coefficient $k_{\max}$. Consequently for trihedral towers the formula is used similarly to the equation (4.4):

$$A_0 = (1 + k_{\min}) / 2, \quad A_1 = (1 - k_{\min}) / 2 . \tag{4.5}$$

Formulae (4.3) − (4.5) can be used in the absence of experimental data on aerodynamic coefficient. In this work constant A_0, A_1 for some values of solidity ratio of triangular and square towers are resulted in the table 3.

Table 3

Value of constants A_0 and A_1

	ψ for square tower						
	circular members				flat-sided members		
	0,132	0,163	0,349	0,535	0,164	0,365	0,55
A_0	1,13	1,14	1,12	1,15	1,15	1,16	1,19
A_1	0,03	0,04	0,08	0,15	0,05	0,09	0,17
	ψ for triangular tower						
	circular members			flat-sided members			
	0,163	0,364	0,547	0,159	0,168	0,345	0,531
A_0	0,945	0,92	0,935	0,875	0,87	0,925	0,915
				0,893	0,878	0,94	0,9
A_1	0,055	0,08	0,065	0,125	0,13	0,075	0,085
				0,108	0,122	0,06	0,1

It would be desirable to note also that offered formulae in some cases describe experimental data with a small error. We think it is not error, and there is a consequence of statistical scatter of values of aerodynamic coefficient received experimentally. Therefore further it is necessary to give formulae (4.1) - (4.5) corresponding probabilistic description.

5. Stochastic Stress State. Probabilistic model of stress stochastic process $\tilde{S}_\varphi(z,t,\varphi)$ is based on the Eq. (3.5). For estimate of distribution density of this process it is necessary to know distribution density of three random quantities in the Eq. (3.5). As shown above, the distribution density $\tilde{S}_\varphi(z,t,\varphi)$ submits to Weibull's law and distribution density $\tilde{A}_\varphi$ is given by formula (3.4). For distribution density $\tilde{C}^\varphi_{aer}(\bullet)$ equation similar to the formula (3.4) is received:

$$f_C(C^\varphi_{aer}) = \frac{C^0_{aer}}{\pi \cdot \sqrt{A_1^2 - (C^\varphi_{aer} / C^0_{aer} - A_0)^2}} . \tag{5.1}$$

Owing to noncorrelated of random components $\tilde{N}_{\max}(z,t)$, $\tilde{A}_\varphi$, $\tilde{C}^\varphi_{aer}(\bullet)$ their combined distribution density looks like:

$$f_\varphi(S_\varphi) = \frac{1}{\pi} \int_0^{\zeta(S_\varphi)} 1 \bigg/ \sqrt{\ln\left(\frac{1}{Z}\right)^{\frac{2\alpha_{w,\varphi}}{\beta_w}} - S_\varphi^2}\, dZ ; \tag{5.2}$$

$$\zeta(S_\varphi) = \exp\left[-|S_\varphi|^{\beta_w} / \alpha_{w,\varphi}\right]; \tag{5.3}$$

$$\alpha_{w,\varphi} = \alpha_{w,ref} A_0^{\beta_w} [\alpha_{\max}(z) C^0_{aer}(z) b_h(z)]^{\beta_w} . \tag{5.4}$$

The frequency structure of process $\tilde{S}_\varphi(z,t,\varphi)$ can be characterized by its effective frequency and a power spectrum. For these purposes it is possible to use one of three lines of table 1.

6. Reliability of Tower Elements

Let's examine the steel element of lattice tower takes sign-variable stochastic stress of compression and a stretching. The principal ideas of applied method were developed in works [6-9]. The failure of an element takes place when a stochastic stress $\tilde{S}_\varphi(z,t)$ under equivalent static wind load exceeds the random limit of carrying capacity of the tower element. Failure of the element is defined by the equation:

$$\tilde{S}_R - \tilde{S}(t) < 0 . \tag{6.1}$$

The non-failure probability of tower element in general case is estimated by the equation (Pichugin-Makhinko, 2006) [6, 8]:

$$P(t) = \int_0^\infty \exp[-N_+(S_R \mid T_{ef})] f_R(S_R) dS_R \; ; \tag{6.2}$$

$$N_+(S_R \mid T_{ef}) = \hat{S}_\varphi \frac{\omega_{e,m} T_{ef}}{\sqrt{2\pi}} f_\varphi(S_R) = \frac{f_\varphi(S_R)}{f_\varphi(S_{0,\varphi})} . \tag{6.3}$$

Here T_{ef} is lifetime of lattice tower; $S_{0,\varphi}$ is characteristic maximum of stochastic stress $\tilde{S}_\varphi(z,t)$; $f_R(\bullet)$ is density distribution of carrying capacity of the tower element; $\hat{N}_\varphi$ is standard $\tilde{S}_\varphi(z,t)$:

$$\hat{N}_\varphi = \frac{\hat{N}_{\max}}{\sqrt{2}} \sqrt{1 + V_m^{-2}} \; . \tag{6.4}$$

Equations (6.2) - (6.3) consider spatiotemporal structure of stochastic process $\tilde{S}_\varphi(\bullet)$, its distribution law and it completely solves a problem of reliability of individual elements of the lattice tower. However its application in practice is related with bulky and inconvenient procedures of numerical integration. Therefore hypothesis is applied to quantity $N_+(\bullet)$: for any stochastic process with distribution density which correspond to exponential type it is possible to present $N_+(\bullet)$ as (Pichugin-Makhinko, 2005):

$$N_+(S_R \mid T_{ef}) = \exp\left[-\lambda_{0,\varphi}\left(S_R / \hat{S}_\varphi - \gamma_{0,\varphi}\right)\right], \tag{6.5}$$

where $\gamma_{0,\varphi}$, $\lambda_{0,\varphi}$ are normalised characteristic maximum and normalised characteristic intensity of stochastic stress $\tilde{S}_\varphi(z,t)$ accordingly [9]:

equation root: $\omega_{e,m} T_{ef} f_{n\varphi}(\gamma_{0,\varphi}) = \sqrt{2\pi}$; (6.6)

$$\lambda_{0,\varphi} = -\frac{1}{f_{n\varphi}(\gamma_{0,\varphi})} \frac{d}{d\gamma} [f_{n\varphi}(\gamma) \mid \gamma = \gamma_{0,\varphi}] . \tag{6.7}$$

Here $f_{n\varphi}(\bullet)$ distribution density of normalised type of process $\tilde{S}_\varphi(z,t)$. It is given by

$$f_{n\varphi}(\gamma) = \frac{\psi}{\pi} \int_0^{\zeta(\gamma)} 1 \Bigg/ \sqrt{\ln\left(\frac{1}{Z}\right)^{\frac{2}{\beta_w}} - \gamma^2 \psi^2}\, dZ \,; \tag{6.8}$$

$$\psi = \Gamma(1+\beta_w^{-1})\sqrt{(1+V_m^2)/2} \,; \tag{6.9}$$

$$\zeta(\gamma) = \exp[-(|\gamma|\psi)^{\beta_w}] \,. \tag{6.10}$$

The used hypothesis allows to formulate non-failure probability of tower element as

$$\begin{gathered} P(t) = \frac{1}{\sqrt{2\pi}} \int_{-V_R^{-1}}^{\infty} \exp\left\{-\exp\left[E(Z)\right] - 0.5Z^2\right\} dZ; \\ E(Z) = -\lambda_{0,\varphi}\left(p_R\left(Z + \frac{1}{V_R}\right) - \gamma_{0,\varphi}\right), \end{gathered} \tag{6.11}$$

where V_R is coefficient of variation of carrying capacity of tower element; $p_R = \hat{N}_R / \hat{N}_\varphi$ is relation of standard.

7. Reliability of Tower Spatial Section. The reliability of a lattice tower is defined by reliability of tower spatial sections. The reliability of tower spatial sections depends on their structural form. Therefore procedure of reliability estimation of the most widespread spatial sections of square and triangular towers is examined further. Failure section is interpreted as failure of any section element. It is considered, that reliability of elements identical to designation (chord, girder, diagonal strut) is equal. This statement is entered on the basis of hypothesis about equal probability of different wind directions. Let's go into designations: $P_{ch}(t)$ − chord reliability; $P_g(t)$ − girder reliability; $P_{ds}(t)$ − diagonal strut reliability.

1. Triangular tower with cross bracing (see Fig. 8): $P_{\sec}(t) = P_{ch}^3(t) P_{ds}^6(t)$.
2. Triangular tower with single lattice and girders: $P_{\sec}(t) = P_{ch}^3(t) P_g^3(t) P_{ds}^3(t)$.
3. Triangular tower with tension bracing: $P_{\sec}(t) = P_{ch}^3(t) P_{ds}^6(t) P_g^3(t)$.
4. Triangular tower with cross bracing and girders:
$P_{\sec}(t) = P_{ch}^3(t) P_g^3(t) \{1 - [1 - P_{ds}(t)]^2\}^3$.
5. Square tower with a cross bracing: $P_{\sec}(t) = P_{ch}^2(t) P_{ds}^8(t) [2 - P_{ch}^2(t)]$.

6. Square tower with single lattice and girders:

$$P_{\sec}(t) = P_{ch}^3(t) P_{ds}^4(t) P_g^4(t)[2 - P_{ch}(t)].$$

7. Square tower with tension bracing: $P_{\sec}(t) = P_{ch}^4(t) P_{ds}^8(t) P_g^4(t)$.

8. Square tower with a cross bracing and girders:

$$P_{\sec}(t) = P_{ch}^3(t) P_g^4(t) P_{ds}^4(t)[2 - P_{ch}(t)][2 - P_{ds}(t)]^4.$$

If to accept $P_{ch}(t) = P_g(t) = P_{ds}(t) = p$ it's possible to make some remarks about sections reliability. Section 4 has greatest reliability, the least - section 7 as it can be only stretched. The section 8 is more reliable, than the section 4, and section 7 is less reliable, than section 3. It is possible to explain that square sections contain more elements than triangular sections. At tension bracing in square and triangular sections failure of any elements breaks geometrical stability of tower section. Hence reliability of these sections that above, than smaller elements quantity is contained by sections. At cross bracing with girders of square section contain more reserve elements, than triangular sections. Therefore they are more reliable.

Conclusions

This paper formulates a mathematical method for the reliability estimation of lattice towers to gust-excited along-wind vibrations. The method, integrally in closed form and simple to apply, leads to analytical expressions of the equivalent static wind load, of the gust factor, of the aerodynamic coefficient and reliability of tower spatial section. Relative simplicity, wide field of applications, the precision, and the reliability of this method make it very suitable for rapid engineering design of lattice tower and guyed masts also.

References

[1] Solari G. and Piccardo G., 2001, Probabilistic 3-D Turbulence Modelling for Gust Buffeting of Structures, Probabilistic Engineering Mechanics, № 16. – P. 73 – 86.

[2] Pichugin S. and Makhinko A. 2005, Wind Load on Building Structures, Monograph, Poltava, Ukraine. – 342 p.

[3] Piccardo G. and Solari G., 2000, 3D Wind-Excited Response of Slender Structures: Closed-Form Solution, Journal of Structural Enginee-ring, ASCE, Vol. 126, № 8. – P. 936 – 943.

[4] Solari G. and Pagnini L.C., 1999, Gust Buffeting and Aeroelastic Behaviour of Poles and Monotubular Towers, Journal of Fluids and Structures, № 13. – P. 877 – 905.

[5] Brian W. Smith, 2007, "Communication Structures", ASCE Press, Published by Thomas Telford Publishing, Thomas Telford L. – 338 p.

[6] Makhinko A., 2006, Reliability of Steel Elements of Building Structures under Stochastic Variable Loads, Dissertation Poltava National Technical University. – 348 p.

[7] Pichugin S., 1997, Probabilistic Analysis of Wind Load and Reliability of Structures, Proc. 2-nd European and African Conf. Wind Eng., Genoa, Italy. – P. 1883 – 1890.

[8] Pichugin S., Makhinko A. and Makhinko N., 2007, Reliability Estimation of Steel Structures under Wind and Snow Loads, Lublin University of Technology, Lublin, Poland. – P. 343 – 356.

[9] Pichugin S., Makhinko A., 2006, Probabilistic Model of Wind Load Maxima, Proc. 3-rd Inter-national Conf. Problems of the Technical Meteorology, Lviv, Ukraine.

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